Quantifying uncertainty in hydrodyamic simulations of heavy-ion collisions attributable to imperfect knowledge of the QCD equation of state

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I. INTRODUCTION

Hydrodynamic simulations are a popular tool to model the spacetime evolution of the quark-gluon plasma (QGP) produced in relativistic heavy-ion collisions. Lattice regularization is the only reliable method to calculate the QCD equation of state equation of state in the vicinity of a phase transition and hence constitutes a critical component of modern computer simulations. While lattice techniques are rigorous in their treatment of the underlying QCD Lagrangian, they are subject to numerical errors inherent in the lattice discretization procedure. These errors are manifest in differences in the continuum extrapolated QCD trace anomaly predicted by different lattice collaborations and lead to an overall uncertainty in the true value of the QCD equation of state at zero baryochemical potential.

For the purposes of hydrodynamic simulations, the equation of state is typically treated as a theoretically constrained quantity in contrast to e.g. the QGP specific shear viscosity η/s which is varied and tuned to optimally replicate experimental data. Consequently, numerical discrepancies between different lattice collaborations introduce an inherent systematic bias in the best fit values of underconstrained QGP properties determined from systematic model-to-data comparison. A notable exception to this convention is a recent model-to-data analysis which parameterized the QGP equation of state and used a Bayesian, data driven approach to constrain its functional form [1].

Uncertainties in the equation of state have been studied both at low temperature, by comparing lattice predictions to results from a hadron resonance gas model [2], and at high temperature by comparing hydrodynamic predictions obtained using different parameterizations of the QCD trace anomaly [2, 3]. Large differences were

Recent calculations by the HotQCD and Wuppertal-Budapest collaborations of the QCD trace anomaly in the continuum limit now show good agreement within errors. This signals an important convergence in lattice descriptions of the QCD equation of state which previously exhibited a tension in the peak of the trace anomaly near the QGP phase transition. It is not yet clear however, if current lattice errors are under sufficient control for hydrodynamic transport models or if further improvement is needed.

In this work, we analyze the current status of lattice gauge calculations in the continuum limit by comparing simulation predictions using different lattice calculations for the QGP equation of state. We study the latest HotQCD and Wuppertal-Budapest parametrizations as well as the depreciated s95 parametrization based on older HotQCD lattice results.

We embed each equation of state in an event-by-event hydrodynamic model with a hadronic afterburner and measure spectra, flows and Bertsch-Pratt radii predicted by the simulations to quantify systematic differences between the calculations. We also assess the uncertainty introduced by continuum extrapolation when constructing the best fit parametrization by sampling splines from the bootstrap coefficients used in the HotQCD error analysis. Using these results, we comment on the resolving power of hydrodynamic simulations and assess the need for improved lattice calculations at zero baryo-chemical potential.

II. EQUATIONS OF STATE

III. HYBRID MODEL

The equations of state are compared using the VISHNU transport model which couples boost invariant viscous fluid dynamics [**?**] for the hot and dense early phase of the collision with a microscopic, kinetic description of late hadronic rescattering and freeze-out [**?**]. The mock particle data generated by each simulated event are then stored and analyzed using the same methods applied in the experiment.

A. Initial Conditions

We generate hydrodyamic initial conditions using a standard two-component Monte Carlo Glauber model which deposits entropy proportional to a linear combination of nucleon participants and binary nucleon-nucleon collisions,

$$dS/dy\,|_{y=0} \propto \frac{(1-\alpha)}{2} N_{\rm part} + \alpha N_{\rm coll}. \qquad (1)$$

The entropy is localized about each nucleon's transverse parton density $T_p(\mathbf{x})$,

$$dS/dy\,|_{y=0} \propto \sum_{i=0}^{N_{\rm part,A}} w_i\, T_p(\mathbf{x}-\mathbf{x}_i)(1-\alpha+\alpha\, N_{\rm coll,i}) + \sum_{j=0}^{N_{\rm part,B}} w_j\, T_p(\mathbf{x}-\mathbf{x}_i)(1-\alpha+\alpha\, N_{\rm coll,j}) \quad (2)$$

where the summations run over the participants in each nucleus, $N_{\rm coll,i}$ denotes the number of binary collisions suffered by the $i^{\rm th}$ nucleon and the proton density $T_p(\mathbf{x})$ is described by a Gaussian

$$T_p(\mathbf{x}) = \frac{1}{\sqrt{2\pi B}} \exp\left(-\frac{x^2+y^2}{2B}\right) \quad (3)$$

with transverse area $B = 0.36$ fm^2.

