

Part IV: State

Marco T. Morazán

Seton Hall University

Outline

- 1 State
- 2 Language with Explicit References
- 3 Language with Implicit References
- 4 Mutable Pairs
- 5 Parameter Passing Variations

State

Language with
Explicit
References

Language with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- Computations may also have effects
 - print
 - change a memory location
 - change a file

State

Language with
Explicit
References

Language with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- Computations may also have effects
 - print
 - change a memory location
 - change a file
- An effect is global: affects the entire computation

State

Language with
Explicit
References

Language with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- Computations may also have effects
 - print
 - change a memory location
 - change a file
- An effect is global: affects the entire computation
- We will now study assignment (aka mutation)

State

Language with
Explicit
References

Language with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- Assignment is about sharing values/information between unrelated parts of a computation
- CSAS 1115: telephone book, bank account

State

Language with
Explicit
References

Language with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- Assignment is about sharing values/information between unrelated parts of a computation
- CSAS 1115: telephone book, bank account
- Memory model
 - A finite map of locations to storable values (aka store or heap)
 - A place in memory where values can be stored

State

Language with
Explicit
References

Language with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- Assignment is about sharing values/information between unrelated parts of a computation
- CSAS 1115: telephone book, bank account
- Memory model
 - A finite map of locations to storable values (aka store or heap)
 - A place in memory where values can be stored
- Implementation
 - Typically, storable values are the same as the expressed values
 - A data structure that represents a location is called a reference (aka pointer)

State

Language with
Explicit
References

Language with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- Two ways to design a language with store
- Explicit references
- programmer allocates, dereferences, and mutates locations/memory

State

Language with
Explicit
References

Language with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- Two ways to design a language with store
- Explicit references
- programmer allocates, dereferences, and mutates locations/memory
- Implicit references
- language packages common patterns of allocation, dereferencing, and mutation

Language with Explicit References

State

Language with
Explicit
References

Language with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- $\text{expval} = \text{int} + \text{bool} + \text{proc} + \text{ref}(\text{expval})$
- $\text{denval} = \text{expval}$

Language with Explicit References

State

Language with
Explicit
ReferencesLanguage with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- $\text{expval} = \text{int} + \text{bool} + \text{proc} + \text{ref}(\text{expval})$
- $\text{denval} = \text{expval}$
- 3 new ops needed:
 - newref** allocates a new location and returns a reference to the new location
 - deref** returns the content of a reference
 - setref** changes the content of a referenced location

Language with Explicit References

- Value?

```
let temp = newref(0)
in let x = newref(5)
   in let mystery = proc (y)
       begin
         setref(temp, deref(x));
         setref(x, deref(y));
         setref(y, deref(temp));
         deref(x)
       end
   in (mystery newref(10))
```

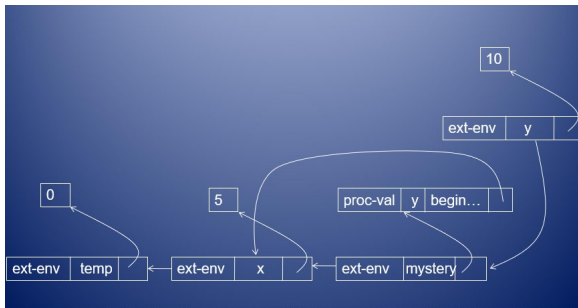
Language with Explicit References

- Value?

```

let temp = newref(0)
in let x = newref(5)
   in let mystery = proc (y)
       begin
         setref(temp, deref(x));
         setref(x, deref(y));
         setref(y, deref(temp));
         deref(x)
       end
   in (mystery newref(10))

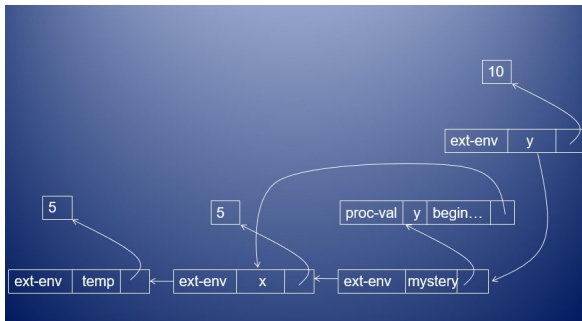
```



Language with Explicit References

```

• let temp = newref(0)
  in let x = newref(5)
    in let mystery = proc (y)
      begin
        setref(temp, deref(x));
        setref(x, deref(y));
        setref(y, deref(temp));
        deref(x)
      end
    in (mystery newref(10))
  
```



Language with Explicit References

State

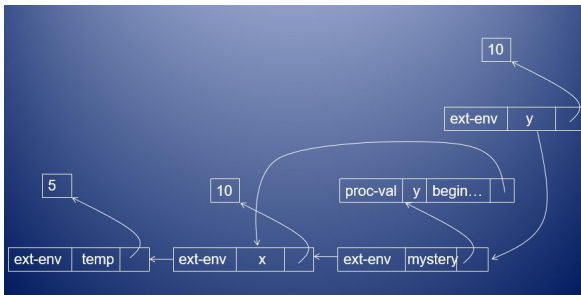
Language with
Explicit
ReferencesLanguage with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

```

• let temp = newref(0)
  in let x = newref(5)
    in let mystery = proc (y)
      begin
        setref(temp, deref(x));
        setref(x, deref(y));
        setref(y, deref(temp));
        deref(x)
      end
    in (mystery newref(10))
  
```

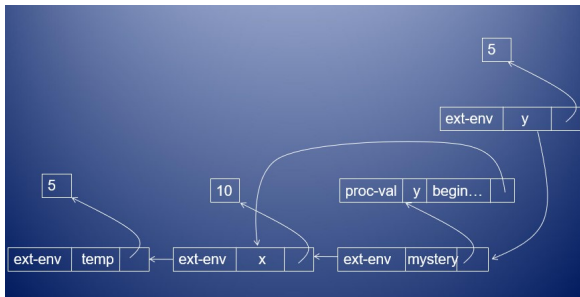


Language with Explicit References

- ```

let temp = newref(0)
in let x = newref(5)
 in let mystery = proc (y)
 begin
 setref(temp, deref(x));
 setref(x, deref(y));
 setref(y, deref(temp));
 deref(x)
 end
 in (mystery newref(10))

```



- Returns: 10

# Language with Explicit References

State

Language with  
Explicit  
ReferencesLanguage with  
Implicit  
References

Mutable Pairs

Parameter  
Passing  
Variations

- Is this a valid program? If so, what does it evaluate to?

```
let x = newref(newref(10))
in begin
 setref(deref(x), 20));
 +(20, deref(deref(x)))
end
```

# Language with Explicit References

## State

Language with  
Explicit  
ReferencesLanguage with  
Implicit  
References

## Mutable Pairs

Parameter  
Passing  
Variations

- ```
let x = newref(newref(10))  
in begin  
  setref(deref(x), 20);  
  +(20, deref(deref(x)))  
end
```



10

Language with Explicit References

State

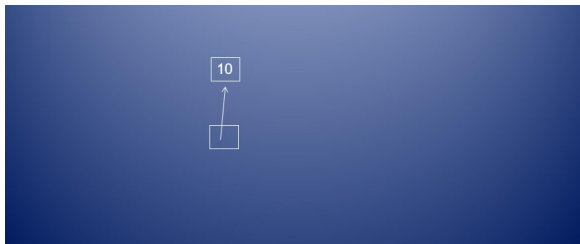
Language with
Explicit
ReferencesLanguage with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- Is this a valid program? If so, what does it evaluate to?

```
let x = newref(newref(10))  
in begin  
  setref(deref(x), 20);  
  +(20, deref(deref(x)))  
end
```



Language with Explicit References

State

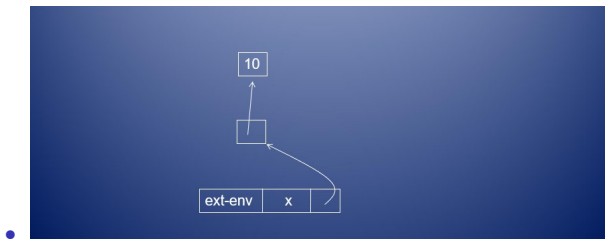
Language with
Explicit
ReferencesLanguage with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- Is this a valid program? If so, what does it evaluate to?

```
let x = newref(newref(10))  
in begin  
  setref(deref(x), 20);  
  +(20, deref(deref(x)))  
end
```



Language with Explicit References

State

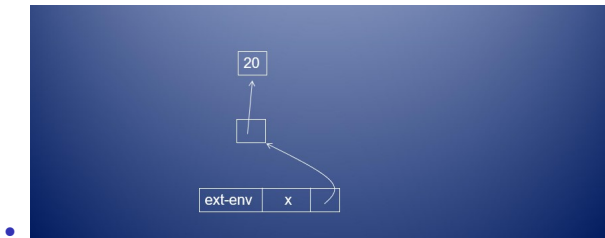
Language with
Explicit
ReferencesLanguage with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- Is this a valid program? If so, what does it evaluate to?

```
let x = newref(newref(10))  
in begin  
  setref(deref(x), 20);  
  +(20, deref(deref(x)))  
end
```



Language with Explicit References

State

Language with
Explicit
ReferencesLanguage with
Implicit
References

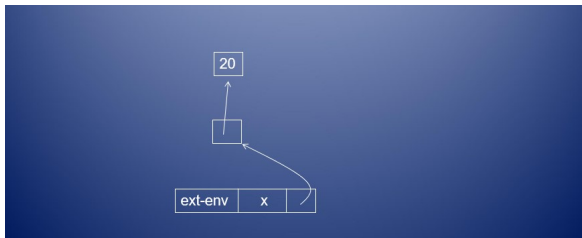
Mutable Pairs

Parameter
Passing
Variations

- Is this a valid program? If so, what does it evaluate to?

```
let x = newref(newref(10))  
in begin  
  setref(deref(x), 20);  
  +(20, deref(deref(x)))  
end
```

