Quantifying uncertainty in hydrodyamic simulations of heavy-ion collisions attributable to imperfect knowledge of the QCD equation of state

J. Scott Moreland[1] and Ron A. Soltz[2]

[1] *Department of Physics, Duke University, Durham, NC 27708-0305*
[2] *Lawrence Livermore National Laboratory, Livermore, CA 94551-0808*

(Dated: July 30, 2015)

I. INTRODUCTION

Hydrodynamic simulations are a popular tool to model the spacetime evolution of the quark-gluon plasma (QGP) produced in relativistic heavy-ion collisions. Lattice regularization is the only reliable method to calculate the QCD equation of state equation of state in the vicinity of a phase transition and hence constitutes a critical component of modern computer simulations. While lattice techniques are rigorous in their treatment of the underlying QCD Lagrangian, they are subject to numerical errors inherent in the lattice discretization procedure. These errors are manifest in differences in the continuum extrapolated QCD trace anomaly predicted by different lattice collaborations and lead to an overall uncertainty in the true value of the QCD equation of state at zero baryochemical potential.

For the purposes of hydrodynamic simulations, the equation of state is typically treated as a theoretically constrained quantity in contrast to e.g. the QGP specific shear viscosity η/s which is varied and tuned to optimally replicate experimental data. Consequently, numerical discrepancies between different lattice collaborations introduce an inherent systematic bias in the best fit values of underconstrained QGP properties determined from systematic model-to-data comparison. A notable exception to this convention is a recent model-to-data analysis which parameterized the QGP equation of state and used a Bayesian, data driven approach to constrain its functional form [1].

Uncertainties in the equation of state have been studied both at low temperature, by comparing lattice predictions to results from a hadron resonance gas model [2], and at high temperature by comparing hydrodynamic predictions obtained using different parameterizations of the QCD trace anomaly [2, 3]. Large differences were

Recent calculations by the HotQCD and Wuppertal-Budapest collaborations of the QCD trace anomaly in the continuum limit now show good agreement within errors. This signals an important convergence in lattice descriptions of the QCD equation of state which previously exhibited a tension in the peak of the trace anomaly near the QGP phase transition. It is not yet clear however, if current lattice errors are under sufficient control for hydrodynamic transport models or if further improvement is needed.

In this work, we analyze the current status of lattice gauge calculations in the continuum limit by comparing simulation predictions using different lattice calculations for the QGP equation of state. We study the latest HotQCD and Wuppertal-Budapest parametrizations as well as the depreciated s95 parametrization based on older HotQCD lattice results.

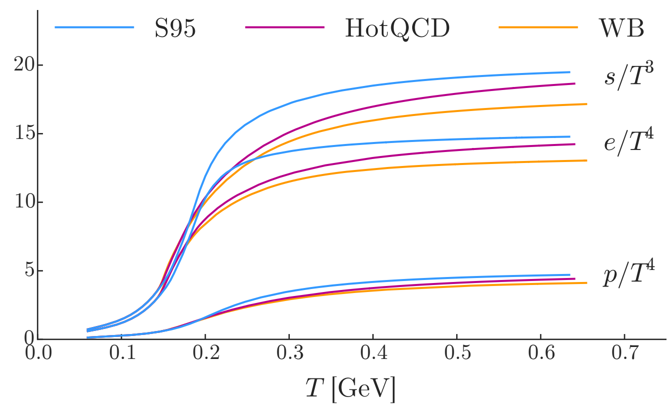


FIG. 1.

We embed each equation of state in an event-by-event hydrodynamic model with a hadronic afterburner and measure spectra, flows and Bertsch-Pratt radii predicted by the simulations to quantify systematic differences between the calculations. We also assess the uncertainty introduced by continuum extrapolation when constructing the best fit parametrization by sampling splines from the bootstrap coefficients used in the HotQCD error analysis. Using these results, we comment on the resolving power of hydrodynamic simulations and assess the need for improved lattice calculations at zero baryo-chemical potential.

