Analog to Digital Audio:

The array of sampled values from the electrical signal is digital audio.

Sampling rate = $44,100$ samples per second (CD quality).

Analog signal $s(t) \rightarrow$ digital signal $\{y_0, y_1, \ldots, y_n\}$.

- y_i: array of 16-bit (*bit depth*); one bit for sign, 15 bits for value.

Period, Amplitude and Frequency:

A signal that exhibits the same repeated pattern (cycle) is periodic.

$$\text{period} = \frac{1}{\text{frequency}}$$

Sinusoids:

Both $y = \sin t$ and $y = \cos t$ completes one cycle in 2π seconds:

- $y = \sin t,\ t \in [0, 2\pi]$ (one cycle in 2π seconds);
- $y = \sin(2\pi t),\ t \in [0, 1]$ (one cycle in 1 second = 1 Hz).
- $y = \sin(2\pi \boldsymbol{f} t), t \in [0, 1]$ (f cycles in 1 second = f Hz).

Real Sinusoids:

$y = a\sin(2\pi f t + \phi)$ and $y = a\cos(2\pi f t + \phi)$.

- a : amplitude;
- f : frequency (Hz);
- $\frac{1}{f}$: period (s);
- ϕ : phase (radians).
- $\omega = 2\pi f$: angular frequency.

Example: Middle C (or C4) has frequency 261.6 Hz.

- $y(t) = \sin(2\pi \cdot 261.6 \cdot t),\ t \in [0, 2]$.

C major:

The C major chord (C-E-G) has frequencies 261.6 Hz, 329.6 Hz and 392 Hz.

The analog signal for this chords is:

$$y(t) = \sin(2\pi \cdot 261.6 \cdot t) + \sin(2\pi \cdot 329.6 \cdot t) + \sin(2\pi \cdot 392.0 \cdot t).$$

Loudness vs Pitch:

Loudness is our brain's perception of amplitude.

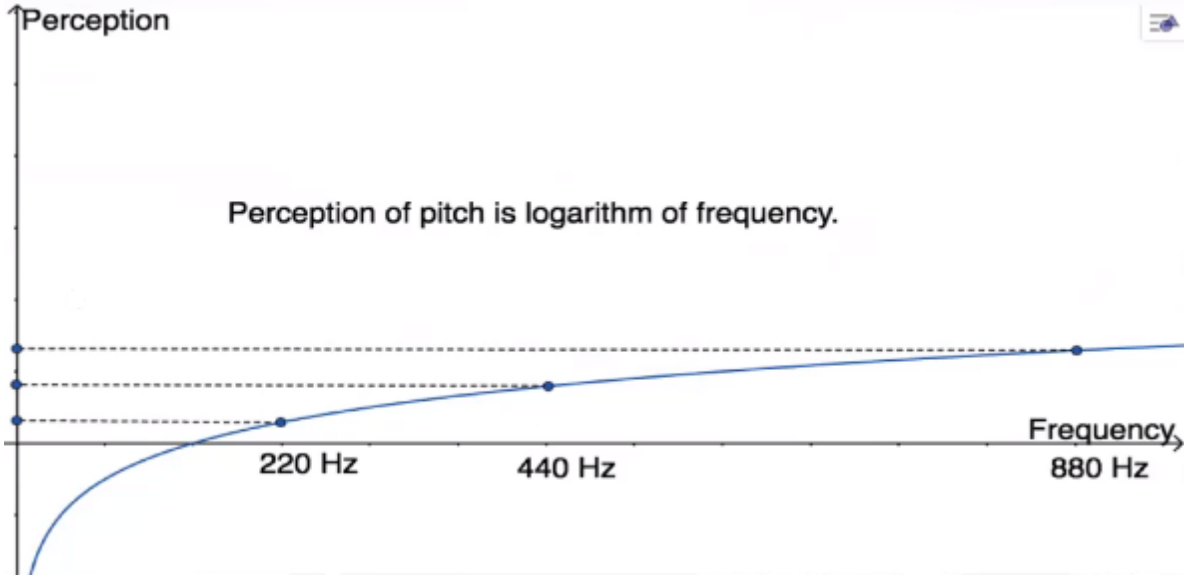
Pitch is our brain's perception of frequency.