•



- Returns 40

Language with Explicit References

State

Language with
Explicit
ReferencesLanguage with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- Expressed Values

```
(define-datatype expval expval?  
  (num-val  
    (value number?))  
  (bool-val  
    (boolean boolean?))  
  (proc-val  
    (proc proc?))  
  (ref-val  
    (ref reference?)))
```


Language with Explicit References

State

Language with
Explicit
ReferencesLanguage with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- Expressed Values

```
(define-datatype expval expval?
```

```
  (num-val
```

```
    (value number?))
```

```
  (bool-val
```

```
    (boolean boolean?))
```

```
  (proc-val
```

```
    (proc proc?))
```

```
  (ref-val
```

```
    (ref reference?)))
```

- ```
;; expval --> ref throws error
```

```
;; Purpose: Extract ref from given expval
```

```
(define (expval2ref v)
```

```
 (cases expval v
```

```
 (ref-val (ref) ref)
```

```
 (else (expval-extractor-error 'reference v))))
```

# Language with Explicit References

State

Language with  
Explicit  
References

Language with  
Implicit  
References

Mutable Pairs

Parameter  
Passing  
Variations

- To have *effects* values must be stored somewhere

# Language with Explicit References

State

Language with  
Explicit  
References

Language with  
Implicit  
References

Mutable Pairs

Parameter  
Passing  
Variations

- To have *effects* values must be stored somewhere
- $\sigma$  ranges over the store (or heap)
- $[l = v]\sigma \rightarrow$  the store  $\sigma$  extended with location  $l$  mapped to  $v$

# Language with Explicit References

## State

Language with  
Explicit  
ReferencesLanguage with  
Implicit  
References

## Mutable Pairs

Parameter  
Passing  
Variations

- To have *effects* values must be stored somewhere
- $\sigma$  ranges over the store (or heap)
- $[l = v]\sigma \rightarrow$  the store  $\sigma$  extended with location  $l$  mapped to  $v$
- We shall think of the store as an argument to value-of
- $(\text{value-of } \text{exp1 } \rho \ \sigma_0) = (\text{val1}, \sigma_1)$
- $\sigma_0$  may or may not be the same as  $\sigma_1$

# Language with Explicit References

## Specification

### State

### Language with Explicit References

### Language with Implicit References

### Mutable Pairs

### Parameter Passing Variations

- $(\text{value-of } (\text{const-exp } n) \rho \sigma) = ((\text{numval } n), \sigma)$

## Language with Explicit References

## Specification

State

Language with  
Explicit  
ReferencesLanguage with  
Implicit  
References

Mutable Pairs

Parameter  
Passing  
Variations

- $(\text{value-of } (\text{const-exp } n) \rho \sigma) = ((\text{numval } n), \sigma)$
- $$\frac{(\text{value-of } \text{exp1 } \rho \sigma_0) = (\text{val1}, \sigma_1) \wedge (\text{value-of } \text{exp2 } \rho \sigma_1) = (\text{val2}, \sigma_2)}{(\text{value-of } (\text{diff-exp } \text{exp1 } \text{exp2}) \rho \sigma_0) = ((\text{num-val } \text{val1} - \text{val2}) \sigma_2)}$$

## Language with Explicit References

## Specification

## State

Language with  
Explicit  
ReferencesLanguage with  
Implicit  
References

## Mutable Pairs

Parameter  
Passing  
Variations

- $(\text{value-of } (\text{const-exp } n) \rho \sigma) = ((\text{numval } n), \sigma)$
- $$\frac{(\text{value-of } \text{exp1 } \rho \sigma_0) = (val1, \sigma_1) \wedge (\text{value-of } \text{exp2 } \rho \sigma_1) = (val2, \sigma_2)}{(\text{value-of } (\text{diff-exp } \text{exp1 } \text{exp2}) \rho \sigma_0) = ((\text{num-val } val1 - val2) \sigma_2)}$$

$$\frac{(\text{value-of } e1 \rho \sigma_0) = (v1, \sigma_1)}{(\text{value-of } (\text{if-exp } e1 \ e2 \ e3) \rho \sigma_0) = \begin{cases} ((\text{value-of } e2 \rho \sigma_1), \sigma_2) & \text{if } (\text{exp} \rightarrow \text{val } v1) = \#t \\ ((\text{value-of } e3 \rho \sigma_1), \sigma_2) & \text{otherwise} \end{cases}}$$

## Language with Explicit References

## Specification

## State

Language with  
Explicit  
ReferencesLanguage with  
Implicit  
References

## Mutable Pairs

Parameter  
Passing  
Variations

- $$\frac{(value-of\ exp\ \rho\ \sigma_0) = (val1, \sigma_1) \ l \notin dom(\sigma_0)}{(value-of\ (newref-exp\ exp)\ \rho\ \sigma_0) = ((ref-val\ l), [l=val1]\sigma_1)}$$
- $l$  is a new store location



## Language with Explicit References

## Specification

## State

Language with  
Explicit  
ReferencesLanguage with  
Implicit  
References

## Mutable Pairs

Parameter  
Passing  
Variations

- $$\frac{(value-of\ exp\ \rho\ \sigma_0) = (val1, \sigma_1) \ l \notin dom(\sigma_0)}{(value-of\ (newref-exp\ exp)\ \rho\ \sigma_0) = ((ref-val\ l), [l=val1]\sigma_1)}$$
- $\mathbf{l}$  is a new store location
- $$\frac{(value-of\ exp\ \rho\ \sigma_0) = (l, \sigma_1)}{(value-of\ (deref-exp\ exp)\ \rho\ \sigma_0) = (\sigma_1(l), \sigma_1)}$$

## Language with Explicit References

## Specification

## State

Language with  
Explicit  
ReferencesLanguage with  
Implicit  
References

## Mutable Pairs

Parameter  
Passing  
Variations

- $$\frac{(value-of\ exp\ \rho\ \sigma_0) = (val1, \sigma_1) \ l \notin dom(\sigma_0)}{(value-of\ (newref-exp\ exp)\ \rho\ \sigma_0) = ((ref-val\ l), [l=val1]\sigma_1)}$$
- $\mathbf{l}$  is a new store location
- $$\frac{(value-of\ exp\ \rho\ \sigma_0) = (l, \sigma_1)}{(value-of\ (deref-exp\ exp)\ \rho\ \sigma_0) = (\sigma_1(l), \sigma_1)}$$
- $$\frac{(value-of\ exp1\ \rho\ \sigma_0) = (l, \sigma_1) \wedge (value-of\ exp2\ \rho\ \sigma_1) = (val, \sigma_2)}{(value-of\ (setref-exp\ exp1\ exp2)\ \rho\ \sigma_0) = (\emptyset, [l=val]\sigma_2)}$$

# Language with Explicit References

## Implementation

State

Language with  
Explicit  
ReferencesLanguage with  
Implicit  
References

Mutable Pairs

Parameter  
Passing  
Variations

- Grammar

```
(expression ("begin" expression (arbno ";" expression) "end")
 begin-exp)
```

```
(expression ("newref" "(" expression ")") newref-exp)
```

```
(expression ("deref" "(" expression ")") deref-exp)
```

```
(expression ("setref" "(" expression "," expression ")")
 setref-exp)
```

# Language with Explicit References

## Implementation

- Design choice: the store is a global variable
- Design choice: Represent the store as a (listof expval)

State

Language with  
Explicit  
References

Language with  
Implicit  
References

Mutable Pairs

Parameter  
Passing  
Variations

# Language with Explicit References

## Implementation

- Design choice: the store is a global variable
- Design choice: Represent the store as a (listof expval)
- `;; reference? : RacketVal --> Bool`  
`(define (reference? v) (and (integer? v) (>= v 0)))`

State

Language with  
Explicit  
References

Language with  
Implicit  
References

Mutable Pairs

Parameter  
Passing  
Variations

# Language with Explicit References

## Implementation

### State

### Language with Explicit References

### Language with Implicit References

### Mutable Pairs

### Parameter Passing Variations

- Design choice: the store is a global variable
- Design choice: Represent the store as a (listof expval)
- `;; reference? : RacketVal --> Bool`  
`(define (reference? v) (and (integer? v) (>= v 0)))`
- `;; the-store: the current state of the store`  
`(define the-store 'uninitialized)`

# Language with Explicit References

## Implementation

### State

### Language with Explicit References

### Language with Implicit References

### Mutable Pairs

### Parameter Passing Variations

- Design choice: the store is a global variable
- Design choice: Represent the store as a (listof expval)
- `;; reference? : RacketVal --> Bool`  
`(define (reference? v) (and (integer? v) (>= v 0)))`
- `;; the-store: the current state of the store`  
`(define the-store 'uninitialized)`
- `;; empty-store : --> store`  
`(define (empty-store) '())`

# Language with Explicit References

## Implementation

### State

### Language with Explicit References

### Language with Implicit References

### Mutable Pairs

### Parameter Passing Variations

- Design choice: the store is a global variable
- Design choice: Represent the store as a (listof expval)
- `;; reference? : RacketVal --> Bool`  
`(define (reference? v) (and (integer? v) (>= v 0)))`
- `;; the-store: the current state of the store`  
`(define the-store 'uninitialized)`
- `;; empty-store : --> store`  
`(define (empty-store) '())`
- `;; initialize-store! : --> store`  
`(define (initialize-store!) (set! the-store (empty-store)))`