II. HYBRID MODEL

The equations of state are compared using the VISHNU transport model which couples boost invariant viscous fluid dynamics for the hot and dense early phase of the collision with a microscopic, kinetic description of late hadronic rescattering and freeze-out.

A. Initial Conditions

We generate hydrodyamic initial conditions using a standard two-component Monte Carlo Glauber model which deposits entropy proportional to a linear combination of nucleon participants and binary nucleon-nucleon collisions,

$$dS/dy\,|_{y=0} \propto \frac{(1-\alpha)}{2} N_{\rm part} + \alpha N_{\rm coll}. \quad (1)$$

The entropy is localized about each nucleon's transverse parton density $T_p(\mathbf{x})$,

$$\begin{aligned} dS/dy\,|_{y=0} \propto & \sum_{i=0}^{N_{\rm part,A}} w_i\, T_p(\mathbf{x}-\mathbf{x}_i)(1-\alpha+\alpha\, N_{\rm coll,i}) \\ & + \sum_{j=0}^{N_{\rm part,B}} w_j\, T_p(\mathbf{x}-\mathbf{x}_i)(1-\alpha+\alpha\, N_{\rm coll,j}) \end{aligned} \quad (2)$$

where the summations run over the participants in each nucleus, $N_{\rm coll,i}$ denotes the number of binary collisions suffered by the $i^{\rm th}$ nucleon and the proton density $T_p(\mathbf{x})$ is described by a Gaussian

$$T_p(\mathbf{x}) = \frac{1}{\sqrt{2\pi B}} \exp\left(-\frac{x^2+y^2}{2B}\right) \quad (3)$$

with transverse area $B = 0.36$ fm^2.

The random nucleon weights w_i in equation (2) are sampled independently from a Gamma distribution with unit mean

$$P_k(w) = \frac{k^k}{\Gamma(k)} w^{k-1} e^{-kw}, \quad (4)$$

and shape parameter $k = \mathrm{Var}(P)^{-1}$ which modulates the variance of the distribution. These fluctuations are typically added [?] to reproduce the large multiplicity fluctuations observed in minimum bias proton-proton collisions. In this work the shape parameter is fixed to $k = 1$ to fit the 200 GeV UA5 data [?].

It is important to note that the aforementioned Monte Carlo Glauber model is not a state of the art model for initializing hydrodynamic simulations. It makes many simplifying assumptions such as ignoring pre-equillibrium dynamics and asserting wounded nucleon and binary collision scaling, an assertion which is questioned by a number of recent works. Nevertheless, the model provides a good description of observed particle multiplicities, flows and spectra. In this work, we are primarily interested in the *sensitivity* of hydrodynamic observables to differences in the QGP equation of state and *not* the overall best fit of model to data. Hence, the Monte Carlo Glauber model serves as suitable surrogate for more accurate physical models.

III. HYDRODYAMICS AND BOLTZMANN TRANSPORT

The initial condition profiles, which provide the entropy density $dS/(d^2r_\perp\, d\eta\, \tau_{\rm therm})$ at the QGP thermalization time, are then used to initialize the boost invariant viscous hydrodynamics code `VISH2+1` [?].

IV. EQUATIONS OF STATE

The hybrid model approach used in this study

Within the hybrid model approach used in this study, the low temperature behaviour of the QCD equation of state is fixed by the hadronic transport model.

We concentrate on the intermediate and high temperature dependence of the QCD equation of state and fix the.

QCD equation of state is described by the hadron resonance gas model, which is to good approximation

focus on the intermediate and high temperature dependence of the QGP equation of state

study three parameterizations for the QGP equations of state, the state of the art HotQCD and Wuppertal-Budapest parameterizations [?] as well as the older s95-v1 parametrization based on older HotQCD results. To perform

V. RESULTS

VI. ACKNOWLEDGEMENTS

JSM acknowledges support by the DOE/NNSA Stockpile Stewardship Graduate Fellowship under grant no. DE-FC52-08NA28752.

[1] J. Novak, K. Novak, S. Pratt, J. Vredevoogd, C. Coleman-Smith, *et al.*, Phys.Rev. **C89**, 034917 (2014), arXiv:1303.5769 [nucl-th].

