Lecture 19

Review on:

Instrumental variables using two-stage least squares there is heteroskedasticity

For a model:

$$y_1 = \beta_1 + \beta_2 y_2 + \beta_3 x_3 + u$$

When modeling the variance, it is possible to use OLS if the endogenous variable is left out:

$$\sigma_i^2 = \alpha_1 + \alpha_2 x_3 + \text{error}$$

In the original model it is still necessary to use instrumental variables, though.

What a large sample size on White test is

It will depend on the context of the data but:

- Short of 30 will never be a large sample;
- At least 100 to 200 observations, for most applications, in order to be able to exploit the large samples properties, for instance, calculating the rate of convergence.

When to use probit and logit models

Probit: useful when trying to model an unobservable variable, and what is actually observed is only a choice (0 or 1).

Logit: an alternative to a linear probability model.

- In the linear probability model, the estimator has 1) heteroskedasticity and 2) non-normal errors and 3) there is no guarantee the result will be between 0 and 1.

Proofs of bias and consistency

Model:

$$y_i = x_i^* \beta + v_i, \quad \text{where} \quad x_i = x_i^* + e_i$$

Substituting:

$$\begin{aligned} y_i &= (x_i - e_i)\beta + v_i \\ &= x_i \beta + u_i \end{aligned}$$

where $u_i = (v_i - \beta e_i)$.

Then

$$\begin{aligned} \text{plim}\ \hat{\beta} &= \beta + \frac{\text{Cov}(x, u)}{\text{Var}(x)} \\ &= \beta + \frac{\text{Cov}(x^* + e,\ v - \beta e_i)}{\text{Var}(x^* + e)} \\ &= \beta + \frac{-\beta \sigma_e^2}{\sigma_{x^*}^2 + \sigma_e^2} \\ &= \beta \left(1 - \frac{\sigma_e^2}{\sigma_{x^*}^2 + \sigma_e^2} \right) \end{aligned}$$

For a model:

$$y = \beta_1 + \beta_2 x + u$$

Apropos β_2:

$$\text{plim } \hat{\beta}_2 = \beta_2 + \frac{\text{Cov}(x,u)}{\text{Var}(x)}$$

As for β_1:

$$\begin{gathered} \hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x} \\ \therefore \text{plim } \hat{\beta}_1 = \mu_y - \text{plim } \hat{\beta}_2 \mu_x \end{gathered}$$

And from that we conclude that any bias in $\hat{\beta}_2$ will introduce bias in $\hat{\beta}_1$.

Ways of *biasing* the constant $\hat{\beta}_1$:

- Non-zero measurement errors (which affect μ_y or μ_x);
- Bias in $\hat{\beta}_2$.

Structural model estimation

When we do 2SLS, implicitly, what we are doing is:

- Starting with a structural model.
- Then, deriving the reduced form;
- Estimating the reduced form;
- And, finally, using the reduced form to recover the structural model.

The identification question is all about the possibility of recovering the structural parameters through the estimated reduced form.

For instance, for a structural model

$$\begin{aligned} Q_i &= \beta_0 + \beta_1 P_i + \beta_2 Y_i + u_i \\ Q_i &= \alpha_0 + \alpha_1 P_i + v_i \end{aligned}$$

we obtain the reduced form:

$$\begin{aligned} \alpha_0 + \alpha_1 P_i + v_i &= \beta_0 + \beta_1 P_i + \beta_2 Y_i + u_i \\ \therefore P_i &= \frac{1}{\alpha_1 - \beta_1} \Big[(\beta_0 - \alpha_0) + (\beta_1 - \alpha_1) P_i + \beta_2 Y_i + (u_i - v_i) \Big] \end{aligned}$$

which can be rewritten as:

$$P_i = \lambda_0 + \lambda_1 P_i + \lambda_2 Y_i + \text{error}$$

where $\lambda_0 = \dfrac{1}{\alpha_1 - \beta_1}(\beta_0 - \alpha_0)$, $\lambda_1 = \dfrac{1}{\alpha_1 - \beta_1}(\beta_1 - \alpha_1)$ and $\lambda_2 = \dfrac{1}{\alpha_1 - \beta_1}\beta_2$.

Analogously:

$$Q_i = \gamma_0 + \gamma_1 P_i + \gamma_2 Y_i + \text{error}$$

Once we estimate λ's and γ's, we can supposedly retrieve α's and β's.

If there are not enough λ and γ equations to solve for α's and β's, the system is under-identified.

If there are more than enough λ and γ equations to solve for α's and β's, the system is over-identified.

In that model:

- Endogenous: Q, P.
- Exogenous: constants, Y.

$G = \text{num. exog. variables} = 2.$

- In the first structural equation there are **no** excluded variables, then, since $G-1>0$, it is under-identified.
- In the second structural equation there is **one** excluded variable (Y), then, since $G-1=1$, it is exactly identified.
- Were $G-1 > \text{num. excluded variables}$, it would be over-identified.