

Quantifying uncertainty in hydrodynamic simulations of heavy-ion collisions attributable to imperfect knowledge of the QCD equation of state

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I. INTRODUCTION

Fluid dynamics is a useful framework to study the collective behaviour of hot and dense nuclear matter produced in relativistic heavy-ions. Quantum Chromodynamics (QCD) predicts that at sufficiently high energies these collisions form a new state of matter consisting of deconfined quarks and gluons known as a quark-gluon plasma (QGP). Simulations based on relativistic viscous hydrodynamics play a central role in extracting properties of the QGP which expands and freezes into hadrons too quickly for direct observation.

The hydrodynamic transport equations require two essential ingredients to specify the full time evolution of the QGP fireball: initial conditions which describe the thermal profile of the QGP droplet at some early starting time and a QCD equation of state which interrelates energy density, pressure and temperature of each fluid cell in local thermal equilibrium.

Lattice discretization is the only reliable method to calculate the QCD equation of state in the vicinity of the QGP phase transition and hence constitutes a critical component of hydrodynamic simulations. While lattice techniques are rigorous in their treatment of the underlying QCD Lagrangian, they are subject to numerical errors inherent in the lattice discretization procedure. These errors are manifest in differences in the continuum extrapolated QCD trace anomaly predicted by different lattice collaborations and lead to an overall uncertainty in the true value of the QCD equation of state at zero baryochemical potential.

Simulations using a lattice equation of state thus inherit all forms of numeric and systematic uncertainty associated with the underlying lattice methodology. These uncertainties have been studied both at low temperature, by comparing lattice predictions to results from a hadron resonance gas model [1], and at high temperature by comparing hydrodynamic predictions obtained using different parameterizations of the QCD trace anomaly [1, 2].

Recent calculations by the HotQCD and Wuppertal-Budapest collaborations of the QCD trace anomaly in the continuum limit now show good agreement within errors. This signals an important convergence in lattice descriptions of the QCD equation of state which previously exhibited a tension in the peak of the trace anomaly near the QGP phase transition. It is not yet clear however, if current lattice errors are under sufficient control for hydrodynamic transport models or if further improve-

ment is needed.

In this work, we analyze the current status of lattice gauge calculations in the continuum limit by comparing simulation predictions using different lattice calculations for the QGP equation of state. We study the latest HotQCD and Wuppertal-Budapest parametrizations as well as the depreciated s95 parametrization based on older HotQCD lattice results.

We embed each equation of state in an event-by-event hydrodynamic model with a hadronic afterburner and measure spectra, flows and Bertsch-Pratt radii predicted by the simulations to quantify systematic differences between the calculations. We also assess the uncertainty introduced by continuum extrapolation when constructing the best fit parametrization by sampling splines from the bootstrap coefficients used in the HotQCD error analysis. Using these results, we comment on the resolving power of hydrodynamic simulations and assess the need for improved lattice calculations at zero baryo-chemical potential.

II. EQUATIONS OF STATE

We examine the most recent HotQCD and Wuppertal-Budapest equations of state [?] as well as an older HotQCD parameterization calculated using a coarser lattice spacing [?]. The hybrid model approach used in this study switches from a fluid dynamic description of the medium for the liquid-like, QGP phase of the collision to a microscopic Boltzmann description once the system cools and freezes into a hadron resonance gas. While the equation of state enters the hydrodynamic phase of the simulation as an explicit function interrelating energy density, entropy density, pressure and temperature, the equation of state in the microscopic, kinetic phase of the simulation is not simple to change as it depends on the complex interplay of scattering kinematics and resonance accounting.

Consequently, the hadronic equation of state used in this study is regarded as a fixed quantity, determined by the list of particles and particle resonances included in the UrQMD model which is used to simulate the hadronic

III. HYBRID MODEL

The equations of state are compared using the VISHNU transport model which couples boost invariant viscous fluid dynamics [?] for the hot and dense early phase of the collision with a microscopic, kinetic description of late hadronic rescattering and freeze-out [?]. The mock particle data generated by each simulated event are then stored and analyzed using the same methods applied in the experiment.

