CS 532: 3D Computer Vision
4th Set of Notes

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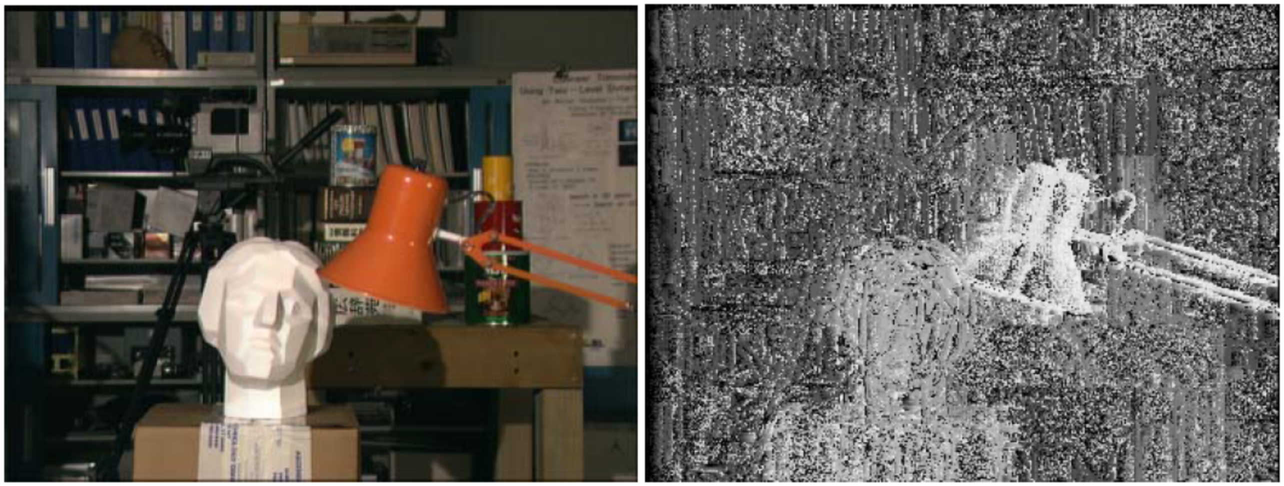
Lecture Outline

- Binocular Stereo
  - Continued
- Confidence for stereo
- Stereo beyond the Winner-Take-All algorithm

Partially based on slides by M. Bleyer,
R. Szeliski, S. Seitz and R. Zabih

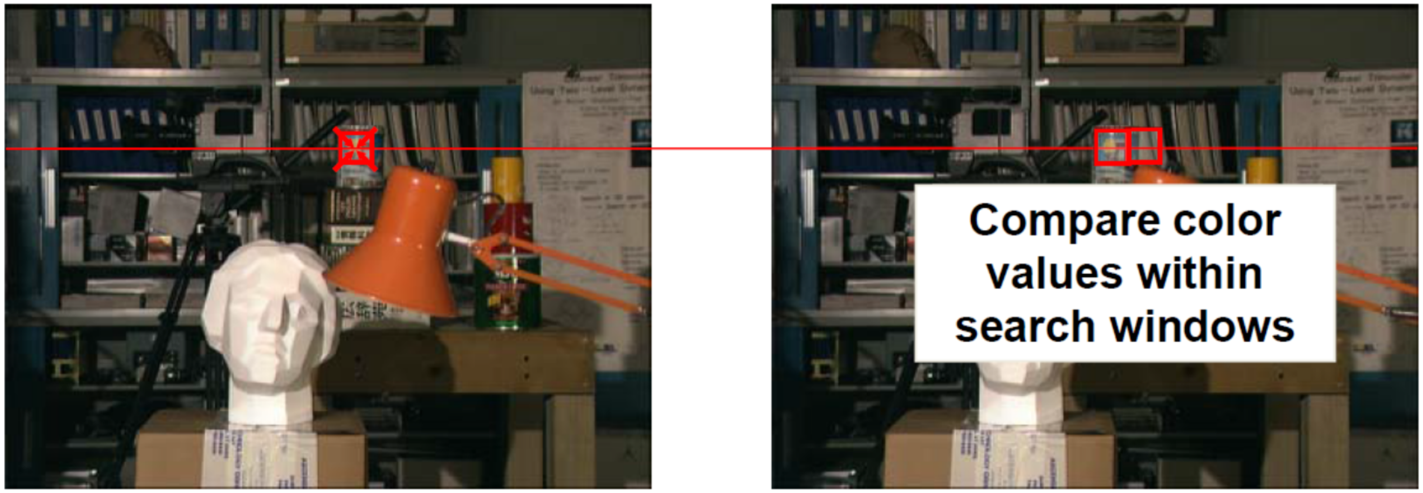
Naïve Stereo Algorithm

- For each pixel p of the left image:
  - Compare color of p against the color of each pixel on the same horizontal scanline in the right image.
  - Select the pixel of most similar color as matching point



Window-Based Matching

- Instead of matching single pixels, center a small window on a pixel and match the whole window in the right image



Compare color values within search windows

(a) Left image

(b) Right image

Window-Based Matching

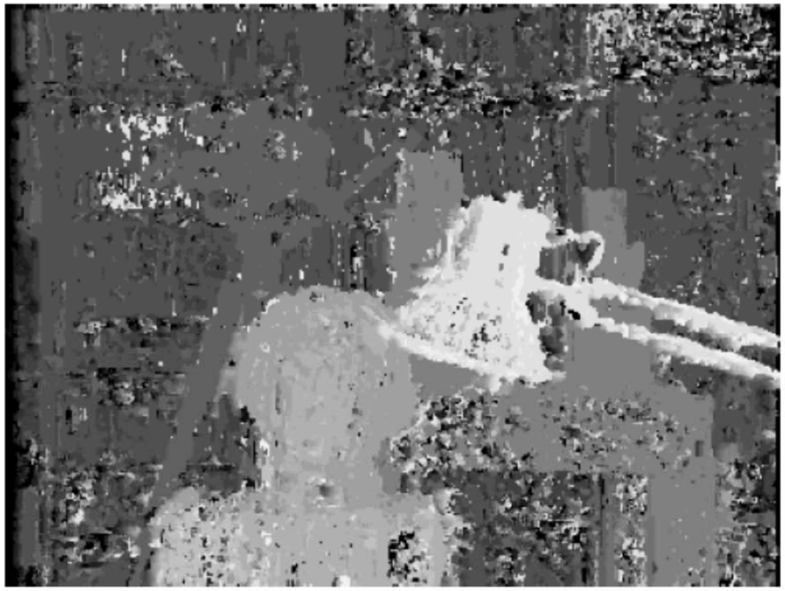
- the disparity d_p of a pixel p in the left image is computed as

$$d_p = \underset{0 \le d \le d\max}{\arg\min} \sum_{q \in Wp} c(q, q-d)$$

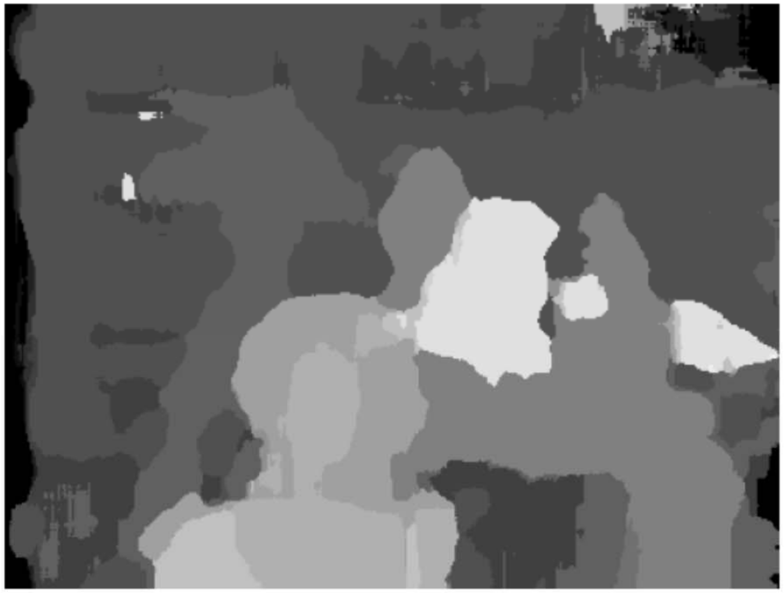
- where
  - argmin returns the value at which the function takes a minimum
  - d_{max} is a parameter defining the maximum disparity (search range)
  - W_p is the set of all pixels inside the window centered on p
  - c(p,q) is a function that computes the color difference between a pixel p of the left and a pixel q of the right image

Results

- The window size is a crucial parameter

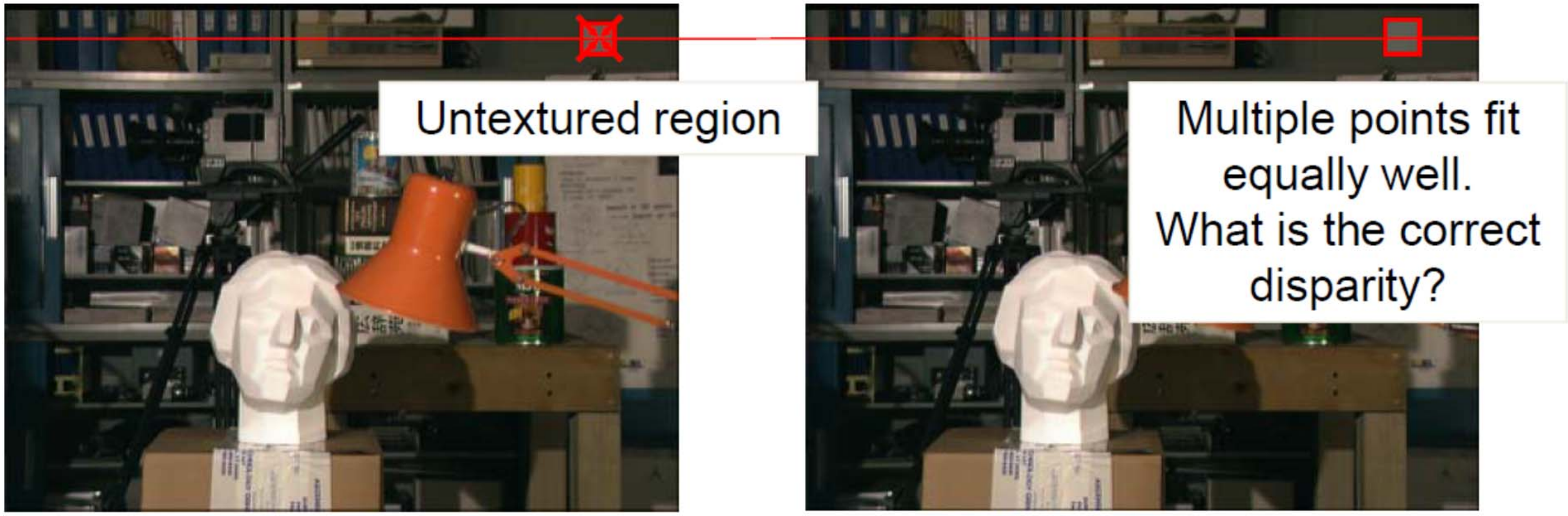


Window size = 3x3 pixels



Window size = 21x21 pixels

Untextured Regions



Untextured region

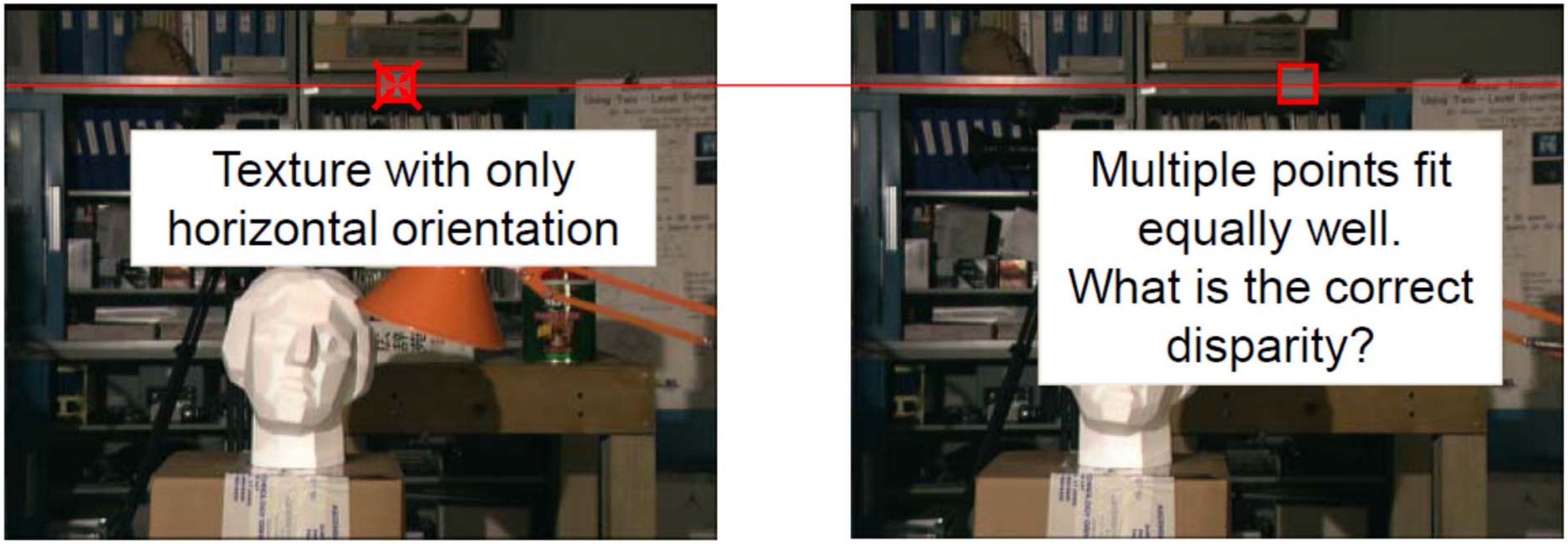
Multiple points fit equally well. What is the correct disparity?

(a) Left image

(b) Right image

Aperture Problem

- There needs to be a certain amount of texture with vertical orientation



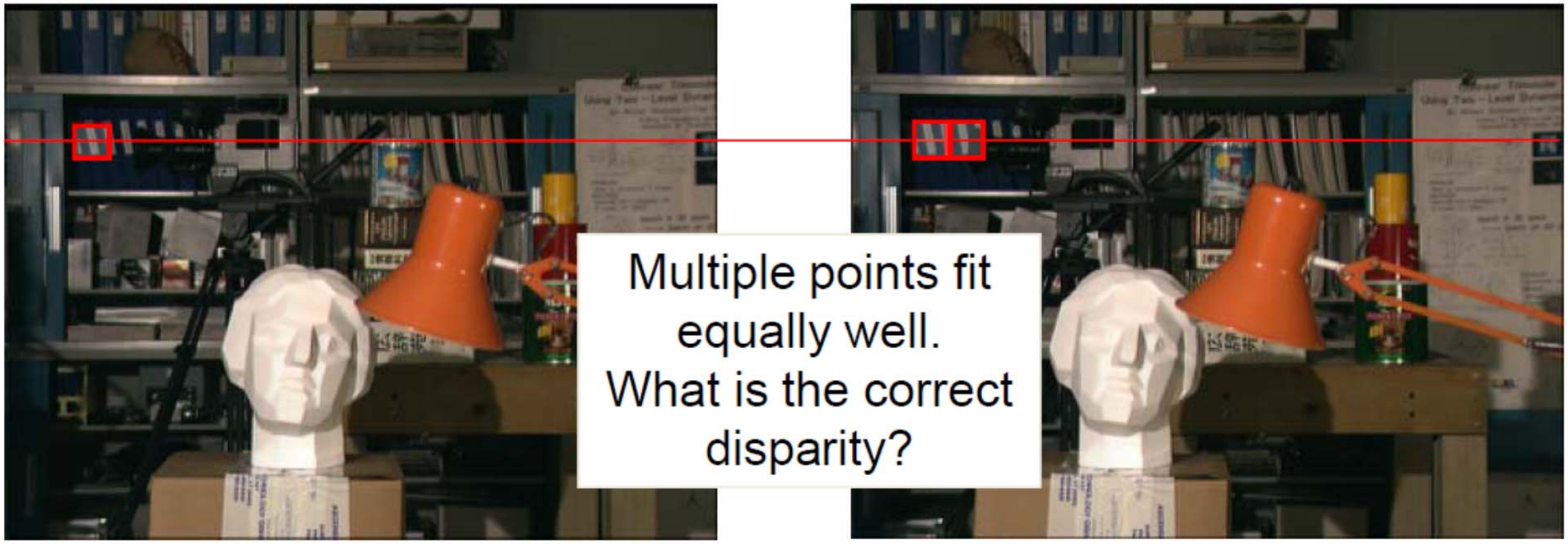
Texture with only horizontal orientation

Multiple points fit equally well. What is the correct disparity?

(a) Left image

(b) Right image

Repetitive Patterns

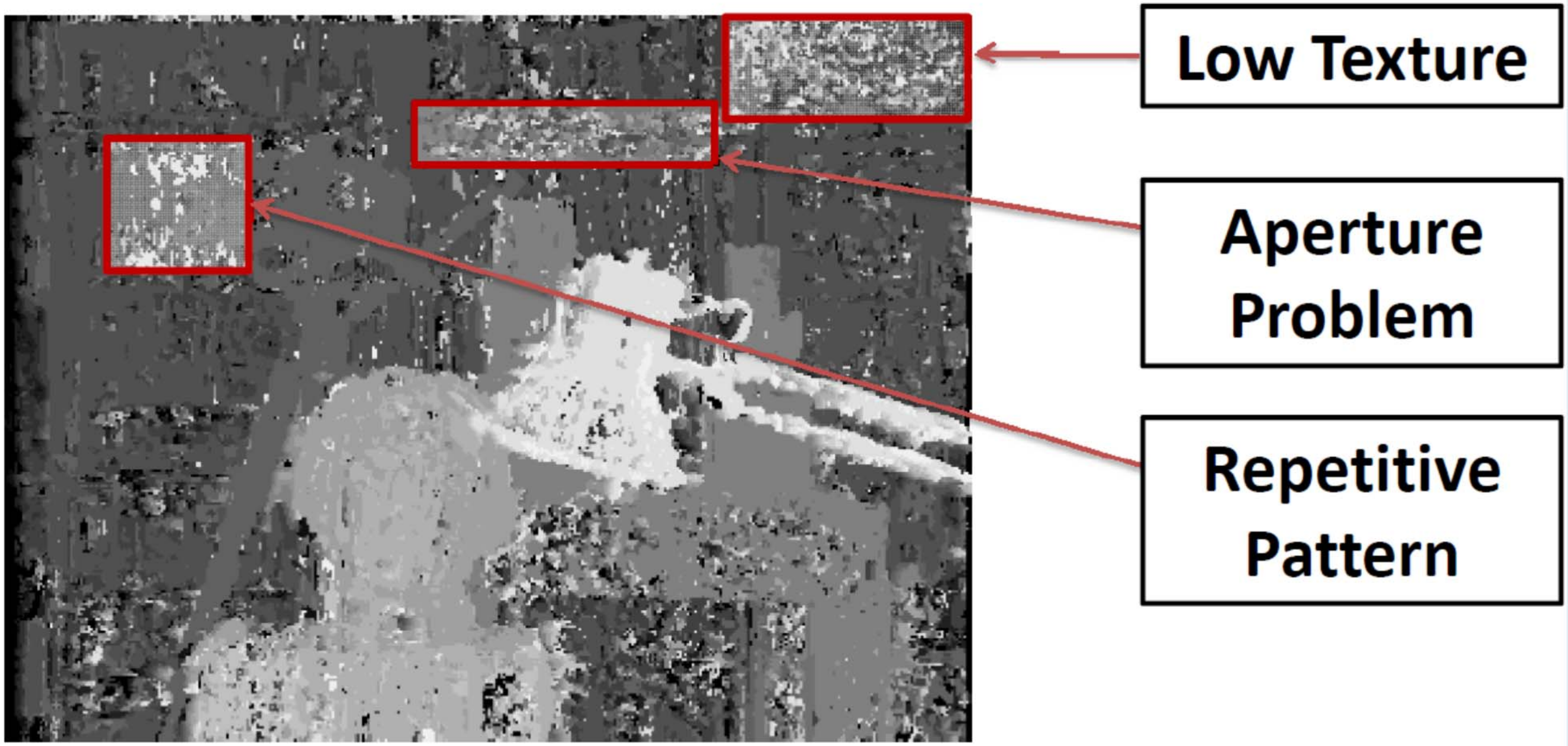


Multiple points fit equally well. What is the correct disparity?

