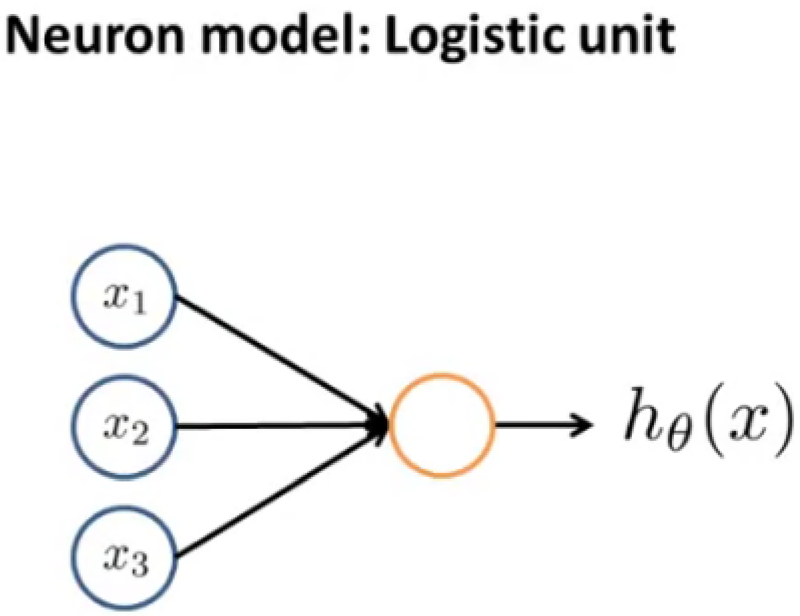
Neural Networks

- Origins: algorithms that try to mimic the brain;
- Widely used in 80s and early 90s:
  - Popularity diminished in the late 90s;
- Recent resurgence: State-of-the-art technique for many applications.

Model representation



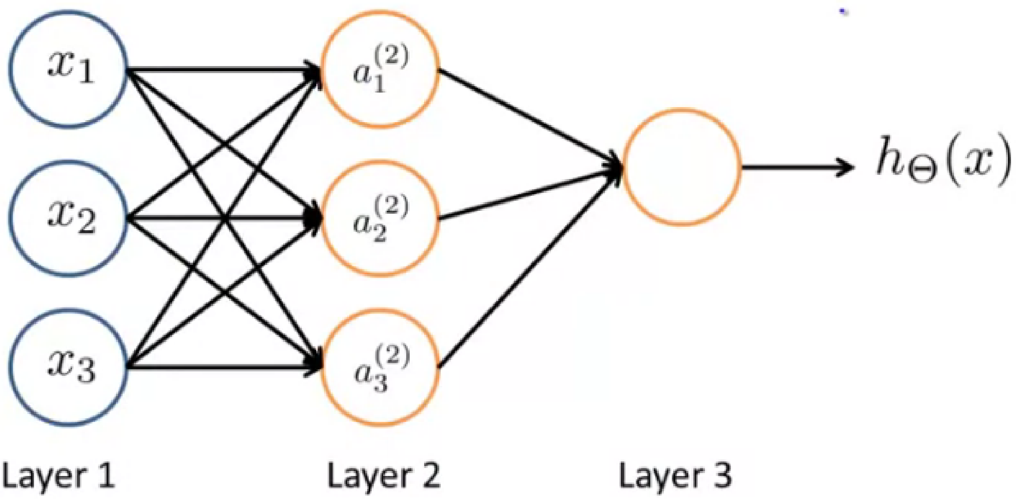
where $h_\theta(x) = \dfrac{1}{1+e^{-\theta^T x}}$, with:

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}.$$

$x_0 = 1$ is also called **bias**. It is sometimes omitted in the diagrams.

The sigmoid function works as the **activation function**.

Neural Network



Layer 1 is called the **input layer**.

Layer 2 is the **hidden layer**.

Layer 3 is also known as the **output layer**.

$a_i^{(j)}$: 'activation' of unit i in layer j ;

$\Theta^{(j)}$: matrix of weights controlling function mapping from layer j to layer $j+1$.

Since the previous model has three input units (four with the bias) and three hidden units, $\Theta^{(1)} \in \mathbb{R}^{3\times 4}$:

- In this case, the matrix of weights is supposed to be pre-multiplied by the input vector (without transposition).