## Language with Explicit References

## Implementation

## State

Language with  
Explicit  
ReferencesLanguage with  
Implicit  
References

## Mutable Pairs

Parameter  
Passing  
Variations

- Design choice: the store is a global variable
- Design choice: Represent the store as a (listof expval)
- `;; reference? : RacketVal --> Bool`  
`(define (reference? v) (and (integer? v) (>= v 0)))`
- `;; the-store: the current state of the store`  
`(define the-store 'uninitialized)`
- `;; empty-store : --> store`  
`(define (empty-store) '())`
- `;; initialize-store! : --> store`  
`(define (initialize-store!) (set! the-store (empty-store)))`
- `;; newref : expval --> ref`  
`(define (newref val)`  
`(let ((next-ref (length the-store)))`  
`(set! the-store (append the-store (list val)))`  
`next-ref))`

## Language with Explicit References

## Implementation

## State

Language with  
Explicit  
ReferencesLanguage with  
Implicit  
References

## Mutable Pairs

Parameter  
Passing  
Variations

- Design choice: the store is a global variable
- Design choice: Represent the store as a (listof expval)
- `;; reference? : RacketVal --> Bool`  
`(define (reference? v) (and (integer? v) (>= v 0)))`
- `;; the-store: the current state of the store`  
`(define the-store 'uninitialized)`
- `;; empty-store : --> store`  
`(define (empty-store) '())`
- `;; initialize-store! : --> store`  
`(define (initialize-store!) (set! the-store (empty-store)))`
- `;; newref : expval --> ref`  
`(define (newref val)`  
 `(let ((next-ref (length the-store)))`  
 `(set! the-store (append the-store (list val)))`  
 `next-ref))`
- `;; deref : ref --> expval`  
`(define (deref ref) (list-ref the-store ref))`

## Language with Explicit References

## Implementation

## State

Language with  
Explicit  
ReferencesLanguage with  
Implicit  
References

## Mutable Pairs

Parameter  
Passing  
Variations

- Design choice: the store is a global variable
- Design choice: Represent the store as a (listof expval)
- `;; reference? : RacketVal --> Bool`  
`(define (reference? v) (and (integer? v) (>= v 0)))`
- `;; the-store: the current state of the store`  
`(define the-store 'uninitialized)`
- `;; empty-store : --> store`  
`(define (empty-store) '())`
- `;; initialize-store! : --> store`  
`(define (initialize-store!) (set! the-store (empty-store)))`
- `;; newref : expval --> ref`  
`(define (newref val)`  
 `(let ((next-ref (length the-store)))`  
 `(set! the-store (append the-store (list val)))`  
 `next-ref))`
- `;; deref : ref --> expval`  
`(define (deref ref) (list-ref the-store ref))`
- `;; setref : ref expval --> expval`  
`(define (setref! ref new-expval)`  
 `(set! the-store (append (take the-store ref)`  
 `(list new-expval)`  
 `(drop the-store (add1 ref)))))`

# Language with Explicit References

## Implementation

State

Language with  
Explicit  
ReferencesLanguage with  
Implicit  
References

Mutable Pairs

Parameter  
Passing  
Variations

- ```
;; value-of-program : program --> expval
(define (value-of-program pgm)
  (begin
    (initialize-store!)
    (cases program pgm
      (a-program (exp1)
        (value-of exp1 (empty-env)))))))
```

Language with Explicit References

Implementation

State

Language with
Explicit
References

Language with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- ```
(define (value-of exp env)
 (cases expression exp
 (const-exp (num) (num-val num))
 (true-exp () (bool-val #t))
 (false-exp () (bool-val #f))
```

# Language with Explicit References

## Implementation

### State

### Language with Explicit References

### Language with Implicit References

### Mutable Pairs

### Parameter Passing Variations

- ```
(define (value-of exp env)
  (cases expression exp
    (const-exp (num) (num-val num))
    (true-exp () (bool-val #t))
    (false-exp () (bool-val #f))
    (var-exp (var) (apply-env env var))
```

Language with Explicit References

Implementation

State

Language with Explicit References

Language with Implicit References

Mutable Pairs

Parameter Passing Variations

- ```
(define (value-of exp env)
 (cases expression exp
 (const-exp (num) (num-val num))
 (true-exp () (bool-val #t))
 (false-exp () (bool-val #f))
 (var-exp (var) (apply-env env var))
 (diff-exp (exp1 exp2)
 (let ((num1 (expval2num (value-of exp1 env)))
 (num2 (expval2num (value-of exp2 env))))
 (num-val (- num1 num2))))))
```

# Language with Explicit References

## Implementation

### State

### Language with Explicit References

### Language with Implicit References

### Mutable Pairs

### Parameter Passing Variations

- ```
(define (value-of exp env)
  (cases expression exp
    (const-exp (num) (num-val num))
    (true-exp () (bool-val #t))
    (false-exp () (bool-val #f))
    (var-exp (var) (apply-env env var))
    (diff-exp (exp1 exp2)
      (let ((num1 (expval2num (value-of exp1 env)))
            (num2 (expval2num (value-of exp2 env))))
        (num-val (- num1 num2))))
    (zero?-exp (exp1)
      (let ((val1 (expval2num (value-of exp1 env))))
        (if (zero? val1)
            (bool-val #t)
            (bool-val #f))))))
```


Language with Explicit References

Implementation

State

Language with
Explicit
ReferencesLanguage with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- ```
(define (value-of exp env)
 (cases expression exp
 (const-exp (num) (num-val num))
 (true-exp () (bool-val #t))
 (false-exp () (bool-val #f))
 (var-exp (var) (apply-env env var))
 (diff-exp (exp1 exp2)
 (let ((num1 (expval2num (value-of exp1 env)))
 (num2 (expval2num (value-of exp2 env))))
 (num-val (- num1 num2))))
 (zero?-exp (exp1)
 (let ((val1 (expval2num (value-of exp1 env))))
 (if (zero? val1)
 (bool-val #t)
 (bool-val #f))))
 (if-exp (exp1 exp2 exp3)
 (let ((val1 (value-of exp1 env)))
 (if (expval2bool val1)
 (value-of exp2 env)
 (value-of exp3 env))))))
```

## Language with Explicit References

## Implementation

## State

Language with  
Explicit  
ReferencesLanguage with  
Implicit  
References

## Mutable Pairs

Parameter  
Passing  
Variations

- ```
(define (value-of exp env)
  (cases expression exp
    (const-exp (num) (num-val num))
    (true-exp () (bool-val #t))
    (false-exp () (bool-val #f))
    (var-exp (var) (apply-env env var))
    (diff-exp (exp1 exp2)
      (let ((num1 (expval2num (value-of exp1 env)))
            (num2 (expval2num (value-of exp2 env))))
        (num-val (- num1 num2))))
    (zero?-exp (exp1)
      (let ((val1 (expval2num (value-of exp1 env))))
        (if (zero? val1)
            (bool-val #t)
            (bool-val #f))))
    (if-exp (exp1 exp2 exp3)
      (let ((val1 (value-of exp1 env)))
        (if (expval2bool val1)
            (value-of exp2 env)
            (value-of exp3 env))))
    (let-exp (vars exps body)
      (let [(vals (map (lambda (e) (value-of e env)) exps))]
        (value-of body
          (foldr (lambda (var val acc)
                    (extend-env var val acc))
                  env
                  vars
                  vals))))))
```

⋮

Language with Explicit References

Implementation

State

Language with Explicit References

Language with Implicit References

Mutable Pairs

Parameter Passing Variations

- ```
(define (value-of exp env)
 (cases expression exp
 :
 :
 (proc-exp (params body)
 (proc-val (procedure params body (vector env))))))
```

# Language with Explicit References

## Implementation

## State

Language with  
Explicit  
ReferencesLanguage with  
Implicit  
References

## Mutable Pairs

Parameter  
Passing  
Variations

- ```
(define (value-of exp env)
  (cases expression exp
    :
    :
    (proc-exp (params body)
               (proc-val (procedure params body (vector env))))))
```
- ```
(call-exp (rator rands)
 (let [(proc (expval2proc (value-of rator env)))]
 (args (map (lambda (rand) (value-of rand env)) rands)))
 (apply-procedure proc args)))
```

# Language with Explicit References

## Implementation

## State

Language with  
Explicit  
ReferencesLanguage with  
Implicit  
References

## Mutable Pairs

Parameter  
Passing  
Variations

- ```
(define (value-of exp env)
  (cases expression exp
    :
    :
    (proc-exp (params body)
               (proc-val (procedure params body (vector env))))))
```
- ```
(call-exp (rator rands)
 (let [(proc (expval2proc (value-of rator env)))]
 (args (map (lambda (rand) (value-of rand env)) rands)))
 (apply-procedure proc args)))
```
- ```
(letrec-exp (names params bodies letrec-body)
  (value-of letrec-body (mk-letrec-env names params bodies env)))
```