(a) Left image

(b) Right image

Effects of these Problems



Low Texture

Aperture Problem

Repetitive Pattern

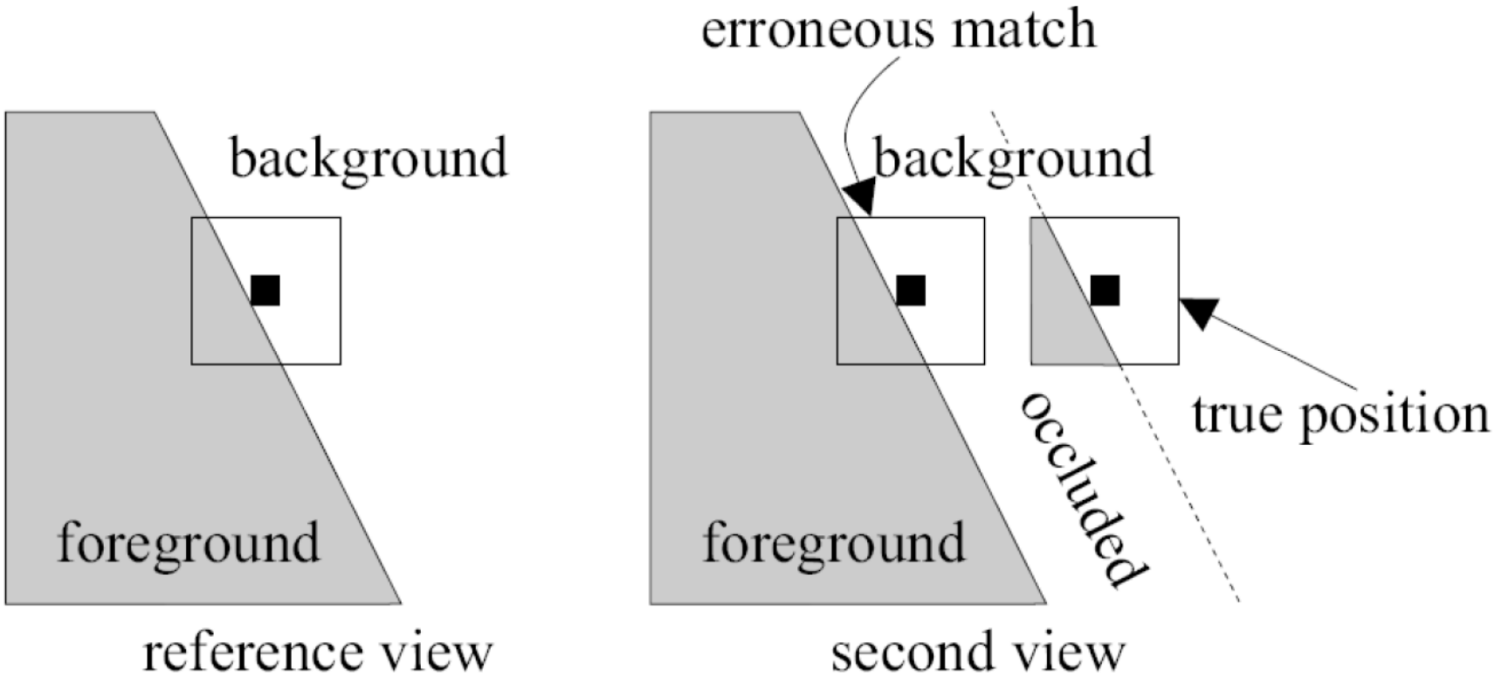
Window size = 3x3 pixels

Foreground Fattening

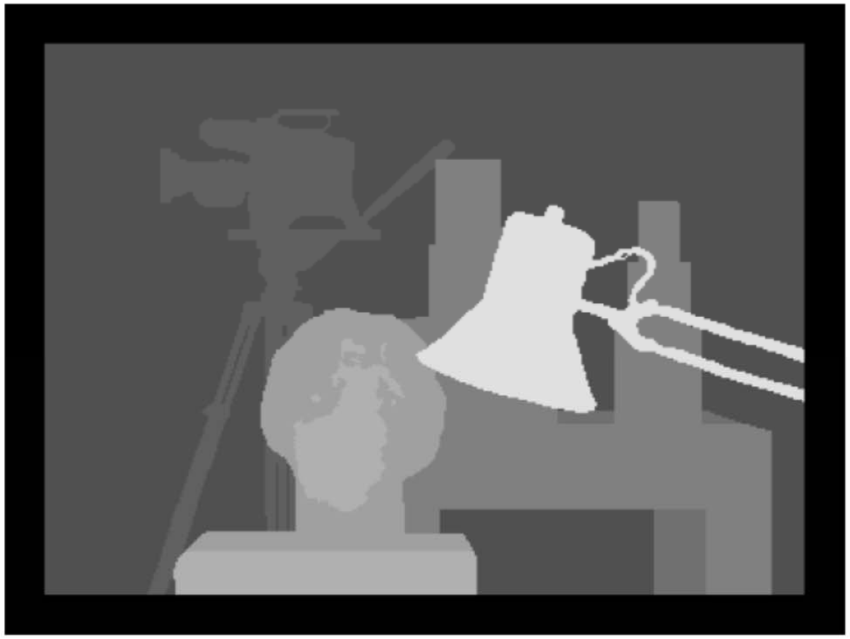
- By using a window as matching primitive, we have made an implicit smoothness assumption:
  - All pixels within the window are assumed to have the same disparity
- This leads to a systematic error in regions close to disparity discontinuities

Foreground Fattening

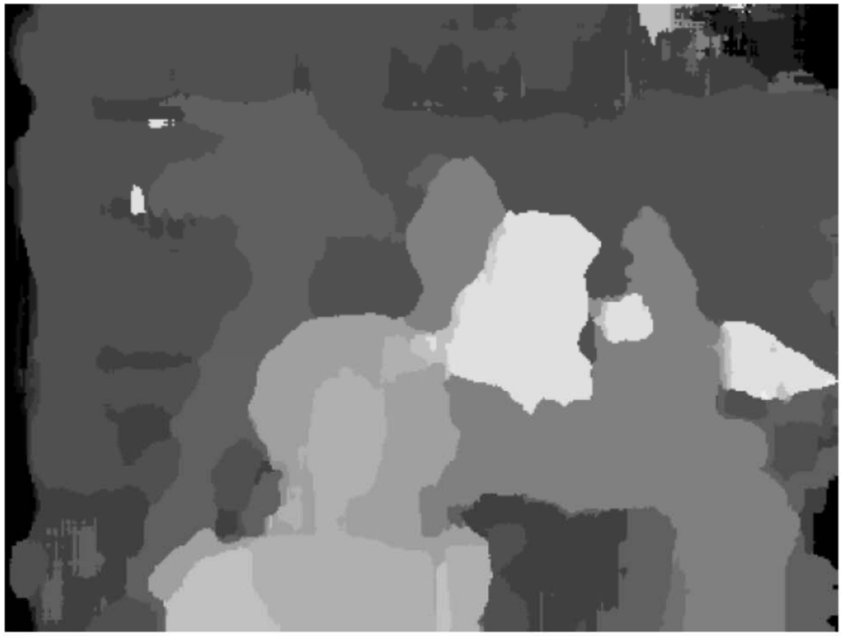
- Background regions close to disparity discontinuities tend to be erroneously assigned to the foreground disparity



Foreground Fattening



Ground Truth Disparities



Window size = 21x21 pixels

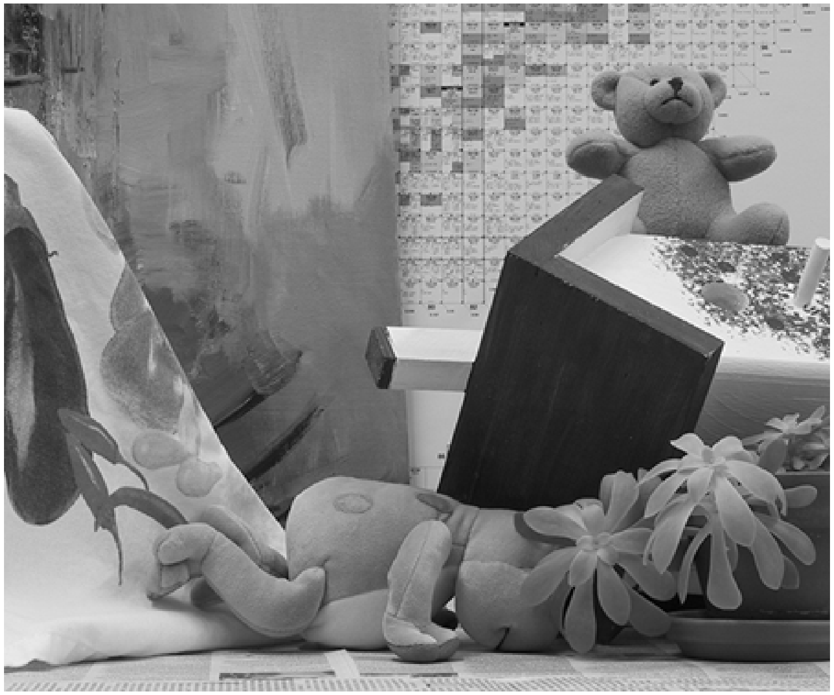
Large vs. Small Windows

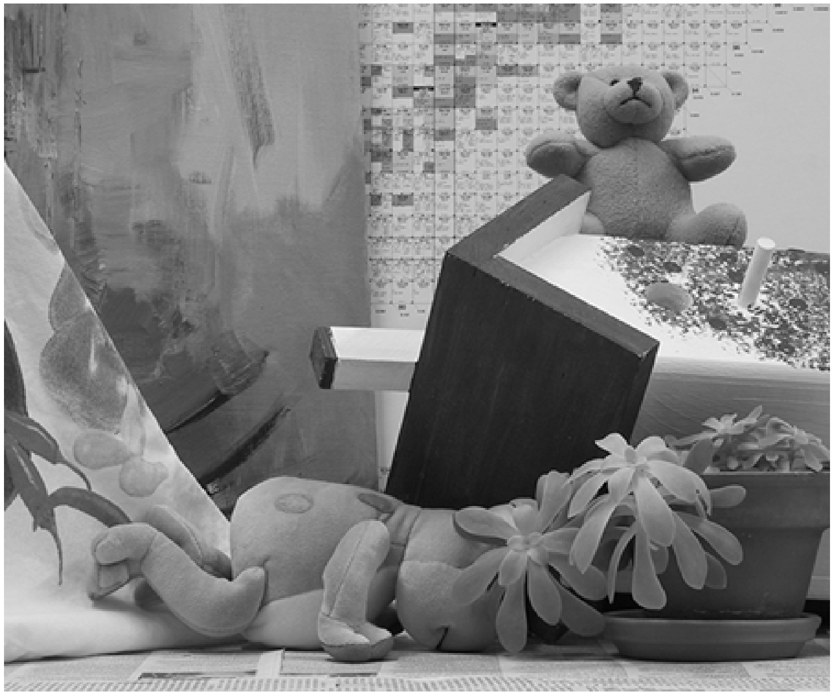
- Large windows are better for:
  - Untextured Regions
  - Aperture Problem Why?
  - Repetitive Patterns
- Small windows reduce:
  - Foreground Fattening Effect
- Problem:
  - There is no ‘optimal’ window size that can handle all these problems at once

Matching Costs

- See slides 42-51 from previous set of notes

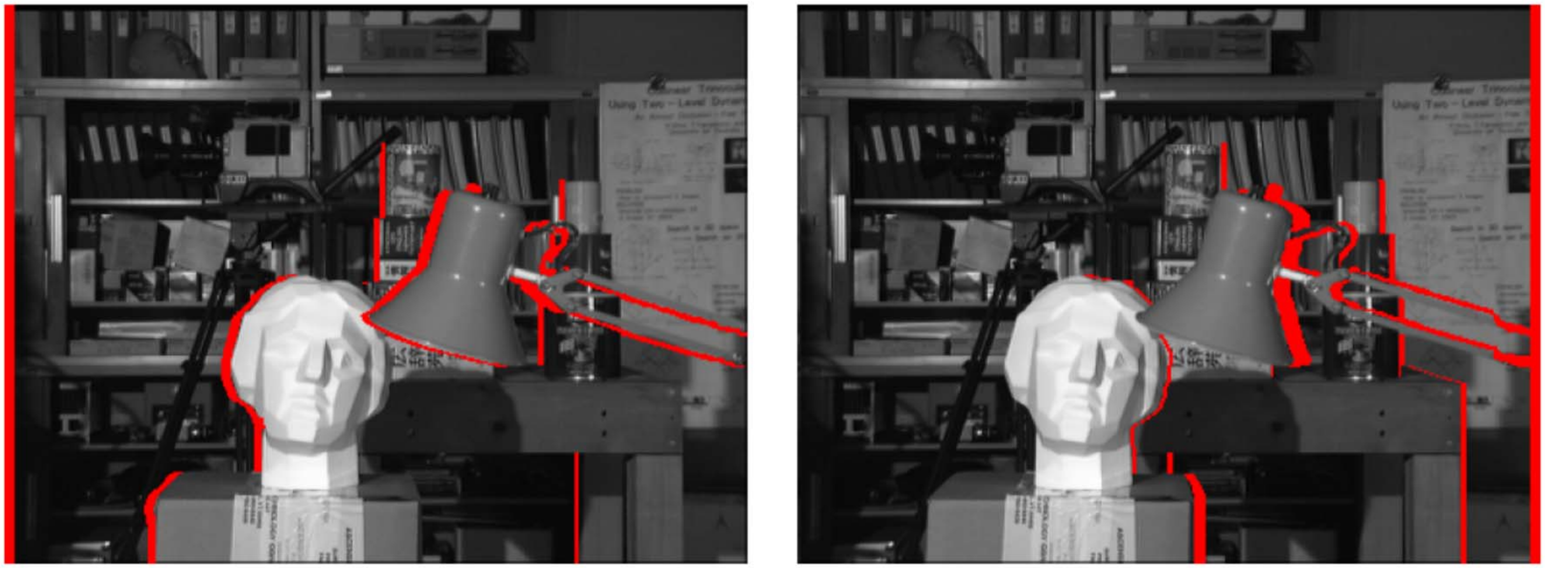
Implement SAD



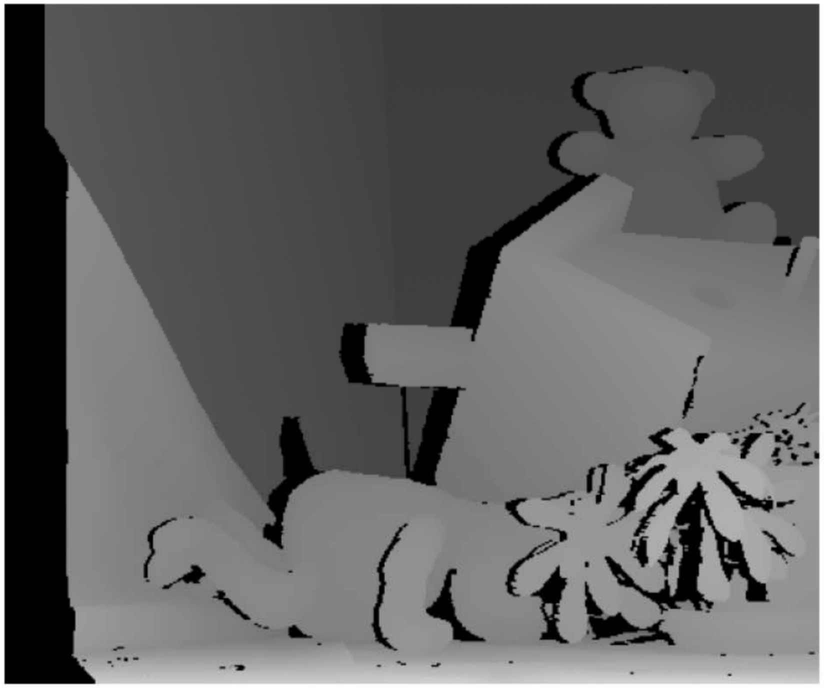


Occlusion

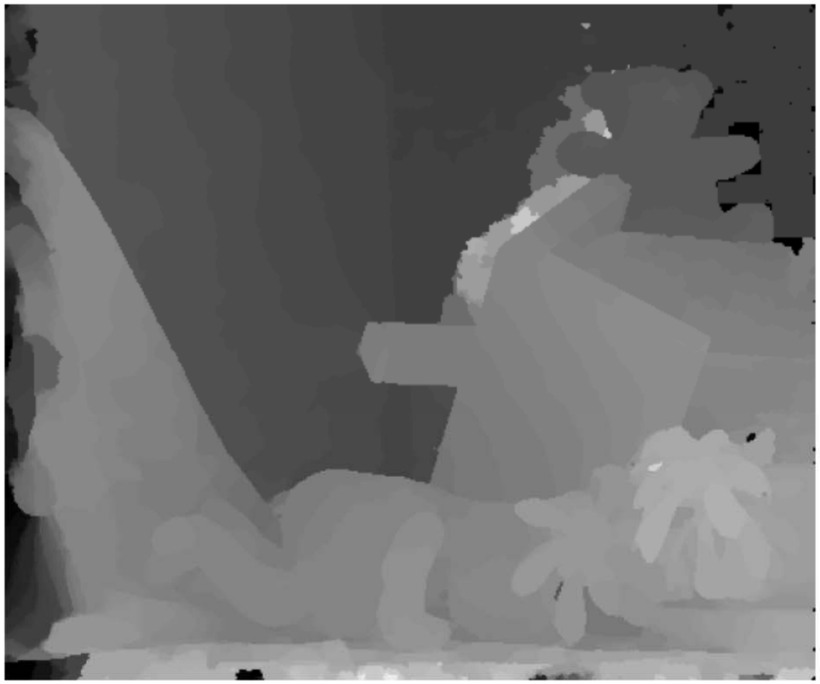
- There are pixels that are only visible in one of the two views (red pixels in the images)
- For occluded, pixels there exists no correspondence => We cannot estimate disparity



