Determinism Beyond Time Evolution

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Physicists are increasingly beginning to take seriously the possibility of laws which may be non-local, global, atemporal, retrocausal, or in some other way outside the traditional time-evolution paradigm. Yet our understanding of determinism is still predicated on a forwards time-evolution picture, making it manifestly unsuited to the diverse range of research programmes in modern physics. In this article, we set out a generalization of determinism which does not presuppose temporal directedness, and we explore some of the consequences of this generalization for the the philosophy of determinism and chance.

We begin in section 1 by identifying several problems with the Laplacean definition of determinism. First, Laplacean determinism fails to deliver meaningful verdicts when applied to theories with non-standard approaches to time-evolution. Second, Laplacean determinism gives a special status to the initial state of the universe, which seems unjustified given the manifest time-symmetry of nearly all physical laws. And third, Laplacean determinism fails to distinguish clearly between the related but distinct notions of predictability and determinism. Indeed, these problems are linked: if we elide determinism with predictability then naturally we will think of determinism in terms of the determination of the future by the past, but if we consider determinism to be a property of the world which is distinct from facts about predictive capabilities, we will be more inclined to seek definitions of determinism which make sense from a global, external point of view.

In section 3 we use a constraint-based framework to provide several such definitions, distinguishing between strong, weak and hole-free global determinism. Then in section 4 we discuss some interesting consequences of these generalized notions of determinism. In section 5, we show that this approach sheds new light on the long-standing debate surrounding the nature of objective chance, because it transpires that in a globally deterministic world it is possible to have events which appear probabilistic from the local point of view but which nonetheless don't require us to invoke 'objective chance' from the external point of view. Finally, in section 6 we discuss how global determinism relates to several other relevant research programmes.

1 Problems for Laplacean Determinism

1.1 Temporal Direction

The notion of determinism was famously first articulated by Laplace, who suggested that '*we ought to regard the present state of the universe as the effect of its antecedent state and as the cause of the state that is to follow.*' [1] That is, a theory is deterministic if according to that theory, the present state of the universe, together with the laws of nature, is sufficient to determine everything that happens in the future. We will refer to this approach as 'Laplacean determinism.' Although the terminology has evolved somewhat, our modern notion of determinism is still essentially a version of Laplacean determinism: in order to reconcile determinism with relativity's denial of absolute simultaneity it is necessary to update the definition to refer to the state on any given hyperplane of simultaneity, rather than simply 'the present state,' but the fundamental idea of the present determining the future is unchanged. For example, ref [2] provides a careful classification of three different approaches to defining determinism for physical theories: we may require that the solution to the differential equations should always be unique, we may require that if two linear temporal realizations can be mapped at one time they can always be mapped at all future times, or we may require

that the theory's models are not branching. However, as the authors note, all three of their approaches are based on the core idea *'that given the way things are at present, there is only one possible way for the future to turn out'* and therefore all three approaches are different expressions of the temporally directed picture associated with Laplacean determinism.

This made sense for Laplace, who was working in the context of Newtonian physics wherein the fundamental role of laws is to give rise to time evolution: the Newtonian universe is a kind of computer which takes in an initial state and evolves it forward in time [3]. But there have been several major scientific revolutions since the time of Newton, and thus definitions of determinism based on a Newtonian time-evolution picture may not be well-suited to the realities of modern physics. Ref [4] details a large variety of research programmes in modern physics which are moving away from the time evolution paradigm, so here we will limit ourselves to mentioning a few examples. First, although it is most common to conceptualise classical mechanics in terms of the Newtonian schema [5] in which laws act on states to produce time evolution, there is also an alternative Lagrangian description of classical mechanics in which systems are required to take the path which optimizes a quantity known as the Lagrangian [6]. Path integrals - the analogue of the Lagrangian method within quantum mechanics [7] - have become so important to quantum field theory that increasingly we are seeing calls to take the Lagrangian description more seriously [8, 9]. And as argued by Wharton [3], taking Lagrangian methods seriously leads to a novel 'all-at-once' approach to lawhood in which we think of laws applying externally and atemporally to the whole of history at once [10], so the time evolution paradigm is no longer appropriate. In a similar vein, retrocausal approaches to the interpretation of quantum mechanics have been attracting significant attention in recent years; see for example the two-state vector interpretation [11], the transactional interpretation [12], Kent's approach to Lorentzian beables [13], Wharton's retrocausal path integral approach [14], and Sutherland's causally symmetric Bohmian model [15]. The proliferation of such models provides good reason to suppose that a correct understanding of quantum mechanics may require us to rethink some of our ideas about time and temporal evolution. Likewise, large sectors of the growing field of quantum information science are concerned with discovering general constraints on what information-processing tasks can be achieved using quantum systems. For example, there is the 'no-signalling principle' [16], 'information causality' [17], 'monogamy,' and 'no-cloning' [18]. These sorts of constraints are regarded by many researchers in the field as being deep and fundamental features of physical reality, and yet they are certainly not dynamical laws or time evolution laws in the usual sense, since they are primarily concerned with describing what is possible or impossible. In response to these developments, refs [4, 19] have argued that we should move away from the time evolution approach to lawhood and instead conceptualize laws in terms of global cosntraints which are understood to constrain the whole of spacetime at once.

Clearly we will encounter difficulties if we try to apply the definition of Laplacean determinism to worlds governed by laws like these which fall outside the time evolution paradigm. For example, consider a non-Markovian world which is governed by laws such that the evolution at a given time depends not only on the present state, but also on some facts about the past which are not recorded in the present state. The non-Markovian world fails to satisfy Laplacean determinism, since the state at a given time does not suffice to determine all future states. But it seems odd to refer to such a world as 'indeterministic' - after all, the evolution at a given time is entirely determined by the past. Similarly, consider the example of a world whose ontology is something like the Bell flash ontology [20], i.e. reality is composed of 'a constellation of flashes,' which are to be regarded as pointlike events, so there are no 'states' in this world. Clearly in such a world the present state of the universe can't determine the future, since there *is* no present state, but nonetheless it might be the case that the distribution of future flashes is fully determined by the distribution of past flashes[1] and under those circumstances one might think it seems reasonable to describe this world as 'deterministic' even though it doesn't satisfy the standard definition of Laplacean determinism. The problem is that the Laplacean approach to determinism presupposes 'temporal locality,' i.e. it assumes that events at a given time can depend only on the state of the world at that time [10], and therefore worlds like the non-Markovian world and the flash world are necessarily classified as indeterministic despite exhibiting perfect determination of the future by the past.

In response to this problem, Dowe has proposed several alternative definitions of determinism which allow that worlds exhibiting temporal nonlocality may still be deterministic - for example, his modal-nomic definition proposes that a world W is deterministic iff *'for any time t and any other physically possible world W', if W and W' agree up until t then they agree for all times.'* [21] Both the retrocausal world and the flash world can potentially be deterministic

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[1]This is not the case in the actual GRW flash model, but one can imagine a similar theory where it would be the case.

according to this definition. But Dowe's approach retains the temporally-directed features of Laplacean determinism, and thus it still does not accommodate the full spectrum of non-Newtonian laws appearing in modern physics. For example, consider a retrocausal world in which there is both a forwards-evolving state and a backwards-evolving state, and the initial and final states of the universe together with the laws of nature suffice to determine everything which happens over the whole course of history. In this world the evolution at a given time is not wholly determined by the past, nor indeed by the future, so the world will be judged indeterministic by Laplacean determinism and Dowe's criterion, and yet there seems to be a sense in which it is not really indeterministic - after all, the events in it are fixed once and for all by the forwards and backwards evolving states, so nothing that happens is random and we have no need to invoke anything that looks like an objective chance. [2] Similarly, a world governed by 'all-at-once' laws in the style of Wharton will not typically be judged as deterministic by either Laplacean determinism or Dowe's criterion, because in the all-at-once picture past and future events depend on one another mutually and reciprocally and therefore we will not always be able to write these dependence relations wholly in terms of determination of the future by the past. Yet 'all-at-once' laws could in principle fix the whole course of history uniquely, and therefore there is a sense in which worlds governed by such laws don't really look indeterministic. Thus it seems very reasonable to think that we ought to have a definition of determinism which allows retrocausal and all-at-once worlds and other worlds like them to be deterministic.

At this point we should reinforce that it is not our intention to have an argument over terminology - one could certainly make a case that determinism should be defined in terms of evolution forwards in time because that is the way the term has always been used, and proponents of this way of thinking might then be inclined to suggest that the non-Markovian world, flash world, retrocausal world and all-at-once world should not be considered deterministic despite some intuitions to the contrary. We have no particular quarrel with this position: our intention here is simply to argue that there is a meaningful distinction between worlds like the non-Markovian/flash/retrocausal/all-at-once world, and worlds in which some events are not determined by anything, so there is a need for terminology which recognises that distinction. For convenience and familiarity we will describe the former as 'deterministic' and the latter as 'indeterministic,' but readers who prefer to use the term determinism to refer only to determination by forwards-evolving states are welcome to adopt whatever alternative terminology they prefer.

1.2   The Initial State

A further problem for the traditional definition of Laplacean determinism (and other temporally directed approaches like Dowe's) is that it suggests the initial state of the universe should be thought of as an 'input' which is then evolved forwards in time by the laws of nature to produce the rest of history. But there is a tension between this traditional picture and the manifest time-symmetry of the laws of physics[3], because from a mathematical point of view we could equally well start at the end of time and evolve backwards, or indeed start in the middle of time and evolve both forwards and backwards. So it doesn't necessarily seem that the traditional picture's metaphysical commitment to a special role for the initial state is really supported by the empirical evidence: of course there is certainly a *phenomenological* arrow of time pointing in the forward direction, but the way in which it emerges out of largely time-symmetric fundamental physics remains controversial [23, 24], so it is certainly not straightforward to argue that the phenonemological arrow supports the Newtonian picture of initial conditions plus forward time evolution.