Language with Explicit References

Implementation

State

Language with
Explicit
ReferencesLanguage with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- ```
(define (value-of exp env)
 (cases expression exp
 :
 :
 (proc-exp (params body)
 (proc-val (procedure params body (vector env))))))
```
- ```
(call-exp (rator rands)
  (let [(proc (expval2proc (value-of rator env)))]
    (args (map (lambda (rand) (value-of rand env)) rands))]
    (apply-procedure proc args)))
```
- ```
(letrec-exp (names params bodies letrec-body)
 (value-of letrec-body (mk-letrec-env names params bodies env)))
```
- ```
(begin-exp (exp exps)
  (foldl (lambda (e v) (value-of e env)) (value-of exp env) exps))
```

Language with Explicit References

Implementation

State

Language with
Explicit
ReferencesLanguage with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- ```
(define (value-of exp env)
 (cases expression exp
 :
 :
 (proc-exp (params body)
 (proc-val (procedure params body (vector env))))))
```
- ```
(call-exp (rator rands)
  (let [(proc (expval2proc (value-of rator env)))]
    (args (map (lambda (rand) (value-of rand env)) rands))]
    (apply-procedure proc args)))
```
- ```
(letrec-exp (names params bodies letrec-body)
 (value-of letrec-body (mk-letrec-env names params bodies env)))
```
- ```
(begin-exp (exp exps)
  (foldl (lambda (e v) (value-of e env)) (value-of exp env) exps))
```
- ```
(newref-exp (exp1)
 (let ((v1 (value-of exp1 env)))
 (ref-val (newref v1))))
```

# Language with Explicit References

## Implementation

## State

Language with  
Explicit  
ReferencesLanguage with  
Implicit  
References

## Mutable Pairs

Parameter  
Passing  
Variations

- ```
(define (value-of exp env)
  (cases expression exp
    :
    :
    (proc-exp (params body)
      (proc-val (procedure params body (vector env))))))
```
- ```
(call-exp (rator rands)
 (let [(proc (expval2proc (value-of rator env)))]
 (args (map (lambda (rand) (value-of rand env)) rands)))
 (apply-procedure proc args)))
```
- ```
(letrec-exp (names params bodies letrec-body)
  (value-of letrec-body (mk-letrec-env names params bodies env)))
```
- ```
(begin-exp (exp exps)
 (foldl (lambda (e v) (value-of e env)) (value-of exp env) exps))
```
- ```
(newref-exp (exp1)
  (let ((v1 (value-of exp1 env)))
    (ref-val (newref v1))))
```
- ```
(deref-exp (exp1)
 (let ((v1 (value-of exp1 env)))
 (let ((ref1 (expval2ref v1)))
 (deref ref1))))
```



# Language with Explicit References

## Implementation

State

Language with  
Explicit  
ReferencesLanguage with  
Implicit  
References

Mutable Pairs

Parameter  
Passing  
Variations

- ```
(define (value-of exp env)
  (cases expression exp
    :
    :
    (proc-exp (params body)
      (proc-val (procedure params body (vector env))))))
```
- ```
(call-exp (rator rands)
 (let [(proc (expval2proc (value-of rator env)))]
 (args (map (lambda (rand) (value-of rand env)) rands)))
 (apply-procedure proc args)))
```
- ```
(letrec-exp (names params bodies letrec-body)
  (value-of letrec-body (mk-letrec-env names params bodies env)))
```
- ```
(begin-exp (exp exps)
 (foldl (lambda (e v) (value-of e env)) (value-of exp env) exps)))
```
- ```
(newref-exp (exp1)
  (let ((v1 (value-of exp1 env)))
    (ref-val (newref v1))))
```
- ```
(deref-exp (exp1)
 (let ((v1 (value-of exp1 env)))
 (let ((ref1 (expval2ref v1)))
 (deref ref1))))
```
- ```
(setref-exp (exp1 exp2)
  (let ((ref (expval2ref (value-of exp1 env))))
    (let ((v2 (value-of exp2 env)))
      (begin
        (setref! ref v2)
        (num-val -1))))))
```

Language with Explicit References

Homework

State

Language with
Explicit
References

Language with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- 4.1, 4.2, 4.4, 4.8, 4.9

Language with Implicit References

State

Language with
Explicit
References

Language with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- Most modern PLs package common patterns of allocation, dereferencing, and mutation
- Programmers do not need to worry about these operations

Language with Implicit References

State

Language with
Explicit
References

Language with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- Most modern PLs package common patterns of allocation, dereferencing, and mutation
- Programmers do not need to worry about these operations
- Every variable denotes a reference
- References are no longer expressed values and exist only as bindings of vars

```
expval = int + bool + proc  
denval = ref(expval)
```

Language with Implicit References

State

Language with Explicit References

Language with Implicit References

Mutable Pairs

Parameter Passing Variations

- Most modern PLs package common patterns of allocation, dereferencing, and mutation
- Programmers do not need to worry about these operations
- Every variable denotes a reference
- References are no longer expressed values and exist only as bindings of vars

```
expval = int + bool + proc
denval = ref(expval)
```
- Locations are created with each binding operation: procedure call, let, and letrec

Language with Implicit References

State

Language with
Explicit
References

Language with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- Most modern PLs package common patterns of allocation, dereferencing, and mutation
- Programmers do not need to worry about these operations
- Every variable denotes a reference
- References are no longer expressed values and exist only as bindings of vars

```
expval = int + bool + proc
denval = ref(expval)
```
- Locations are created with each binding operation: procedure call, let, and letrec
- What happens when the interpreter encounters a var-exp?
 - env look-up to find the location to which it's bound
 - look-up in the store to find the value at that location
 - two-level system for var-exps

- The content of a location can be changed (or mutated)
- expression \rightarrow set identifier = expression
- the identifier is *not* an expression; not evaluated
- vars are mutable

State

Language with Explicit References

Language with Implicit References

Mutable Pairs

Parameter Passing Variations

- The content of a location can be changed (or mutated)
- expression \rightarrow set identifier = expression
- the identifier is *not* an expression; not evaluated
- vars are mutable
- Extend LETREC language and implement call-by-value semantics
- *Values* are passed to every function
- Formal parameters bound to locations of operand values
- It is the most common form of parameter passing

- The content of a location can be changed (or mutated)
- expression \rightarrow set identifier = expression
- the identifier is *not* an expression; not evaluated
- vars are mutable
- Extend LETREC language and implement call-by-value semantics
- *Values* are passed to every function
- Formal parameters bound to locations of operand values
- It is the most common form of parameter passing
- Why are chains of references not possible?

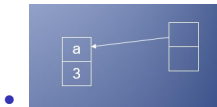
- The content of a location can be changed (or mutated)
- expression \rightarrow set identifier = expression
- the identifier is *not* an expression; not evaluated
- vars are mutable
- Extend LETREC language and implement call-by-value semantics
- *Values* are passed to every function
- Formal parameters bound to locations of operand values
- It is the most common form of parameter passing
- Why are chains of references not possible?
- Refs are not expressed values

- Consider

```
let a = 3
in let p = proc (x) set x = 4
    in begin (p a); a end
```

- Consider

```
let a = 3
in let p = proc (x) set x = 4
    in begin (p a); a end
```

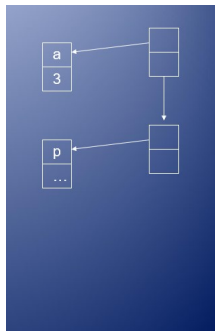


- Consider

```
let a = 3
```

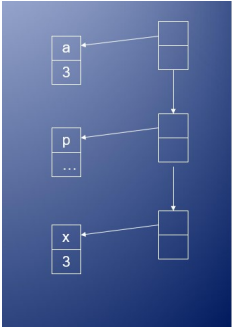
```
in let p = proc (x) set x = 4
```

```
in begin (p a); a end
```



- Consider

```
let a = 3
in let p = proc (x) set x = 4
    in begin (p a); a end
```



State

State

Language with
Explicit
References

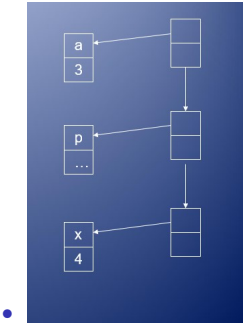
Language with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- Consider

```
let a = 3
in let p = proc (x) set x = 4
  in begin (p a); a end
```



- Returns 3

State Specification

State

Language with Explicit References

Language with Implicit References

Mutable Pairs

Parameter Passing Variations

- $(\text{value-of}(\text{var-exp } v) \rho \sigma) = (\sigma(\rho(v)), \sigma)$
- Get v 's binding (a reference) and access store for v 's expval
- The store is unchanged

State
Specification

State

Language with
Explicit
ReferencesLanguage with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- $(\text{value-of}(\text{var-exp } v) \rho \sigma) = (\sigma(\rho(v)), \sigma)$
- Get v 's binding (a reference) and access store for v 's expval
- The store is unchanged
- $$\frac{(\text{value-of } \text{exp1 } \rho \sigma_0) = (\text{val1}, \sigma_1)}{(\text{value-of}(\text{set-exp } v \text{ exp1}) \rho \sigma_0) = (\emptyset, [\sigma(v)=\text{val1}]\sigma_1)}$$
- The location of v is changed to store val1
- The original value stored in $\sigma(v)$ is lost forever

- $(\text{value-of}(\text{var-exp } v) \rho \sigma) = (\sigma(\rho(v)), \sigma)$
- Get v 's binding (a reference) and access store for v 's expval
- The store is unchanged
- $$\frac{(\text{value-of } \text{exp1 } \rho \sigma_0) = (\text{val1}, \sigma_1)}{(\text{value-of}(\text{set-exp } v \text{ exp1}) \rho \sigma_0) = (\emptyset, [\sigma(v)=\text{val1}]\sigma_1)}$$
- The location of v is changed to store val1
- The original value stored in $\sigma(v)$ is lost forever
- $$\begin{aligned} &(\text{apply-procedure} (\text{procedure } v \text{ b } \rho) \text{ val } \sigma) \\ &= (\text{value-of } b \ [v = l] \rho \ [l = \text{val}] \sigma) \end{aligned}$$
- The body is evaluated in a store where l contains the value of the parameter and an environment that binds the parameter to l

