# Random Walk Probability Script

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[1]:
```
import math
```

Let $\{S_n\} = \sum_{i=1}^{n} X_i$ where $\{X_i\}$ are iid random variables with distribution $\mathbf{P}(X_i = 1) = \mathbf{P}(X_i = -1) = \frac{1}{2}$.

Calculate $\mathbf{P}(\frac{S_{35}}{\sqrt{50}} \geq 1)$:

Note that $\mathbf{P}(\frac{S_{35}}{\sqrt{50}} \geq 1) \equiv \mathbf{P}(S_{35} \geq \sqrt{50})$ $\sqrt{50} \approx 7.07$, but this random walk can only take on interger values, as each increment is either -1 or 1. This tells us $\mathbf{P}(S_{35} \geq \sqrt{50}) = \mathbf{P}(S_{35} \geq 7.07) = \mathbf{P}(S_{35} \geq 8)$. At time index 35, the maximum value the random walk could attain would be 35 (i.e. if $\forall i \in \{1, 2, \ldots, 35\}, X_i = 1, S_{35} = 35$). In addition, note that $\mathbf{P}(S_n = x) > 0 \iff n \geq x, \frac{1}{2}(n + x) \in \mathbb{Z}$, e.g. the probability that $S_2 = 3$ is zero bceause it's not possible to reach the value 3 in 2 time instants, and similarly the probability that $S_2 = 1$ is 0, as the possible values at that time instant are -2, 0, and 2.

[2]:
```
# Find the possible values of the random walk at time index 35 using list␣
↪comprehension
vals = [val for val in range(8, 36) if (35+val)%2 ==0]
vals
```

[2]: `[9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35]`

So the values of x for which $\mathbf{P}(S_{35} = x) > 0$ are the odd numbers between 9 and 35, inclusive. Denote this set as $\mathcal{S}$. Then $\mathbf{P}(S_{35} \geq 8) = \sum_{x \in \mathcal{S}} \binom{35}{\frac{1}{2}(n+x)} (\frac{1}{2})^{\frac{1}{2}(35+x)} (\frac{1}{2})^{\frac{1}{2}(35-x)}$

[5]:
```
p = 0
for x in vals:
    p += math.comb(35, int(((1/2)*(35+x))))*(1/2)**((1/2)*(35+x))*(1/2)**((1/
↪2)*(35-x))
p
```

[5]: `0.08773262449540198`

Therefore, $\mathbf{P}(\frac{S_{35}}{\sqrt{50}} \geq 1) \approx 0.0877$.