- Our perception of pitch is based on the logarithm of frequency.
- The interval between two notes is the perceived difference between the two pitches.
- As a result, the interval we hear from two notes depends on the ratio of their frequencies, not the difference.

Octave:

- A3: 220 Hz; A4: 440 Hz; A5: 880 Hz.

- Perceived interval A4, A3 $= \log(440) - \log(220) = \log(440/220) = \log 2$.
- Perceived interval A5, A4 $= \log(880) - \log(440) = \log(880/440) = \log 2$.



Piano keys:

An 88-key piano has frequencies given by:

$$f(n) = 440\left(\sqrt[12]{2}\right)^{n-49}, \quad n = 1, 2, \ldots, 88$$

- $f(1) \approx 27.5$ Hz (A0).
- $f(88) \approx 4186$ Hz (C8).

Fundamental frequency and overtones:

Pure tone: one frequency.

- Most sounds are more complex.
  - E4 piano note = supposition of many frequencies: 330 Hz, 660 Hz, 990 Hz, 1320 Hz and 1650 Hz.
  - Fundamental frequency: 330 Hz.
  - Overtones or harmonics: 660 Hz, 990 Hz, 1320 Hz and 1650 Hz.

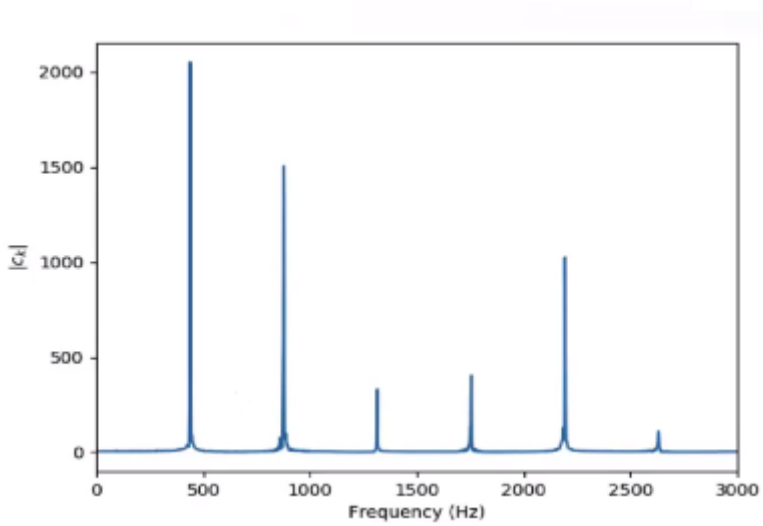
Time vs Frequency Domain:

In the time domain, the voltage/current signal is a function of time.

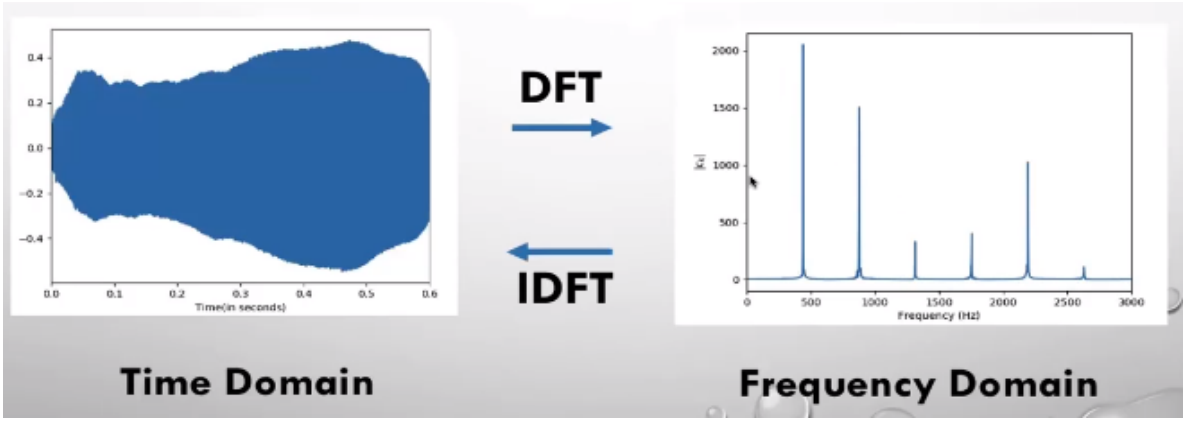
In the frequency domain, the signal is represented as a function of frequencies that are present in the signal.