[2] P. Huovinen and P. Petreczky, Nucl. Phys. **A837**, 26 (2010), arXiv:0912.2541 [hep-ph].

[3] P. Huovinen, Nucl. Phys. **A761**, 296 (2005), arXiv:nucl-th/0505036 [nucl-th].

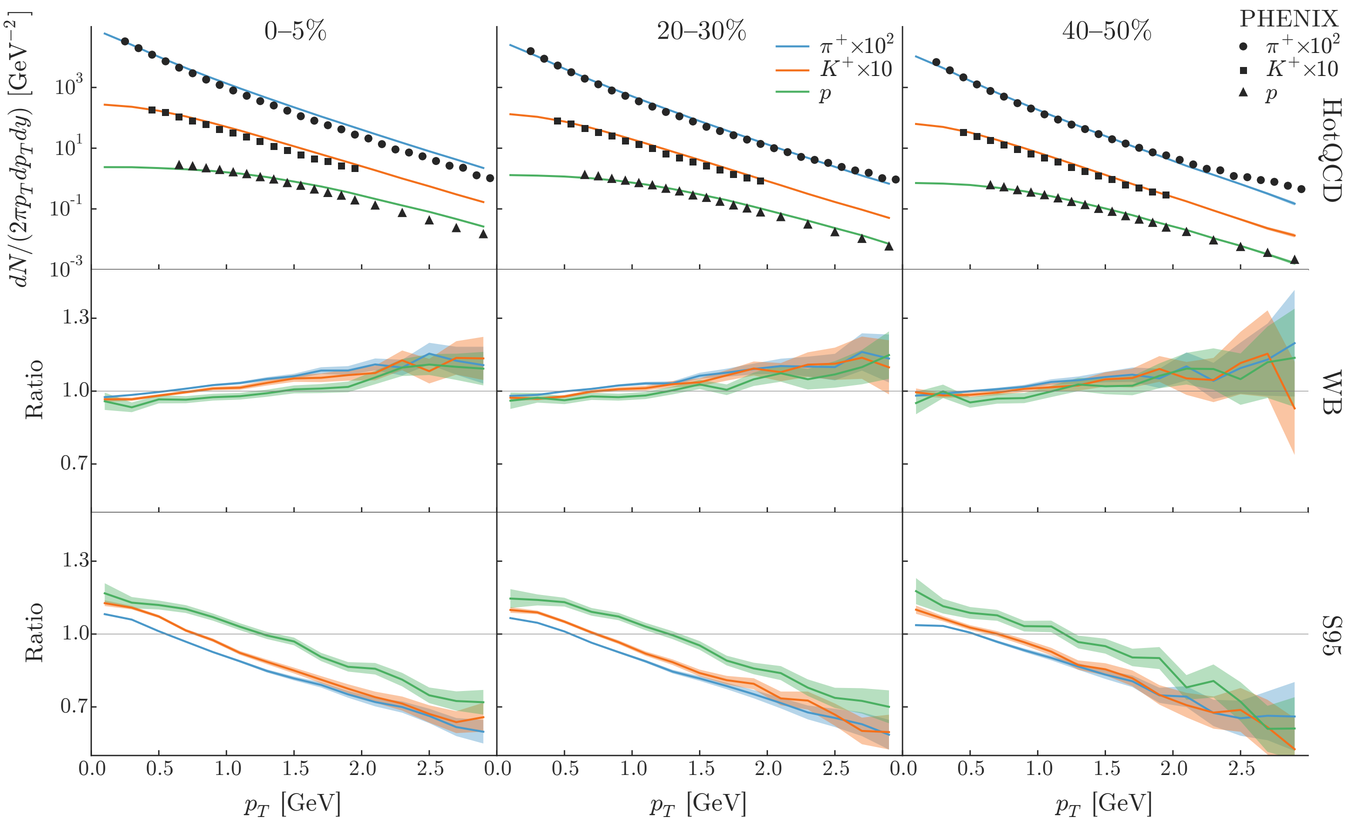

0–5%
20–30%
40–50%
$dN/(2\pi p_T dp_T dy)$ [GeV^{-2}]
$\pi^+\times10^2$
$K^+\times10$
p
PHENIX
$\pi^+\times10^2$
$K^+\times10$
p
HotQCD
WB
S95
Ratio
Ratio
p_T [GeV]
p_T [GeV]
p_T [GeV]

FIG. 2.

