Homework 2 Problem 3 Bonus

Truth measurement: $\vec{b} = A(\vec{q})\vec{r} = A(\delta q)A(\bar{q})\vec{r} \overset{\text{small angles}}{\approx} (I_{3\times3} - [\delta\alpha^\times])A(\bar{q})\vec{r}$

Expected Measurement: $\hat{\vec{b}}^- = A(\bar{q})\vec{r}$

$$\tilde{y}_{err} = \Delta\vec{b} = \vec{b} - \hat{\vec{b}}^- = A(\vec{q})\vec{r} - A(\bar{q})\vec{r} \overset{\text{small angles}}{\approx} (I_{3\times3} - [\delta\alpha^\times])A(\bar{q})\vec{r} - A(\bar{q})\vec{r} = \left(I_{3\times3} - [\delta\alpha^\times] - I_{3\times3}\right)A(\bar{q})\vec{r}$$

$$\Delta\vec{b} = -[\delta\alpha^\times]\underbrace{A(\bar{q})\vec{r}}_{\text{vector}}; \quad \text{let vector} \triangleq \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$\Delta\vec{b} = \begin{bmatrix} 0 & +\delta\alpha_3 & -\delta\alpha_2 \\ -\delta\alpha_3 & 0 & +\delta\alpha_1 \\ +\delta\alpha_2 & -\delta\alpha_1 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} v_2\delta\alpha_3 - v_3\delta\alpha_2 \\ -v_1\delta\alpha_3 + v_3\delta\alpha_1 \\ v_1\delta\alpha_2 - v_2\delta\alpha_1 \end{bmatrix} = \begin{bmatrix} 0 & -v_3 & +v_2 \\ +v_3 & 0 & v_1 \\ -v_2 & +v_1 & 0 \end{bmatrix}\begin{bmatrix} \delta\alpha_1 \\ \delta\alpha_2 \\ \delta\alpha_3 \end{bmatrix}$$

$$\left(A(\hat{\bar{q}}^-)\vec{r}\right)^\times$$

Skew Symmetric

$$\Delta\vec{b} = \left([A(\bar{q}^-)\vec{r}]^\times\right)\delta\vec{\alpha}$$

$$H = \frac{\partial\Delta\vec{b}}{\partial\vec{\alpha}} = [A(\bar{q}^-)\vec{r}]^\times$$