Effects of Occlusion



Ground Truth with Occlusions in Black



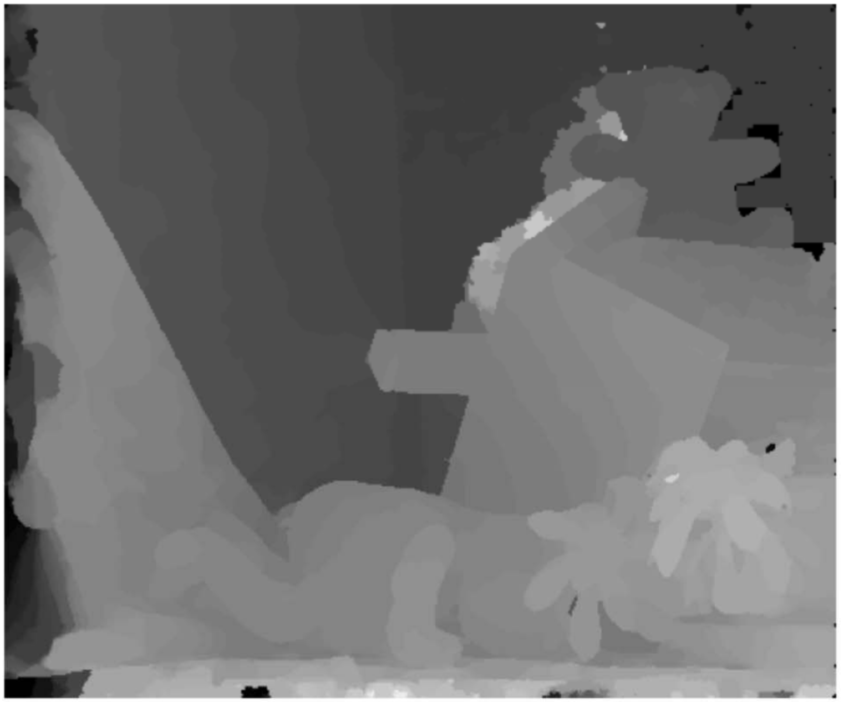
Core Algorithm of [Hosni,ICIP09]

Approximate adaptive support weight implementation

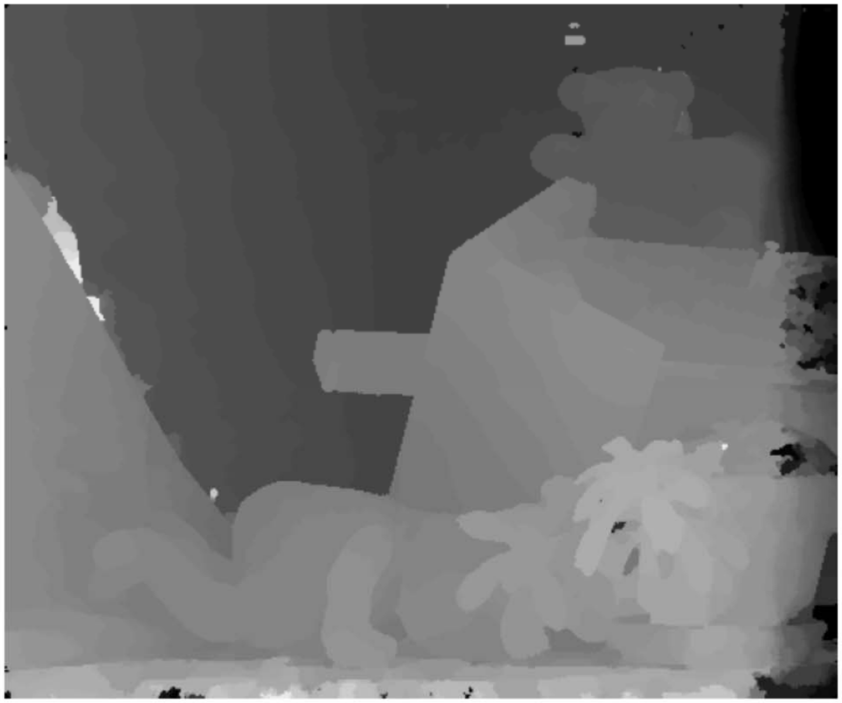
Left-Right Consistency

- Compute 2 disparity maps
  - Using the left image as reference frame
  - Using the right image as reference frame
- Left-right consistency check:
  - For each pixel p_l of the left view:
  - Lookup p_l's matching point m_r in the right view using the left disparity map
  - For the pixel m_r, lookup its matching point q_l in the left view using the right disparity map
  - If p_l= q_l the disparity is assumed to be correct
  - Otherwise, the disparity is invalidated
- Check typically fails for
  - Occluded pixels
  - Mismatched pixels

Left-Right Consistency

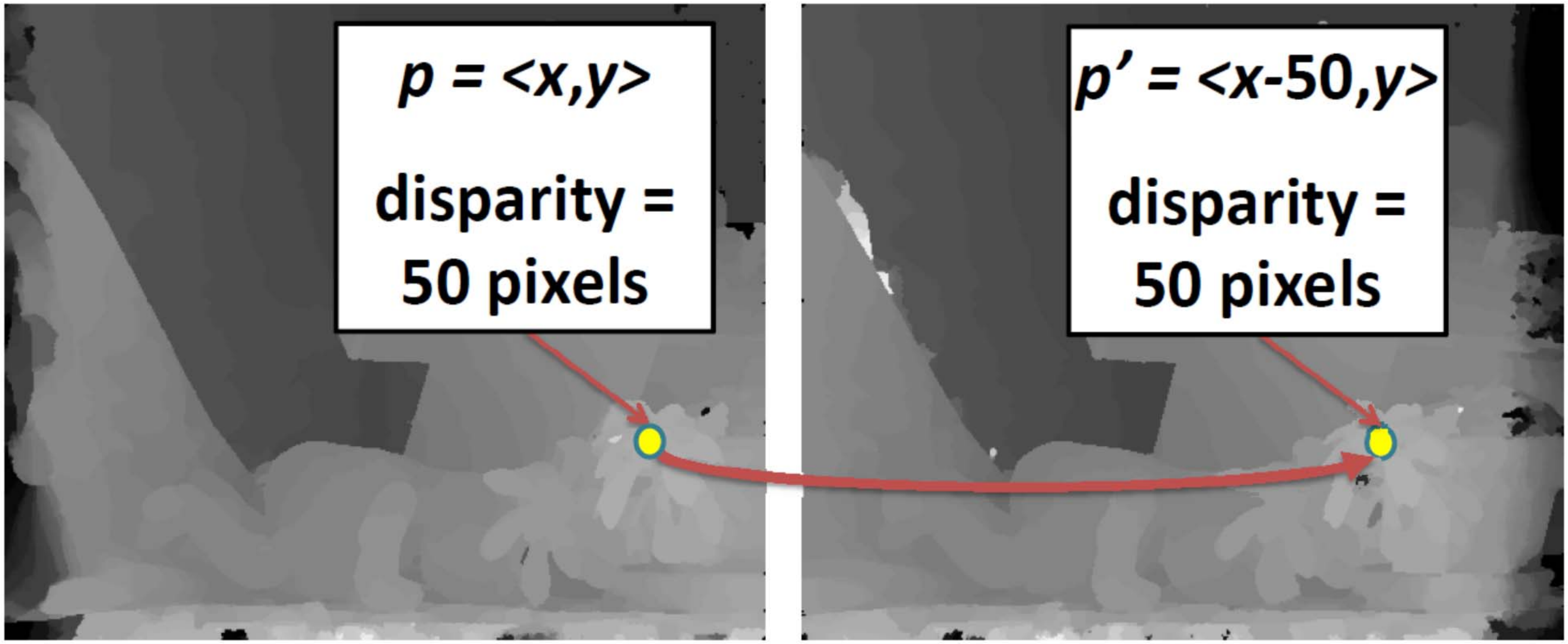


Disparity Map (Left Reference)



Disparity Map (Right Reference)

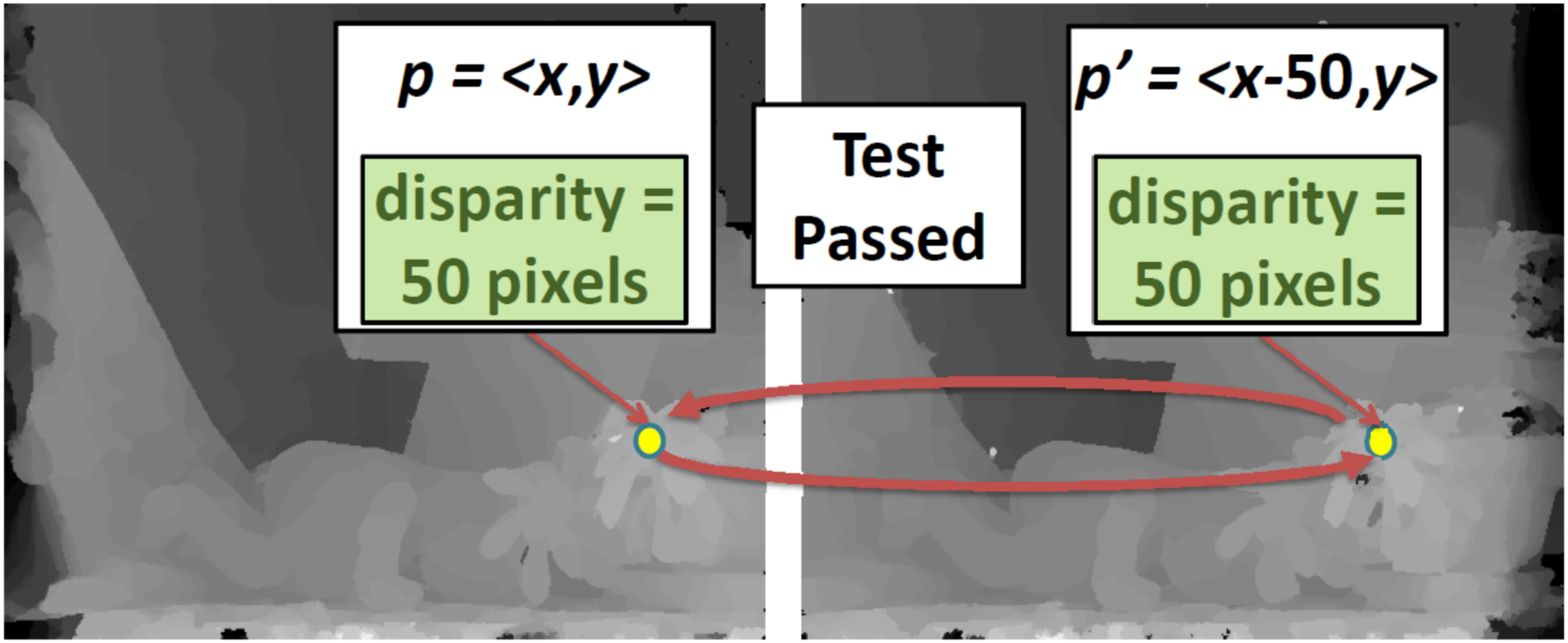
Left-Right Consistency



Disparity Map (Left Reference)

Disparity Map (Right Reference)

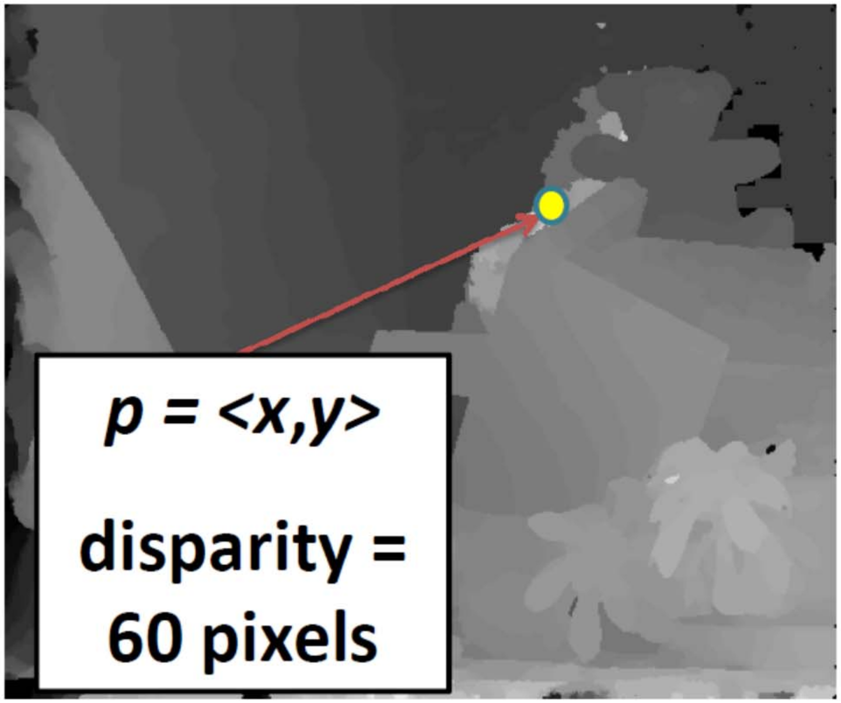
Left-Right Consistency



Disparity Map (Left Reference)

Disparity Map (Right Reference)

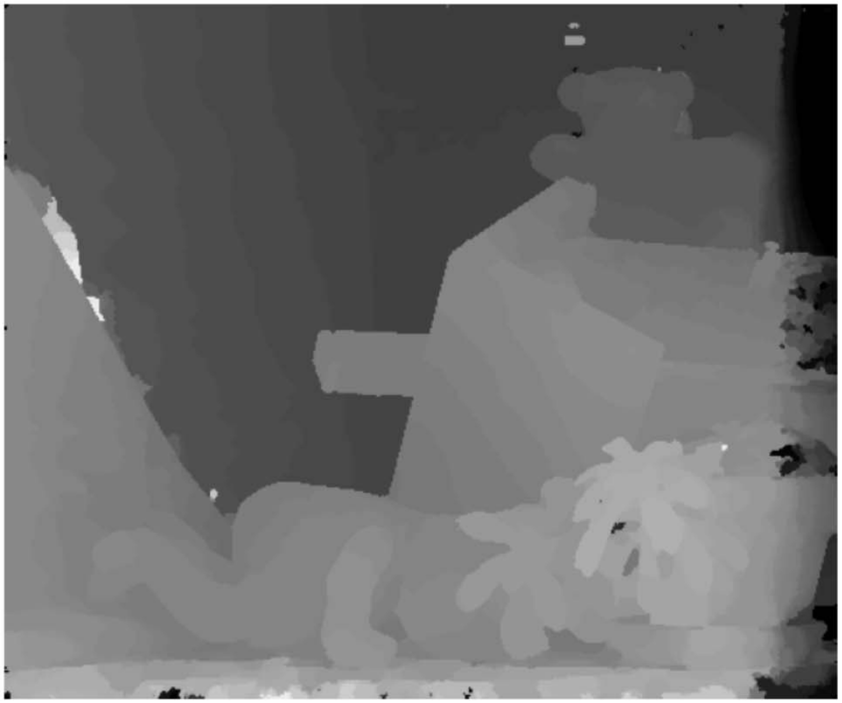
Left-Right Consistency



p = <*x*,*y*>

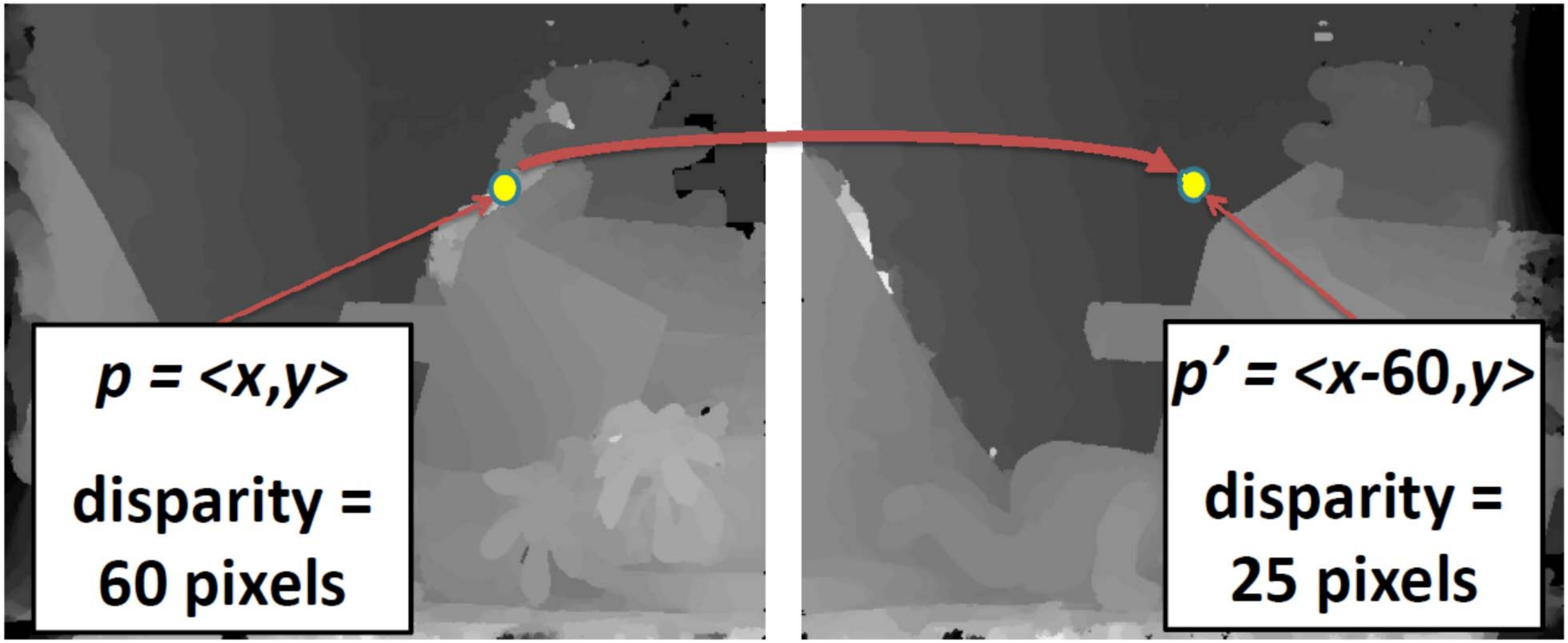
disparity = 60 pixels

Disparity Map (Left Reference)



Disparity Map (Right Reference)