One possible option for restoring time symmetry is to suppose that *both* the initial conditions and the final conditions of the universe are given as inputs to the universe-as-computer and then we get both forwards and backwards time evolution. It would be difficult to make this work in the context of a theory like classical mechanics which obeys Laplacean determinism, since in such a theory the final condition is fully determined by the initial condition, so there is no freedom for the initial and final conditions to be distinct inputs. But such a move is possible within a probabilistic theory like quantum mechanics, and indeed several proposals of this kind have been made, such as the two-state vector

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[2]Note that here and throughout this article we will use 'objective chance' to refer exclusively to chances which arise from the fundamental laws of nature, such as the probabilities arising from the Born rule within indeterministic interpretations of quantum mechanics - i.e. in this article 'objective chance' does not include higher-level emergent chances, chances derived via the method of arbitrary functions, deterministic probabilities or anything else that might in another context be called an objective chance.

[3]It is common to hold up the measurement process in quantum mechanics as an example of laws which are not time-symmetric, but it has recently been observed that in fact the Born rule *is* time-symmetric [22], so it is not clear that any of the laws of physics point clearly to the existence of time asymmetry at a fundamental level.

formalism for quantum mechanics in which we impose an initial condition *and* a final condition and intermediate measurement outcomes are determined by the interaction of the forward-evolving state and the backwards-evolving state [11]. It should be noted, however, that in this sort of picture it's still not the case that the initial and final states can be regarded as completely independent 'inputs', for they must be at least minimally consistent with one another: for example, if it is the case that conservation laws apply to the whole universe, then presumably quantities like the total amount of mass-energy must be the same in both the initial and final states

Moreoever, although this approach does solve the asymmetry problem, it still gives rise to underdetermination, because the physics does not compel us to choose boundary points for our inputs: we could equally well choose some spacelike hyperlane at some point in the middle of time and evolve forwards or backwards from it. Thus the insistence that the world contains some form of directed time evolution at the metaphysical level actually leads to a strong form of underdetermination, since any spacelike hyperplane could in principle be *the* hyperplane from which everything else in the universe is generated via forwards and backwards time evolution, and there is no possible way to tell which one in the real 'initial input.' Underdetermination is not always bad news, but when the underdetermination is generated only by our metaphysical preconceptions (as for example in the hole argument [25]) physicists and philosophers alike tend to be suspicious, and so it is natural to ask if there is an alternative metaphysical picture we could adopt which would make this extreme form of underdetermination go away (such as moving to sophisticated substantivalism in the case of the hole argument [26]). One possible response would be to adopt a best-systems approach to lawhood [27, 28], in order to argue that these apparently distinct possibilities are merely different systematizations of the Humean mosaic so there is no fact of the matter about which one of them is correct; but that route sidesteps all the interesting questions about what lawhood and determinism might look like if we don't presume the usual input-output form, and thus there is a clear mandate for a new definition of determinism which addresses those questions and avoids singling out some 'input' from which the rest of history is generated.

1.3 Predictability

A final difficulty for the Laplacean definition of determinism is that it fails to properly disentangle the concepts of determinism and predictability. Laplace's original comments on the matter conflated the two, describing determinism metaphorically in terms of the predictive abilities of *'an intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed,'* [1] and this way of thinking is still evident in many modern analyses of determinism [29]. Yet at least in theory, predictability and determinism are supposed to be quite distinct - predictability is an epistemic matter, whereas determinism is metaphysical, which is to say, it is supposed to capture something about the way the world really is, independent of our ability to find out things about it. Thus for example Clark, considering the possibility that the functional dependence of the future on the present state might fail to be effectively computable, asserts that *'there is no reason at all to tie the claim of determinism to a thesis of global predictability.'* [30] The Laplacean definition doesn't seem to do justice to this intuition.

Clearly this problem is linked to the first two, because the temporally directed definition of Laplacean determinism is to a large extent a consequence of conflating predictability and determinism: traditionally we say that a theory or world is deterministic if facts about the present state fully determine the future, and non-coincidentally, we ourselves have a strong practical interest in predicting the future from facts about the present. But if deteminism is genuinely to be regarded as a property of the world rather than as a function of our practical interests, there's no reason it should be defined in this way: from the point of view of the universe as a whole, it need not be the case that things are always determined in the particular temporal direction in which human observers usually want to predict them. Of course, the temporal prejudice written into the definition of Laplacean determinism is harmless so long as we are dealing with laws of nature which always take the standard dynamical, forward-evolving form, since in that picture determination of the future by the present state is the only possible type of determination, but since some parts of physics are now beginning to move away from the time-evolution picture, we are in need of a new way of thinking about determinism which decouples it from time-evolution.

Now, one might perhaps respond to this criticism of Laplacean determinism by pushing back on the inuition that determinism is supposed to say something about the world itself independently of any theory. Indeed, although the notion of determinism as a property of the world itself is widespread and intuitive, philosophers of physics are often disinclined to use the term in this way, for as Butterfield notes [31], the idea that the world as a whole may be deter-

ministic is often predicated on ideas about 'events', 'causation' and 'prediction' which '*are vague and controversial notions, and are not used (at least not univocally) in most physical theories.*' Thus in the philosophy of physics it is common to limit talk about determinism to discussions about whether some particular theory is deterministic.

In fact, we agree whole-heartedly with Butterfield's criticisms. However, we contend that the problem arises not because the notion of determinism as a property of the world is incoherent, but because current definitions of determinism don't have the conceptual resources to capture this notion. In particular, given that the definition of Laplacean determinism is so focused on the practical interests of human observers, it is hardly surprising that things get murky when we try to apply that definition in an observer-independent way. The solution, therefore, is not to abandon the idea of determinism as a property of the world, but rather to set out a precise way of talking about determinism which is perspective-neutral and independent of the specific epistemic concerns of observers like ourselves. Thus we hope to mitigate Butterfield's concerns by offering an improved, obsever-independent definition of determinism which makes no implicit or explicit appeal to issues of predictability.

2   The Constraint Framework

Determinism and objective chance are patently modal notions - a world is deterministic if it is the case that the course of history in that world *could not* have gone any other way. Thus we submit that a meaningful notion of determinism for *worlds*, rather than merely theories, must be predicated on a fairly robust approach to modality - for every possible world there must be some well-defined modal structure which we can invoke to determine whether or not that world is deterministic. Therefore we will henceforth take for granted the existence of a well-defined modal structure, although we will not comment further on the metaphysical nature of that modal structure - it might be defined by the axioms of the systematization of that world's Humean mosaic which is robustly better than all other systematizations [28], or it could result from governing laws within any one of the realist approaches to lawhood [32–34], or it could be understood in terms of Lewis' modal realism [35], or it could be something else altogether.

Objective modal structure is most commonly discussed in the context of ontic structural realism, whose proponents often characterise it in terms of *causal* structure [36–38]. But as argued in ref [4], due to the asymmetrical nature of the standard notion of causation this approach doesn't work well when we're dealing with possibilities outside the time evolution paradigm, so in this paper we will make use of the more general approach to characterising modal structure developed in ref [4], which is based on *constraints*. Recall that the Humean mosaic for a possible world is the set of all local matters of particular fact in that world, i.e. all the instantiations of categorical properties across the spacetime of that world, including facts about the structure of spacetime itself if necessary. We will define a constraint as a set of Humean mosaics. In some cases, a constraint can be expressed in simple English as as a requirement like 'no process can send information faster than light,' such that the constraint corresponds to exactly the set of mosaics in which the requirement is satisfied. But clearly there are many sets of mosaics which will not have any straightforward English characterisation, so we will not be able to state the corresponding constraint in simple terms. Nonetheless, each set still defines a unique constraint.

Using these definitions, we postulate that every world has some set of laws of nature which are an objective fact about the modal structure of that world, and we characterise these laws in terms of *probability distributions over constraints*. For example, a law which prohibits superluminal signalling induces a probability distribution which assigns probability 1 to the constraint consisting of all the Humean mosaics in which no superluminal signalling occurs, and 0 to all other constraints. Within this picture, we can imagine that the laws of nature operate as follows: first, for each law a constraint is drawn according to the associated probability distribution, and then the constraints *govern* by singling out a set of mosaics from which the Humean mosaic of the actual world must be drawn - i.e. the actual mosaic must be in the intersection of all the chosen constraints. As shown in ref [4], a very large class of possible laws can be written in this framework - it can accommodate the various non-Newtonian laws discussed in section 1.1 but it can also accommodate standard dynamical laws by singling out sets of dynamically possible histories. Thus we can use these probability distributions over constraints as proxies for the laws of nature without needing to choose any particular metaphysical account of lawhood, and this makes it possible for us to give a general definition of determinism which does not depend on any ontological commitments other than the commitment to the existence of objective modal structure.

We could of course have given a similar definition in terms of probability distributions over mosaics rather than

constraints, but we have chosen to use constraints here in order to acknowledge that a law may require (deterministically or probabilistically) that the actual Humean mosaic belongs to a given set, but then say nothing further about which particular mosaic within the set will be selected - that is, the law prescribes no distribution within the set, not even the uniform distribution. This point will be a crucial feature of our approach to determinism.