A. Initial Conditions

We generate hydrodynamic initial conditions using a standard two-component Monte Carlo Glauber model which deposits entropy proportional to a linear combination of nucleon participants and binary nucleon-nucleon collisions,

$$dS/dy|_{y=0} \propto \frac{(1-\alpha)}{2} N_{\text{part}} + \alpha N_{\text{coll}}. \quad (1)$$

The entropy is localized about each nucleon's transverse parton density $T_p(\mathbf{x})$,

$$dS/dy|_{y=0} \propto \sum_{i=0}^{N_{\text{part,A}}} w_i T_p(\mathbf{x} - \mathbf{x}_i) (1 - \alpha + \alpha N_{\text{coll,i}}) + \sum_{j=0}^{N_{\text{part,B}}} w_j T_p(\mathbf{x} - \mathbf{x}_j) (1 - \alpha + \alpha N_{\text{coll,j}}) \quad (2)$$

where the summations run over the participants in each nucleus, $N_{\text{coll,i}}$ denotes the number of binary collisions suffered by the i^{th} nucleon and the proton density $T_p(\mathbf{x})$ is described by a Gaussian

$$T_p(\mathbf{x}) = \frac{1}{\sqrt{2\pi B}} \exp\left(-\frac{x^2 + y^2}{2B}\right) \quad (3)$$

with transverse area $B = 0.36 \text{ fm}^2$.

The random nucleon weights w_i in equation (2) are sampled independently from a Gamma distribution with unit mean

$$P_k(w) = \frac{k^k}{\Gamma(k)} w^{k-1} e^{-kw}, \quad (4)$$

and shape parameter $k = \text{Var}(P)^{-1}$ which modulates the variance of the distribution. These fluctuations are typically added [?] to reproduce the large multiplicity fluctuations observed in minimum bias proton-proton collisions. In this work the shape parameter is fixed to $k = 1$ to fit the 200 GeV UA5 data [?].

For the binary collision fraction in equation (1), we choose the value $\alpha = 0.14$ used in reference [?].

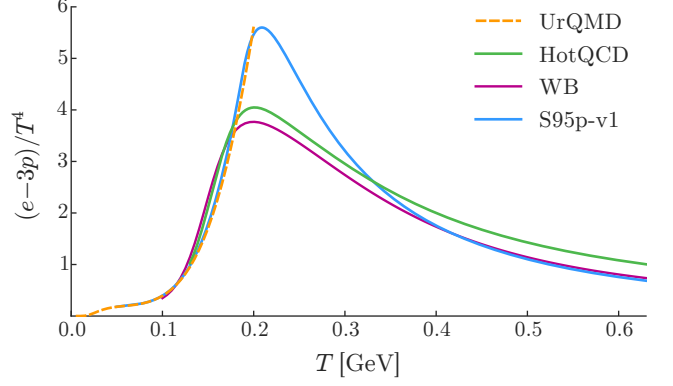


FIG. 1. Trace anomaly plotted for each equation of state along with the UrQMD trace

B. Hydrodynamics and Boltzmann Transport

The initial condition profiles, which provide the entropy density $dS/(d^2r_\perp d\eta \tau_{\text{therm}})$ at the QGP thermalization time, are rescaled by an overall normalization factor which is tuned to fit the centrality dependence of charged particle production in 200 GeV gold-gold collisions.

We follow the work in reference [?] and fix the hydrodynamic specific shear viscosity to a constant value $(\eta/s)_{\text{QGP}} = 0.08$ which has been shown to provide a reasonable description of measured spectra and flows in 200 GeV gold-gold collisions. For the purposes of this study, we assume vanishing bulk viscosity $\zeta/s = 0$, although it would be interesting to account for finite bulk viscosity in future work, as its functional form is sensitive to the value of the QGP trace anomaly near the QCD phase transition [?].

In order to switch from hydrodynamic field equations to microscopic transport, the VISHNU model asserts a sudden switching temperature T_{sw} at which the hydrody-

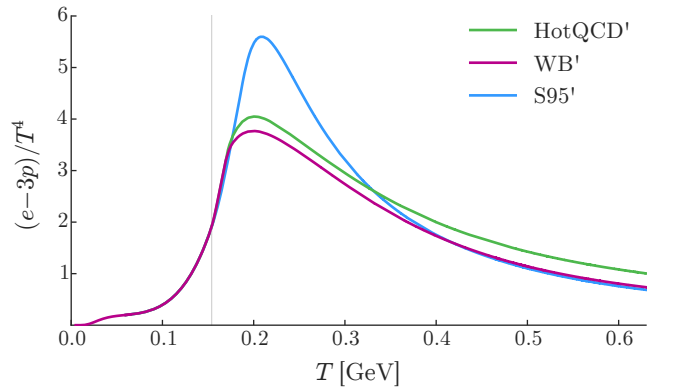


FIG. 2. Comparisons of the entropy density s , energy density e and pressure p divided by powers of the temperature T for the three equations of state used in this study.