Left-Right Consistency



p = <*x*,*y*>

disparity = 60 pixels

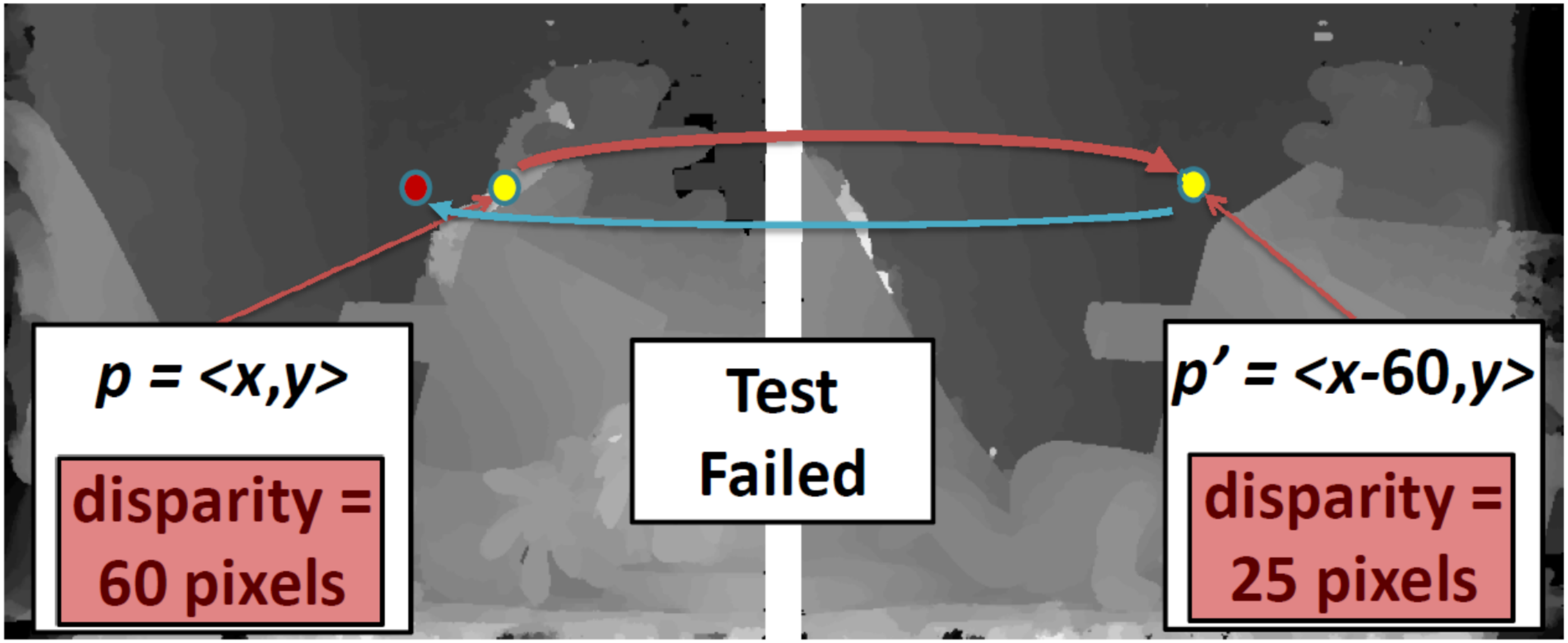
p' = <*x*-60,*y*>

disparity = 25 pixels

Disparity Map (Left Reference)

Disparity Map (Right Reference)

Left-Right Consistency



Disparity Map (Left Reference) Disparity Map (Right Reference)

Confidence Measures for Stereo Matching

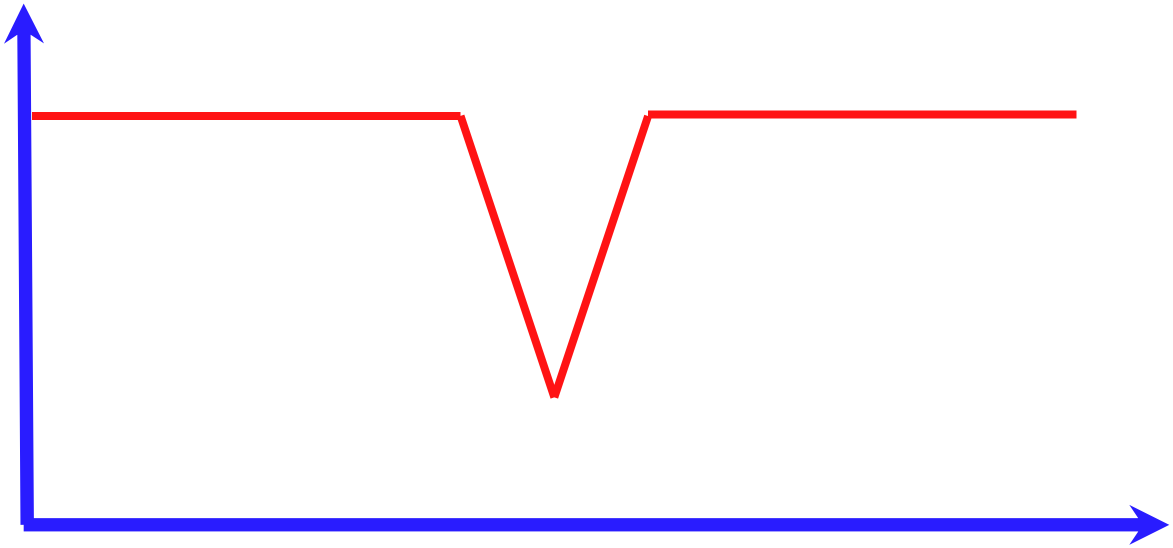
Cost Functions

$$SAD(x, y, d) = \sum_{i \in W} |I_L(x_i, y_i) - I_R(x_i - d, y_i)|$$

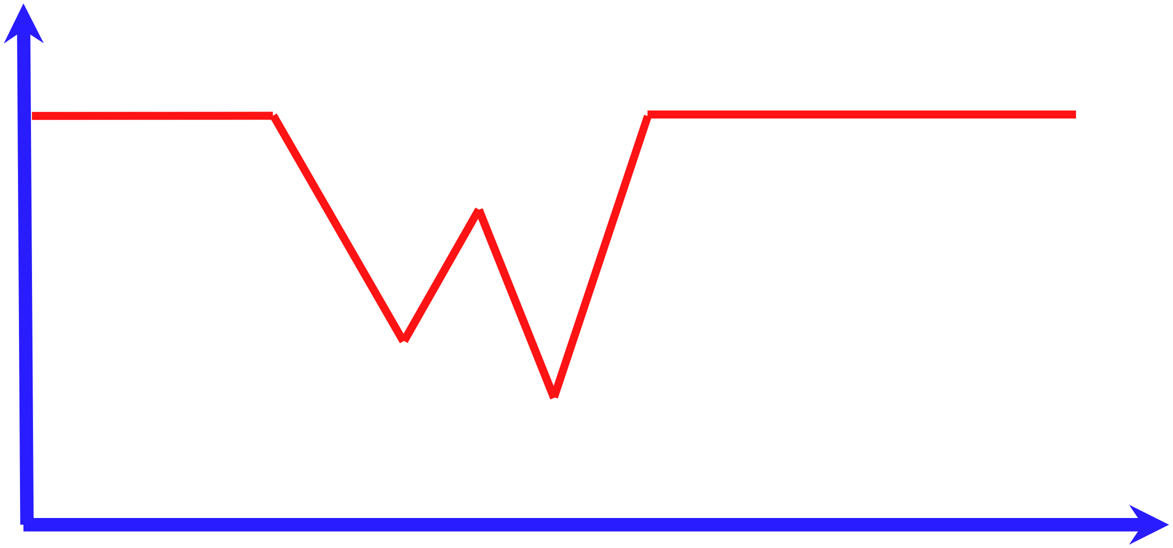
$$NCC(x, y, d) = \frac{\sum_{i \in W}(I_L(x_i, y_i) - \mu_L)(I_R(x_i - d, y_i) - \mu_R)}{\sigma_L \sigma_R}$$

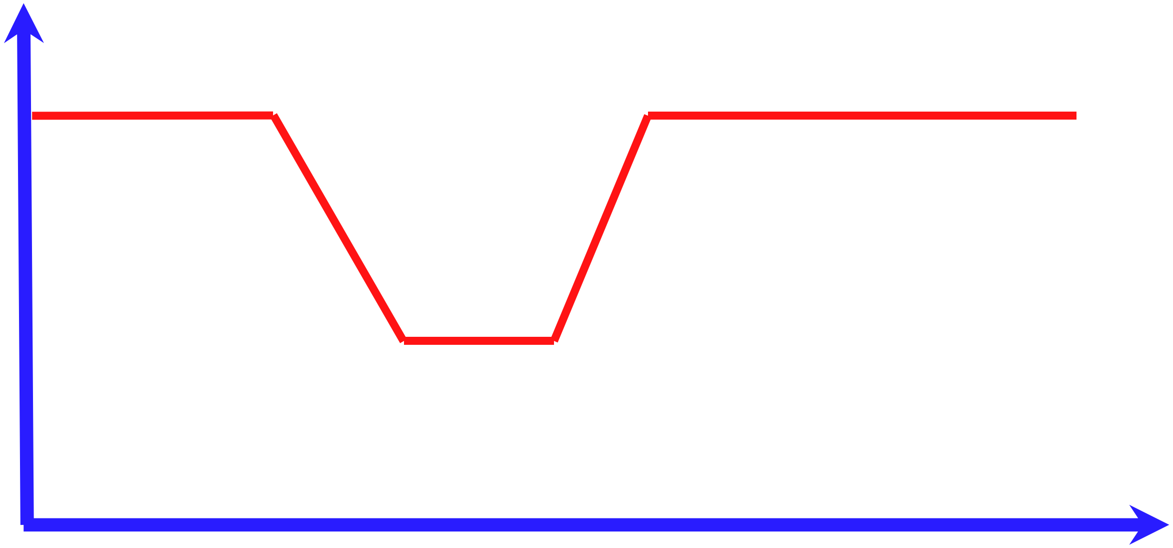
- Functions of disparity d
  - $d=x_L-x_R$

Ideal Cost Curve



Non-Ideal Cost Curves





Correspondence Uncertainty Measures

1. Matching Cost

2. Local Properties of the Cost Curve

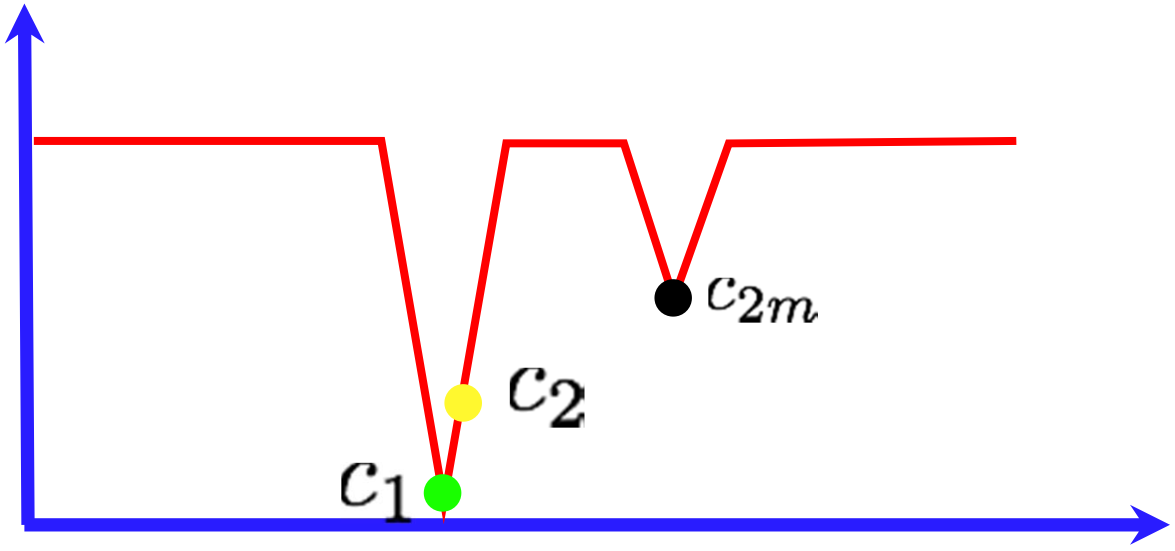
3. Local Minima of the Cost Curve

4. The Entire Cost Curve

6. Consistency Between the Left and Right Disparity Maps

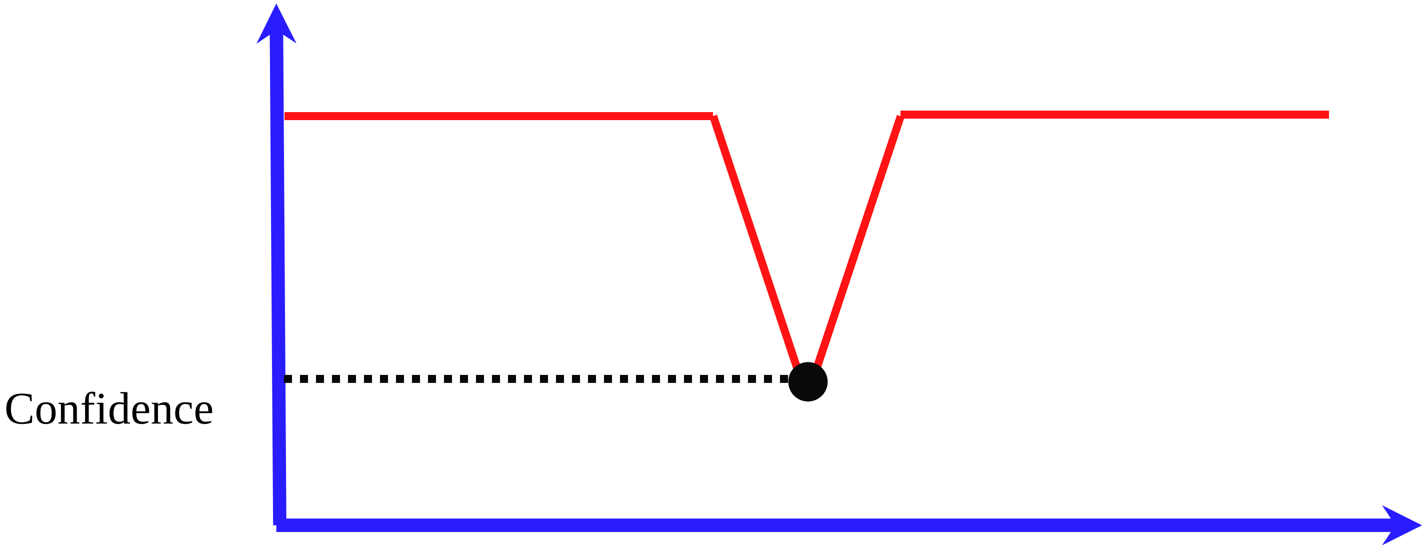
7. More…

Notation



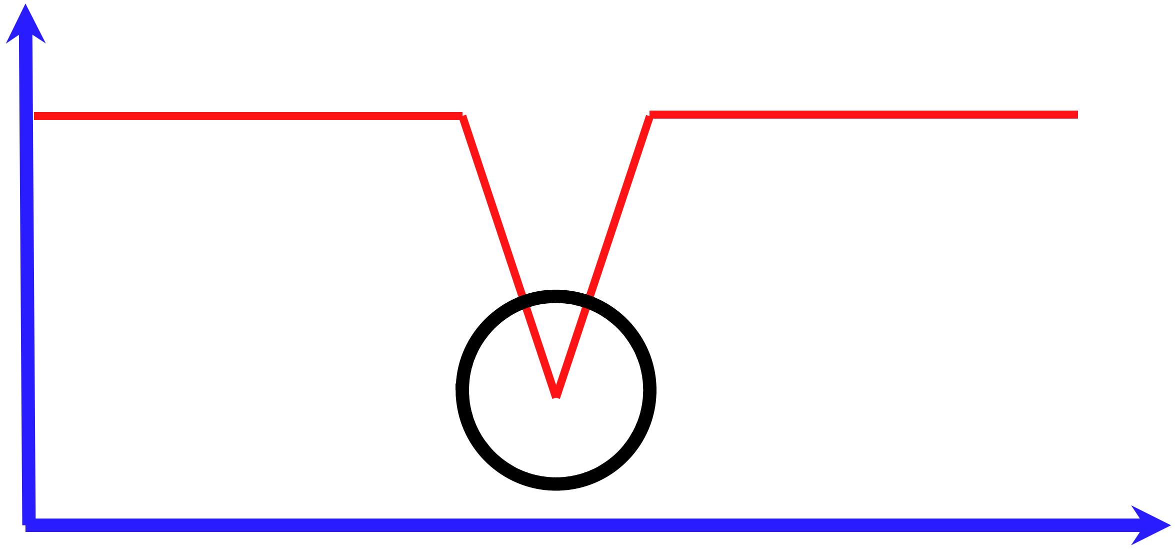
c_1 global minimum of the cost curve
c_2 second smallest value of the cost curve
c_{2m} second smallest value of the cost curve that is also
a local minimum

Matching Cost



Simply using matching cost:
Low cost values correspond to high confidence
high cost values correspond to low confidence