The random nucleon weights w_i in equation (2) are sampled independently from a Gamma distribution with unit mean

$$P_k(w) = \frac{k^k}{\Gamma(k)} w^{k-1} e^{-kw}, \quad (4)$$

and shape parameter $k = \mathrm{Var}(P)^{-1}$ which modulates the variance of the distribution. These fluctuations are typically added [**?**] to reproduce the large multiplicity fluctuations observed in minimum bias proton-proton collisions. In this work the shape parameter is fixed to $k = 1$ to fit the 200 GeV UA5 data [**?**].

For the binary collision fraction in equation (1), we choose the value $\alpha = 0.14$ used in reference [**?**].

It is important to note that the aforementioned Monte Carlo Glauber model is not a state of the art model for initializing hydrodynamic simulations. It makes many simplifying assumptions such as ignoring pre-equillibrium dynamicsand asserting wounded nucleon and binary collision scaling, an assertion which is questioned by a number of recent works. Nevertheless, the model provides a good description of observed particle multiplicities, flows and spectra. In this work, we are primarily interested in the *sensitivity* of hydrodynamic observables to differences in the QGP equation of state and *not* the overall best fit of model to data. Hence, the Monte Carlo Glauber model serves as suitable surrogate for more accurate physical models.

B. Hydrodyamics and Boltzmann Transport

The initial condition profiles, which provide the entropy density $dS/(d^2r_\perp\, d\eta\, \tau_{\rm therm})$ at the QGP thermalization time, are rescaled by an overall normalization factor which is tuned to fit the centrality dependence of charged particle production in 200 GeV gold-gold collisions.

We follow the work in reference [**?**] and fix the hydrodynamic specific shear viscosity to a constant value $(\eta/s)_{\rm QGP} = 0.08$ which has been shown to provide a reasonable description of measured spectra and flows in 200 GeV gold-gold collisions. For the purposes of this study, we assume vanishing bulk viscosity $\zeta/s = 0$, although it would be interesting to account for finite bulk viscosity in future work, as its functional form is sensitive to the value of the QGP trace anomaly near the QCD phase transition [**?**].

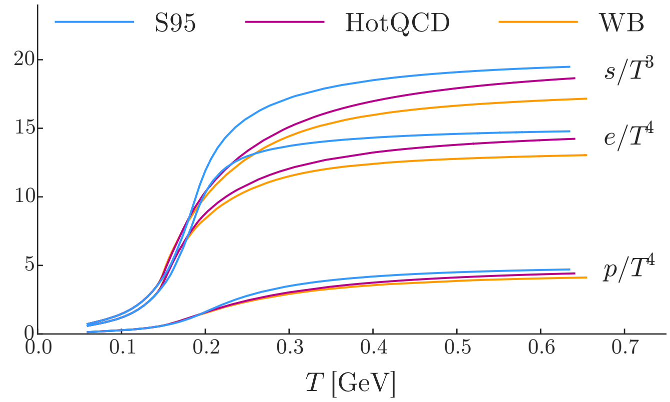

FIG. 1. Comparisons of the entropy density s, energy density e and pressure p divided by powers of the temperature T for the three equations of state used in this study.

In order to switch from hydrodyamic field equations to microscopic transport, the VISHNU model asserts a sudden switching temperature $T_{\rm sw}$ at which the hydrodynamic energy-momentum tensor is particlized using the Cooper-Frye freezeout prescription,

$$E\frac{dN_i}{d^3p} = \int_\sigma f_i(x,p) p^\mu d^3\sigma_\mu \quad (5)$$

where f_i is the distribution function of particle species i, p^μ is its four-momentum and $d^3\sigma_\mu$ characterizes an element of the isothermal freezeout hypersurface defined by $T_{\rm sw}$.

For a simplistic physical system with a first order phase transition, one would typically fix the hydro to kinetic switching temperature to a value just below the phase transition temperature. This allows the hydrodyamic phase of the model to account for complex dynamics of the phase transition which are difficult to model microscopically. Unfortunately, the prescription for fixing the switching temperature is not so clear in the case of relativistic heavy-ion collisions where the phase transition is a smooth cross over and cannot be identified with a single transition temperature.