TABLE I. Switching energy density and temperature for transition from fluid dynamics to Boltzmann transport.

	S95	HotQCD	WB
e_{sw} [GeV/fm ³]	0.335	0.335	0.335
T_{sw} [GeV]	0.157	0.154	0.154

namic energy-momentum tensor is particlized using the Cooper-Frye freezeout prescription,

$$E \frac{dN_i}{d^3p} = \int_{\sigma} f_i(x, p) p^{\mu} d^3\sigma_{\mu} \quad (5)$$

where f_i is the distribution function of particle species i , p^{μ} is its four-momentum and $d^3\sigma_{\mu}$ characterizes an element of the isothermal freezeout hypersurface defined by T_{sw} .

For a simplistic physical system with a first order phase transition, one would typically fix the hydro to kinetic switching temperature to a value just below the phase transition temperature. This allows the hydrodynamic phase of the model to account for complex dynamics of the phase transition which are difficult to model microscopically. Unfortunately, the prescription for fixing the

switching temperature is not so clear in the case of relativistic heavy-ion collisions where the phase transition is a smooth cross over and cannot be identified with a single transition temperature.

In this work we switch from fluid dynamics to microscopic kinetics using a hypersurface of constant temperature corresponding to a fixed energy density of $e_{\text{sw}} = 335$ MeV. This leads to a slightly different transition temperatures for each equation of state as shown in table I.

IV. POSTPROCESSING AND ANALYSIS

V. RESULTS

VI. CONCLUSION

VII. SUMMARY

VIII. ACKNOWLEDGEMENTS

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 [2] P. Huovinen, Nucl. Phys. **A761**, 296 (2005), arXiv:nucl-th/0505036 [nucl-th].