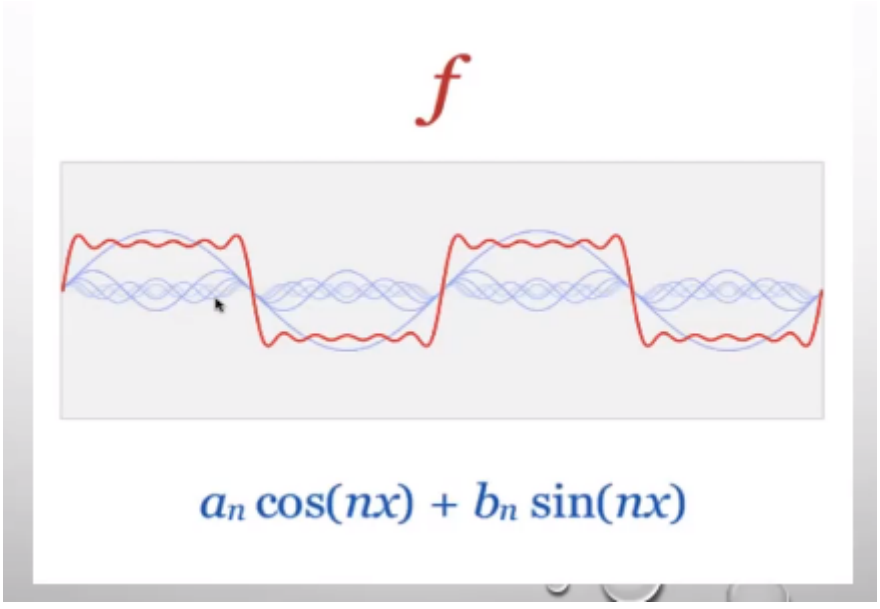
- The set of frequencies in a signal and their magnitude is called spectrum of the signal.

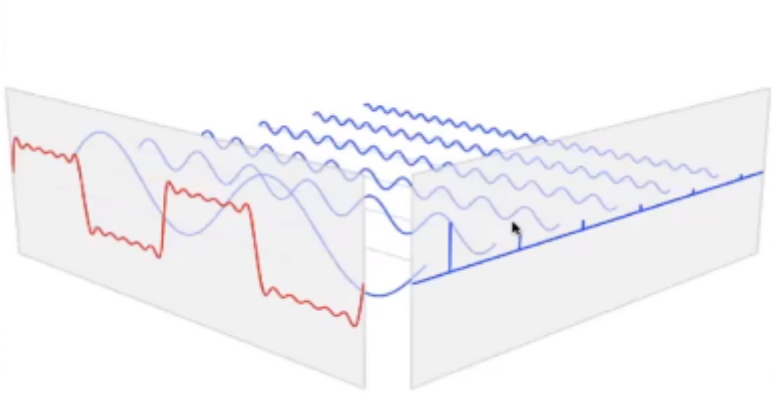
A major chord:



Discrete Fourier Transform:







Important take-aways:

- Digital audio is simply sampling from a continuous, analog function at some sampling rate producing an array of numbers.
- Sinusoids take on the form $y = a\sin(2\pi ft + \phi)$.
- Pitch is our brain's perception of frequency (log scale).
    - Fundamental frequency vs harmonics.
- Time and frequency domains are two equivalent representation of a signal.
- The DFT and the Inverse Discrete Fourier Transform (IDFT) allows us to go back and forth between representations.