In this work we switch from fluid dynamics to microscopic kinetics using a hypersurface of constant temperature corresponding to a fixed energy density of $e_{\rm sw} = 335$ MeV. This leads to a slightly different transition temperatures for each equation of state as shown in table I.

TABLE I. Switching energy density and temperature for transition from fluid dynamics to Boltzmann transport.

	S95	HotQCD	WB
e_{sw} [GeV/fm^3]	0.335	0.335	0.335
T_{sw} [GeV]	0.157	0.154	0.154

IV. POSTPROCESSING AND ANALYSIS

V. RESULTS

VI. CONCLUSION

VII. SUMMARY

VIII. ACKNOWLEDGEMENTS

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[2] P. Huovinen and P. Petreczky, Nucl. Phys. **A837**, 26 (2010), arXiv:0912.2541 [hep-ph].
[3] P. Huovinen, Nucl. Phys. **A761**, 296 (2005), arXiv:nucl-th/0505036 [nucl-th].

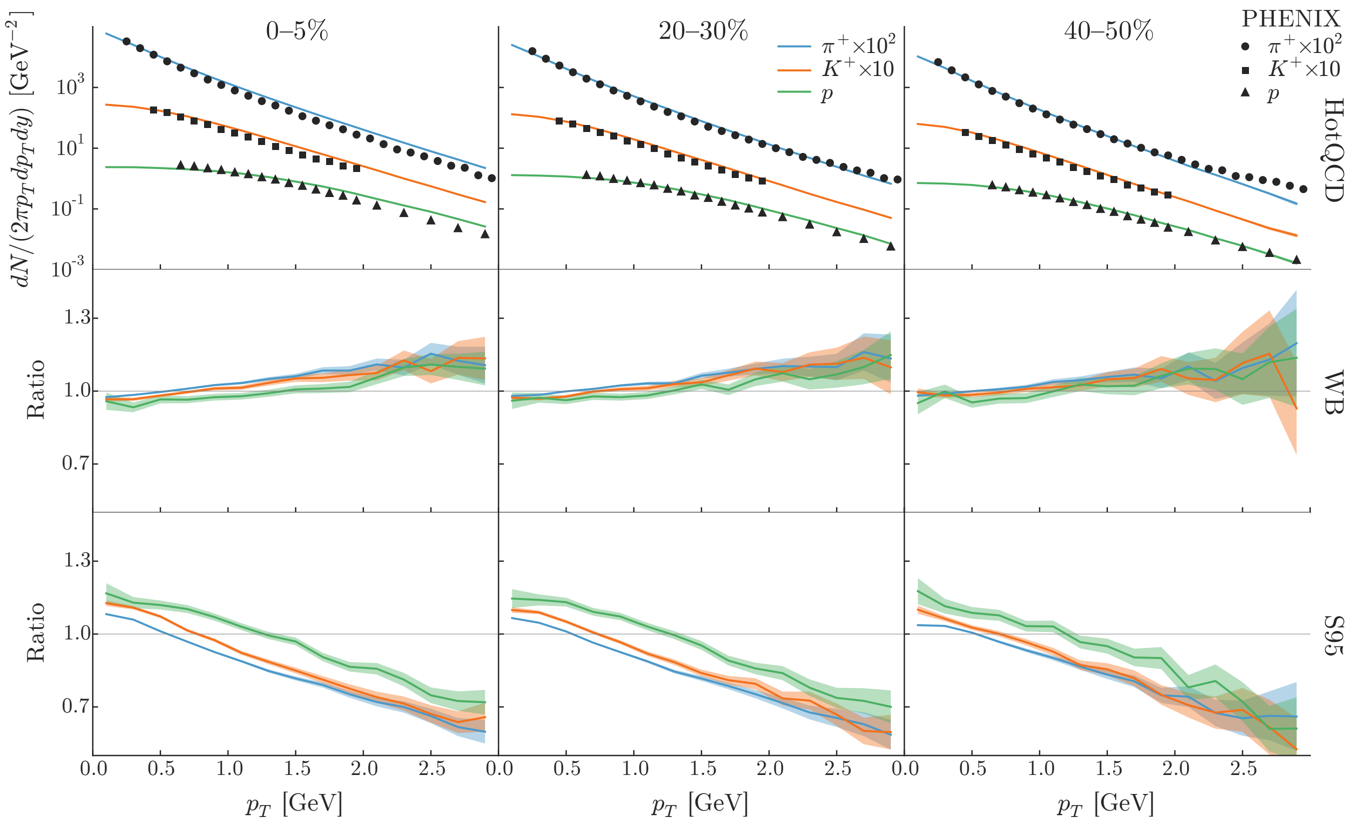

0–5%
20–30%
40–50%
$dN/(2\pi p_T dp_T dy)$ [GeV^{-2}]
10^3
10^1
10^{-1}
10^{-3}
$\pi^+ \times 10^2$
$K^+ \times 10$
p
PHENIX
$\pi^+ \times 10^2$
$K^+ \times 10$
p
HotQCD
Ratio
1.3
1.0
0.7
WB
Ratio
1.3
1.0
0.7
S95
0.0
0.5
1.0
1.5
2.0
2.5
p_T [GeV]

FIG. 2.