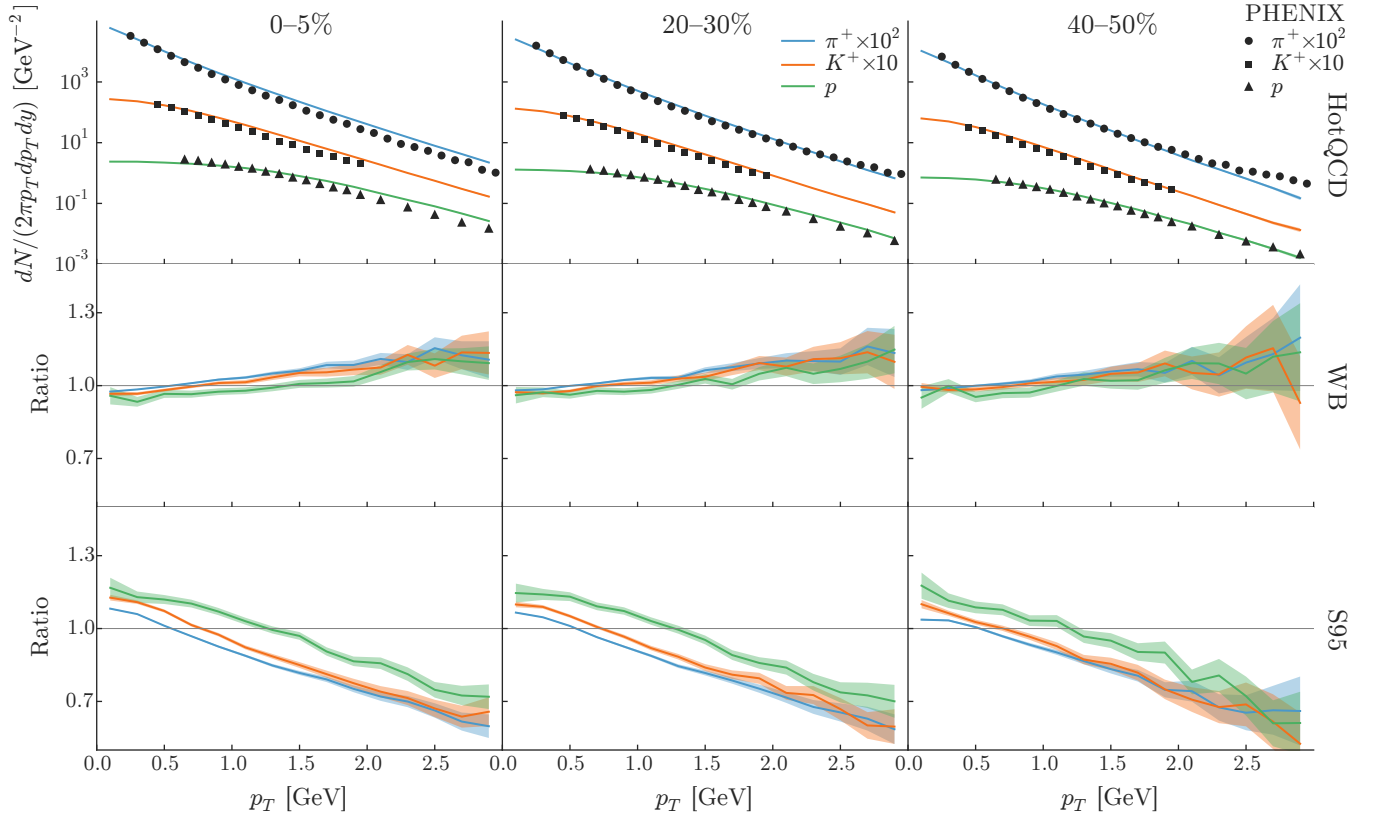


FIG. 3.