State

Language with
Explicit
ReferencesLanguage with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- $(\text{value-of}(\text{var-exp } v) \rho \sigma) = (\sigma(\rho(v)), \sigma)$
- Get v 's binding (a reference) and access store for v 's expval
- The store is unchanged
- $$\frac{(\text{value-of } \text{exp1 } \rho \sigma_0) = (val1, \sigma_1)}{(\text{value-of}(\text{set-exp } v \text{ exp1}) \rho \sigma_0) = (\emptyset, [\sigma(v)=val1]\sigma_1)}$$
- The location of v is changed to store $val1$
- The original value stored in $\sigma(v)$ is lost forever
- $$\begin{aligned} &(\text{apply-procedure} (\text{procedure } v \text{ b } \rho) \text{ val } \sigma) \\ &= (\text{value-of } b \text{ } [v = l] \rho \text{ } [l = val] \sigma) \end{aligned}$$
- The body is evaluated in a store where l contains the value of the parameter and an environment that binds the parameter to l
- $$\frac{(\text{value-of } \text{exp1 } \rho \sigma) = (val, \sigma_1)}{(\text{value-of} (\text{let-exp } var \text{ exp1 exp2}) \rho \sigma) = (\text{value-of } \text{exp2 } [var=l] \rho \text{ } [l=val] \sigma_1)}$$
- Evaluate the body of the let-exp in a store where l contains the value of the local variable and the local variable is bound to l

State

Language with
Explicit
ReferencesLanguage with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- $$\frac{(value-of\ e0\ \rho\ \sigma_0)=(p,\ \sigma_1) \wedge (value-of\ e1\ \rho\ \sigma_1)=(v1,\sigma_2) \wedge (value-of\ e2\ \rho\ \sigma_2)=(v2,\ \sigma_3)\wedge \dots}{(value-of\ (call-exp\ e0\ e1\dots en)\ \rho\ \sigma_0)=(apply-procedure\ p\ v1\dots vn\ \sigma_{n+1})}$$
- Evaluate all expressions using the given environment
- Evaluate e_i using σ_i
- Apply the proc to the args using the store state after evaluating all expressions

- $$\frac{(\text{value-of } e0 \ \rho \ \sigma_0)=(p, \ \sigma_1) \wedge (\text{value-of } e1 \ \rho \ \sigma_1)=(v1, \sigma_2) \wedge (\text{value-of } e2 \ \rho \ \sigma_2)=(v2, \ \sigma_3) \wedge \dots}{(\text{value-of } (\text{call-exp } e0 \ e1 \dots en) \ \rho \ \sigma_0)=(\text{apply-procedure } p \ v1 \dots vn \ \sigma_{n+1})}$$
- Evaluate all expressions using the given environment
- Evaluate ei using σ_i
- Apply the proc to the args using the store state after evaluating all expressions
- $$\frac{\rho_n=[n_1=l_1 \dots n_n=l_n] \rho \wedge p1=(\text{proc-val } n_1 \ p1 \ e1 \ \rho_n) \wedge \dots \wedge pn=(\text{proc-val } n_n \ pn \ en \ \rho_n)}{(v-o \ (\text{letrec-exp } n1 \dots nn \ p1 \dots pn \ e1 \dots en \ en_{n+1}) \ \rho \ \sigma)=(v-o \ en_{n+1} \ \rho_n \ [l_1=p1 \dots l_n=pn] \sigma)}$$
- v-o = value-of
- All procs are allocated in the store

State

Implementation

State

Language with
Explicit
ReferencesLanguage with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- `(expression`
 `("begin" expression (arbno ";" expression) "end")`
 `begin-exp)`

 `(expression ("set" identifier "=" expression) set-exp)`

State

Implementation

State

Language with
Explicit
ReferencesLanguage with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- `(expression`
 `("begin" expression (arbno ";" expression) "end")`
 `begin-exp)`

 `(expression ("set" identifier "=" expression) set-exp)`
- The store is the same as with Explicit Refs

- `(expression`
 `("begin" expression (arbno ";" expression) "end")`
 `begin-exp)`

 `(expression ("set" identifier "=" expression) set-exp)`
- The store is the same as with Explicit Refs
- `(define-datatype expval expval?`
 `(num-val`
 `(value number?))`
 `(bool-val`
 `(boolean boolean?))`
 `(proc-val`
 `(proc proc?)))`
- Unlike Explicit Refs, no `ref-val`

- `(expression`
`("begin" expression (arbno ";" expression) "end")`
`begin-exp)`
- `(expression ("set" identifier "=" expression) set-exp)`
- The store is the same as with Explicit Refs
- `(define-datatype expval expval?`
`(num-val`
`(value number?))`
`(bool-val`
`(boolean boolean?))`
`(proc-val`
`(proc proc?)))`
- Unlike Explicit Refs, no `ref-val`
- Same as Explicit Refs
- `(define (value-of-program pgm)`
`(begin`
`(initialize-store!)`
`(cases program pgm`
`(a-program (exp1)`
`(value-of exp1 (empty-env))))))`

State

Implementation

State

Language with
Explicit
ReferencesLanguage with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- ```
(check-equal? (eval "if zero?(1) then 1 else 2")
 (num-val 2))

(check-equal? (eval "-(15, 10)")
 (num-val 5))

(check-equal?
 (eval "let x = 10 in if zero?(-(x, x)) then x else 2")
 (num-val 10))

(check-equal? (eval "let decr = proc (a) -(a, 1) in (decr 30)")
 (num-val 29))

(check-equal? (eval "(proc (g) (g 30) proc (y) -(y, 1))")
 (num-val 29))

(check-equal? (eval "let x = 200
 in let f = proc (z) -(z, x)
 in let x = 100
 in let g = proc (z) -(z, x)
 in -((f 1), (g 1))")
 (num-val -100))
```

- ```
(check-equal?  
  (eval "let sum = proc (x) proc (y) -(x, -(0, y)) in ((sum 3) 4)")  
  (num-val 7))  
  
(check-equal?  
  (eval "let sum = proc (x) proc (y) -(x, -(0, y))  
        in letrec sigma (n) = if zero?(n)  
                                then 0  
                                else ((sum n) (sigma -(n, 1)))  
        in (sigma 5)")  
  (num-val 15))  
  
(check-equal? (eval "letrec even(n) = if zero?(n)  
                                then zero?(n)  
                                else if zero?(-(n, 1))  
                                    then zero?(n)  
                                    else (even -(n, 2))  
                                in (even 501)")  
  (bool-val #f)))
```

- ```
(check-equal? (eval "let a = 3
 in let p = proc (x) set x = 4
 in begin
 (p a);
 a
 end")
 (num-val 3))

(check-equal? (eval "let x = 0
 in letrec f (x) = set x = +(x, 1)
 g (a) = set x = +(x, 2)
 in begin
 (f x);
 (g x);
 x
 end")
 (num-val 2))
```

- ```
(define (value-of exp env)
  (cases expression exp

    (const-exp (num) (num-val num))

    (true-exp () (bool-val #t))

    (false-exp () (bool-val #f))

    (var-exp (var) (deref (apply-env env var)))

    (diff-exp (exp1 exp2)
      (let ((num1 (expval2num (value-of exp1 env)))
            (num2 (expval2num (value-of exp2 env))))
        (num-val (- num1 num2))))

    (zero?-exp (exp1)
      (let ((val1 (expval2num (value-of exp1 env))))
        (if (zero? val1)
            (bool-val #t)
            (bool-val #f)))))
```

- ```

(if-exp (exp1 exp2 exp3)
 (let ((val1 (value-of exp1 env)))
 (if (expval2bool val1)
 (value-of exp2 env)
 (value-of exp3 env))))

(let-exp (vars exps body)
 (let [(vals (map (lambda (e) (value-of e env)) exps))]
 (value-of body
 (foldr (lambda (var val acc)
 (extend-env var (newref val) acc))
 env
 vars
 vals))))

(proc-exp (params body)
 (proc-val (procedure params body (vector env))))

(call-exp (rator rands)
 (let [(proc (expval2proc (value-of rator env)))]
 (args (map (lambda (rand) (value-of rand env))
 (apply-procedure proc args))))

```

# State

## Implementation

- ```
(letrec-exp (names params bodies rec-body)
  (value-of rec-body (mk-letrec-env names params bodies env)))

(begin-exp (exp exps)
  (foldl (lambda (e v) (value-of e env))
    (value-of exp env)
    exps))

(set-exp (var exp1)
  (begin
    (setref! (apply-env env var) (value-of exp1 env))
    (num-val 31))))
```

```

• (define (mk-letrec-env names params bodies env)
  (let* [(temp-proc-vals
          (map (lambda (p b)
                 (proc-val (procedure p b (vector (empty-env))))))
          params
          bodies))
        (new-env (foldl (lambda (name proc env)
                          (extend-env name
                                      (newref proc)
                                      env)
                          names
                          temp-proc-vals))]
    (begin
      (for-each (lambda (p)
                   (cases proc p
                     (procedure (p b ve)
                               (vector-set! ve 0 new-env))))
                (map (lambda (p) (expval2proc p))
                     temp-proc-vals))
      new-env)))

```


