CSCI 2824 – Spring 2024

Homework 7

Name: XXXXXXX XXXXXXX Student ID: ZZZZZZZZZ

This assignment is due on Friday, April 5^{th} to Gradescope by 6PM. There are 6 questions on this homework. You are expected to write or type up your solutions neatly. Remember that you are encouraged to discuss problems with your classmates, but you must work and write your solutions on your own.

Important: Make sure to clearly write your full name and your student ID number at the top of your assignment. You may **neatly** type your solutions in LaTeX for extra credit on the assignment. Make sure that your images/scans are clear or you will lose points/possibly be given a 0. Additionally, please be sure to match the problems from the Gradescope outline to your uploaded images.

Do not just give the answer, show the steps or give the reasoning behind all your answers.

1. (a) (2 points) In a group of 100 people, everyone shakes hands with everyone else once. How many handshakes occur?

   (b) (3 points) Mia wants to guess her friend's laptop password when she is away with the following hints.

   - It only has lowercase letters (a-z).
   - It is six characters long.
   - The keypad has smudges over two different letters after her friend typed the password.

   How many distinct passwords are possible with the above conditions.

2. Answer the following: (6 points)

   (a) (3 points)How many solutions in non-negative integers are there to the equation $x+y+z+w+u = 26$?

   (b) (3 points) Find the minimum number of students required in an Algorithms class to be sure that at least six will receive the same grade, if the five possible grades are A+, A, B+, B, F ?(Hint: Pigeon hole principle)

3. (6 points) Consider all strings of length 12, consisting of all uppercase letters. Letters may be repeated. Please do not simplify your answers.

   (a) (1 point) How many such strings are there?

   (b) (2 points) How many such strings contain the word "SCOOBY"?

   (c) (3 points) How many such strings contain neither the word "SCOOBY" nor the word "DAPHNE"?

4. Solve each of the following:

   (a) (3 points) Using Binomial Theorem, give the closed-form expression for: $\sum_{k=0}^{n} \binom{n}{k} 9^n \cdot 8^k$

   (b) (3 points) What is the coefficient of $x^{12}y^{13}$ in the expansion of $(3x - 2y)^{25}$.

   Note: For part (b), You can keep your final answer in exponential and factorial form and need not simplify it.

5. (2 points) State and prove Pascal's identity using the formula for $\binom{n}{k}$.

6. The English alphabet contains 21 consonants and 5 vowels. How many strings of **five** lowercase letters can be formed using the following constraints? Give two answers for each of the following - one where repetition is allowed in the string and one where repetition is not allowed.

   (a) (2 points)Only one vowel (placed anywhere)

   (b) (3 points)Maximum two consonants (placed anywhere)