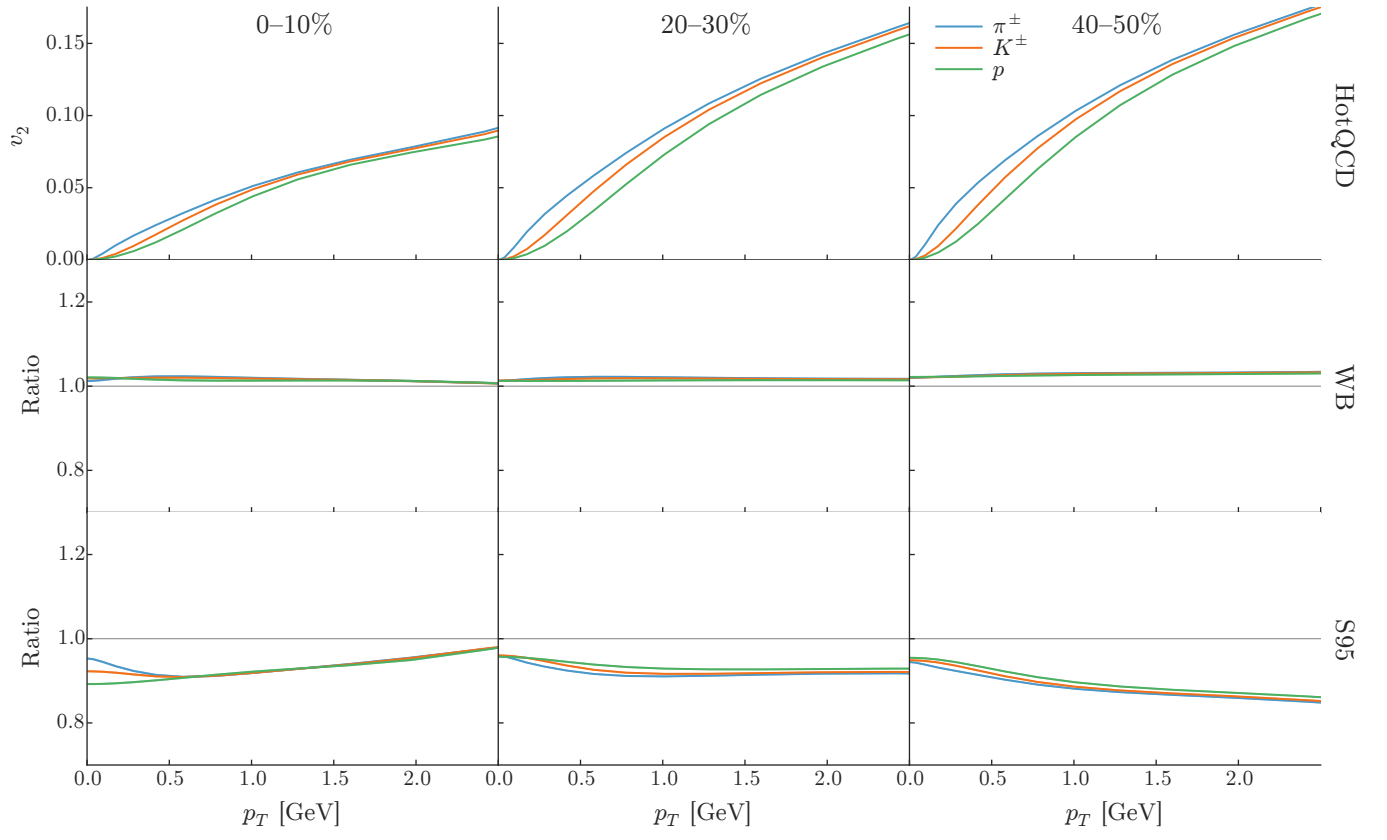


FIG. 4.