Thus, the model represented in the preceding diagram can be written as:

$$\begin{aligned}
a_1^{(2)} &= g(\Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3) \\
a_2^{(2)} &= g(\Theta_{20}^{(1)}x_0 + \Theta_{21}^{(1)}x_1 + \Theta_{22}^{(1)}x_2 + \Theta_{23}^{(1)}x_3) \\
a_3^{(2)} &= g(\Theta_{30}^{(1)}x_0 + \Theta_{31}^{(1)}x_1 + \Theta_{32}^{(1)}x_2 + \Theta_{33}^{(1)}x_3) \\
h_\Theta(x) = a_1^{(3)} &= g(\Theta_{10}^{(2)}a_0^{(2)} + \Theta_{11}^{(2)}a_1^{(2)} + \Theta_{12}^{(2)}a_2^{(2)} + \Theta_{13}^{(2)}a_3^{(2)})
\end{aligned}$$

If a network has s_j units in layer j , s_{j+1} in layer $j+1$, then $\Theta^{(j)}$ will be of dimension $s_{j+1} \times (s_j + 1)$.

- The unit added to the second dimension accounts for the bias (which is inputted to every layer).

Forward propagation: vectorized implementation

$$z^{(2)} = \begin{bmatrix} z_1^{(2)} \\ z_2^{(2)} \\ z_3^{(2)} \end{bmatrix} = \begin{bmatrix} \Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3 \\ \Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3 \\ \Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3 \end{bmatrix}$$

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow \begin{matrix} z^{(2)} = \Theta^{(1)} x \\ a^{(2)} = g(z^{(2)}) \end{matrix}$$

The input units x may as well be understood as the activation of the first layer, that is, $a^{(1)}$.

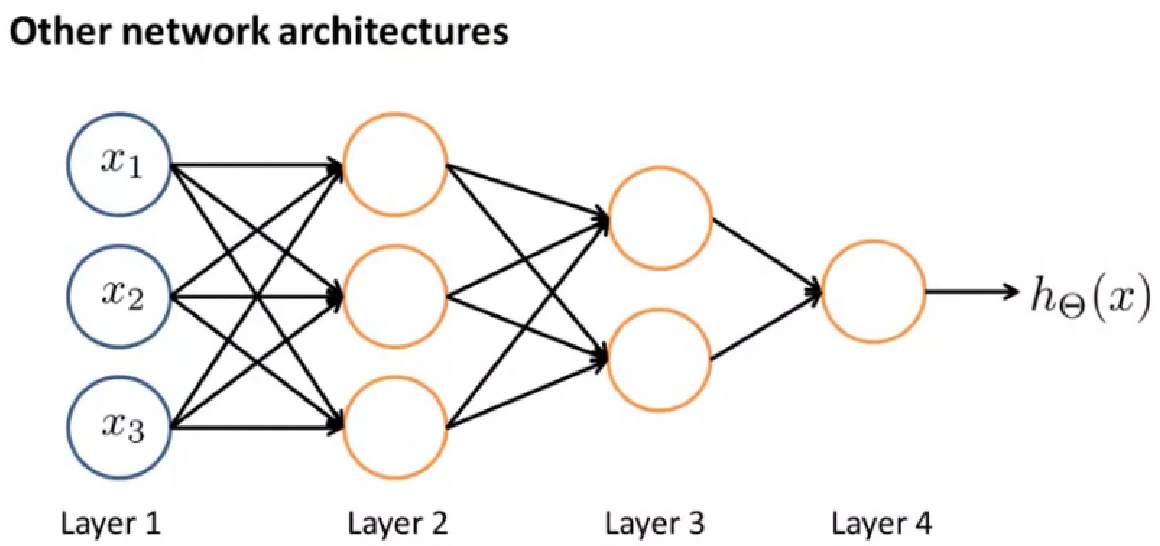
$$z^{(2)} = \Theta^{(1)} a^{(1)}$$
$$a^{(2)} = g(z^{(2)})$$

Then, **add** $a_0^{(2)} = 1$, and proceed:

$$z^{(3)} = \Theta^{(2)} a^{(2)}$$
$$h_\Theta(x) = a^{(3)} = g(z^{(3)})$$

More generally:

$$z^{(j+1)} = \Theta^j a^{(j)}$$
$$a^{(j+1)} = g(z^{(j+1)})$$



Multi-class classification

Multiple output units: One-vs-all.

Thus, for the first class (out of three): $h_\Theta(x) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$;

The second and third classes: $h_\Theta(x) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $h_\Theta(x) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$;

And so on.

The training set will then be: $(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})$,

where:

$$y^{(i)} \in \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}.$$