Part I: Introduction

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Outline

1. Introduction
2. Local Variables
3. Scoping
4. Lexical Analysis

Recursively Specified Data

- When writing a function we must know precisely what kinds of values may occur as arguments

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-
  - $0 \in S$
  - $x+3 \in S$, where $x \in S$
- What has been defined?

Introduction

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  - $\frac{}{'() \in (listofnumber)}$
  - $\frac{L \in (listofnumber)}{(consnumberL) \in S}$

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- A third way to specify data is using `BNF` (Backus Naur Form)
- Grammar rules used to specify programming languages
- Elements of rules

  Terminal symbols Characters in the external representation

  Nonterminal Symbols Names of sets being defined usually in angle brackets (syntactic categories)

  Productions Rules with a RHS and a LHS
  - LHS ::= RHS
  - Read as: LHS can be/is RHS

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  - <lon> ::= '()
  - <lon> ::= (cons <number> <lon>)
- Syntactic Derivation: (2 4)

<lon>	::=	(<number> <list-of-numbers>)
	::=	(2 <list-of-numbers>)
	::=	(2 <number> <list-of-numbers>)
	::=	(2 4 <list-of-numbers>)
	::=	(2 4)

- Order of substitutions does not matter

Introduction

- Symbolic Lists
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    - <slist> : := (<sexp>*)
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  - <slist> ::= (<sexp>*)
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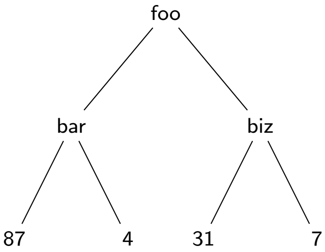
- Derivation of (a (b c) d)

<slist>	::=	(<sexp>*)
	::=	(<sexp> <sexp> <sexp>)
	::=	(a <sexp> <sexp>)
	::=	(a <sexp> d)
	::=	(a <slist> d)
	::=	(a (<sexp>*) d)
	::=	(a (<sexp> <sexp>) d)
	::=	(a (b <sexp>) d)
	::=	(a (b c) d)

Introduction

- Define a binary tree with numeric leaves



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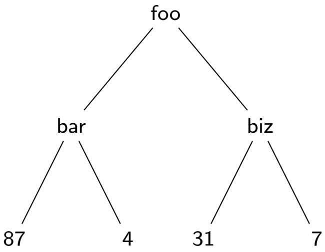

-
-
    - <bintree> : := <number>
    - <bintree> : := (<symbol> <bintree> <bintree>)

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- A simple language used to study the theory of programming languages
- Contains:
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  - lambda expressions with a single parameter
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  - application expressions
<exp>	::=	<id>
	::=	(lambda (<id>) <exp>)
	::=	(<exp> <sexp>)

Introduction

- HOMEWORK: 1.1, 1.3

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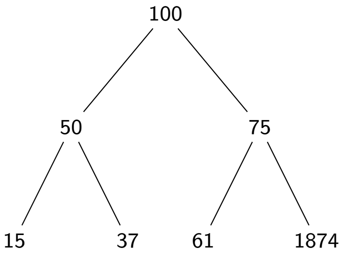
- 

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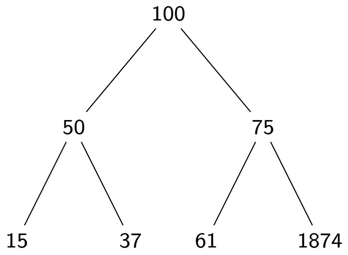
- 

- ```
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- Any problems with this?
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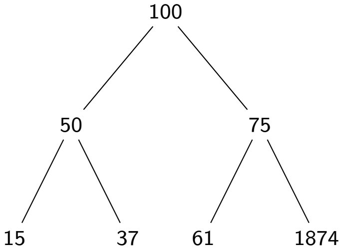
- 

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- Such constraints are context-sensitive: valid LSTs and RSTs depend on the root value

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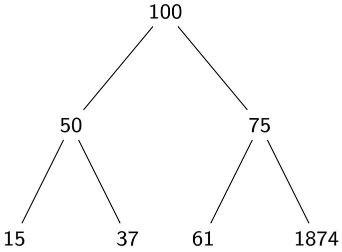

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- Any problems with this?
- Does not capture: everything in the LST must be $\leq$ to the root number and everything in the RST
- Such constraints are context-sensitive: valid LSTs and RSTs depend on the root value
- Arise in some PLs: variables must be declared before using them

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- Proof by induction on, h, the height of T
- Base case: h = 0