- ```
(define (apply-procedure f vals)
 (cases proc f
 (procedure (params body envv)
 (let [(saved-env (vector-ref envv 0))]
 (value-of body
 (foldr (lambda (binding acc)
 (extend-env (car binding)
 (newref (cadr binding))
 acc))
 saved-env
 (map (lambda (p v) (list p v))
 params
 vals))))))))
```

- We will add mutable pairs to IMPLICIT-REFS

Marco T.  
Morazán

### State

Language with  
Explicit  
References

Language with  
Implicit  
References

### Mutable Pairs

Parameter  
Passing  
Variations

- We will add mutable pairs to IMPLICIT-REFS
- $\text{expval} = \text{int} + \text{bool} + \text{proc} + \text{mutpair}$
- $\text{mutpair} = \text{ref}(\text{expval}) \times \text{ref}(\text{expval})$

Marco T.  
Morazán

State

Language with  
Explicit  
References

Language with  
Implicit  
References

Mutable Pairs

Parameter  
Passing  
Variations

- We will add mutable pairs to IMPLICIT-REFS
- $\text{expval} = \text{int} + \text{bool} + \text{proc} + \text{mutpair}$
- $\text{mutpair} = \text{ref}(\text{expval}) \times \text{ref}(\text{expval})$
- $\text{DenVal} = \text{ref}(\text{expval})$

- We will add mutable pairs to IMPLICIT-REFS
- $\text{expval} = \text{int} + \text{bool} + \text{proc} + \text{mutpair}$
- $\text{mutpair} = \text{ref}(\text{expval}) \times \text{ref}(\text{expval})$
- $\text{DenVal} = \text{ref}(\text{expval})$
- Specification
  - $\text{newpair: expval expval} \rightarrow \text{mutpair}$
  - $\text{left: mutpair} \rightarrow \text{expval}$
  - $\text{right: mutpair} \rightarrow \text{expval}$
  - $\text{setleft: mutpair expval} \rightarrow \emptyset$
  - $\text{setright: mutpair expval} \rightarrow \emptyset$

- We will add mutable pairs to IMPLICIT-REFS
- $\text{expval} = \text{int} + \text{bool} + \text{proc} + \text{mutpair}$
- $\text{mutpair} = \text{ref}(\text{expval}) \times \text{ref}(\text{expval})$
- $\text{DenVal} = \text{ref}(\text{expval})$
- Specification
  - $\text{newpair: expval expval} \rightarrow \text{mutpair}$
  - $\text{left: mutpair} \rightarrow \text{expval}$
  - $\text{right: mutpair} \rightarrow \text{expval}$
  - $\text{setleft: mutpair expval} \rightarrow \emptyset$
  - $\text{setright: mutpair expval} \rightarrow \emptyset$
- (define-datatype expval expval?
  - (num-val  
  (value number?))
  - (bool-val  
  (boolean boolean?))
  - (proc-val  
  (proc proc?))
  - (mutpair-val ;; new for mutable pairs  
  (p mutpair?))

- Grammar

- (expression ("newpair" "(" expression "," expression ")")  
newpair-exp)
- (expression ("left" "(" expression ")") left-exp)
- (expression ("setleft" expression "=" expression)  
setleft-exp)
- (expression ("right" "(" expression ")") right-exp)
- (expression ("setright" expression "=" expression)  
setright-exp)

- Let's trace

```
(eval "let p = newpair(4, 5)
 in begin
 setleft p = 15;
 setright p = 15;
 -(left(p), right(p))
 end")
```



## • Let's trace

```
(eval "let p = newpair(4, 5)
 in begin
 setleft p = 15;
 setright p = 15;
 -(left(p), right(p))
 end")
```

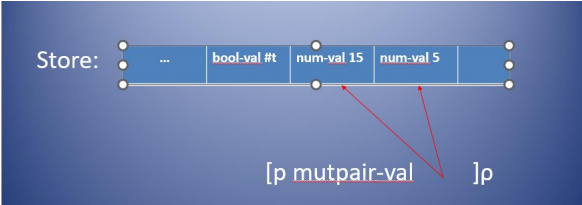
Store:

|     |             |           |           |  |
|-----|-------------|-----------|-----------|--|
| ... | bool-val #t | num-val 4 | num-val 5 |  |
|-----|-------------|-----------|-----------|--|

[p mutpair-val]p

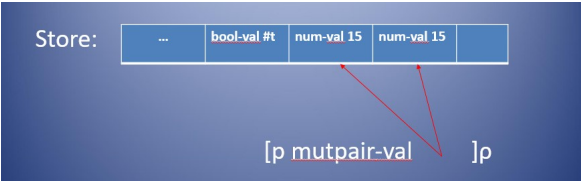
- Let's trace

```
(eval "let p = newpair(4, 5)
 in begin
 setleft p = 15;
 setright p = 15;
 -(left(p), right(p))
 end")
```



- Let's trace

```
(eval "let p = newpair(4, 5)
 in begin
 setleft p = 15;
 setright p = 15;
 -(left(p), right(p))
 end")
```



- Returns (num-val 0)

- How can we represent a mutable pair?

- How can we represent a mutable pair?
- `(define-datatype mutpair mutpair?  
 (a-pair (left-loc reference?)  
 (right-loc reference?)))`
- Is this a good implementation choice?

- How can we represent a mutable pair?
- ```
(define-datatype mutpair mutpair?  
  (a-pair (left-loc reference?)  
          (right-loc reference?)))
```
- Is this a good implementation choice?
- Does not take into account everything we know about mutable pairs
 - The two locations are independently assignable
 - Not independently allocated

- How can we represent a mutable pair?
- `(define-datatype mutpair mutpair?
 (a-pair (left-loc reference?)
 (right-loc reference?)))`
- Is this a good implementation choice?
- Does not take into account everything we know about mutable pairs
 - The two locations are independently assignable
 - Not independently allocated
- Consider `newpair(4, 5)` and σ
 - $\sigma = (\dots)$
 - $\sigma = (\dots 4)$
 - $\sigma = (\dots 4 \ 5)$

- How can we represent a mutable pair?
- `(define-datatype mutpair mutpair?
 (a-pair (left-loc reference?)
 (right-loc reference?)))`
- Is this a good implementation choice?
- Does not take into account everything we know about mutable pairs
 - The two locations are independently assignable
 - Not independently allocated
- Consider `newpair(4, 5)` and σ
 - $\sigma = (\dots)$
 - $\sigma = (\dots 4)$
 - $\sigma = (\dots 4 \ 5)$
- If the left is in position p in σ , where is the right?

- How can we represent a mutable pair?
- ```
(define-datatype mutpair mutpair?
 (a-pair (left-loc reference?)
 (right-loc reference?)))
```
- Is this a good implementation choice?
- Does not take into account everything we know about mutable pairs
  - The two locations are independently assignable
  - Not independently allocated
- Consider `newpair(4, 5)` and  $\sigma$ 
  - $\sigma = ( \dots )$
  - $\sigma = ( \dots 4 )$
  - $\sigma = ( \dots 4 \ 5 )$
- If the left is in position  $p$  in  $\sigma$ , where is the right?
- What does this tell you?

- How can we represent a mutable pair?
- `(define-datatype mutpair mutpair?  
 (a-pair (left-loc reference?)  
 (right-loc reference?)))`
- Is this a good implementation choice?
- Does not take into account everything we know about mutable pairs
  - The two locations are independently assignable
  - Not independently allocated
- Consider `newpair(4, 5)` and  $\sigma$ 
  - $\sigma = ( \dots )$
  - $\sigma = ( \dots 4 )$
  - $\sigma = ( \dots 4 \ 5 )$
- If the left is in position  $p$  in  $\sigma$ , where is the right?
- What does this tell you?
- We can implement mutable pairs using a single reference

Marco T.  
Morazán

State

Language with  
Explicit  
References

Language with  
Implicit  
References

Mutable Pairs

Parameter  
Passing  
Variations

- ```
;; expval --> reference throws error
(define (expval->mutpair v)
  (cases expval v
    (mutpair-val (ref) ref)
    (else (expval-extractor-error 'mutable-pair v))))
```

- ```
;; expval --> reference throws error
(define (expval->mutpair v)
 (cases expval v
 (mutpair-val (ref) ref)
 (else (expval-extractor-error 'mutable-pair v))))
```
- ```
;; mutpair? : X -> Boolean
(define (mutpair? v) (reference? v))
```

- ```
;; expval --> reference throws error
(define (expval->mutpair v)
 (cases expval v
 (mutpair-val (ref) ref)
 (else (expval-extractor-error 'mutable-pair v))))
```
- ```
;; mutpair? : X -> Boolean
(define (mutpair? v) (reference? v))
```
- ```
;; make-pair : expval expval -> mutpair
(define (make-pair val1 val2)
 (let ((ref1 (newref val1)))
 (let ((ref2 (newref val2)))
 ref1)))
```

- ```
;; expval --> reference throws error
(define (expval->mutpair v)
  (cases expval v
    (mutpair-val (ref) ref)
    (else (expval-extractor-error 'mutable-pair v))))
```
- ```
;; mutpair? : X -> Boolean
(define (mutpair? v) (reference? v))
```
- ```
;; make-pair : expval expval -> mutpair
(define (make-pair val1 val2)
  (let ((ref1 (newref val1)))
    (let ((ref2 (newref val2)))
      ref1)))
```
- ```
;; left : mutpair -> expval
(define (left p) (deref p))

;; right : mutpair -> expval
(define (right p) (deref (+ 1 p)))
```

- ```
;; expval --> reference throws error
(define (expval->mutpair v)
  (cases expval v
    (mutpair-val (ref) ref)
    (else (expval-extractor-error 'mutable-pair v))))
```
- ```
;; mutpair? : X -> Boolean
(define (mutpair? v) (reference? v))
```
- ```
;; make-pair : expval expval -> mutpair
(define (make-pair val1 val2)
  (let ((ref1 (newref val1)))
    (let ((ref2 (newref val2)))
      ref1)))
```
- ```
;; left : mutpair -> expval
(define (left p) (deref p))

;; right : mutpair -> expval
(define (right p) (deref (+ 1 p)))
```
- ```
;; setleft : mutpair expval -> Unspecified
(define (setleft p val) (setref! p val))

;; setright : mutpair expval -> Unspecified
(define (setright p val) (setref! (+ 1 p) val))
```