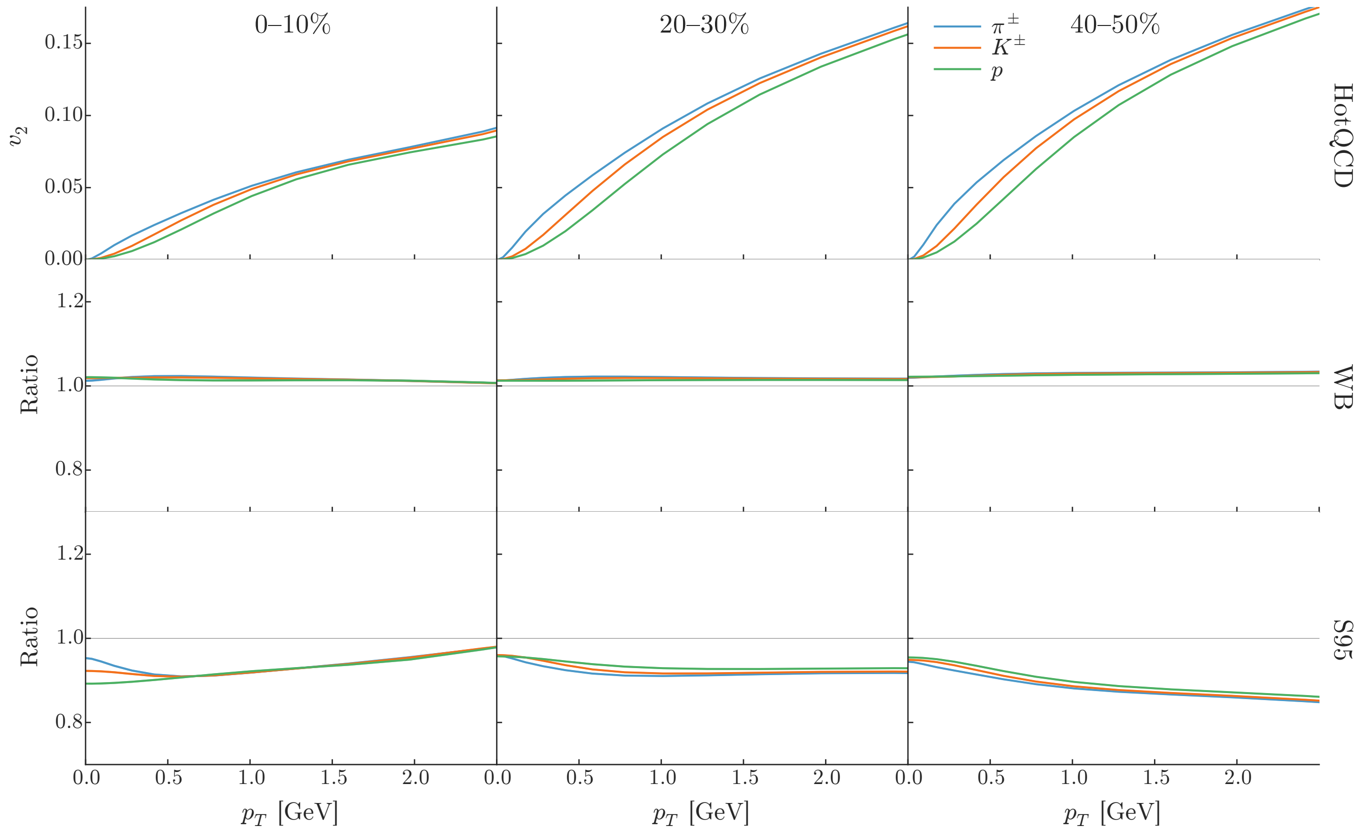


FIG. 3.