3 Global Determinism

Now our task is to find a way of decoupling the notion of determinism from time evolution. One obvious way to achieve this would be to appeal to the intuitive idea that determinism and objective chance are mutually exclusive: that is, either it is the case that the fundamental laws of nature are deterministic, or it is the case that they involve some sort of 'intrinsic randomness.' This line of thought leads to the idea that we should simply define determinism as the absence of objective chance. But the problem with this route is that we don't really have a strong grasp on what objective chances are either - the only generally agreed-upon characterisation is that they should obey the Principal Principle [27].

The constraint framework provides a precise way of formulating this idea. Indeed, it allows us to disambiguate several different ways in which a world may be deterministic. First, we will say a world (which, recall, can be associated with some set of laws of nature) obeys *global determinism* iff the probability distributions induced by the laws are trivial:

Definition 3.1. A world satisfies *global determinism* iff every one of its fundamental laws induces a probability distribution which assigns probability 1 to a single constraint and zero to all disjoint constraints.

When this condition is satisfied, we can get rid of the probability distributions altogether and simply assign to each law of nature a constraint, and the actual mosaic must lie in the intersection of all of these constraints. This is the analogue in the constraint framework of the idea that determinism is associated with the absence of objective chance: 'no objective chance' is translated as 'there are no objective probability distributions over constraints.' Of course the actual mosaic must still be selected from the intersection, but this selection is only 'arbitrary' rather than chancy, since as noted in section 2 the laws do not define any particular probability distribution over the mosaics in the intersection, not even the uniform distribution.

However, it might be argued that defining determinism as the absence of objective chance is in some cases too broad. Consider for example Einstein's hole argument, which points out that the laws of general relativity allow us to define a region of spacetime or 'hole' such that the distribution of metric and matter fields inside the hole are not fixed by the fields outside the hole [25]. A common response to this argument is to adopt 'sophisticated substantivalism' which allows us to say that the various possible distributions inside the hole are not really physically distinct; but for now let us consider an 'Einstein Hole World' where the configurations that may exist inside the hole really are all physically distinct[4]. Intuitively, it seems clear that this world is not deterministic, since a large variety of physically inequivalent configurations inside the hole are all compatible with the laws of nature. But the theory defines no probability distribution over the different configurations inside the hole, so as far as we know there aren't really any 'chances' here, not even the uniform distribution - some configuration must occur, and so some configuration does, but there is nothing that can be said about how likely different configurations are to occur. We will henceforth refer to cases of this kind, involving events which are not determined by anything but which also are not chancy events, as 'arbitrary.' Thus it seems that a generalized definition of determinism should allow that worlds containing arbitrarinesss of this kind may be indeterministic even in the absence of objective chance.

In light of this possibility, we distinguish further between strong and weak global determinism:

Definition 3.2. A world satisfies *strong global determinism* iff it satisfies global determinism and there is only one mosaic in the intersection of the set of constraints associated with its laws of nature

Definition 3.3. A world satisfies *weak global determinism* iff it satisfies global determinism and there is more than one mosaic in the intersection of the set of constraints associated with the laws of nature

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[4]This is intended only as an example; we don't mean to claim that this is the right take on the Einstein hole argument, or even that a world like this is actually possible.

In the case of strong global determinism the actual mosaic is singled out uniquely by the laws and we have neither chance nor arbitrariness; whereas weak global determinism allows arbitrariness but not chance. Thus Einstein hole worlds satisfy only weak global determinism and not strong global determinism. One might perhaps question whether 'weak global determinism,' really ought to count as a form of determinism, given that it allows events to occur which are not determined by anything. However, even worlds satisfying Laplacean determinism exhibit some arbitrariness - the initial state of the universe is 'arbitrary' in just the same way as the configuration inside the hole in an Einstein hold world, since we don't assign objective chance distributions over initial states but we don't usually take them to be determined by anything either. So when we set out to generalize determinism in a way that does not give any special status to the initial state of the universe, we have two options - either we allow arbitrariness elsewhere as well, or we allow no arbitrariness whatsoever. Weak global determinism takes the former route and strong global determinism the latter, so both have at least some claim to be the spiritual heir of Laplacean determinism.

That said, there does seem to be a sense in which a world satisfying Laplacean determinism is more strongly deterministic than an Einstein hole world, since the arbitrariness involved in the former is limited to the start of time, whereas in the latter we can have undetermined holes popping into existence all over the place. So ideally we would like to offer some further distinction which distinguishes between worlds satisfying Laplacean determinism and worlds like the Einstein hole world. Of course, we could add a special exception for the initial state into our definition of determinism, but this would seem like a relic of the older temporally directed approach - if we're no longer insisting that laws must take the forward time-evolution form there's no good reason to treat the initial state as special. So instead we suggest the following definition:

Definition 3.4. A world associated with a set of laws of nature satisfies *hole-free global determinism* iff it satisfies global determinism, and there is no pair of mosaics in the intersection of the set of constraints associated with its laws of nature which are identical everywhere except on some small subregion of spacetime

The point of this definition is that it ensures the arbitrariness involved in selecting the actual mosaic can't be localised in any particular region, which means we can't have any indeterministic 'holes.' To see how the definition works, consider a world satisfying Laplacean determinism in which the laws of nature are time-reversal invariant. In the constraint framework, this world has a set of laws of nature which are each associated with a single constraint, and the intersection of these constraints is a set of mosaics such that each mosaic in the set has a different initial condition. Moreover, because the laws of nature of this world are deterministic and time-reversal invariant, it must be the case that no pair of mosaic in the set are the same on any time-slice after the beginning of time, since otherwise reversing the direction of time would take a single state into two different states, in violation of the assumption that the laws of this world satisfy Laplacean determinism. Thus when the actual mosaic is drawn from this set of mosaics, this has the effect of fixing the state of the world on the initial state, but since the mosaics differ at all subsequent times as well one could equally well regard this process as fixing the state of the world on any other time slice. So there is no fact of the matter about which *particular* time-slice is responsible for the rest of history: when we select one of the allowed mosaics we determine all the time-slices at once. Therefore the arbitrariness involved in a universe satisfying Laplacean determinism can't be located in any specific spacetime region, so the Laplacean universe does indeed satisfy weak hole-free global determinism. On the other hand, Einstein hole worlds are ruled out, since in Einstein hole worlds the arbitrariness is localised to the areas inside the holes. Thus this definition captures what is distinctive about worlds satisfying Laplacean determinism without needing to give special significance to the initial condition - the important thing about such worlds is not that the arbitrariness occurs only at the start of time, but that the arbitrariness is not localised anywhere, and thus in general such worlds have a high degree of predictability and consistency even though the laws of nature do not fix everything in the world uniquely. Note that according to this definition strong global determinism is a form of hole-free global determinism, since in the case of strong global determinism there is only one mosaic in the intersection of the set of constraints associated with the laws of nature and therefore there can't be any pair of mosaics in that intersection.

3.1 Solutions

We now demonstrate that these new definitions of determinism resolve the problems that we set out in section 1. First, none of these definitions of determinism has any intrinsic temporal direction: rather than treating the initial

conditions as an input and asking if they are sufficient to determine the rest of the course of history, we have the laws induce constraints which pick out entire mosaic, and then we ask if these laws involve any objective chances and if they suffice to pick out the course of history uniquely. Indeed, the definition of hole-free global determinism prohibits the localisation of 'arbitrariness' in specific spacetime regions, so it explicitly rules out approaches which treat some particular region of spacetime as an input or initial condition. Thus these definitions for determinism work much better than the Laplacean approach when applied to the problem cases we considered in section 1.1: the non-Markovian world, the flash world, the retrocausal world and the all-at-once world all satisfy weak global determinism, and they may also satisfy hole-free or even strong global determinism, depending on the specific details of the models in question. Moreover, as shown in ref [4], Lagrangian laws, retrocausal laws, constraint-based laws and many other types of laws outside the time-evolution paradigm can be expressed in the constraint form, and therefore this constraint-based approach to determinism can be meaningfully applied to all of them.

It is also straightforward to see that the constraint-based approach resolves the underdetermination problem set out in section 1.2. For in order to apply the constraint approach to a universe governed by time-evolution laws satisfying Laplacean determinism and time-reversal invariance, we need only select a mosaic from the set of all mosaics in which those laws are satisfied: there is no evolution and thus no need to pick a privileged point in history from which the evolution is understood to begin. Of course there will be some arbitrariness associated with this choice since time-evolution laws do not fix the course of history uniquely, but as noted in section 3 this arbitrariness is not localised anywhere so we have no underdetermination.

Indeed, this approach has the interesting consequence of dissolving the distinction between initial conditions and parameter values. Wallace has observed that the physical content of a theory has three aspects: the qualitative form of its dynamical equations (in which the coefficients are unspecified parameters); the actual, numerical values of the parameters (expressed as dimensionless ratios); and the initial conditions [39]. As Wallace notes, it is common to consider that the parameter values are 'lawlike' while the initial conditions are merely 'contingent' - that is, parameters and initial conditions are usually thought to have importantly different modal status. But in the constraint framework the case for this different status is significantly weaker. If the universe satisfies strong global determnism then both the initial conditions and the numerical values of the parameters must be determined by the laws of nature - that is, we must have state rigidity and parameter rigidity, in Wallace's terms. But if the universe is only weakly deterministic, it could well be the case that across the set of mosaics in the intersection of the constraints there is some variation in the initial conditions of the universe and also in the numerical value of the parameters, so drawing a mosaic from the set entails arbitrarily selecting both initial conditions and parameter values. In this context both initial conditions and parameters are simply different facets of the degrees of freedom left by the laws of nature, and thus they have precisely the same modal status. Of course, it *could* also be the case that the parameters are in fact fixed by the laws of nature while the initial conditions are not, but likewise it *could* be the case that the initial conditions are in fact fixed by the laws of nature while the parameters are not: until we have some specific evidence for either parameter rigidity or state rigidity, parameters and initial conditions should be afforded the same modal status.