- ```
(check-equal? (eval "let p = newpair(4, 5)
 in left(p)")
 (num-val 4))

(check-equal? (eval "let p = newpair(4, 5)
 in right(p)")
 (num-val 5))

(check-equal? (eval "let p = newpair(4, 5)
 in begin
 setleft p = 15;
 setright p = 15;
 -(left(p), right(p))
 end")
 (num-val 0))
```



- ```
(define (value-of exp env)
  (cases expression exp
    :
    (newpair-exp (exp1 exp2)
      (let ((v1 (value-of exp1 env))
            (v2 (value-of exp2 env)))
        (mutpair-val (make-pair v1 v2))))))
```

- ```
(define (value-of exp env)
 (cases expression exp
 :
 (newpair-exp (exp1 exp2)
 (let ((v1 (value-of exp1 env))
 (v2 (value-of exp2 env)))
 (mutpair-val (make-pair v1 v2))))))
```
- ```
(left-exp (exp1)
  (let ((v1 (value-of exp1 env)))
    (let ((p1 (expval->mutpair v1)))
      (left p1))))
(right-exp (exp1)
  (let ((v1 (value-of exp1 env)))
    (let ((p1 (expval->mutpair v1)))
      (right p1))))
```

- (define (value-of exp env)
 - (cases expression exp
 - :
 - :
 - (newpair-exp (exp1 exp2)
 - (let ((v1 (value-of exp1 env))
 (v2 (value-of exp2 env)))
 (mutpair-val (make-pair v1 v2))))
- (left-exp (exp1)
 - (let ((v1 (value-of exp1 env)))
 (let ((p1 (expval->mutpair v1)))
 (left p1))))
- (right-exp (exp1)
 - (let ((v1 (value-of exp1 env)))
 (let ((p1 (expval->mutpair v1)))
 (right p1))))
- (setleft-exp (exp1 exp2)
 - (let ((v1 (value-of exp1 env))
 (v2 (value-of exp2 env)))
 (let ((p (expval->mutpair v1)))
 (begin (setleft p v2)
 (num-val 82)))))) ;; this is a don't care value.
- (setright-exp (exp1 exp2)
 - (let ((v1 (value-of exp1 env))
 (v2 (value-of exp2 env)))
 (let ((p (expval->mutpair v1)))
 (begin (setright p v2)
 (num-val 83)))))) ;; this is a don't care value.

- Homework: 4.28–4.30

Parameter Passing Variations

Call by Reference

State

Language with Explicit References

Language with Implicit References

Mutable Pairs

Parameter Passing Variations

- In call-by-value semantics the callee is isolated from the caller
- Assignments by the callee to its parameters can not be seen by the caller

Parameter Passing Variations

Call by Reference

State

Language with Explicit References

Language with Implicit References

Mutable Pairs

Parameter Passing Variations

- In call-by-value semantics the callee is isolated from the caller
- Assignments by the callee to its parameters can not be seen by the caller
- Sometimes it is desirable to pass in variables expecting the callee to make assignments to them
- This can be done by passing references to the callee instead of actual values
- This is known as *call-by-reference*

Parameter Passing Variations

Call by Reference

State

Language with Explicit References

Language with Implicit References

Mutable Pairs

Parameter Passing Variations

- In call-by-value semantics the callee is isolated from the caller
- Assignments by the callee to its parameters can not be seen by the caller
- Sometimes it is desirable to pass in variables expecting the callee to make assignments to them
- This can be done by passing references to the callee instead of actual values
- This is known as *call-by-reference*
- If an operand is a variable, then a reference to the variable's location is passed
- The formal parameter is bound to this location

Parameter Passing Variations

Call by Reference

State

Language with Explicit References

Language with Implicit References

Mutable Pairs

Parameter Passing Variations

- In call-by-value semantics the callee is isolated from the caller
- Assignments by the callee to its parameters can not be seen by the caller
- Sometimes it is desirable to pass in variables expecting the callee to make assignments to them
- This can be done by passing references to the callee instead of actual values
- This is known as *call-by-reference*
- If an operand is a variable, then a reference to the variable's location is passed
- The formal parameter is bound to this location
- If the operand is some other type of expression, then the formal parameter is bound to a new location containing the value of the operand
- Just like in call-by-value

Parameter Passing Variations

Call by Reference

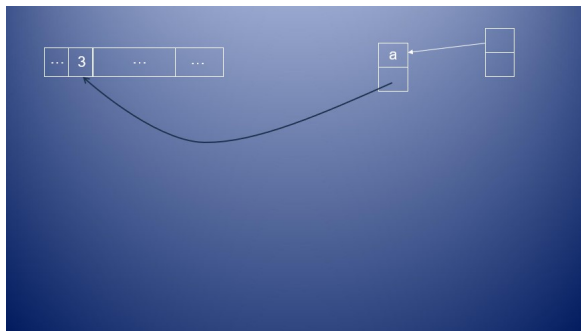
State

Language with
Explicit
ReferencesLanguage with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- let **a = 3**
p = proc (x) set x = 4
in begin
 (p a);
 a
end



Parameter Passing Variations

Call by Reference

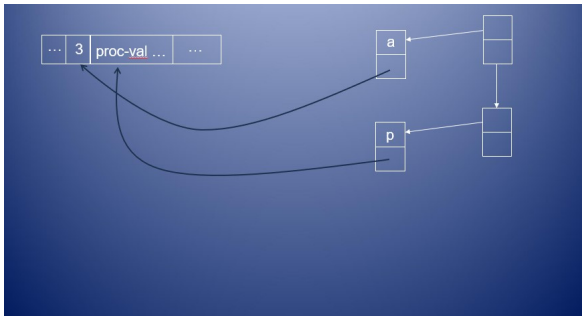
State

Language with
Explicit
ReferencesLanguage with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- ```
let a = 3
 p = proc (x) set x = 4
in begin
 (p a);
 a
end
```



# Parameter Passing Variations

## Call by Reference

### State

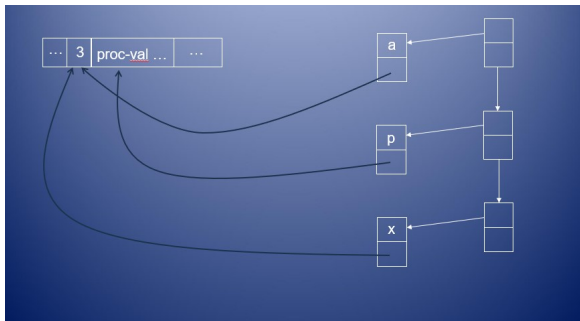
#### Language with Explicit References

#### Language with Implicit References

#### Mutable Pairs

#### Parameter Passing Variations

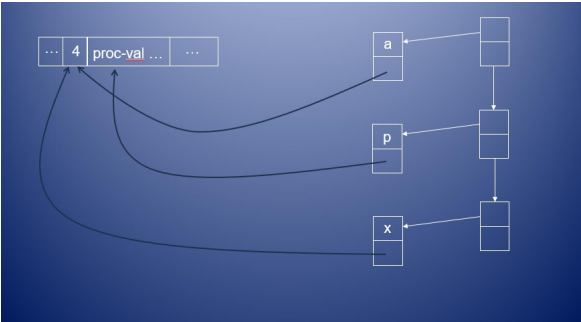
- ```
let a = 3
  p = proc (x) set x = 4
in begin
  (p a);
  a
end
```



Parameter Passing Variations

Call by Reference

- ```
let a = 3
 p = proc (x) set x = 4
in begin
 (p a);
 a
end
```



- Returns 4

# Parameter Passing Variations

## Call by Reference

State

Language with  
Explicit  
References

Language with  
Implicit  
References

Mutable Pairs

Parameter  
Passing  
Variations

- Why use call-by-reference?
  - Return multiple values (by making assignments to parameters)
  - Implementation of common operations

# Parameter Passing Variations

## Call by Reference

### State

#### Language with Explicit References

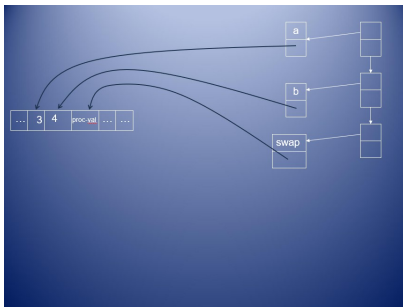
#### Language with Implicit References

#### Mutable Pairs

#### Parameter Passing Variations

- Call-by-Value

```
let a = 3
 b = 4
 swap = proc (x, y)
 let temp = x
 in begin
 set x = y
 set y = temp
 end
in begin
 swap(a b)
 -(a, b)
end
```

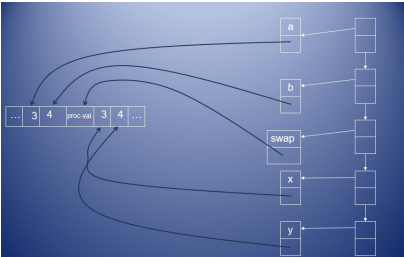


# Parameter Passing Variations

## Call by Reference

- Call-by-Value

```
let a = 3
 b = 4
 swap = proc (x, y)
 let temp = x
 in begin
 set x = y
 set y = temp
 end
in begin
 swap(a b)
 -(a, b)
end
```

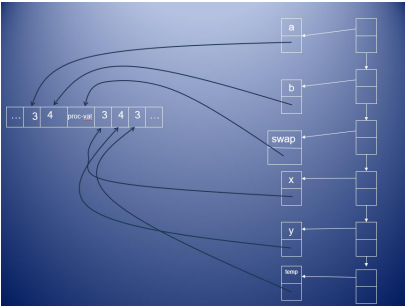


# Parameter Passing Variations

## Call by Reference

- Call-by-Value

```
let a = 3
 b = 4
 swap = proc (x, y)
 let temp = x
 in begin
 set x = y
 set y = temp
 end
in begin
 swap(a b)
 -(a, b)
end
```