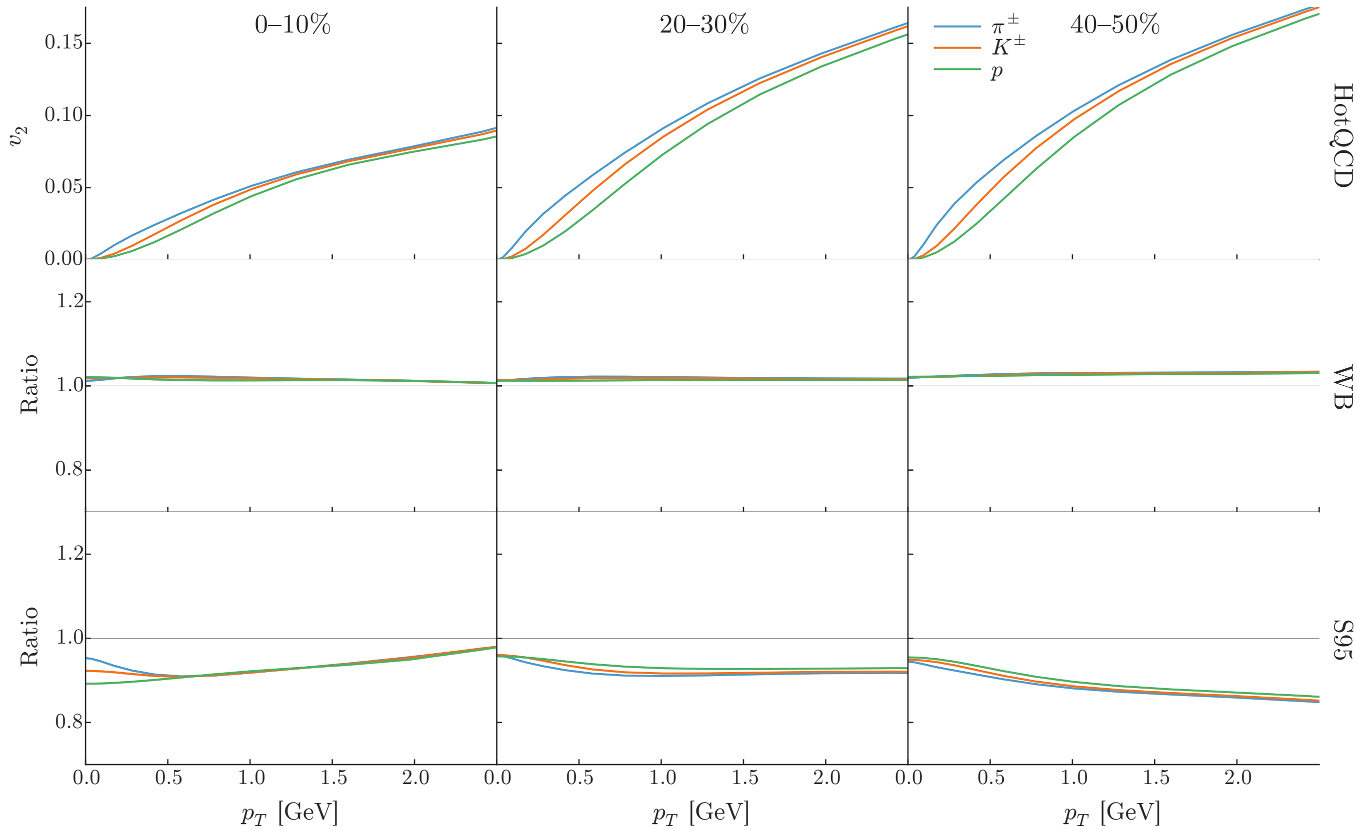
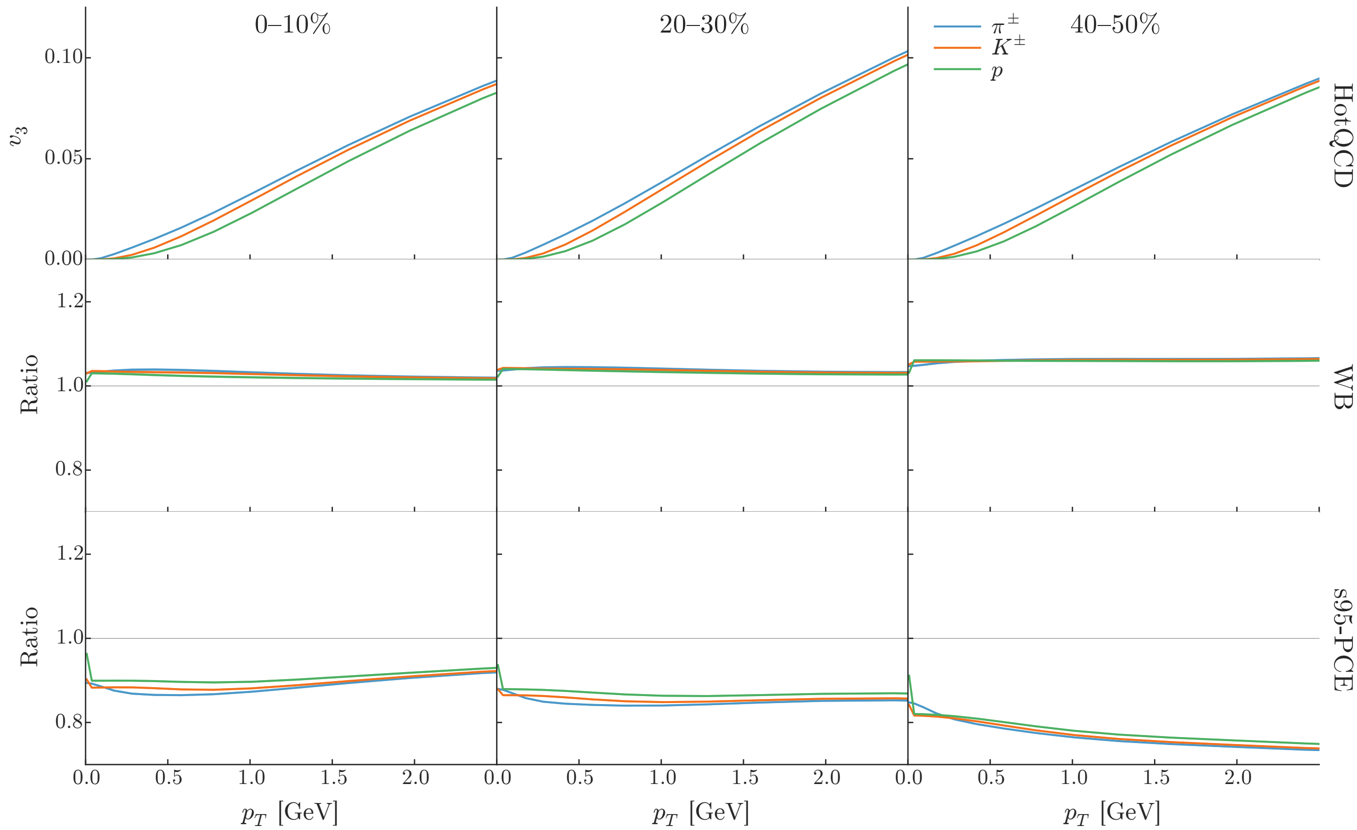

0–10%
20–30%
40–50%
$\pi^\pm$
$K^\pm$
p
HotQCD
WB
S95
v_2
Ratio
Ratio
0.00
0.05
0.10
0.15
1.2
1.0
0.8
1.2
1.0
0.8
0.0
0.5
1.0
1.5
2.0
p_T [GeV]

FIG. 3.