Finally, the constraint-based approach realises our desideratum of making a clean distinction between determinism and predictability, as the definition we have provided here characterises determinism entirely in terms of objective modal structure, with no appeal to any facts about the practical interests or epistemic limitations of human scientists. A world which is deterministic in the global sense may indeed exhibit a high degree of predictability, but it also may not - the way in which the laws determine the course of events may be highly non-local in space and time, meaning that it may be impossible for limited local agents to gather enough data to see the whole picture. Thus this framework provides a precise way of talking about determinism as a property of a *world* rather than merely a theory, opening the door for that notion to be applied to a variety of ongoing metaphysical discussions.

3.2 Comments

1. The distinction we have made here is similar to one employed by Penrose, who distinguished between determinism, i.e. what we have referred to as Laplacean determinism (*'if the state of the system is known at any one time, then it is completely fixed at all later (or indeed earlier) times by the equations of the theory'*) and strong determinism, i.e. what we have referred to as strong global determinism (*' it is not just a matter of the future being determined by the past; the entire history of the universe is fixed, according to some precise mathematical*

scheme, for all time.') [40]. However, Penrose's scheme does not seem to accommodate the intermediate possibility of worlds which satisfy weak global determinism but not Laplacean determinism, such as worlds which have some degree of arbitrariness at some point other than the initial state of the universe but nonetheless have no objectively chancy events. Moreover, Penrose is rather vague about what it would take for the entire history of the universe to be fixed, but the constraint framework allows us to be precise: the history of the universe is fixed if and only the laws of nature induce trivial probability distributions over constraints, and the intersection of the constraints associated with these laws contains exactly one mosaic.

2. In one sense, weak global determinism, strong global determinism and hole-free global determinism are all weaker than Laplacean determinism, since worlds like the retrocausal world and the flash world could satisfy either weak, strong or hole-free global determinism whilst failing to satisfy Laplacean determinism. However, in another sense strong global determinism is much stronger than Laplacean determinism, because Laplacean determinism allows for the existence of arbitrariness (in the selection of initial conditions and the selection of parameter values) while strong global determinism insists that everything is fixed once and for all by the laws of nature.

3. We reinforce that all of these definitions for determinism depends crucially on the claim that reality has a unique, observer-independent modal structure. Given any Humean mosaic we could come up with an infinite number of sets of constraints whose intersection contains only this mosaic - trivially, for a mosaic A we can pick two random mosaics B and C and choose the constraints $\{A, B\}$ and $\{A, C\}$. So simply finding such a set which is consistent with our observations doesn't tell us that the world must be deterministic: we must make the further claim that these constraints are *in fact* the constraints which feature in the objective modal structure of reality, and therefore we must believe that there is some fact of the matter about what this objective modal structure really is. So for example, assessing a claim that the world satisfies strong global determinism has several steps: first we must verify that the proposed constraints do indeed single out a unique Humean mosaic, second, we must verify that the proposed constraints are consistent with our observations, and third, we must decide whether the proposed constraints seem like plausible candidates for laws. This last part of the process will naturally be shaped by whatever expectations we have about laws - for example, if we think the laws of nature are simple then we will tend to expect that the constraints induced by the laws can be expressed in a closed form in simple terms.

4. Our definitions also depend crucially on the stipulation that the laws of nature assign no probability distribution over the mosaics in the constraints that they induce, nor over the mosaics in the intersection of all of those constraints. However, this certainly does not entail that we ourselves may not assign any distribution over mosaics in the intersection. For we may have Bayesian priors which lead us to assign some distribution over mosaics before taking any of the laws of nature into account - for example we might simply assign the uniform distribution, or we might choose a distribution which favours simpler mosaics. Thus when we learn about a constraint induced by a law, we will update our beliefs by eliminating all mosaics not consistent with the corresponding constraint and renormalizing our credences, so after conditioning on all of the laws we will be left with a normalized probability distribution which is non-zero only on mosaics in the intersection of the constraints. In the case of strong global determinism, no matter what priors we start out with, after conditioning on the laws we will be left with a distribution that assigns probability 1 to a single mosaic; but in the case of weak global determinism we will in general end up with a non-trivial distribution, which will be the uniform distribution if the original prior distribution was the uniform distribution, but which may be non-uniform if our original priors were non-uniform. But this distribution is a *subjective* probability distribution derived from our original subjective priors, not an objective chance distribution arising from the laws themselves: so in this case we still have determinism in the sense of 'the absence of objective chance,' even though observers may still assign non-trivial probabilities over mosaics compatible with the laws. This is precisely analogous to the way in which we might assign subjective probability distributions over initial conditions in the context of Laplacean determinism, for example in analyses of statistical mechanical systems based on the principle of indifference [41].

5. Our discussion so far has largely left unanalysed the term 'Humean mosaic.' However, it's important to re-

inforce that given our definitions, questions about whether specific theories are compatible with strong, weak and hole-free global determinism will be sensitive to our judgements about what sorts of categorical properties feature in the Humean mosaic. For example, consider the case of electromagnetism [42]: if the electromagnetic potentials are considered to be mere mathematical devices which are not part of the Humean mosaic, then classical electromagnetism satisfies both Laplacean determinism and weak global determinism, whereas if the electromagnetic potentials *are* a part of the Humean mosaic, then it's not possible for a world governed by laws including the laws of classical electromagnetism to satisfy Laplacean determinism or hole-free weak global determinism - though it could still satisfy weak global determinism, since electromagnetism prescribes no probability distribution over gauge-equivalent potential configurations and therefore according to the definitions adopted here they are arbitrary rather than chancy. Quantum mechanics is another example: if we suppose that quantum mechanics is a complete theory of reality and we take it that wavefunction collapse is a part of the Humean mosaic, then the theory does not satisfy weak or strong global determinism or Laplacean determinism, whereas if we say that only the unitarily evolving quantum state is part of the Humean mosaic, then we get the Everett interpretation [43], which satisfies Laplacean determinism and weak global determinism.[5] One might worry that this is a weakness in the definition. However, it is in fact a deliberate choice, because as noted earlier we are focusing in this paper on defining determinism for *worlds*, not theories, so there is no need to specify what is in the Humean mosaic: all local matters of particular fact in a given possible world are in the Humean mosaic for that world, whatever those matters of fact might turn out to be. This means that we can't expect to straightforwardly read off any given theory whether that theory satisfies weak or strong global determinism: we must first specify which elements of the theoretical structure are to be considered part of the Humean mosaic, which is to say, we must specify an ontology for the theory. But this is not unique to global determinism - as noted above, Laplacean determinism can likewise give different conclusions for electromagnetism and quantum mechanics depending on what we take to feature in the ontology.

6. Since we have defined probabilistic laws of nature in terms of probability distributions over deterministic constraints, it will always be possible to make a probabilistic law into a deterministic one by simply shifting the probabilistic part of the process into the definition of the law. For example, if we have a probabilistic law which stipulates that with probability $p(O)$ the actual mosaic will belong to the constraint O, we can turn this into a deterministic law by drawing a constraint O from the probability distribution $p(O)$ and then making a deterministic constraint which requires that the actual mosaic will belong to the selected constraint O. And similarly, we can turn a deterministic law of nature into a probabilistic law of nature by formulating some probability distribution over constraints which assigns nonzero probability to the constraint associated with the deterministic law. So there are always options for moving back and forward between the probabilistic and deterministic pictures, and therefore ultimately the choice between these pictures is an interpretational stance rather than something that can be proven via empirical observations. That said, for certain sorts of constraints there seems a strong presumption in favour of one picture rather than the other. For example, if we observe that all As are Bs, we could in principle regard this as the consequence of a probabilistic law of nature which assigns high probability to the constraint consisting of the set of Humean mosaics in which all As are Bs, but this seems unnecessarily complex, since we could do just as well with a deterministic law of nature which assigns probability 1 to the constraint consisting of all Humean mosaics in which all As are Bs. Conversely, constraints for which we must specify a large number of parameters which approximately match some probability distribution seem most naturally understood within the probabilistic paradigm, otherwise there will be a sense that we are just shifting 'objective chance' into the choice of the laws of nature rather than genuinely eliminating it (we will see an example of this in section A).

4 Applications

We have criticized Laplacean determinism for being inappropriately intertwined with our practical interests; but one might argue that being aligned with our practical interests is precisely what made Laplacian determinism a useful concept, and thus one might worry that generalizing determinism as we have done here serves only to make the concept less useful. Thus in this section we will explain how this generalization of determinism and objective chance

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[5]The Everett interpretation in its usual form does not satisfy strong global determinism, since the initial state of the universe is arbitrary.

may have important consequences for scientific and philosophical thinking. The concept of Laplacean determinism has played a range of different intellectual roles in science and philosophy, so we will proceed by examining how global determinism can contribute to some of these domains of application.

4.1 Scientific Explanation

Determinism has played an important role in shaping our expectations for scientific explanation - for a long time it was standard in science to explain events by showing how they could be deterministically produced by past conditions. This is formalised in the 'deductive-nomological' model for scientific explanation [48, 51], which posits that a valid explanation should be composed of a set of sentences from which the explanandum logically follows, where at least one of these sentences must express a law of nature and that law must be an essential element in the deduction of the explanandum. Most common examples of DN-explanations involve time-evolution laws or at least temporally directed laws, and thus DN explanations usually involve postulating some past conditions and showing that the deterministic forwards time-evolution of these conditions gives rise to the explanandum. This model is sometimes relaxed to allow for explanations involving statistical inference rather than exact logical deduction [52], but deductive-statistical and inductive-statistical explanations are simply a generalisation of forwards-evolving deterministic explanations and thus they typically preserve the expectation that the future will be explained by lawlike evolution from the past.