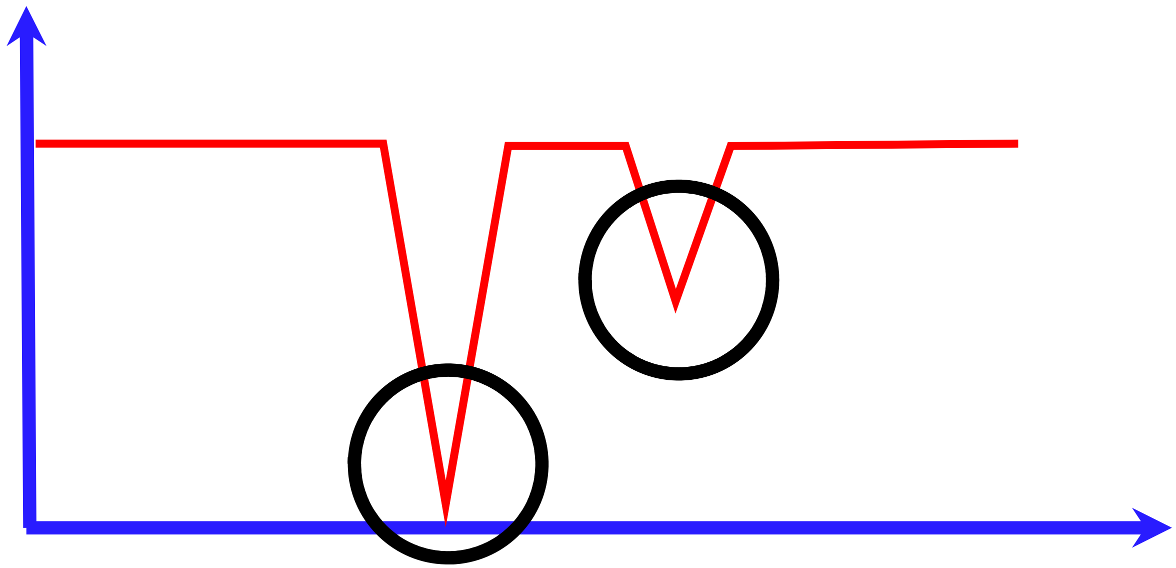
Local Properties of the Cost Curve



$$C_{CUR} = -2c(d_1) + c(d_1 - 1) + c(d_1 + 1)$$

Low curvature indicates flat region around minimum cost

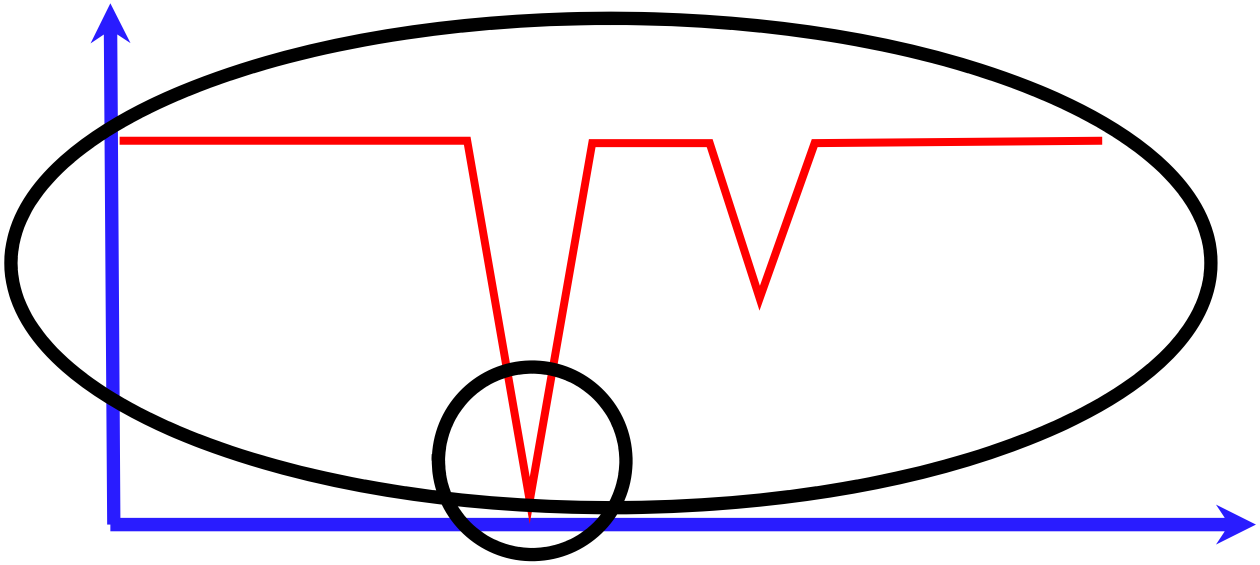
Local Minima of the Cost Curve



$$C_{PKR} = \frac{c_{2m}}{c_1} \qquad C_{\mathrm{PKRN}} = \frac{c_2}{c_1}$$

Match is ambiguous if multiple strong candidates exist

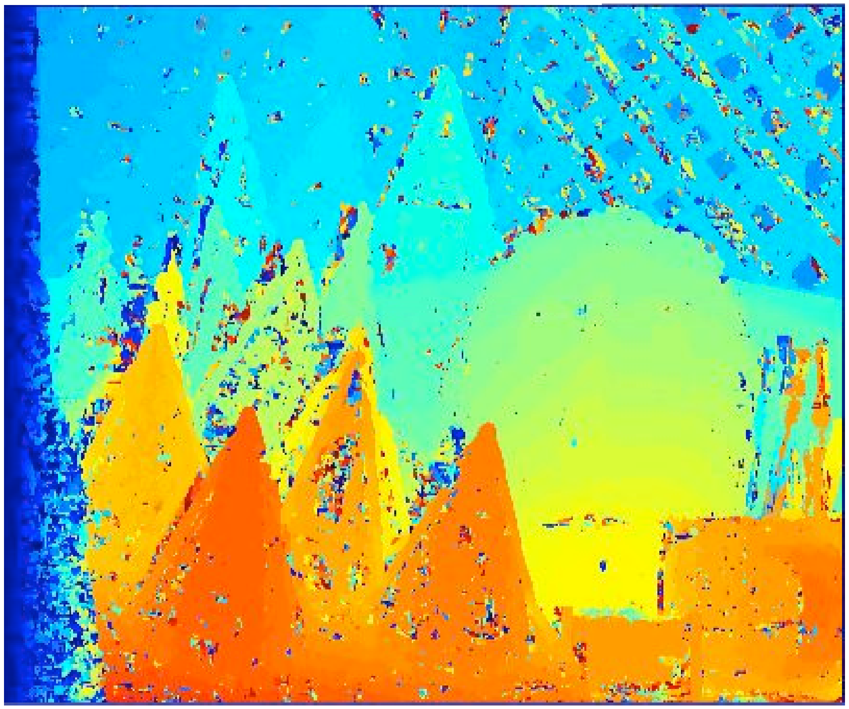
The Entire Cost Curve

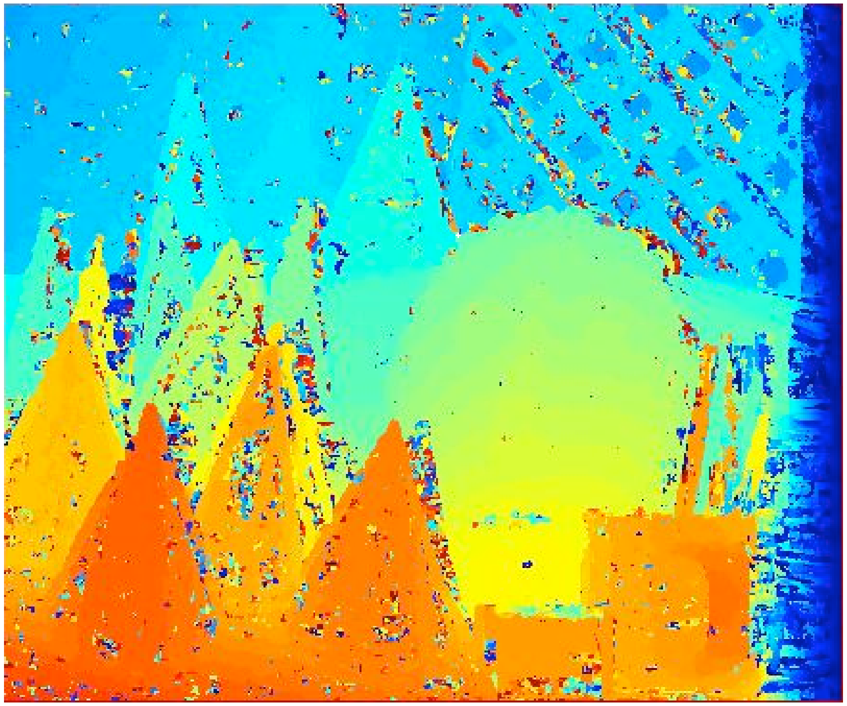


$$C_{AML} = \frac{e^{-\frac{(c_1 - c_1)^2}{2\sigma^2_{AML}}}}{\sum_d e^{-\frac{(c(d) - c_1)^2}{2\sigma^2_{AML}}}}$$

Tests for both flat regions and multiple strong candidates by converting cost curve to probability function and measuring the probability of the best match

Left-Right Consistency



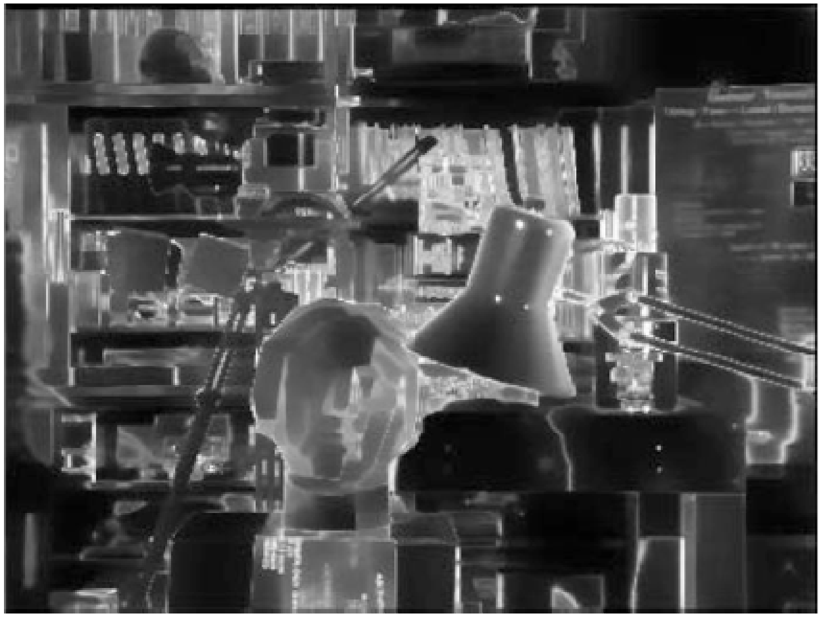


$$C_{LRC}(x,y) = -|d_1 - D_R(x - d_1, y)|$$

LRC is not binary as before, but equal to difference of corresponding disparities

Distinctiveness-based Confidence Measures

- Distinctiveness: Perceptual distance to the most similar other point in the search window in the reference image



Stereo
Beyond Winner-Take-All

Stereo with Non-Linear Diffusion

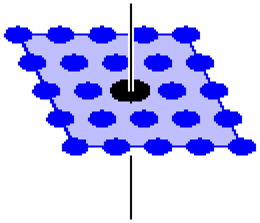
- Problem with traditional approach:
  - gets confused near discontinuities
- Non-Linear Diffusion:
  - use iterative (non-linear) aggregation to obtain better estimate

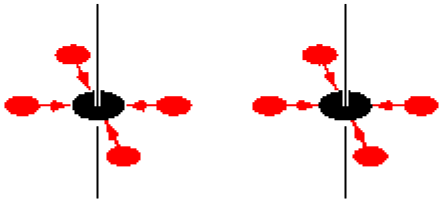
Diffusion

- Average energy with neighbors + starting value

$$E(x,y,d) \;\leftarrow\; (1-4\lambda)E(x,y,d) + \lambda \sum_{(k,l)\in\mathcal{N}_4} E(x+k,y+l,d)$$

$$+\beta(E_0(x,y,d) - E(x,y,d))$$



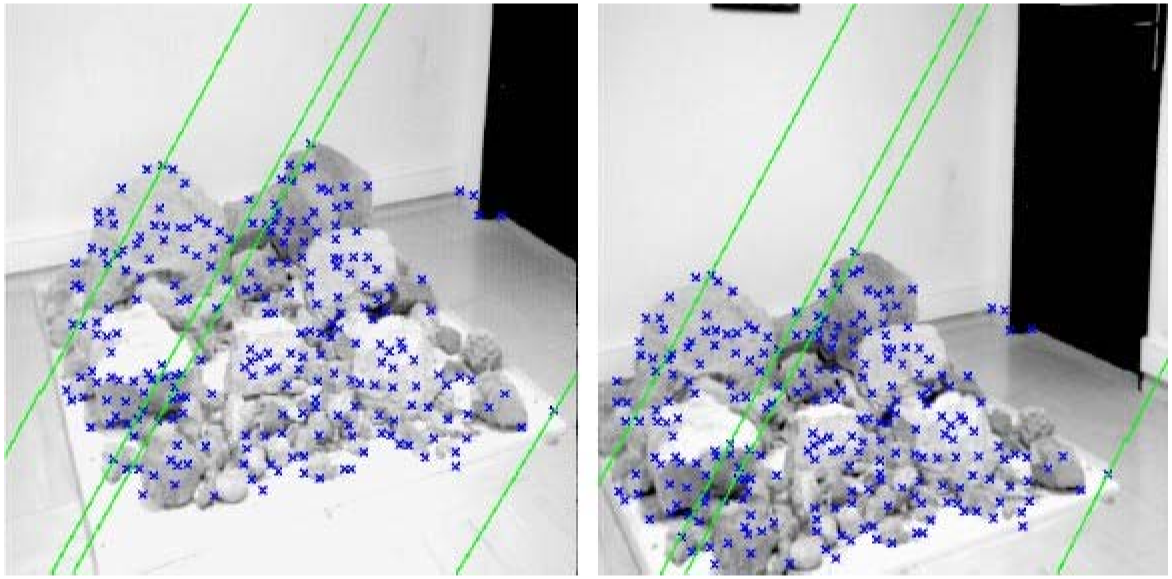


- window   diffusion

Requires appropriate stopping criteria to prevent excessive smoothing

Feature-based Stereo

- Match “corner” (interest) points

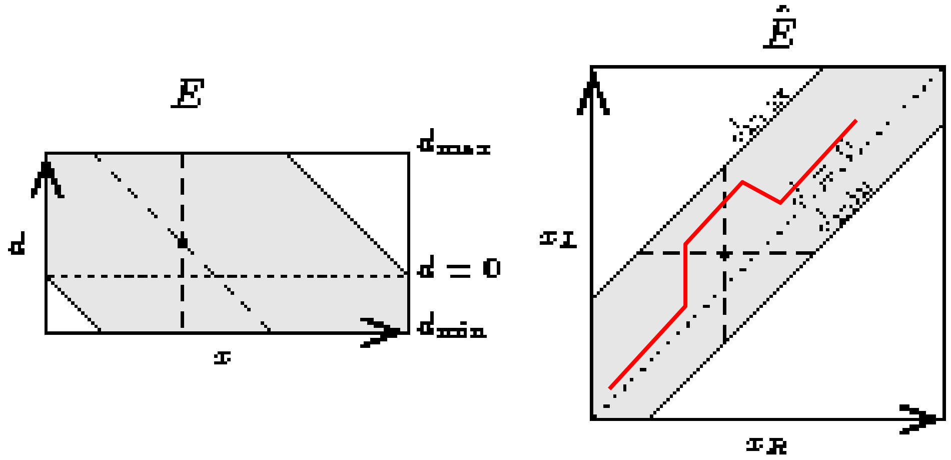


- Interpolate complete solution

Dynamic Programming

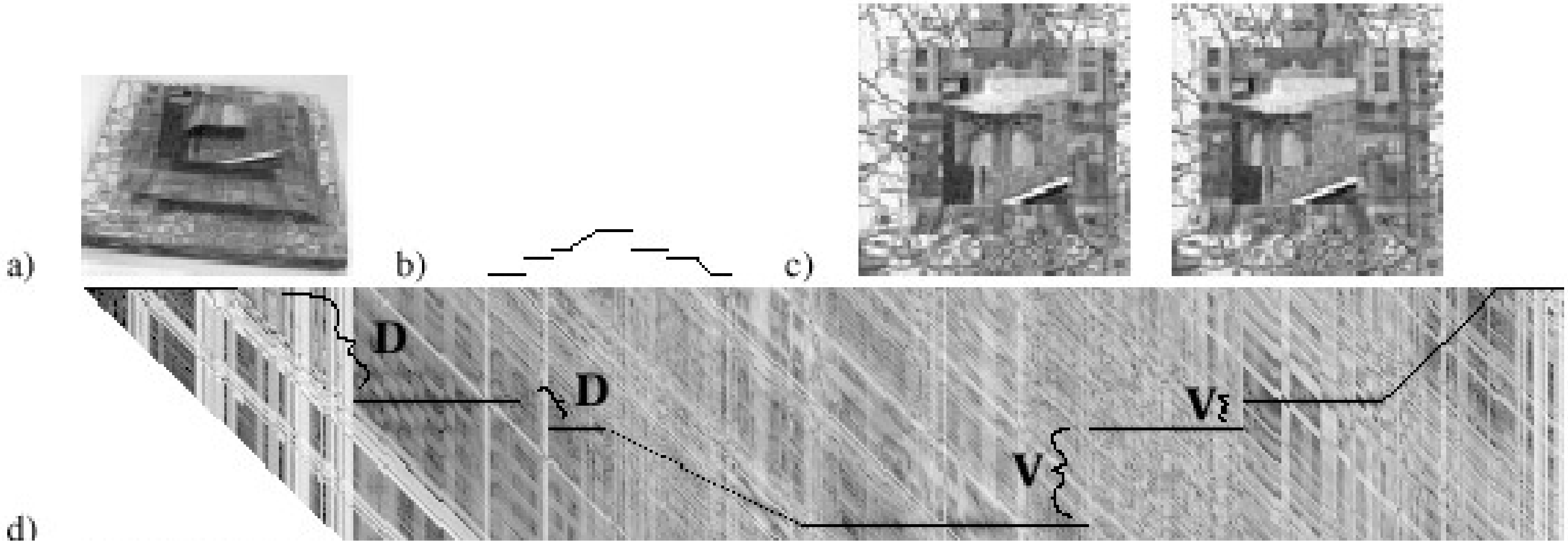
- 1-D cost function

$$\begin{aligned} E(\mathbf{d}) &= \sum_{x,y} \rho_P(d_{x+1,y} - d_{x,y}) + \sum_{x,y} E_0(x,y;d) \\ \tilde{E}(x,y,d) &= E_0(x,y;d) + \\ &\quad \min_{d'} \left(\tilde{E}(x-1,y,d') + \rho_P(d_{x,y} - d'_{x-1,y}) \right) \end{aligned}$$



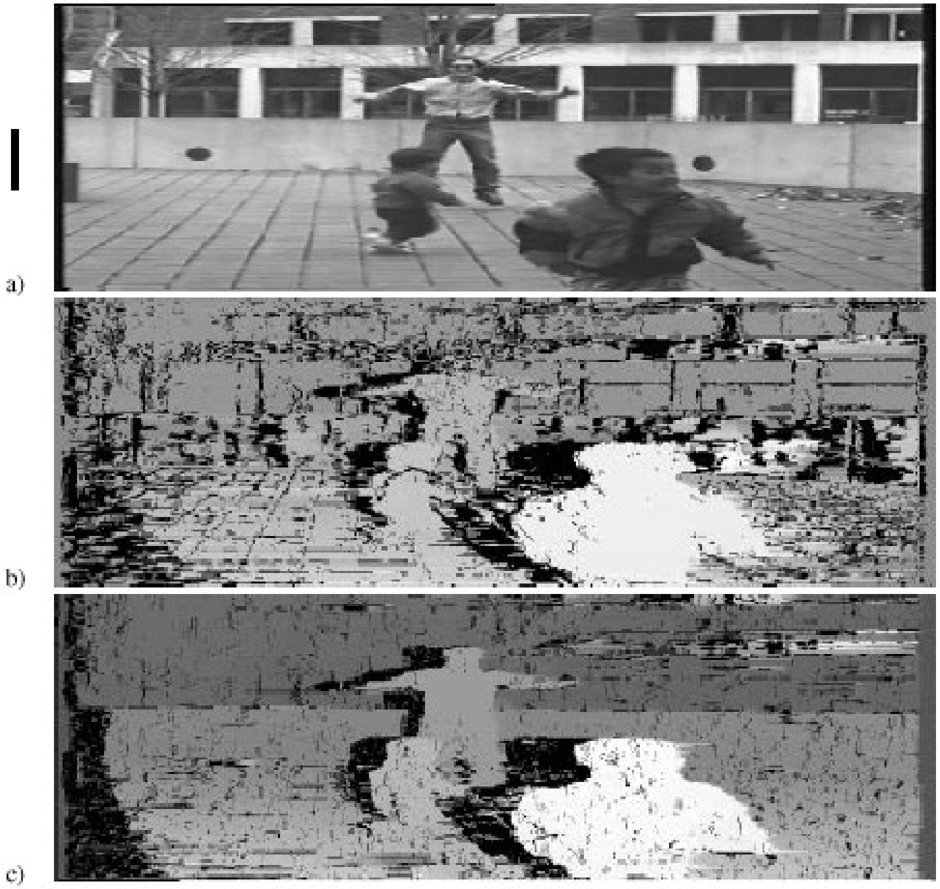
Dynamic Programming

- Disparity space image and min. cost path



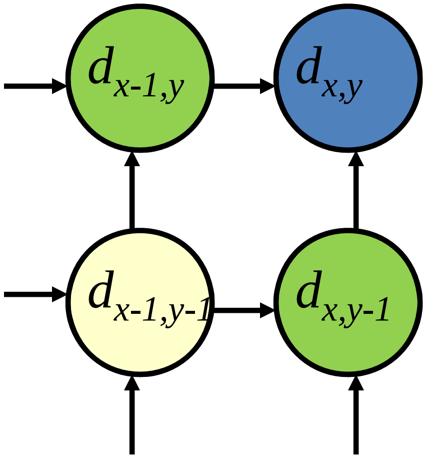
Dynamic Programming

- Sample result (note horizontal streaks)



Dynamic Programming

- Can we apply this trick in 2D as well?