```
h = 0 ⇒ T is a number
      ⇒ T has one node
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- Inductive case

```
Assume: T has an odd number for nodes for h = k
Prove: T has an odd number for nodes for h = k+1

h = k+1 ⇒ T is not a number
        ⇒ T has a root number and two sub-bintrees: lst and rst

Both lst and rst have height at most k
  ⇒ each has an odd number of nodes (by IH)
  ⇒ lst has 2*i+1 nodes and rhs has 2*j+1 nodes
  ⇒ T has 2*i+1 + 2*j+1 + 1 nodes = 2(i+j)+3 nodes
  ⇒ T has an odd number of nodes    □
```

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(check-equal? (cnt-nodes 23) 1)
(check-equal? (cnt-nodes (list 45 88 6561)) 3)
(check-equal? (cnt-nodes (list 45 (list 88 11 99)
                                  (list 6561 -6 42)))
              7)
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  ```
- ```
        (+ 1 (cnt-nodes (cadr s)) (cnt-nodes (caddr s)))))
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Introduction

- **Follow the Grammar**
- At least one function for each syntactic category (i.e., nonterminal)
- Processing an instance of a syntactic category is done by calling a function to process instances of the syntactic category

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(check-equal? (lon? '()) #t)
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  ```
- ```
        (and (number? (car l)) (lon? (cdr l)))))
  ```
- ```
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  ```
- Prove that the function is correct
- Proof by induction on, k, |l|
- Base case: k = 0

```
k = 0 ⇒ l is null
      ⇒ (lon? l) returns #t,
         which correct because l is a lon with 0 numbers
```
```

Introduction

- ```
  ;; (listof X) → Boolean
  ;; Purpose: Determine if the given list is a lon
  (define (lon? l)
    (if (null? l)
        #t
        (and (number? (car l)) (lon? (cdr l)))))
  ```
- Inductive Step

  ```
  Assume: (lon? l) works for |l|=k
    Show: (lon? l) works for |l|=k+1

  |l|=k+1
    ⇒ l = (cons X (listof X))
    ⇒ (lon? (cdr l)) returns #t if (cdr l) is a lon and #f
                                otherwise, by IH

  (lon? (cdr l)) = #t
    ⇒ (lon? l) returns (and (number? (car l)) #t)
    ⇒ (lon? l) returns #t when (number? (car l)) holds, which is
                          correct because l is a lon
                        ∧ #f when (not (number? (car l)))
                          holds, which is correct as l is not a lon  □
  ```
  ```

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- Write a function to extract the nth element

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```
#lang eopl
(require rackunit)

;; (listof X) ::= '() | (cons X (listof X))

;; (listof X) natnum → X
;; Purpose: Extract the nth element of the given list
(define (nthelem l n)
  (cond [(null? l)
         (eopl:error 'nthelem "List too short by ~s elems" (+ n 1))
        [(zero? n) (car l)]
        [else (nthelem (cdr l) (- n 1))]])