Global determinism opens up new possibilities for scientific explanation - for if the definition of Laplacean determinism is largely a function of our practical interests then there is no reason to expect that all valid scientific explanations will be predicated on forwards time evolution, since for scientific realists one of the main purpose of scientific explanation is to achieve *understanding*, to which practical concerns are only tangentially relevant. So when we come upon phenomena for which there seems to be no satisfactory explanation within a forward time-evolution picture, we should be open to the possibility of explanations based on global laws using the constraint framework. A good example of this is the past hypothesis: by definition nothing evolves into the initial state of the universe, so the initial state can't be given a lawlike explanation if we assume that laws always take the forward time-evolution form. But the past hypothesis can straightforwardly be written as a constraint - it is simply the set of all Humean mosaics in which the arrangement of local matters of particular fact at one end of the mosaic has low entropy (or some other more sophisticated characterisation of the desired initial state - see ref [53] for discussion). The past hypothesis can therefore be explained in this framework by hypothesizing that there is some law of nature which induces this particular constraint, thus ensuring that the initial state will necessarily be a low entropy state whilst the selection of one low entropy initial state in particular remains arbitrary.

Similar arguments can be made about parameter values: for example, cosmologists are currently much exercised over the problem of why the cosmological constant is so much smaller than their models suggest it ought to be [54]. It's hard to see how this feature could be explained within the kinematical/dynamical picture, because the cosmological constant is usually considered to be a fundamental constant of nature which has always had the value that it has, and so there is no option to say that it has been produced by evolution from some earlier conditions. Thus most current approaches to explaining the value involve fairly exotic explanatory strategies, such as anthropic arguments in the context of a putative multiverse [55]. However, within the constraint framework there is no reason why we can't simply explain the cosmological constant in a lawlike manner: all we need to do is suggest that there is some law of nature which induces the constraint consisting of all the Humean mosaics in which the cosmological constant is very small. In the constraint framework, this nomological explanation has exactly the same status as more familiar nomological explanations, such as explaining that apples fall due to the law of gravity: the key point is that nomological explanations do not necessarily have to have a temporal character.

Of course, some caution is required with this strategy. Seeking explanations for phenomena that we find surprising or conspiratorial is frequently a good way of making scientific progress, so we don't want to make explanation too cheap: answering every scientific question with 'Because there's a law which makes it so,' isn't likely to lead to any new insights. But of course, the same is true with standard DN and inductive-statistical explanation - not every proposed DN or inductive-statistical explanation is interesting and informative, so we have a variety of criteria which can be applied to judge which explanations have merit. These criteria, including simplicity, unification, the absence of fine-tuning and so on, can equally well be applied to constraint-based explanations. Thus rather than simply imposing a constraint consisting of Humean mosaics in which the initial state is simple or a constraint consisting of Humean

mosaics in which the cosmological constant is small, we might want to come up with some more general feature from which these constraints could be derived - for example, one could imagine deriving the past hypothesis from a more general constraint that singles out Humean mosaics which are sufficiently interesting or varied. This constraint rules out mosaics where everything is in thermal equilibrium throughout the whole of history, which does indeed entail that the initial state must have low entropy, but it may also have *other* interesting consequences, and if the consequences turn out to be varied and powerful enough we would have good reason to accept the proffered explanation. The overarching point is that global determinism offers us a new sort of explanatory framework: there can be good and bad explanations within that framework, and there's work to be done to establish the appropriate criteria for judging these sorts of explanations, but nonetheless this new approach is a promising route to answering questions which seem intractable within standard explanatory paradigms.

4.2 Assessment of Theories

Determinism has long functioned as a gold standard for an ideal scientific theory - for example, the idea that quantum mechanics might be indeterministic was accepted only begrudgingly by many physicists at the time of the theory's formulation, as witnessed by Einstein's complaint that 'He (God) does not play dice.' [56, 57] Over the last century many attempts have been made to 'complete' quantum mechanics so that it obeys Laplacean determinism after all [58], and indeed two of the most popular interpretations of quantum mechanics (the de Broglie-Bohm interpretation [59] and the Everett interpretation [43]) do satisfy Laplacean determinism.

However, if we accept that the standard definition of Laplacean determinism is a function of practical interests rather than a metaphysically meaningful category, there is less justification for regarding Laplacean determinism as the ultimate goal of a scientific theory. Obviously, theories which satisfy Laplacean determinism are still desirable due to their practical utility for predicting the future, but from the point of view of the scientific realist with an interest in understanding how things *really* are, theories which satisfy weak, strong or hole-free determinism may be just as well motivated. After all, all three of these notions of determinism do justice to Einstein's intuition that 'He does not play dice,' in the sense that none of them allows the existence of genuine objective chances from the external, 'god's-eye' point of view.

Thus this approach to determinism suggests new ways of thinking about what a good scientific theory should look like. For example, instead of coming up with theories which postulate a set of states and a set of differential equations, perhaps we should be looking more seriously at theories postulating laws which govern 'all-at-once' from an external point of view. As noted in section 1.1, modern physics does already contain examples of such laws, but in general these examples have been arrived at by starting from a temporal evolution picture and subsequently generalizing it or reformulating it; things might look quite different if we started from the assumption that we are looking for a global, 'all-at-once' theory and then proceeded without insisting on the existence of a time-evolution formulation of the theory. Adjusting our gold standard to more closely reflect the form of the true laws of nature is likely to be a good way to stimulate progress toward understanding those laws, so we have strong practical motivations for rethinking the gold standard.

4.3 Free Will

Laplace's ideas about determinism gave rise to a spirited debate over the possibility of free will in a deterministic universe which has continued to this day [60–62]. The possibility of global determinism certainly raises new questions in this debate. For example, even if you believe that we don't have free will in the context of Laplacean determinism, you might still be willing to say that we could have free will in the context of some sorts of *global* determinism. After all, if the course of history is determined by laws which apply to the whole of history all at once, our actions are determined by other events, but also those events are partly determined by our actions, since each part of the history is optimized relative to all the other parts. So there is a degree of reciprocity which is lacking in the Laplacean context, where the present state determines our actions and our actions do not act back on the present state. This is an interesting direction of enquiry, but somewhat outside the scope of the present paper, so we will leave it as a topic for future research.

5   Objective Chance

Objective chance is famously a murky notion - though a number of analyses of the topic exist, including frequentist and propensity approaches, none has so far garnered universal approval and all seem to have serious problems to overcome [44]. Moreover this lack of clarity is a significant weakness for variety of standard philosophical positions which require some notion of probability or chance - for example, proponents of the Everett interpretation often employ 'no worse off' arguments where they claim that the difficulty of accounting for probabilities in the Everett picture is no reason to reject their approach because accounting for ordinary objective chance is just as difficult [43]. It is tempting to respond to all this by simply insisting that there aren't any objective chances at the level of fundamental laws, but that strategy is hampered the fact that the current scientific evidence seems to be pointing away from Laplacean determinism. As noted earlier, it's common to suppose that we are faced with a dichotomic dilemma - either the world satisfies Laplacean determinism or there exist objective chances - and thus, since it doesn't appear to be the case that quantum mechanics satisfies Laplacean determinism, it may seem inevitable that we accept the existence of objective chances.

But the possibility of global determinism demonstrates that this division of the possibilities is too rigid, for it turns out that we can postulate worlds which do not satisfy Laplacean determinism but which nonetheless are globally deterministic, and therefore the probabilistic features of quantum mechanics do not force us to accept the existence of objective chance from the global, external point of view. There are a variety of ways in which such apparently probabilistic events might emerge within a globally deterministic world - they will appear wherever the world contains some events which depend in part on facts about reality that are not accessible to observers in the local region of those events - but for concreteness, in order to demonstrate how chances can emerge from a globally deterministic world we will henceforth focus on one particularly simple possibility, where probabilistic events arise from frequency constraints which look something like 'the set of mosaics in which eighty percent of events of type E have the outcome A.' The existence of these sorts of global constraints is entirely compatible with strong, weak or hole-free global determinism, so one could equally well say either that this approach reduces objective chance to frequency constraints, or that it simply eliminates objective chance altogether from the global point of view.

Indeed, obtaining objective chances from frequency constraints can also be regarded as a way of reducing them to subjective probabilities: the relevant events are sampled from a set of events with prespecified relative frequencies, so the objective chances attached to these events simply describe a process of sampling without replacement akin to the common textbook example of selecting a ball from a jar containing a specified mixture of black and white balls. And of course the probabilities involved in sampling without replacement can be described in terms of Bayesian credence functions with partially unknown initial conditions, since we know the initial proportions of black and white balls but not the order in which they will be drawn. Thus deriving chances from frequency constraints provides a satisfying explanation of the close conceptual and mathematical relationship between objective chance and subjective probability: objective chances behave like subjective probabilities because they really are just subjective probabilities over a large and temporally extended domain.

In appendix A we discuss in greater detail the form that these frequency constraints might take; but for now, in order to see if frequency constraints can give rise to chances which behave in the way we would expect, we will consider the following desiderata for objective chance:

1. *The Principal Principle:* Objective chances should satisfy the Principal Principle.

2. *Probabilistic confirmation:* We should be able to confirm facts about objective chances by means of observing relative frequencies.

3. *Exchangeability:* Given a sequence of events which are not causally related to one another, the chance for a given sequence should depend only on the relative frequencies in that sequence and not on the order in which the events occur.