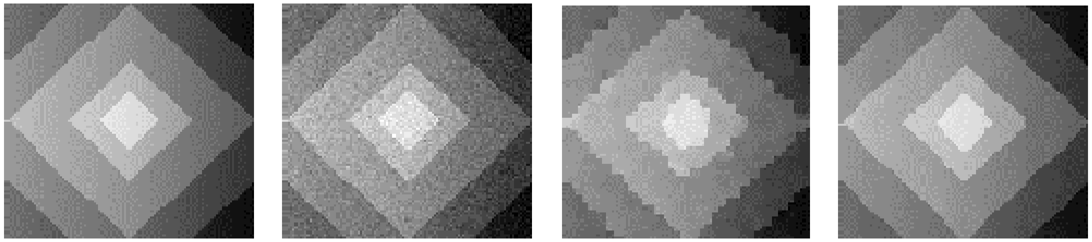


No: $d_{x,y-1}$ and $d_{x-1,y}$ may depend on different values of $d_{x-1,y-1}$

Graph Cuts

- Solution technique for general 2D problem

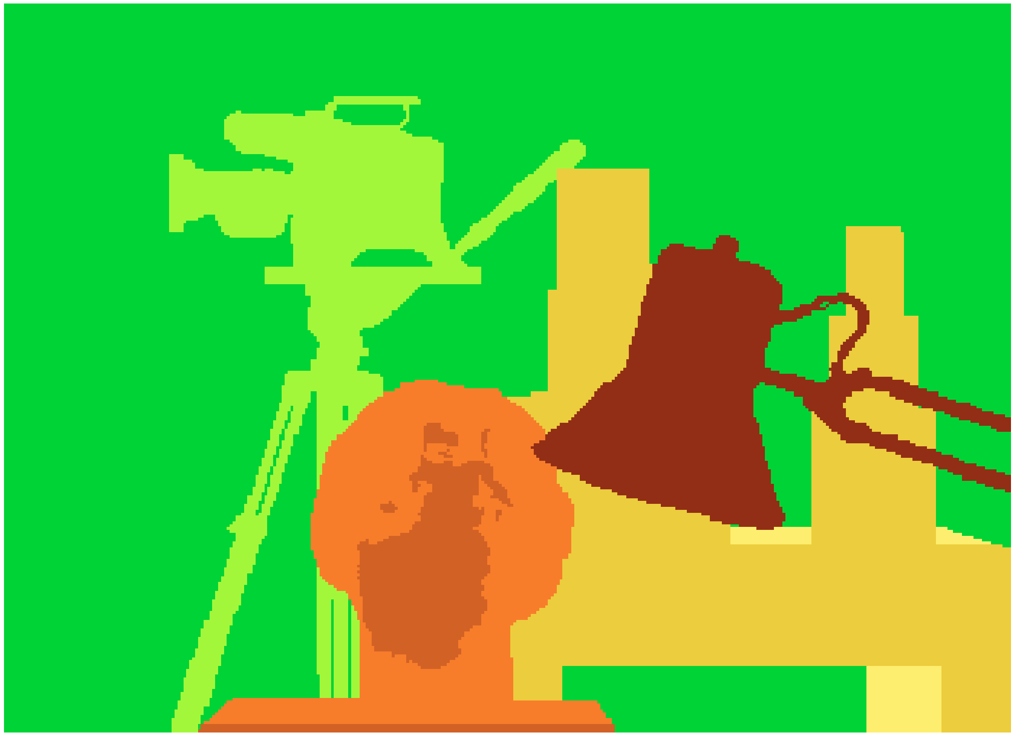
$$\begin{aligned}
E_{\mathsf{total}}(\mathbf{d}) &= E_{\mathsf{data}}(\mathbf{d}) + \lambda E_{\mathsf{smoothness}}(\mathbf{d}) \\
E_{\mathsf{data}}(\mathbf{d}) &= \sum_{x,y} f_{x,y}(d_{x,y}) \\
E_{\mathsf{smoothness}}(\mathbf{d}) &= \sum_{x,y} \rho(d_{x,y} - d_{x-1,y}) \\
&\quad + \sum_{x,y} \rho(d_{x,y} - d_{x,y-1})
\end{aligned}$$



(a) original image (b) observed image (c) local min w.r.t. standard moves (d) local min w.r.t. α-expansion moves

a-expansion moves

In each *a*-expansion a given label “*a*” grabs space from other labels



initial solution

🟢 -expansion

🔴 -expansion

🟢 -expansion

🟠 -expansion

🟡 -expansion

🟠 -expansion

🟢 -expansion

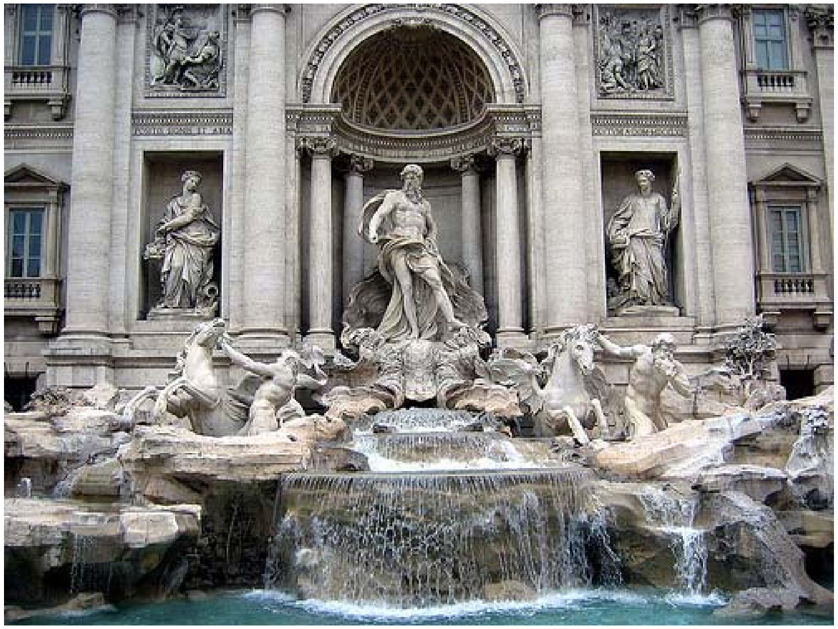
For each move choose the expansion that gives the largest decrease in the energy: **binary optimization problem**

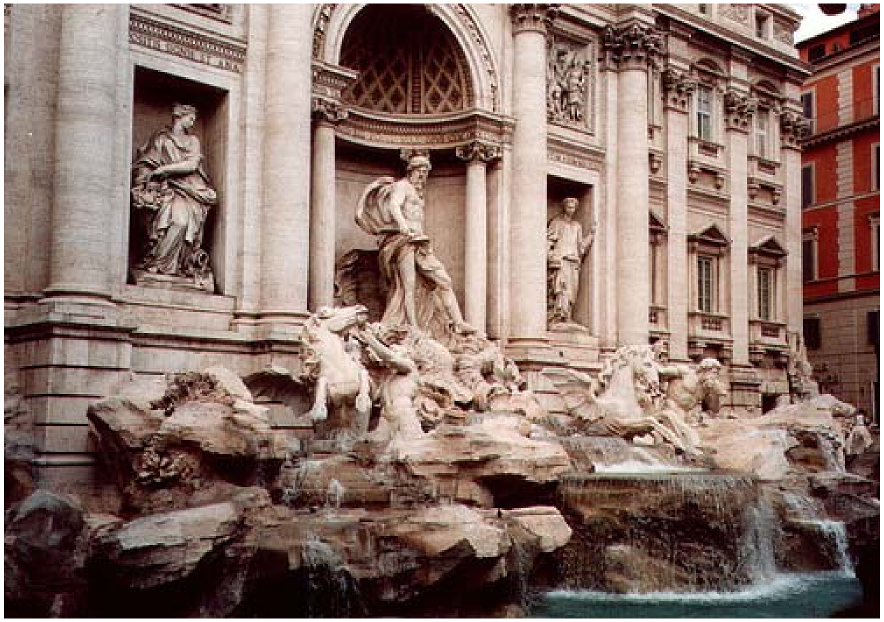
Feature Extraction

Features

- So far, we have assumed that ideal points can be detected in the images and matches across images
  - E.g. for homography and fundamental matrix estimation

What makes a Good Feature?





Uniqueness – at least Distinctiveness

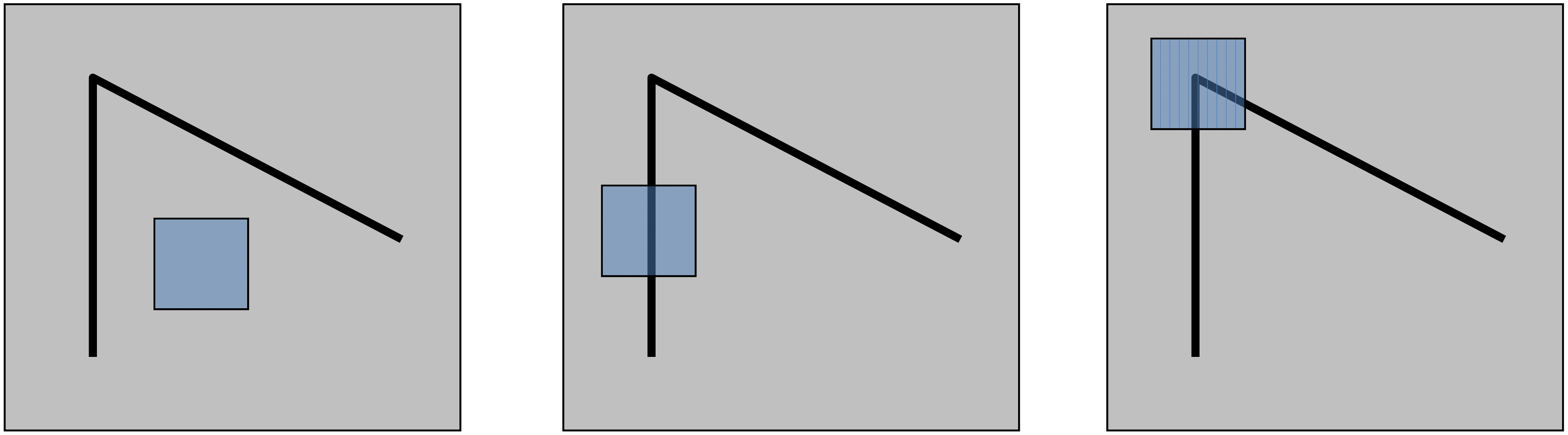
Invariance

Availability - Efficiency

Local Measures of Uniqueness

Suppose we only consider a small window of pixels

- What defines whether a feature is a good or bad candidate?

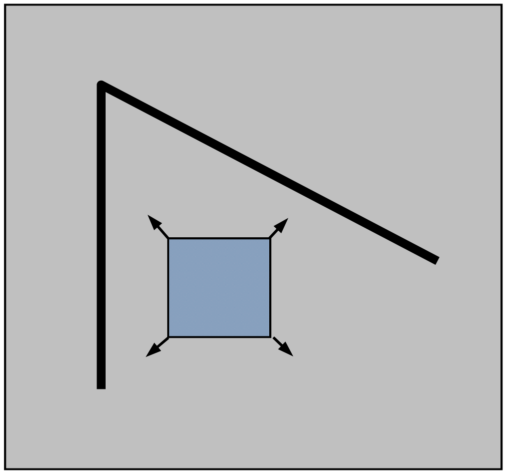


Slide adapted from Darya Frolova, Denis Simakov, Weizmann Institute.

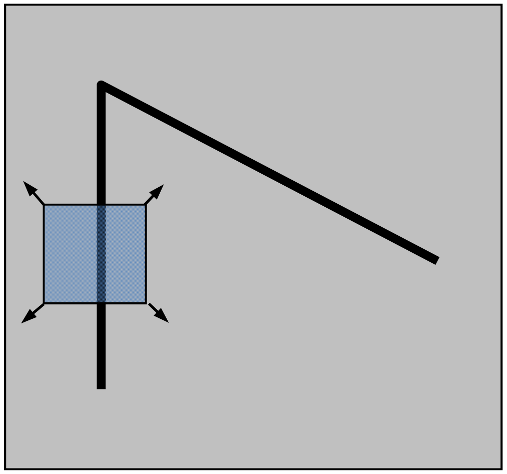
Feature Detection

Local measure of feature uniqueness

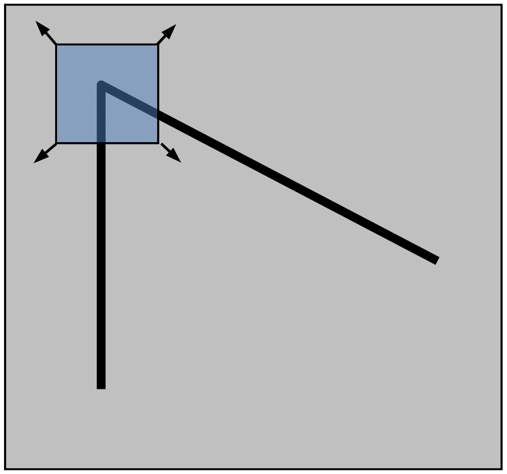
- How does the window change when you shift it?
- Shifting the window in *any direction* causes a *big change*



"flat" region:
no change in all
directions



"edge":
no change along
the edge direction



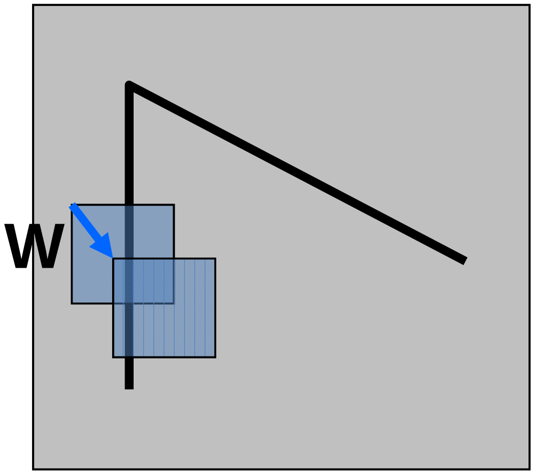
"corner":
significant change
in all directions

Slide adapted from Darya Frolova, Denis Simakov, Weizmann Institute.

Feature Detection: the Math

Consider shifting the window **W** by (u,v)

- how do the pixels in **W** change?
- compare each pixel before and after by summing up the squared differences (SSD)
- this defines an SSD “error” of *E(u,v)*:



$$E(u,v) = \sum_{(x,y)\in W} \left[I(x+u,y+v) - I(x,y)\right]^2$$

Small Motion Assumption

Taylor Series expansion of I:

$$I(x+u,y+v) = I(x,y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

If the motion (u,v) is small, then first order approx is OK

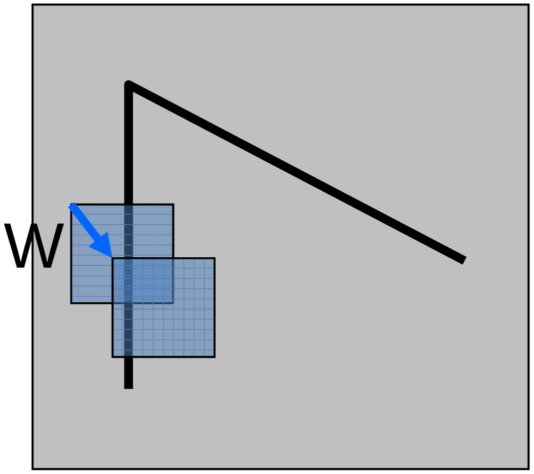
$$I(x+u,y+v) \approx I(x,y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v$$

$$\approx I(x,y) + [I_x \;\; I_y] \begin{bmatrix} u \\ v \end{bmatrix}$$

shorthand: $I_x = \frac{\partial I}{\partial x}$

Plugging this into the formula on the previous slide…

Feature Detection: the Math



$$E(u,v) = \sum_{(x,y)\in W} [I(x+u,y+v) - I(x,y)]^2$$

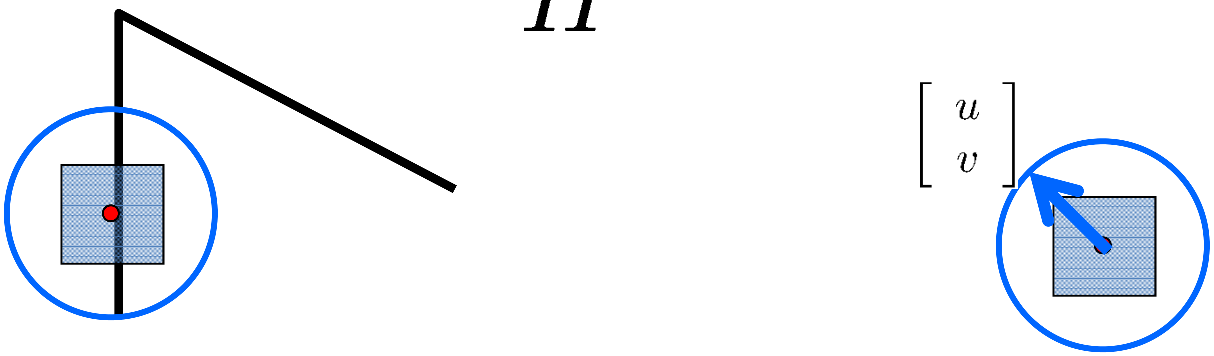
$$\approx \sum_{(x,y)\in W} \left[I(x,y) + [I_x \ I_y] \left[\begin{array}{c} u \\ v \end{array}\right] - I(x,y)\right]^2$$

$$\approx \sum_{(x,y)\in W} \left[[I_x \ I_y] \left[\begin{array}{c} u \\ v \end{array}\right]\right]^2$$

Feature Detection: the Math

This can be rewritten:

$$E(u,v) = \sum_{(x,y)\in W} [u\ \ v] \underbrace{\begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix}}_{H} \begin{bmatrix} u \\ v \end{bmatrix}$$



For the example above

- You can move the center of the window to anywhere on the blue unit circle
- Which directions will result in the largest and smallest E values?
- We can find these directions by looking at the eigenvectors of ***H***

Quick Eigenvalue/Eigenvector Review

The **eigenvectors** of a matrix A are the vectors **x** that satisfy:

$$Ax = \lambda x$$

The scalar λ is the **eigenvalue** corresponding to **x**

- The eigenvalues are found by solving: $det(A - \lambda I) = 0$
- In our case, ***A = H*** is a 2x2 matrix, so we have $det \left[\begin{array}{cc} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{array} \right] = 0$
- The solution: $\lambda_{\pm} = \frac{1}{2} \left[(h_{11} + h_{22}) \pm \sqrt{4h_{12}h_{21} + (h_{11} - h_{22})^2} \right]$

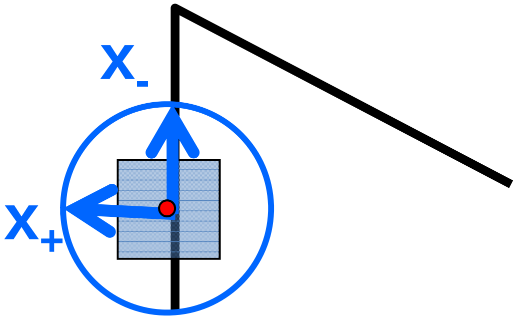
Once you know λ, you find **x** by solving

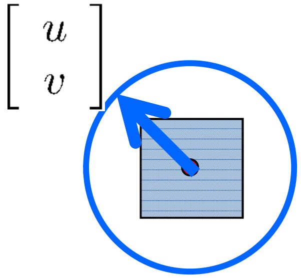
$$\left[\begin{array}{cc} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{array} \right] \left[\begin{array}{c} x \\ y \end{array} \right] = 0$$

Feature Detection: the Math

This can be rewritten:

$$E(u,v) = \sum_{(x,y)\in W} [u\ v] \underbrace{\begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix}}_{H} \begin{bmatrix} u \\ v \end{bmatrix}$$





Eigenvalues and eigenvectors of H

- Define shifts with the smallest and largest change (E value)
- x_+ = direction of **largest** increase in E.
- λ_+ = amount of increase in direction x_+
- x_- = direction of **smallest** increase in E.
- λ- = amount of increase in direction x_+

$$\begin{aligned} Hx_+ &= \lambda_+ x_+ \\ Hx_- &= \lambda_- x_- \end{aligned}$$

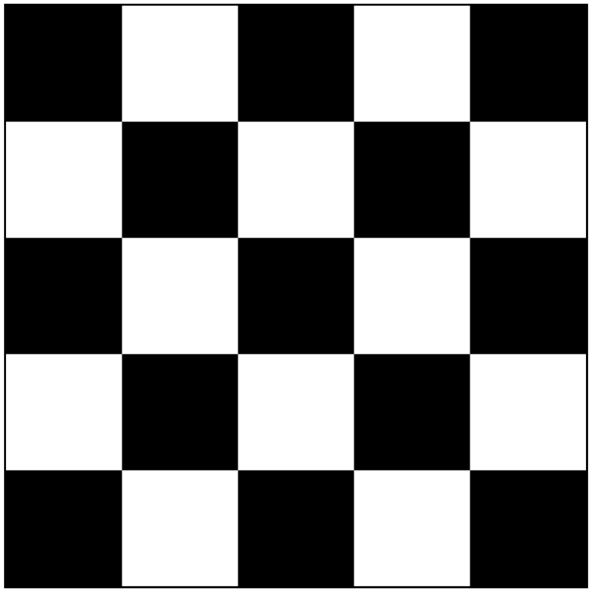
Feature Detection: the Math

How are λ_+, $\mathbf{x}_+$, λ_-, and $\mathbf{x}_-$ relevant for feature detection?