0–10%
20–30%
40–50%
$\pi^{\pm}$
$K^{\pm}$
p
v_3
0.10
0.05
0.00
HotQCD
Ratio
1.2
1.0
0.8
WB
s95-PCE
0.0
0.5
1.0
1.5
2.0
p_T [GeV]

FIG. 4.

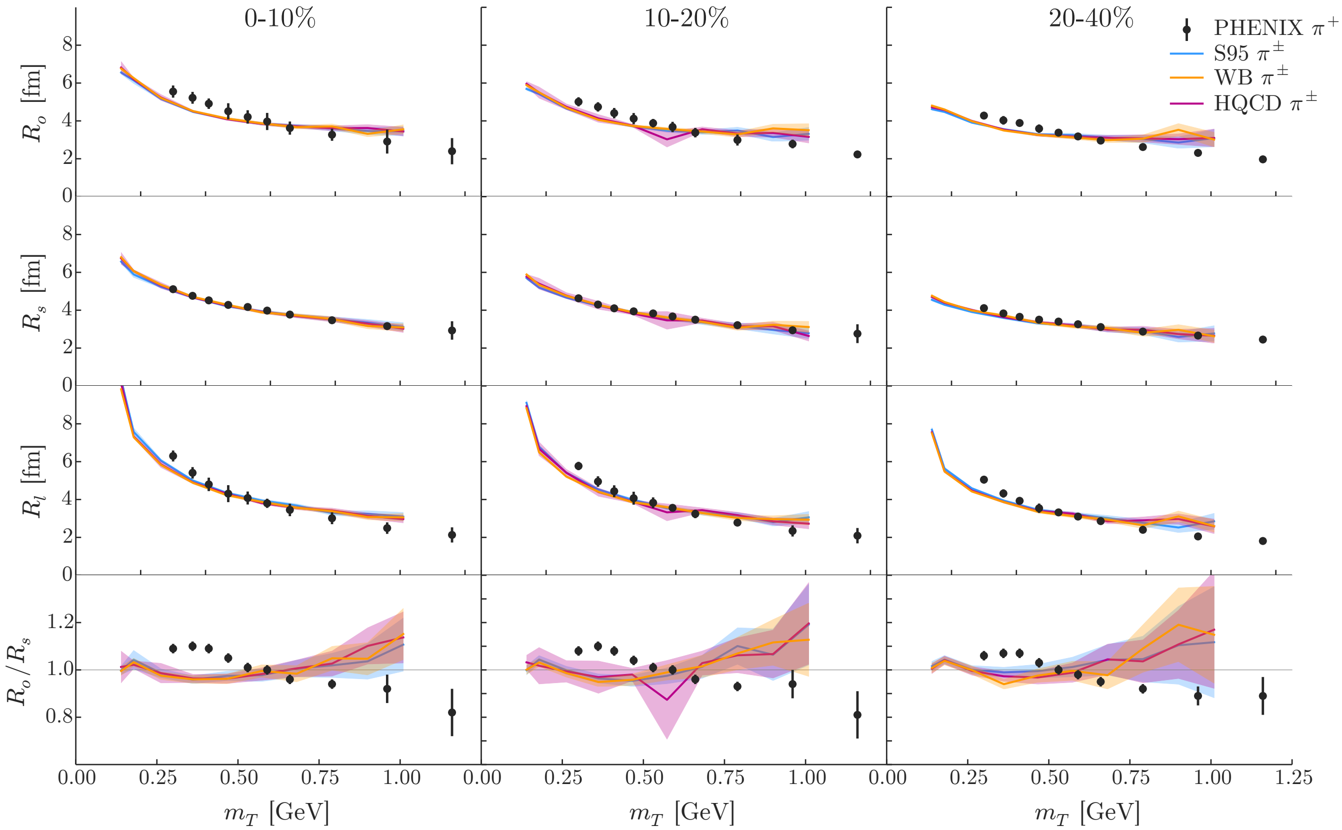

0-10%
10-20%
20-40%
PHENIX π^+
S95 $\pi^\pm$
WB $\pi^\pm$
HQCD $\pi^\pm$
R_o [fm]
R_s [fm]
R_l [fm]
R_o/R_s
m_T [GeV]

FIG. 5.