4. *Counterfactual independence:* Given a sequence of events which are not causally related to one another, the chance for the outcome of an individual event should not depend on the frequency of occurrence of other outcomes in the sequence.

First, do chances derived from frequency constraints satisfy the Principal Principle? Well, in accordance with the method of direct inference [45], if you know that you are in a Humean mosaic which belongs to the set of mosaics in which exactly eighty percent of As are Bs, and you have no other information, you should indeed set your credences for the next observed A to be B to eighty percent. Now in most real cases we will already have observed some As and therefore it is not quite the case that we are sampling at random from the entire class, so the correct probability to assign might actually be infinitesimally different from eighty percent. But provided that the class is large enough, this difference will be so small that it will make no practical difference to the way in which we use this probability in our reasoning processes and thus to all effects and purposes the Principal Principle does indeed pick out the frequencies that appear in the constraints.

Next let us check that it is possible to confirm facts about chances derived from frequency constraints by means of observing relative frequencies: if we observe a set of events of a certain type and find that eighty percent of them have the outcome A, do we have grounds to confirm the hypothesis that we are in a Humean mosaic in which eighty percent of instances of this event type have outcome A? Of course, in general we will be able to observe only a very tiny proportion of the total number of such events occurring across the whole Humean mosaic, so we can't confirm this hypothesis by direct observation; rather we must assume that the As that we have observed are a *representative sample* of the full set of As. Thus the legitimacy of this inference depends crucially on the assumption that the As form a homogenous class which can be expected to exhibit stable relative frequencies across time - it is not the case that just any relative frequencies can be extrapolated in this way, as that would allow us to confirm virtually any hypothesis we like about future relative frequencies by simply labelling events in the right sort of way. Thus from observations of a relatively small subclass of instances of a given event type, we do not have grounds to confirm the hypothesis that eighty percent of instances of this event type have outcome A, but we *do* have grounds to confirm the hypothesis that the laws of nature induce a constraint to the effect that eighty percent of As must be Bs. That is, the inference must include the hypothesis that we have identified a special class of events which appears directly in the laws of nature and that the relative frequencies we have seen are the consequence of a constraint which is induced by the laws of nature. Of course we could always be wrong about this hypothesis, but nonetheless, observing behaviour which is consistent with the hypothesis provides some degree of confirmation for it.

One way of putting this would be to say that we can confirm objective chances derived from frequency constraints only when they pertain to 'natural kinds,' where natural kinds are to be defined as exactly those categories which feature in the constraints induced by the laws of nature. This would be in accordance with the common view that it is (mostly) natural kinds which permit inductive inference [46]. However, across philosophy 'natural kinds' play a variety of roles, and therefore this terminology has a certain amount of baggage - although in principle what counts as a natural kind is subject to the dictates of science, some philosophers have claimed to be able to identify natural kinds purely by perception and/or reflection, and people often have quite specific intuitions about what is and is not a natural kind, so for example it is common to suppose that members of a natural kind should have properties in common and/or be qualitatively similar through the lens of our perceptual equipment [**?**]. But one of the motivations for adopting the constraint framework is to allow us to state more general possibilities for laws which do not necessarily correspond to our existing intuitions, and therefore we reinforce that probabilistic confirmation of this sort is not valid only for categories of the sort which are traditionally regarded as natural kinds - in principle it is legitimate for any category which appears in the laws of nature, regardless of whether that category looks like a natural kind to us. Of course, in practice we have to make decisions about what sort of event classes can be expected to exhibit stable relative frequencies across time, and in the absence of other evidence these decisions are often based on our intuitions about natural kinds. But one can imagine cases where we might make these decisions for other reasons - for example, if a theory which has been successful in one domain were to be unified with another domain in a way that leads naturally to laws governing probabilities over a class of events that we would not otherwise have considered a candidate for a 'natural kind.'

Let us move to the final two desiderata, which are somewhat more mathematical in character. It is clear that chances obtained from frequency constraints will satisfy exchangeability, since these constraints govern only relative frequencies and not order of appearance. But what about counterfactual independence? Well, we have seen that in the frequency constraint picture, observing random events must be understood as a form of sampling without replacement, and so just as we would update our probabilities for the next event after each random draw when performing sampling without replacement, it seems we ought to do the same in the frequency constraint case: if I know that exactly 50

percent of events of a given type must have the outcome A, then if my first one hundred observations all have the outcome B I will conclude that the remaining set of events must contain one hundred more As than Bs, so I will perform the standard sort of inference we see in sampling without replacement, which involves updating my credences to reflect the fact that the next outcome is more likely to be A than B. Note that this remains true for any finite total number of events, so the conclusion does not change if I don't know the total number of events. It also doesn't matter if we have a constraint which prescribes the probability only approximately; there will always be some threshold number n of Bs such that if I have observed n more Bs than As, then the set of unobserved events must contain more As than Bs in order to get final frequencies which are correct to within the limits allowed by the relevant constraint.

This seems problematic, as the absence of counterfactual independence jars with many of our intuitions about probability - for example, it has the consequence that under certain circumstances the gambler's fallacy is not really a fallacy at all. Indeed Hajek argues against standard frequentism on similar grounds, pointing out that defining probabilities as relative frequencies prevents us from saying that the probability for an individual event is counterfactually independent of what happens in other similar events distant in space and time [47]. So we must now ask ourselves whether we can accept an approach which fails to satisfy counterfactual independence as an acceptable analysis of objective chance. One possible response for the proponent of frequency constraints is to point out we will not get failures of counterfactual independence if the total number of instances is infinite, since sampling without replacement from an infinite set is mathematically equivalent to sampling *with* replacement. And after all, we can never know for sure that the number of instances of an event type is finite, since that would require us to have illegitimate knowledge of the future, so it might be argued that we should never make updates to our probabilities which are premised on the number of instances of an event type being finite, meaning that the usual sort of inferences we see in sampling-without-replacement should not be allowed with regard to sequences of chancy events. However, although it is true that we can never know for sure that the number of instances is finite, nonetheless one can imagine cases where our best theories give us good reason to believe that this is so (for example, if we had a theory which implied that spacetime is discrete and the universe has a finite extent and time has both a start and an end), so at least in principle there are circumstances where counterfactual independence really would be violated in this picture.

Another possible response is to point out that we would only ever get violations of counterfactual independence with regard to sequences of chancy events if we had knowledge of frequency constraints that would trump the observed relative frequencies, and one might think that this could never come about. For example, in real life if we performed one hundred measurements and got the result B every time, we would probably form the working hypothesis that there is a constraint requiring that all events of this type must have outcome B, rather than supposing that exactly fifty percent of all events of this type must have outcome B and that therefore subsequent events must be more likely to have outcome A than outcome B in order to make up the total. However, probabilistic laws do not usually exist in isolation, and therefore one can imagine a case where we decide based on considerations of symmetry or coherence with the rest of our theory that the objective chance of B is 0.5 even though so far we have seen significantly more Bs than As. It would seem that under those circumstances, if we accepted the frequency constraint analysis and we had reason to believe that the total number of instances of the relevant event type across all of spacetime would be finite, we would have reason to assign a credence greater than 0.5 to the proposition that the next such event will have outcome A.

So in fact, probably the appropriate response here is to simply say that violations of counterfactual independence *should* sometimes be allowed with respect to sequences of chancy events, because there is little reason to expect our intuitions to be a good guide on this point. For a start, most of our intuitions are based on probabilistic events that involve *subjective* probabilities rather than objective chances of the kind that appear in the fundamental laws of nature - due to decoherence, macroscopic events do not typically depend sensitively on quantum mechanical measurement outcomes, and therefore paradigmatic probabilistic events like 'rolling a die' or 'flipping a coin' can be understood entirely in terms of our ignorance of the specific initial conditions. Provided that the relevant initial conditions are independent (or at least independent enough for practical purposes) these sorts of events do indeed obey counterfactual independence, which is likely the source of our intuitions on this point. Furthermore, even when we do observe events that may involve objective chances, such as the results of quantum measurements, we are presumably observing only a very small subset of the total number of events of this type, so we should not expect these observations to tell us very much about the class as a whole. If we know that the exactly fifty percent of instances of a certain event type have the outcome A, and also that 10^{11} of these events occur across all of spacetime, then in principle the fact that we have

observed one hundred more B outcomes than A outcomes does make it more likely that the next outcome will be an A, but in practice the change in probability is so small that it will not lead to any observable effects.

Therefore, the fact that counterfactual independence conflicts with our intuitions is no reason to reject constraint frequencies as an approach to chance, since our intuitions have been developed in entirely the wrong context of application. Indeed, counterfactual independence can be regarded as a way of expressing the expectation that objective chances should be intrinsic properties of individual entities which are spatially and temporally local; but in many ways chances actually look like properties not of individual entities but rather of large collections of entities. For example, we can't ascertain chances during a single observation, but rather we must observe a large number of events of the same type and then make an inference about what the chances are. A number of popular approaches to the analysis of objective chance - particularly the propensity interpretation, which holds that chances are intrinsic properties of individual entities akin to colours and masses [48] - struggle to make sense of this feature [49]. But frequency constraints do justice to the apparently collective nature of objective chance by turning chances into a form of global coordination across space and time, where events are constrained to coordinate non-locally amongst themselves so as to ensure that outcomes occur with (roughly) the right frequency. Hajek worries that 'it's almost as if the frequentist believes in something like backward causation from future results to current chances,' - and indeed, that is exactly what the proponent of frequency constraints does believe!