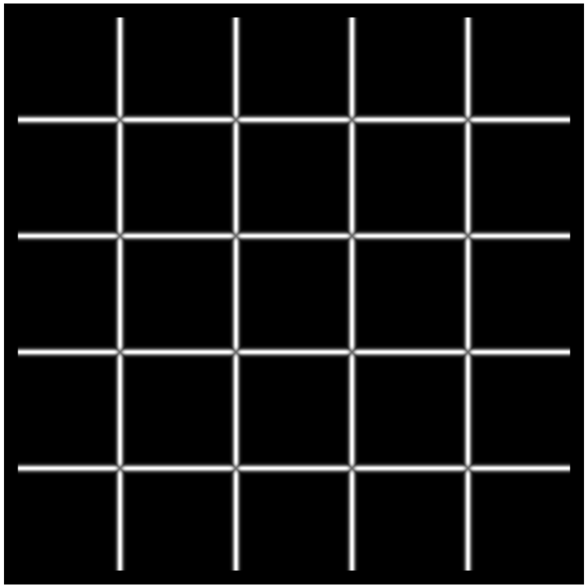
- What's our feature scoring function?

Want *E(u,v)* to be ***large*** for small shifts in ***all*** directions

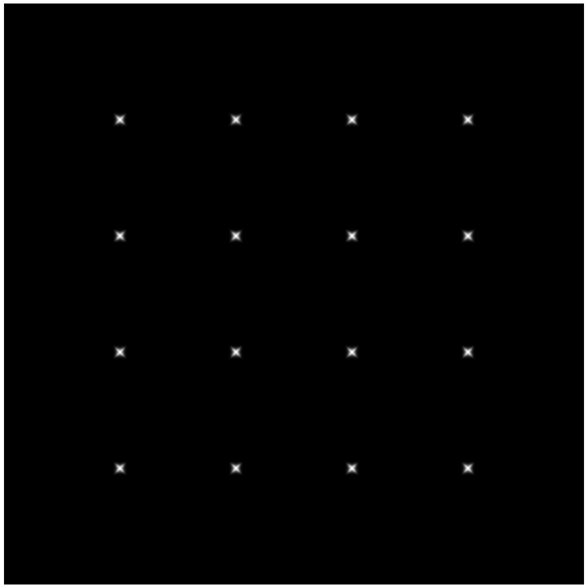
- the *minimum* of *E(u,v)* should be large, over all unit vectors [u v]
- this minimum is given by the smaller eigenvalue (λ_-) of ***H***



I



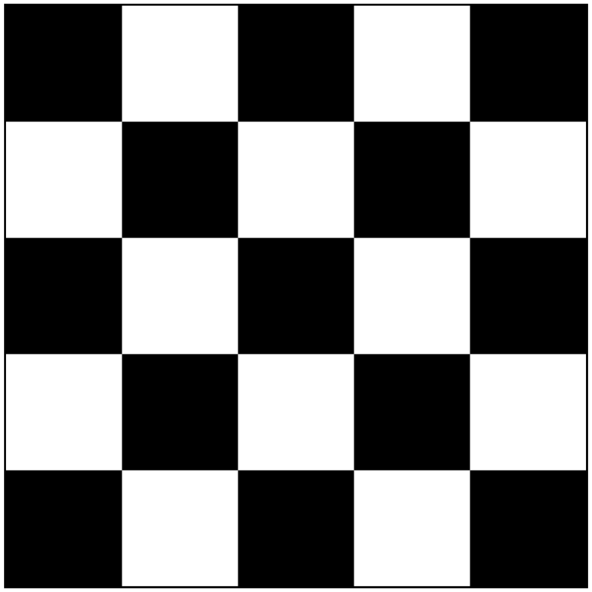
λ_+



λ_-

Feature Detection Summary

- Compute the gradient at each point in the image
- Create the ***H*** matrix from the entries in the gradient
- Compute the eigenvalues
- Find points with large response (λ_- > threshold)
- Choose those points where λ_- is a local maximum as features



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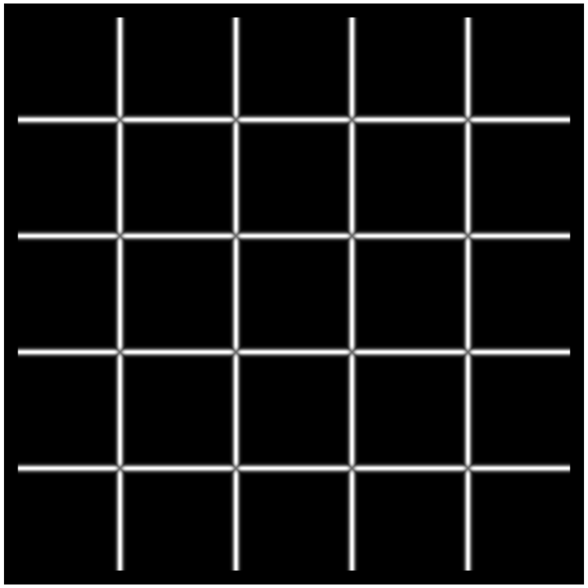
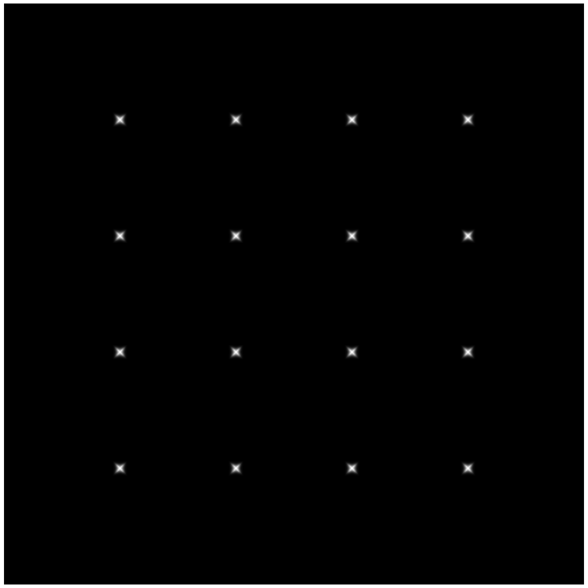


λ_+



λ_-

The Harris Operator

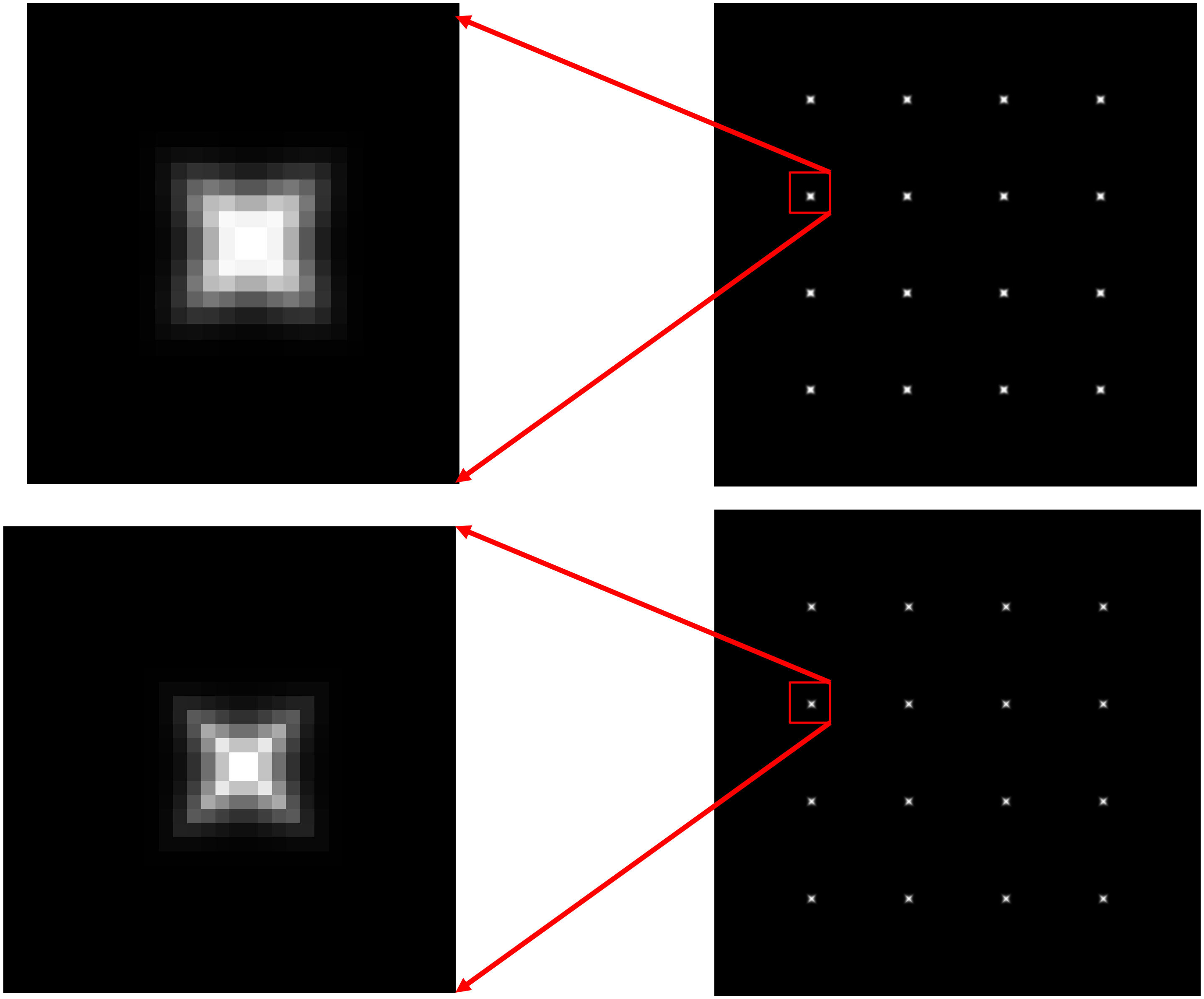
λ_- is a variant of the "Harris operator" for feature detection

$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

$$= \frac{determinant(H)}{trace(H)}$$

- The *trace* is the sum of the diagonals, i.e., $trace(H) = h_{11} + h_{22}$
- Very similar to λ_- but less expensive (no square root)
- Called the "Harris Corner Detector" or "Harris Operator"
- Lots of other detectors, this is one of the most popular