(check-equal? (nthelem '(0 1 2 3) 3) 3)
(check-equal? (nthelem '(a b c d e f g h) 5) 'f)
```

Introduction

- Write a function to substitute in an slist all occurrences of a symbol with another symbol

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```
#| <slist> ::= (<s-exp>*)
    <sexp> ::= <symbol> | <s-list>
|#
;; slist → slist
;; Purpose: Substitute old with new in given slist
(define (subst-slist old new an-slist)
  (map (lambda (s) (subst-sexp old new s)) an-slist))
```
-
```
(check-equal? (subst-slist 'a 'b '()) '())
(check-equal? (subst-slist 'c 'c '(b c c c)) '(b c c c))
(check-equal? (subst-slist 'a 'z '(a (b x a) (f (g a a) b)))
              '(z (b x z) (f (g z z) b)))
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Introduction

- Write a function to substitute in an slist all occurrences of a symbol with another symbol
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  (define (subst-slist old new an-slist)
    (map (lambda (s) (subst-sexp old new s)) an-slist))
  ```
- ```
  (define (subst-sexp old new a-sexp)
    (if (symbol? a-sexp)
        (if (eq? old a-sexp) new a-sexp)
        (subst-slist old new a-sexp)))
  ```
- ```
  (check-equal? (subst-slist 'a 'b '()) '())
  (check-equal? (subst-slist 'c 'c '(b c c c)) '(b c c c))
  (check-equal? (subst-slist 'a 'z '(a (b x a) (f (g a a) b)))
                '(z (b x z) (f (g z z) b)))
  ```
- ```
  (check-equal? (subst-sexp 'a 'b 'a) 'b)
  (check-equal? (subst-sexp 'b 'z 'c) 'c)
  (check-equal? (subst-sexp 'i 'j '(u h (i (j h s (i i i) s) y)))
                '(u h (j (j h s (j j j) s) y)))
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Local Variables

- let-expressions allow you to define local variables
- ```
  (let ((a 10)
        (b 20)
        (c 30))
    (+ a b c))
  ```
- The value of a let-expression is the value of its body
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- The value of a let-expression is the value of its body
- The definitions are not mutually recursive
- What is the value of this expression?

  ```
  (define x 10)

  (let ((x 1)
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    (+ x y))
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  ```

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  (define x 10)

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    (+ x y))
  ```
- $(+\ 1\ 11) = 12$
  ```

Local Variables

- For mutually recursive definitions use a letrec-expression

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- For mutually recursive definitions use a letrec-expression
- What is the value of this expression?

```
(define x 10)

(letrec ((x 1)
         (y (+ x 1)))
  (+ x y))
```

Local Variables

- For mutually recursive definitions use a letrec-expression
- What is the value of this expression?

```
(define x 10)

(letrec ((x 1)
         (y (+ x 1)))
  (+ x y))
```

- (+ 1 2) = 3

Local Variables

- HOMEWORK: 1.4, 1.5, 1.8, 1.11, 1.12, 1.13, 1.15, 1.16, 1.17, 1.24, 1.25, 1.27, 1.28, 1.29

Scoping

- In most PLs variables must appear in two places:
  - declarations
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- In (f x y), f, x, and y are references
- In (lambda (x) (...)), x is a declaration
- The value of a variable is also called its *denotation*
- The denotation must come from some declaration and that bounds the declaration

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  - references
- In (f x y), f, x, and y are references
- In (lambda (x) (. . .)), x is a declaration
- The value of a variable is also called its *denotation*
- The denotation must come from some declaration and that bounds the declaration
- In most PLs, variables have limited scope allowing for multiple uses of the same variable
- The scope of the variable is the part of the program where it is valid

Scoping

- In most PLs, the relationship between a variable reference and its declaration is static
- This relationship can be determined by analyzing the text of the program
- Languages with this property are *statically scoped*

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- In most PLs, the relationship between a variable reference and its declaration is static
- This relationship can be determined by analyzing the text of the program
- Languages with this property are *statically scoped*
- In some PLs, the declaration to which a variable reference refers can not be determined until the program is executed
- These languages are *dynamically scoped*

Scoping

- An extended lambda calculus:

<exp>	::=	<number>
	::=	<Boolean>
	::=	<id>
	::=	(lambda (<id>*) <exp>)
	::=	(<exp> <exp>*)

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<exp>	::=	<number>
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	::=	(lambda (<id>*) <exp>)
	::=	(<exp> <exp>*)

- Binding rule for the extended lambda calculus
  - In (lambda (<id>*) <exp>), the <id>s are declarations
  - These declarations bind all occurrences of these variables in <exp> unless there are intervening declarations of any of the same variables
- In an expression, a variable *occurs bound* if it is referenced and it is bound by a declaration
- In an expression, a variable *occurs free* if it is referenced and it is not bound by a declaration

Scoping

- ((lambda (x z) x) y y)
    - x occurs bound
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- The number of regions crossed is called the *lexical or static depth*
- The position of a variable is the position in the parameter list that declares it
- A variable's lexical depth and position are known as its *lexical address*

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      ((: 1 0) ((: 0 0) (: 1 1))))
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- QUIZ: 1.34, 1.35

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- Due in 1 week