Frequency constraints are similar to a view that has been advanced by Roberts under the name of *nomic frequentism* [50]; much of Roberts' discussion is also relevant here, so we won't dwell on the points of agreement, but in appendix B we discuss a few points of difference. Frequency constraints are also clearly related to finite frequentism, which is which is the view that objective probabilities are by definition equal to the actual relative frequencies of the relevant sort of event across all of spacetime. But the frequency constraint approach avoids several of the main problems encountered by finite frequentism. For example, it has been objected that finite frequentism entails that probabilities can't be defined for processes which as a matter of fact occur only once or not at all [47]. But this is not a problem for the frequency constraint view provided that we take a robust attitude to modality which maintains that constraints are ontologically prior to the Humean mosaic - in that case a constraint exists and is well-defined regardless of how many occurrences of the event type in question actually occur. Another common objection to finite frequentism is that it has the result that probability depends on a choice of reference class, and in the case of macroscopic everyday events this can be very nontrivial, because different descriptions of the event may suggest different natural reference classes which can lead to very different probabilities [47]. However, this is not a problem for the frequency constraint view provided that we take constraints to be objective and mind-independent, because the reference class is defined within the constraint and therefore there is always a definite fact about which reference class is relevant. Of course, there is still an epistemological question about how we as observers can decide which reference class actually appears in the real underlying constraint, and it's always possible that we will get it wrong and will thus make the wrong predictions, but there is no ambiguity in the definition of the objective chance itself. Roberts gives more examples of the advantages of frequency constraints over finite frequentism in ref [50].

6   Related Topics

In this section, we discuss the relationship between global determinism and some other relevant research programmes.

6.1   Superdeterminism

Since the terms 'global determinism' and 'superdeterminism' are superficially similar, it's important to reinforce that these concepts are not identical. Broadly speaking, 'superdeterminism' describes approaches to quantum mechanics which deny the existence of non-locality in quantum mechanics by rejecting the assumption of statistical independence which goes into Bell's theorem, i.e. the idea that our choice of measurement is independent of the state of the system which we are measuring [63–65]. There are two typical ways to achieve this. First, given that our choice of measurement is always determined or at least influenced by facts about the past (e.g. the physical state of our brain in the time leading up to our choice) one could imagine the initial state of the universe being arranged such that our measurement choices are always correlated with the states of the systems we are measuring. Second, one could imagine

there is some sort of retrocausality or 'future outcome dependence' where the state of the system we are measuring is influenced by our future decisions about what measurement to perform.

It is immediately clear that neither of these approaches actually requires determinism: a *probabilistic* dependency between choice of measurement and state is already enough to vitiate statistical independence and thus restore locality. Thus it is actually somewhat misleading to use the term 'superdeterminism' to refer to the violation of statistical independence. The term 'superdeterminism' contains the word 'determinism' because Bell originally proved a theorem ruling out deterministic local hidden variable theories [66], and so at that time superdeterminism was suggested as an approach to restore both locality and determinism; but a later version of Bell's theorem ruled out *all* local hidden variable theories, either probabilistic or deterministic [67], so in fact determinism is not really the key issue here, and modern proponents of superdeterminism are usually motivated more by the desire to preserve locality than any particular interest in preserving determinism.

That said, for the moment let us focus on superdeterministic approaches which *are* also deterministic in some sense. What sort of determinism would that be? Well, if we take the first route where correlations between measurement choices and states are written into the initial state of the universe, then we have just standard Laplacean determinism (and thus also weak global determinism). If we take the second route involving retrocausality, the resulting theory would probably not be compatible with Laplacean determinism, because events which are determined by something in the future usually can't also be fully determined by the past; but such a theory certainly could exhibit either weak or strong global determinism. However, although superdeterminism can be a form of global determinism, the concept we have defined here is significantly more general than superdeterminism. In particular, a major motivation for our approach to was to understand what determinism could look like in a world which includes spatial and/or temporal non-locality, whereas superdeterminism is typically employed as a way of *denying* the existence of either spatial or temporal non-locality, and therefore global determinism also accommodates realist approaches which take a very different approach from superdeterminism.

6.2   Deterministic Probabilities

There is a long-standing philosophical debate around the question of whether it is possible to have chances in a deterministic world [68–70], but this debate usually takes place against the backdrop of Laplacean determinism. Moving to a picture based on global determinism therefore opens up new ways in which we can have 'chancy' events in a deterministic world, since we saw in section 5 that an event can be described by a non-trivial probability distribution when we condition on all of the information available to local observers, even if the event is not objectively chancy when we consider the way in which it is embedded in the complete objective modal structure of reality. This reinforces the point that 'chanciness' is in many cases relative to a perspective: obviously if the outcome of a process is determined by something in the future it is for all practical purposes objectively chancy from the point of view of observers like ourselves who have directly epistemic access only to the past and present, but this does not necessarily mean that the process must be 'chancy' from the point of view of the underlying modal structure. This particular approach does depend on the assumption that there exists some unique observer-independent modal structure, but it's clear that even without that assumption we will get different conclusions about whether a process is 'chancy' depending on which information about reality we are willing to condition on, and therefore questions about the existence of chances in a deterministic universe should be phrased carefully to account for the way in which different perspectives will lead to different conclusions even in very simple cases.

6.3   Closed Causal Loops

Refs [71, 72] employed a notion of 'global determinism' which was similarly intended as a generalization of determinism for non-Newtonian contexts. In this section we consider how that approach to determinism relates to the framework we have set out in this article.

In refs [71, 73], global determinism was used in the form of a prohibition on indeterministic closed loops: the idea is that the values of variables in these closed loops 'come out of nowhere,' which is to say, they are not determined by anything, so closed causal loops are not consistent with a globally deterministic universe. Now, clearly variables which are not determined by anything are incompatible with a universe that obeys strong global determinism, so the argument of refs [71, 73] work perfectly well if 'global determinism' is intended to mean 'strong global determinism.'

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Moreover, indeterministic closed loops are also ruled out by hole-free weak global determinism, because the arbitrariness associated with the variables in a closed causal loop is localised to the events inside the loop. On the other hand indeterministic closed loops are not ruled out by weak global determinism simpliciter, since the values of variables inside the loop could be arbitrary rather than chancy. Thus insofar as the results of refs [71, 73] can be regarded as evidence for the hypothesis of global determinism, they should be regarded as evidence for strong global determinism or hole-free weak global determinism.

7   Conclusion

We have argued that definitions of determinism drawn directly from Laplace's original vision are no longer fit for purpose, since they can't be usefully applied to the diverse range of laws outside the time evolution paradigm that appear in modern physics. However, we contend that once determinism is disentangled from predictability, the intuition behind determinism captures a real and important classification of ways the world could be. Thus in this article we have provided several generalized definitions of determinism which can be applied to a wide variety of nonstandard laws, thus rehabilitating determinism for the post-time-evolution era.

Using these definitions, we have shown that a world governed by global laws may fail to satisfy the standard conception of Laplacean determinism but may nonetheless exhibit a form of determinism on a global scale, where we say we have some form of 'determinism' provided that we are never required to assign probability distributions over entire Humean mosaics or worlds. This has the consequence of dissolving the traditional dichotomic dilemma where we supposedly have to choose between Laplacean determinism and intrinsic randomness - a world governed by global laws may exhibit what appear to be objective chances from the point of view of local observers whilst requiring no irreducible objective chances from the external, global perspective. We have discussed one possible way of implementing this vision by means of constraints prescribing relative frequencies across spacetime, although frequency constraints are only one possible way in which chancy events could arise within a globally deterministic universe.

Several interesting conceptual points have come up along this journey. In particular, the possibility of weak global determinism requires something of a conceptual shift, requiring us to distinguish between events which are chancy and events which are undetermined but nonetheless not chancy. This distinction is not entirely new - for example, it is common to hold that the initial conditions of the world are not determined by anything but also not governed by any objective probabilities - but it is not always well articulated, and often fails to be taken seriously as an option for any phenomena other than the initial conditions of the universe. Weak global determinism thus offers a welcome halfway house for those who find it implausible that everything in the universe is fully determined by the laws of nature, but who are also suspicious of the ill-defined concept of objective chance. We have briefly surveyed some of the consequences of this new perspective, nothing that it offers interesting insight into for long-standing debates in which determinism plays a role within both physics and philosophy.

A   Simple and functional constraint frequentism

The simplest way to define frequency constraints is to say that the actual relative frequency of a certain sort of event across the whole of spacetime must be equal or approximately to some value. That is to say, these constraints are of the form 'the actual mosaic must belong to the set of mosaics in which the actual relative frequency of events of type E is equal to some value.' However, on their own constraints of this kind do not seem capable of explaining the sorts of inferences we make about the relationships between different types of events. Simple frequency constraints are capable of explaining why it is the case that observing a sequence of events of type M could lead us to form expectations about the results of future events of type M; however, they do not explain why it is the case that observing a set of events of various types sometimes leads us to form expectations about the results of events of a type we have not yet observed. For example, consider a fixed measurement with two outcomes, A and B, on a particle in a set of states $sin(\theta)|0\rangle + cos(\theta)|1\rangle$ parametrized by θ. Suppose I perform one thousand measurements for each value of θ in the set $S = \{0, \frac{\pi}{20}, \frac{2\pi}{20}...\frac{9\pi}{20}, \frac{\pi}{2}\}$ and I observe that in each case the relative frequency for outcome A is very close to $\cos^2(\theta)$. This gives me grounds to form expectations about the likely results of future measurements on states with θ in

the set S, but what about a measurement on a state with $\theta = \frac{23\pi}{276}$? According to the simple frequency constraint view, I have no grounds for forming any expectation here, because I have not observed any events of this particular type and therefore I don't have access to any relative frequencies which could inform my expectations. Yet in a real situation of this kind we would clearly be inclined to set our probability for outcome A equal to $\cos^2(\frac{23\pi}{276})$ - indeed, this is exactly what we *do* in fact do in quantum mechanics, since we take it that the theory gives us well-defined probabilities for all quantum measurements, even though the set of states is continuous so we can't ever actually perform all possible measurements. Neither standard frequentism nor simple constraint frequentism view can easily account for this sort of interpolation; indeed in both cases it seems as if we must regard the similarity of the functional form for the relative frequencies across different values of θ as a mere coincidence.