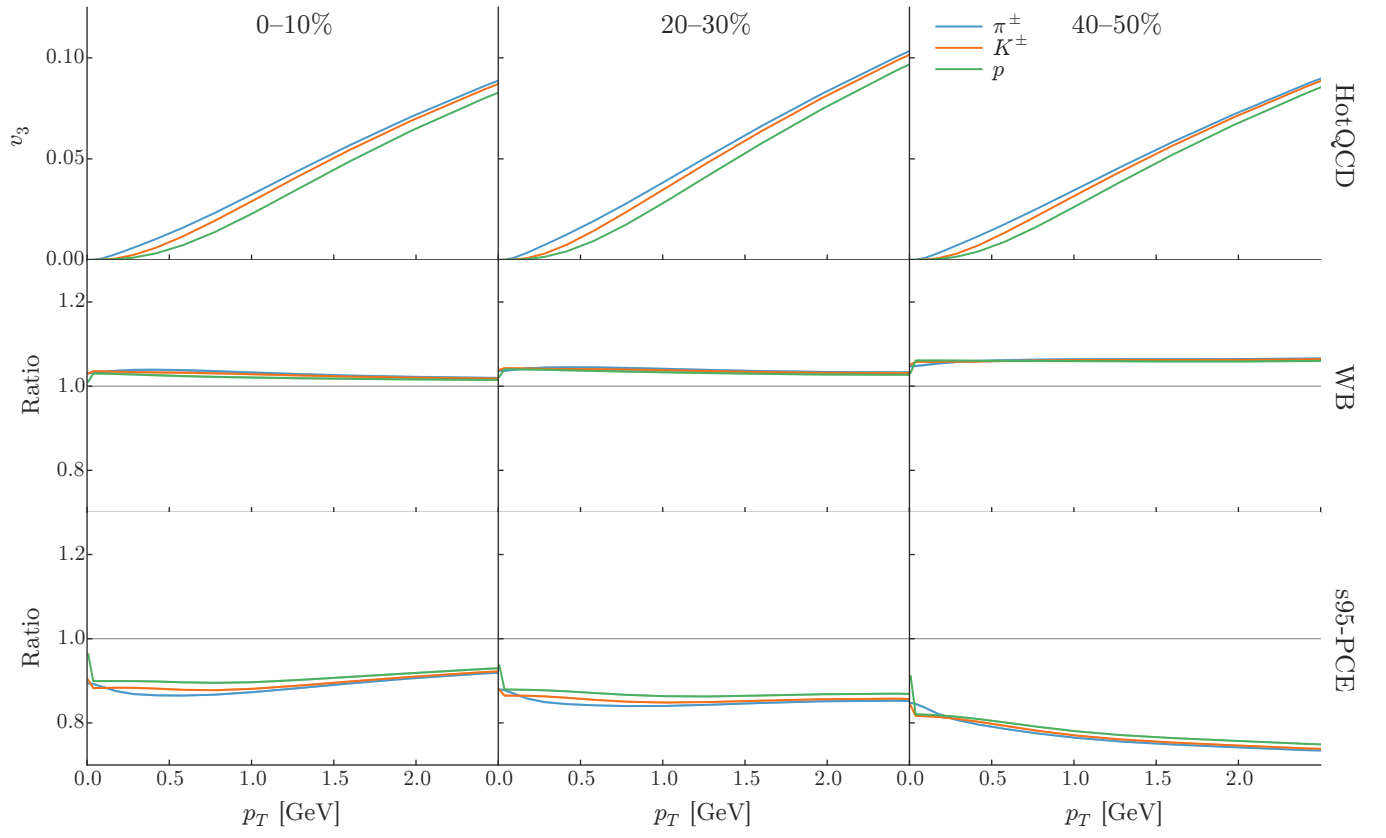


FIG. 5.

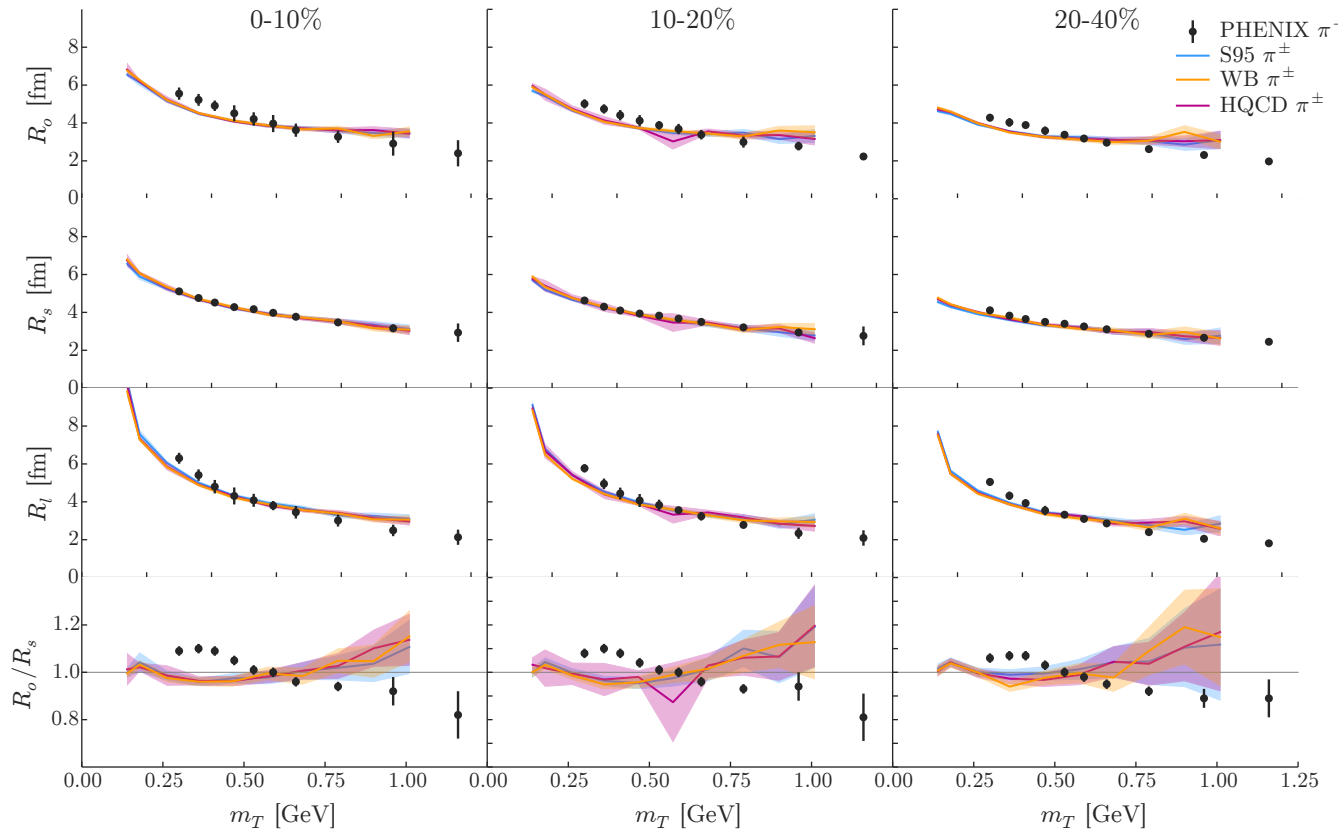


FIG. 6.