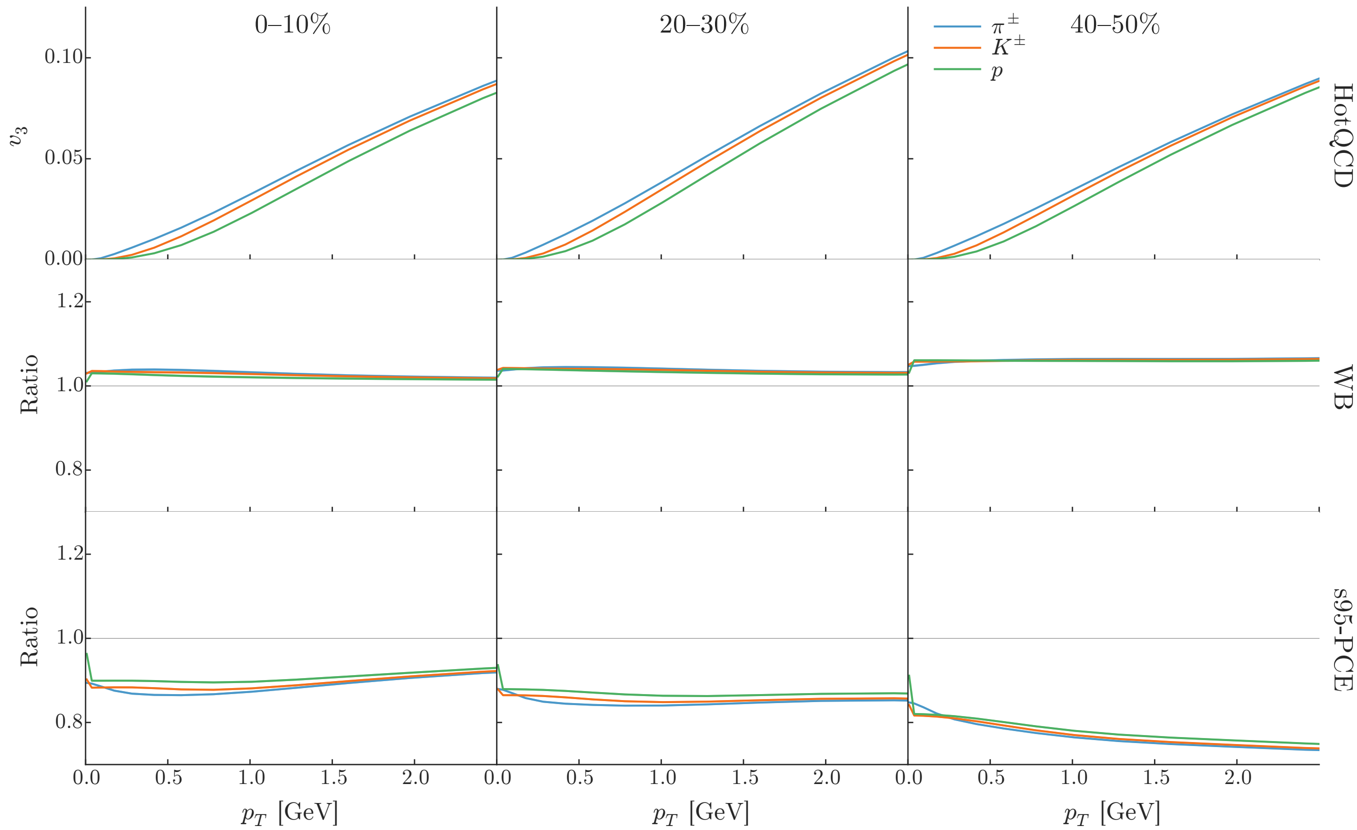
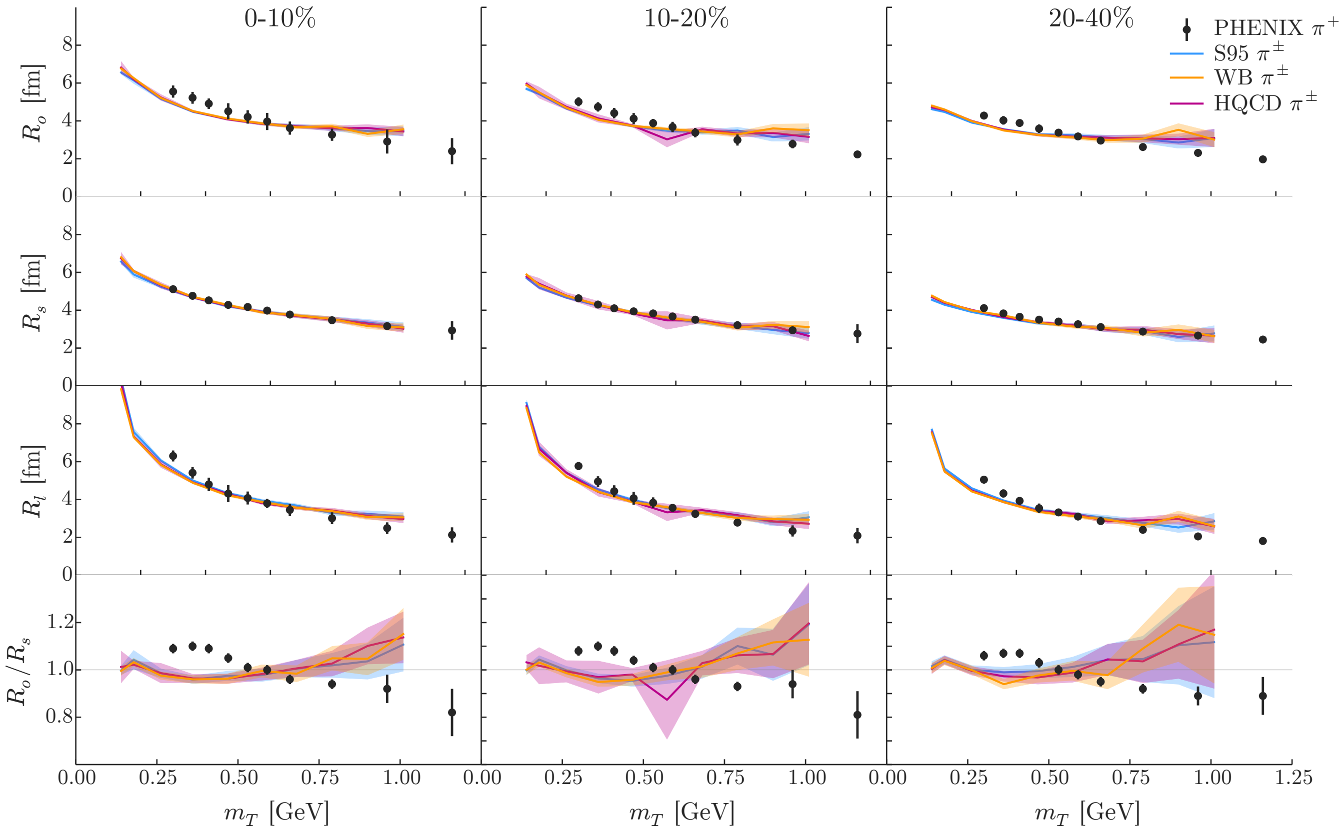

0–10%
20–30%
40–50%
$\pi^{\pm}$
$K^{\pm}$
p
v_3
0.10
0.05
0.00
HotQCD
Ratio
1.2
1.0
0.8
WB
Ratio
1.2
1.0
0.8
s95-PCE
0.0 0.5 1.0 1.5 2.0
p_T [GeV]

FIG. 4.



0-10%
10-20%
20-40%
PHENIX π^+
S95 $\pi^\pm$
WB $\pi^\pm$
HQCD $\pi^\pm$
R_o [fm]
R_s [fm]
R_l [fm]
R_o/R_s
m_T [GeV]

FIG. 5.