The obvious way to fix this problem by moving to functional constraints, where we define relative frequencies not case by case but rather in terms of functional relationships. So for example, we might postulate a constraint which says something like 'for a set of continuous measurements parametrised by θ, the relative frequency of outcome one across all actual measurements for a given θ is given by $f(\theta)$,' where f is some suitable function of θ. But this leads to new problems if the range of values of one or more parameters is continuous - for then if we select our parameters at random, no event type will ever occur more than once, and therefore none of these relative frequencies will not be well defined. Moreover, even if we don't select the parameter at random and instead try to prepare the same value of the parameter many times, one might worry that due to limitations on the accuracy of our state preparations we will never in fact be able to prepare the same parameter twice, so the relative frequencies still won't be well defined. Yet in scenarios of this type the probabilities still exist and have an observable effect - for example, in the case where the functional relationship is $\cos^2(\theta)$, then we would expect to see outcome A more often for values of θ close to zero, and this effect will be observable even if we never measure the same state twice. So simply constraining relative frequencies doesn't seem to give the right results for continuous measurements.

Moreover, relaxing the constraints doesn't help either. Suppose we insist that the actual relative frequencies are always such as to minimize the distance (by some appropriate metric) from the true probability, conditional on the actual number of outcomes. Various metrics are possible, but most sensible metrics (including, for example, the one proposed by Roberts [50]) would seem to have the property that if only one measurement of a given type is ever made, then the outcome which is most likely must occur. This might not be a problem if in fact the vast majority of event types happen a great many times, but it causes difficulties if we consider the case of continuous measurements where each event type happens only once, since then every measurement must always have its most likely outcome - so for example, in the $\cos^2(\theta)$ example, it would necessarily be the case that every measurement with $\theta < \frac{\pi}{3}$ would have outcome A and every measurement with $\theta > \frac{\pi}{3}$ would have outcome B. This would prevent us from ever assigning any non-trivial probabilities to event types with continuous parametrisations, whereas in quantum mechanics we quite frequently do exactly that.

Perhaps the most elegant solution to this problem is to hypothesize that the world is fundamentally discrete, so that no parametrisation is ever really continuous; then we can straightforwardly return to the idea that probabilistic laws correspond to constraints constraining actual relative frequencies. In particular, this would probably involve insisting that spacetime is discrete, since probabilistic measurements in quantum mechanics may be parametrized by spatiotemporal variables. But while there are a few indications from modern physics that discretized spacetime is not out of the question, concluding that spacetime *must* be discrete based solely on a conceptual analysis of probability seems a bit of a reach, so it behoves us to consider alternative possibilities. One option would be to impose an artificial discretization - for example, in the case discussed above, we could divide $[0, \frac{\pi}{2}]$ into N intervals and then insist that the number of A outcomes across the set of all measurements with $\theta \in [\frac{n\pi}{2N}, \frac{(n+1)\pi}{2N}]$ must be (approximately) equal to $\cos^2(\frac{m\pi}{2N})$. Here, N should be regarded as a free parameter which we can set to ensure that this constraint is functionally indistinguishable from the usual continuous probability rule. This allows us to obtain what is in effect a finite set of prepare-measure scenarios from the original continuous set of prepare-measure scenarios, and therefore we can straightforwardly apply simple or functional constraints. However, it must be admitted that insisting on an overlay of discreteness within a continuous theory looks very awkward, and indeed if we really believed that all possible spacetime measurements were discretized in this way, then in a sense it would really be the discretized units rather than the continuous underlying manifold which played the relevant functional role in our spatiotemporal experiences, and at this juncture one might well start to wonder whether there is really any justification for retaining the manifold in the theory at all. An alternative would be to postulate a function $f(\theta, n)$ such that for each pair θ, n, if

n measurements are made with parameter θ across the whole of spacetime, then the total number of A outcomes must be equal to $f(\theta, n)$. However, for the reasons we have just discussed, we can't choose this function such that $f(\theta, n)$ is always equal to the most likely number of A outcomes, since then we would have $f(\theta, 1) = 1$ for every θ where A is the most likely outcome, and we have already seen that this doesn't work out. So either we would have to let $f(\theta, n)$ depend on something else in the universe, or we would have to choose it at random, in which case we seem just to have shifted 'objective chance' into the choice of the constraint rather than eliminating it entirely.

A further problem for both simple and functional constraint frequentism is the possibility that the relevant event type might occur an infinite number of times. In this case, we can't simply impose a frequency constraint of the kind discussed above; we might hope to impose a constraint on the limiting relative frequency of a given outcome, but in an infinite sequence one can obtain any relative frequency one likes by simply rearranging the items in the sequence, so this will not produce well-defined results [50]. What we seem to need is some rule to the effect that in any sufficiently large patch of spacetime (containing a large but finite number of instances of the event type in question), the relative frequencies of the various outcomes must obey the relevant constraint. That is to say, we should impose a constraint which selects only Humean mosaics where the frequencies are approximately uniform across spacetime and where the relevant finite frequency constraint is satisfied within all sufficiently large patches of spacetime. For example, Roberts proposes the requirement that if we select any spacetime point and then move outwards in a series of concentric 4-dimensional shells, counting instances of the relevant event type in each shell as we go, then the resulting sequence must converge to the appropriate relative frequency. Alternatively, note that we have already seen that constraint frequentism seems to work best in a context where spacetime is discretized; and if spacetime is discretized and also bounded (i.e. there is both a start and end of time and space has a finite extent), then no event type can be repeated an infinite number of times, since there are not an infinite number of possible locations at which events can occur. So if we have already accepted the need for discretization to make sense of constraint frequentism, we might also be willing to consider insisting that spacetime is bounded, in which case we have no need to deal with the infinite case.

It's interesting that there appears to be a close connection between global determinism and discreteness/finiteness; we intend to investigate this connection further in future work. Note, however, that the apparent connection holds only if we insist that the fundamental laws of nature include irreducible probabilities, *and* if we insist that those fundamental laws must be analysed as frequency constraints as opposed to some other sort of nonlocal determination. If for example one were to move to an account like the de Broglie-Bohm interpretation of quantum mechanics where at the most fundamental level the theory exhibits local determinism, then one can certainly have global determinism without any need for discretization, since there will be no need to postulate any frequency constraints.

B   Nomic frequentism

Roberts' 'nomic frequentism' proposal [50, 57], where he suggests that laws about probabilities should be understood in terms of laws of the form 'R percent of the Fs are Gs' has much in common with the frequency constraint approach that we have discussed in this article. However, there are a few key differences. First, Roberts presents nomic frequentism as a general analysis of all probabilities that appear in laws, including for example the laws of evolutionary biology, whereas we have suggested it only for the specific case of objective chances appearing in the fundamental laws of nature, and in particular quantum mechanics - we would expect that the laws of evolutionary biology could be satisfactorily understood in terms of subjective probabilities. Moreover, we do not even claim that *all* of the objective chances appearing in the fundamental laws of nature must be attributed to frequency constraints; we merely observe that this is one possible way in which apparently probabilistic events could arise in a deterministic universe. So our account is in that sense considerably less general and ambitious than Roberts'.

Second, Roberts suggests that 'eighty percent of As are Bs' should be regarded as conceptually equivalent to a law like 'All As are Bs,' (or rather, the latter should be regarded a special case of the former). But there is one important conceptual difference: the constraint 'All As are Bs' does not seem to require any 'communication' between the As, as it is enough that each A has the intrinsic property of being a B, whereas 'exactly eighty percent of As are Bs' or even 'approximately eighty percent of As are Bs' does seem to require some sort of coordination, as in order to ensure that the relative frequency is exactly or approximately correct, it seems that each A must 'know' something about what the other As are doing. Thus frequency constraints seem to require some form of non-locality (as Roberts himself later observes), which means we are in quite a different conceptual space from standard laws like 'all As are Bs.'

Third, Roberts' solution to the apparent absence of counterfactual independence within nomic frequentism is to say that 'the way a nomic frequentist will represent independence of distinct fair coin-tosses is by denying the existence of any law that implies that the conditional frequency of heads on one toss given the results of another toss is different from (50 percent).' But as we have seen, it does not seem that this can be entirely correct: constraint frequentism does allow the violation of counterfactual independence in at least certain special cases. Roberts justifies his solution on the basis that knowing facts which violate counterfactual independence 'would require (us) to have advance intelligence from the future,' but this argument seems to depend implicitly on the assumption that we are in a Humean context where the only information which is relevant to inferences about a future event is an actual observation of the event itself. But if the laws which induce the frequency constraints are understood as laws which are ontologically prior to the Humean mosaic, and if we are able to make correct inferences about the laws based on observations of a limited subset of the mosaic, then we would in principle be able to know about violations of counterfactual independence without having any illegitimate information about events which have not yet occurred. This sort of 'knowledge of the future' is not really any different from the knowledge of the future that we get from more familiar sorts of scientific laws: the laws constrain the Humean mosaic, including the future, and thus by figuring out the laws we can make inferences about the future.

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