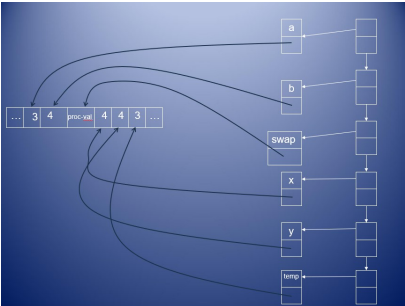


# Parameter Passing Variations

## Call by Reference

- Call-by-Value

```
let a = 3
 b = 4
 swap = proc (x, y)
 let temp = x
 in begin
 set x = y
 set y = temp
 end
in begin
 swap(a b)
 -(a, b)
end
```



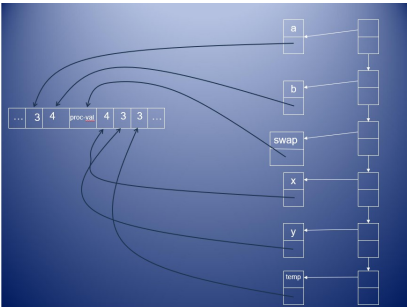
# Parameter Passing Variations

## Call by Reference

- Call-by-Value

```
let a = 3
 b = 4
 swap = proc (x, y)
 let temp = x
 in begin
 set x = y
 set y = temp
 end
in begin
 swap(a b)
 -(a, b)
end
```

Returns -1

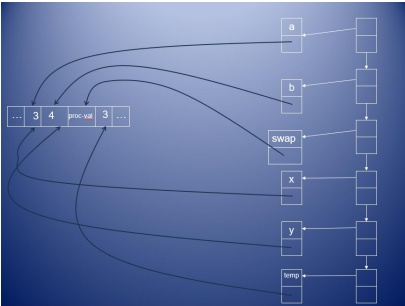


# Parameter Passing Variations

## Call by Reference

- Call-by-Reference

```
let a = 3
 b = 4
 swap = proc (x, y)
 let temp = x
 in begin
 set x = y
 set y = temp
 end
in begin
 swap(a b)
 -(a, b)
end
```

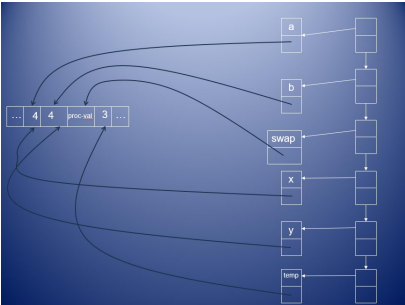


# Parameter Passing Variations

## Call by Reference

- Call-by-Reference

```
let a = 3
 b = 4
 swap = proc (x, y)
 let temp = x
 in begin
 set x = y
 set y = temp
 end
in begin
 swap(a b)
 -(a, b)
end
```



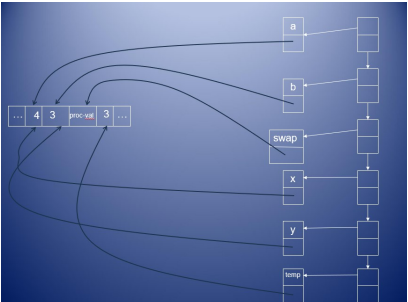
# Parameter Passing Variations

## Call by Reference

- Call-by-Reference

```
let a = 3
 b = 4
 swap = proc (x, y)
 let temp = x
 in begin
 set x = y
 set y = temp
 end
 in begin
 swap(a b)
 -(a, b)
 end
```

Returns 1



# Parameter Passing Variations

## Call by Reference

### State

### Language with Explicit References

### Language with Implicit References

### Mutable Pairs

### Parameter Passing Variations

- Only change is for when new references are created:
  - call-by-value: a new reference is created for every operand evaluated
  - call-by-reference: a new reference is created for evaluation of an operand other than a variable
- Under call-by-reference we need a new location for some operands and not for others

# Parameter Passing Variations

## Call by Reference

- ```
;; apply-procedure : proc (listof expval) -> expval
(define (apply-procedure f vals)
  (cases proc f
    (procedure (params body envv)
      (let [(saved-env (vector-ref envv 0))]
        (value-of body
          (foldr (lambda (binding acc)
                    (extend-env (car binding)
                                (newref (cadr binding))
                                acc))
                  saved-env
                  (map (lambda (p v) (list p v)) params vals)))))))
```

Can't always allocate an argument in the store

Parameter Passing Variations

Call by Reference

- ```
;; apply-procedure : proc (listof expval) -> expval
(define (apply-procedure f vals)
 (cases proc f
 (procedure (params body envv)
 (let [(saved-env (vector-ref envv 0))]
 (value-of body
 (foldr (lambda (binding acc)
 (extend-env (car binding)
 (newref (cadr binding))
 acc))
 saved-env
 (map (lambda (p v) (list p v)) params vals)))))))
```

Can't always allocate an argument in the store

- ```
;; apply-procedure : proc (listof ref) -> expval
(define (apply-procedure f vals)
  (cases proc f
    (procedure (params body envv)
      (let [(saved-env (vector-ref envv 0))]
        (value-of body
          (foldr (lambda (binding acc)
                    (extend-env (car binding)
                                (cadr binding) acc))
                  saved-env
                  (map (lambda (p v) (list p v)) params vals))))))
```

Decision made in the evaluation of a call-exp

Parameter Passing Variations

Call by Reference

State

Language with Explicit References

Language with Implicit References

Mutable Pairs

Parameter Passing Variations

- In value-of

```
(call-exp (rator rands)
  (let [(proc (expval2proc (value-of rator env)))
        (args (map (lambda (rand) (value-of rand env)) rands))]
    (apply-procedure proc args)))
```

apply-procedure must be called with a (listof ref)

Parameter Passing Variations

Call by Reference

State

Language with Explicit References

Language with Implicit References

Mutable Pairs

Parameter Passing Variations

- In value-of

```
(call-exp (rator rands)
  (let [(proc (expval2proc (value-of rator env)))
        (args (map (lambda (rand) (value-of rand env)) rands))]
    (apply-procedure proc args)))
```

apply-procedure must be called with a (listof ref)

- (call-exp (rator rands)
 (let [(proc (expval2proc (value-of rator env)))
 (args (map (lambda (rand) (value-of rand env)) rands))]
 (apply-procedure proc args)))

value-of rand returns a reference

Parameter Passing Variations

Call by Reference

State

Language with
Explicit
ReferencesLanguage with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- In value-of

```
(call-exp (rator rands)
  (let [(proc (expval2proc (value-of rator env)))
        (args (map (lambda (rand) (value-of rand env)) rands))]
    (apply-procedure proc args)))
```

apply-procedure must be called with a (listof ref)

- (call-exp (rator rands)


```
(let [(proc (expval2proc (value-of rator env)))
            (args (map (lambda (rand) (value-of rand env)) rands))]
        (apply-procedure proc args)))
```

value-of-rand returns a reference

- ;; value-of-rand : expression environment -> Ref
 ;; Purpose: For a var-exp return existing reference.
 ;; Otherwise, return reference to a new cell.

```
(define (value-of-rand exp env)
  (cases expression exp
    (var-exp (var) (apply-env env var))
    (else (newref (value-of exp env))))))
```

Parameter Passing Variations

Call by Reference

State

Language with
Explicit
ReferencesLanguage with
Implicit
References

Mutable Pairs

Parameter
Passing
Variations

- ```

(check-equal? (eval "let a = 3
 in let p = proc (x) set x = 4
 in begin (p a); a end")
 (num-val 4))

(check-equal? (eval "let x = 0
 in letrec f (x) = set x = +(x, 1)
 g (a) = set x = +(x, 2)
 in begin (f x);
 (g x);
 x
 end")
 (num-val 3))

(check-equal?
 (eval "let swap = proc (a)
 proc (b)
 let t = a
 in begin set a = b; set b = t end
 in let a = 33
 in let b = 44
 in begin ((swap a) b);
 -(a, b)
 end")
 (num-val 11))

```

# Parameter Passing Variations

## Lazy Evaluation: Call by Name

- Call-by-value and call-by-reference are eager
- Always find the value of each operand

# Parameter Passing Variations

## Lazy Evaluation: Call by Name

State

Language with  
Explicit  
References

Language with  
Implicit  
References

Mutable Pairs

Parameter  
Passing  
Variations

- Call-by-value and call-by-reference are eager
- Always find the value of each operand
- Lazy evaluation
- Operands not evaluated until needed
- Never needed → never evaluated

- R

# Parameter Passing Variations

Lazy Evaluation: Call by Name

- R

# Parameter Passing Variations

Lazy Evaluation: Call by Name



- R

# Parameter Passing Variations

Lazy Evaluation: Call by Name

- R

# Parameter Passing Variations

Lazy Evaluation: Call by Name

- R

# Parameter Passing Variations

Lazy Evaluation: Call by Name

- R

# Parameter Passing Variations

Lazy Evaluation: Call by Need

- R

# Parameter Passing Variations

Lazy Evaluation: Call by Need

- R

# Parameter Passing Variations

Lazy Evaluation: Call by Need

- R

# Parameter Passing Variations

Lazy Evaluation: Call by Need

- R

# Parameter Passing Variations

Lazy Evaluation: Call by Need



- R

# Parameter Passing Variations

Lazy Evaluation: Call by Need

- R

# Parameter Passing Variations

Lazy Evaluation: Call by Need