The Harris Operator

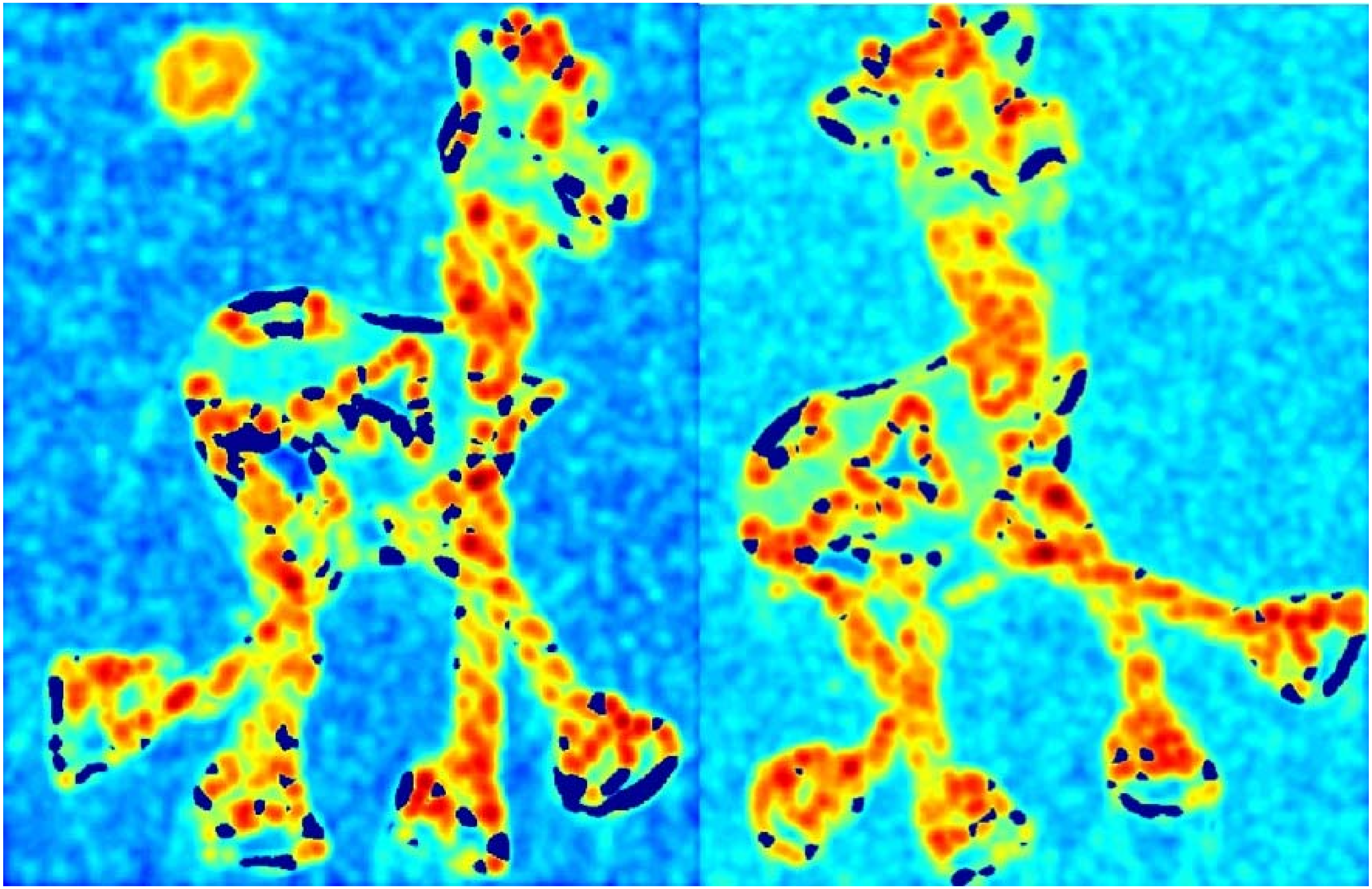


Harris operator

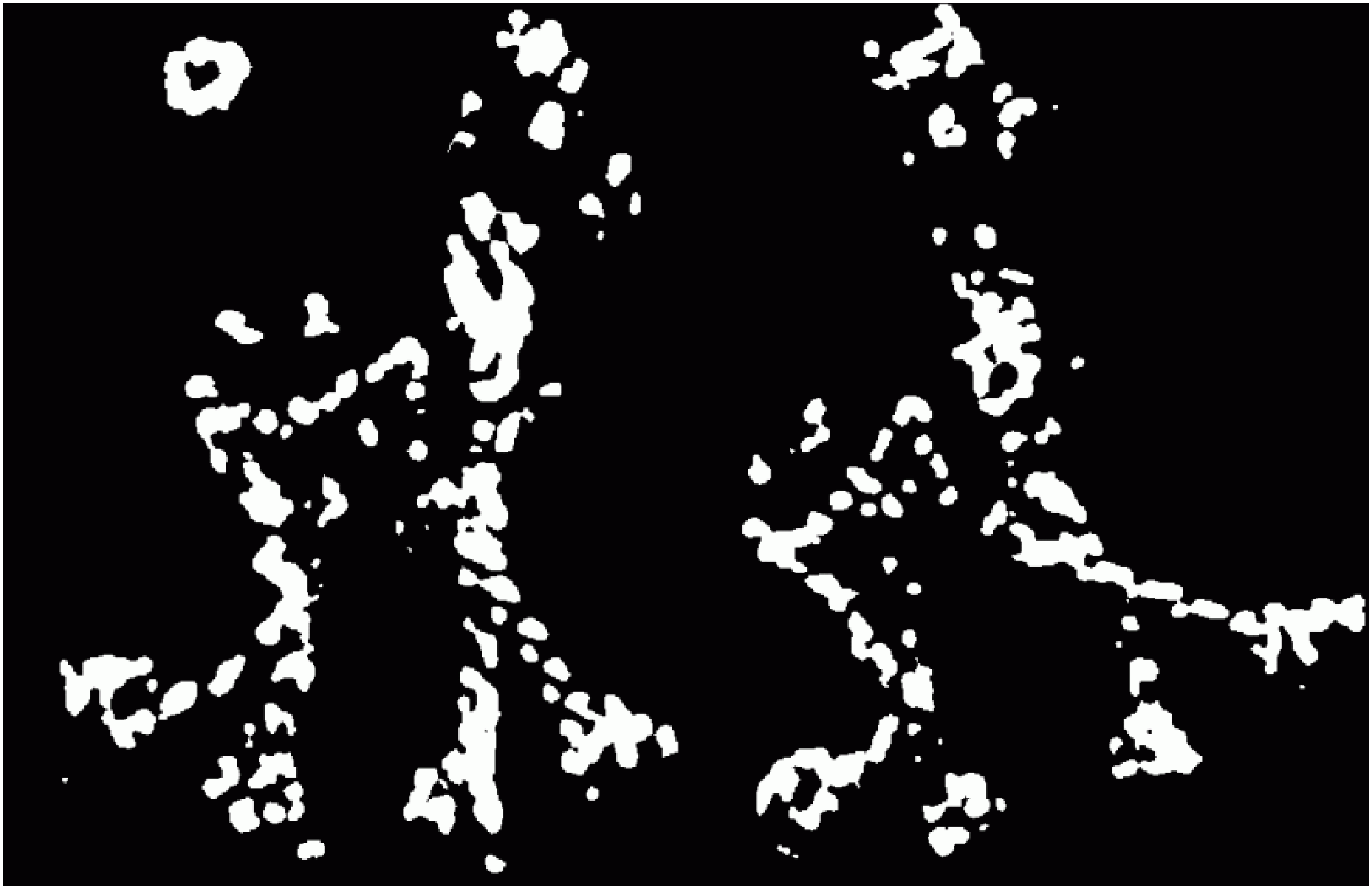
λ_-

Harris Detector Example

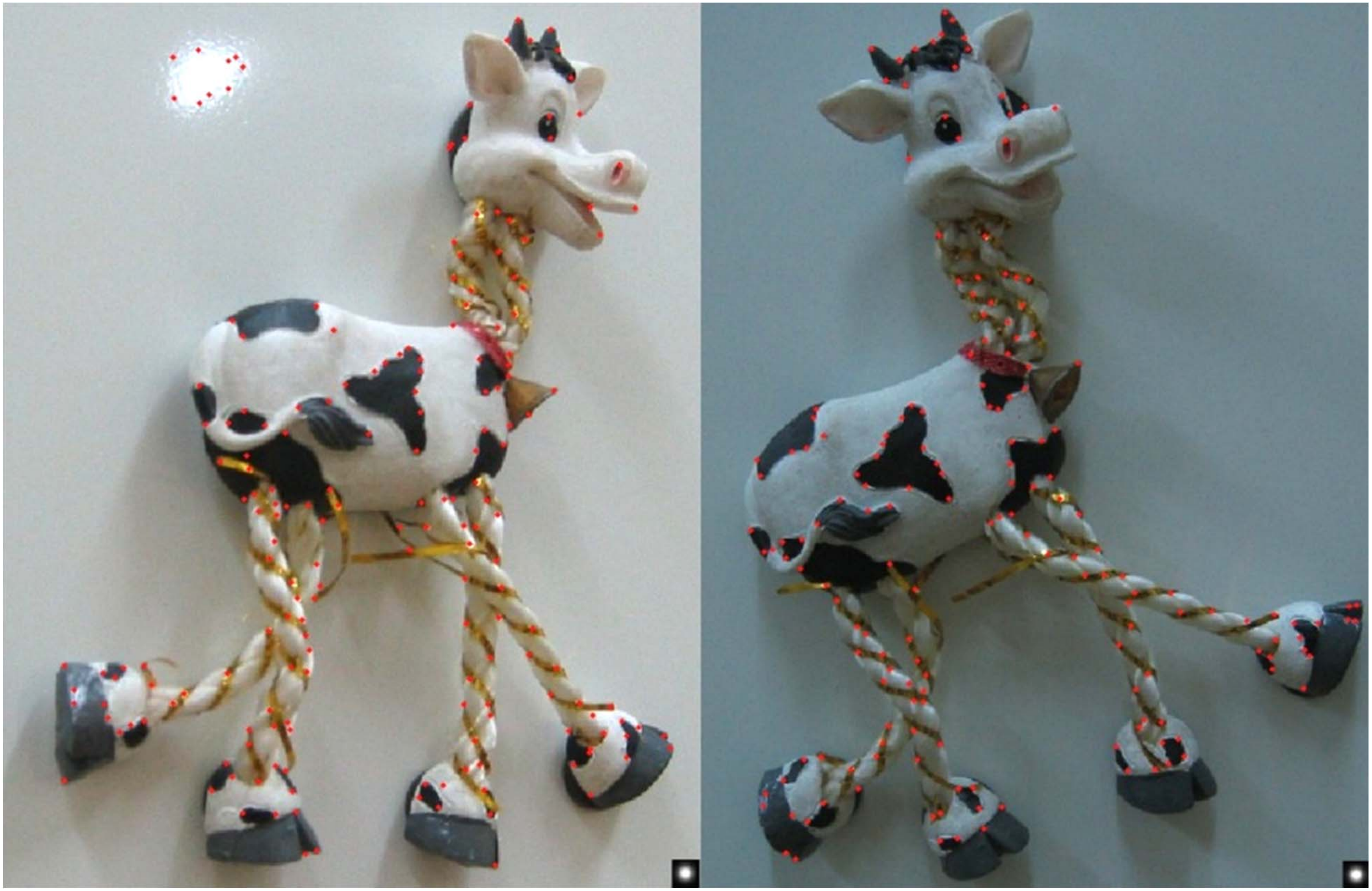
f value (red high, blue low)



Threshold (f > value)



Harris Features (in red)



Invariance

Suppose you **rotate** the image by some angle

- Will you still pick up the same features?

What if you change the brightness?

Scale?

Feature Matching

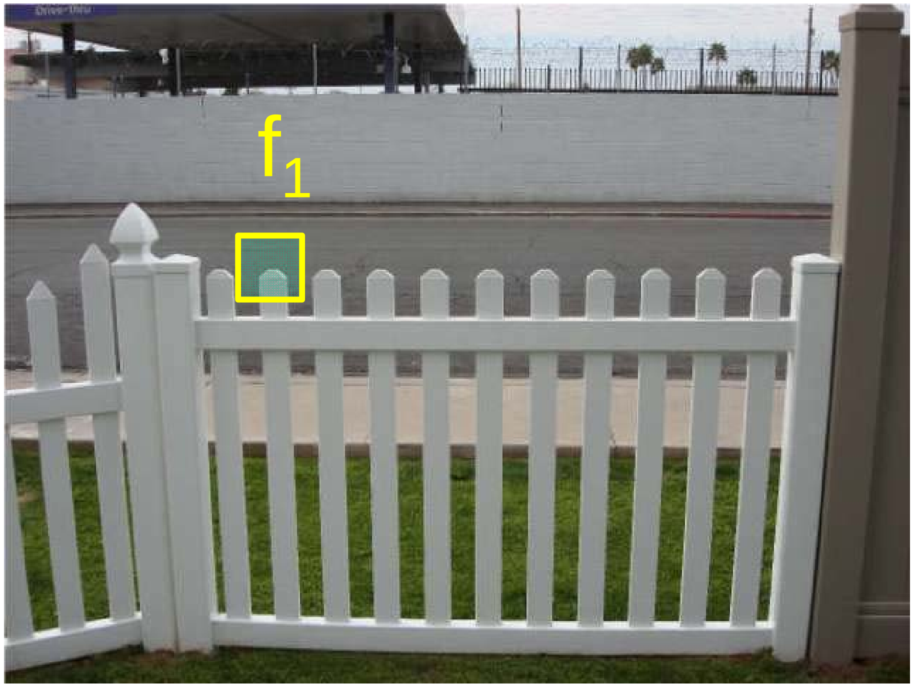
Given a feature in I_1, how to find the best match in I_2?

1. Define distance function that compares two descriptors
2. Test all the features in I_2, find the one with min distance

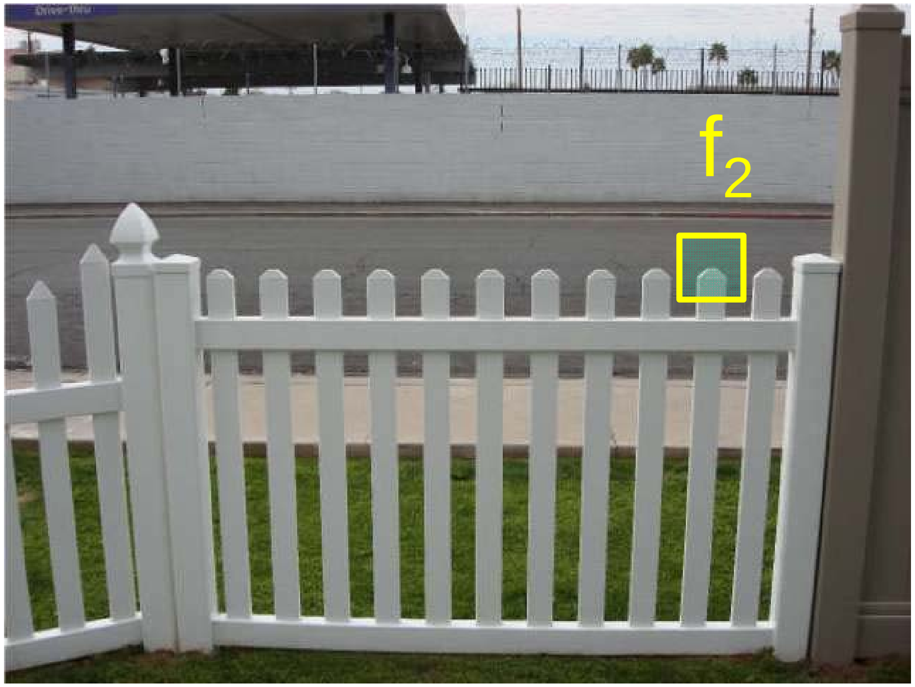
Feature Distance

How to define the difference between two features f_1, f_2?

- Simple approach is SSD(f_1, f_2)
  - sum of square differences between entries of the two descriptors
  - can give good scores to very ambiguous (bad) matches



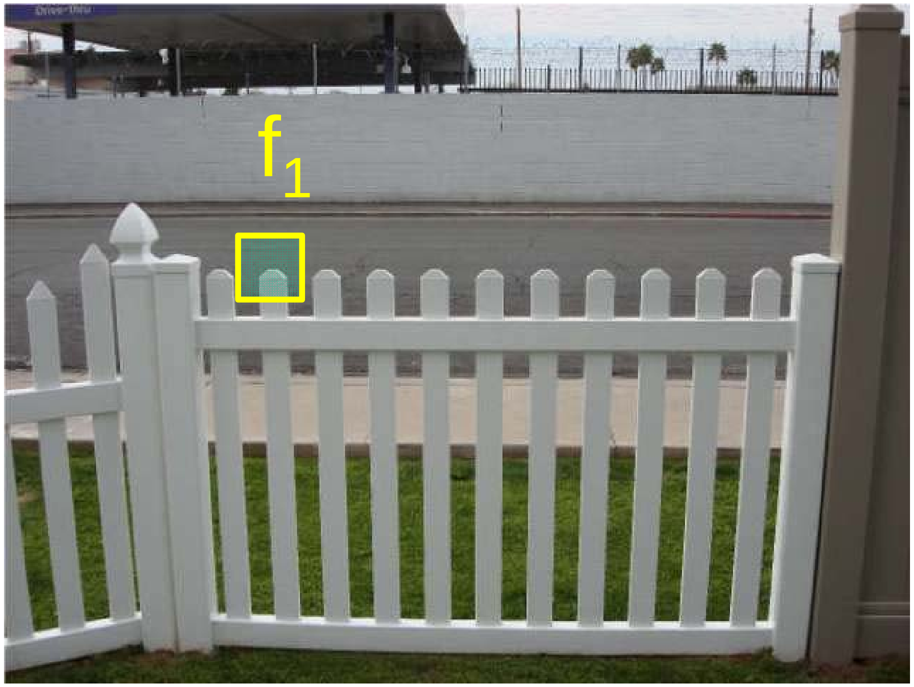
I_1



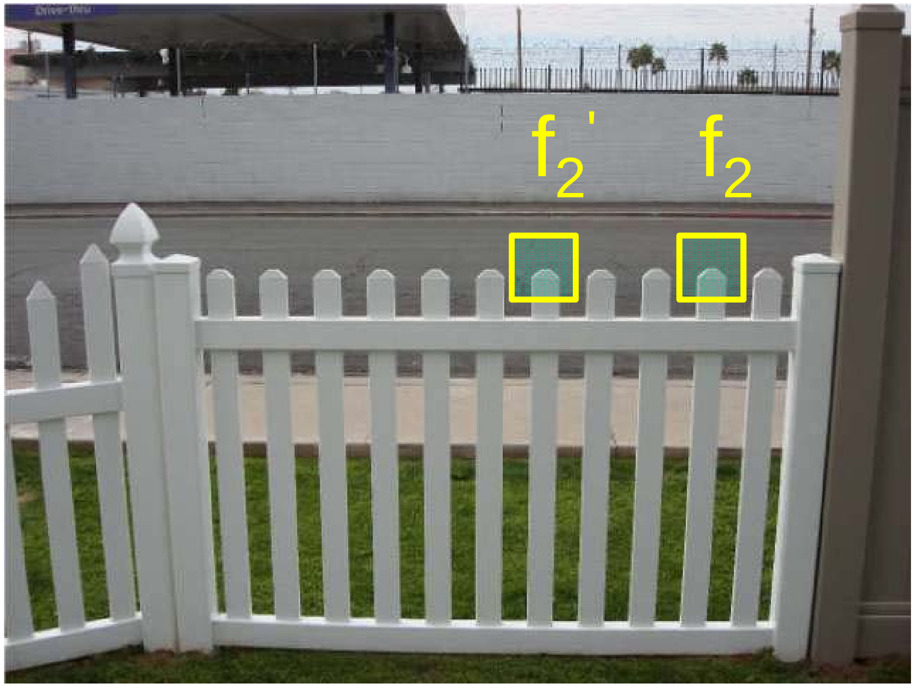
I_2

Feature Distance

- Better approach:
  ratio distance = SSD(f_1, f_2) / SSD(f_1, f_2’)
  - f_2 is best SSD match to f_1 in I_2
  - f_2’ is 2nd best SSD match to f_1 in I_2
  - gives small values for ambiguous matches



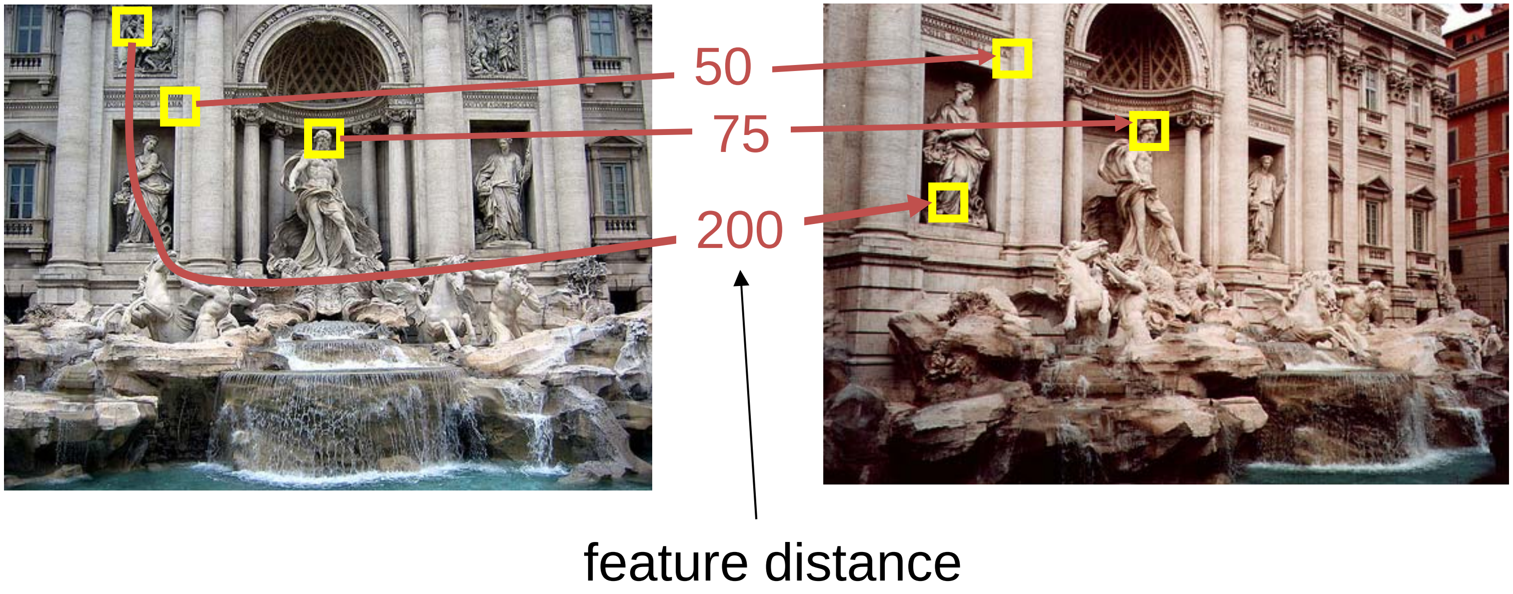
I_1



I_2

Evaluating the Results

How can we measure the performance of a feature matcher?



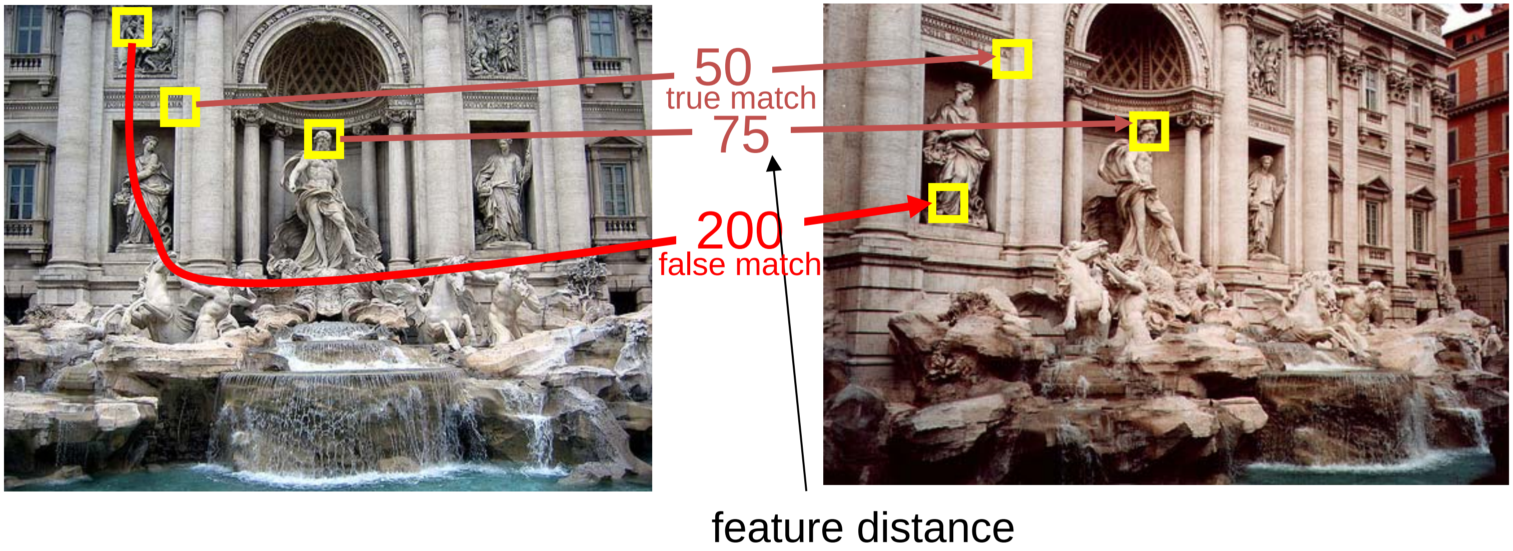
50

75

200

feature distance

True/False Positives



The distance threshold affects performance

- True positives = # of detected matches that are correct
  - Suppose we want to maximize these–how to choose threshold?
- False positives = # of detected matches that are incorrect
  - Suppose we want to minimize these–how to choose threshold?