

# CS109 – Data Science

Joe Blitzstein, Hanspeter Pfister, Verena Kaynig-Fittkau

[vkaynig@seas.harvard.edu](mailto:vkaynig@seas.harvard.edu)

staff@cs109.org

# Announcements

- Grades for HW2 are getting out tonight
- Final Projects:
  - 3-4 persons per team

(10/12-10/18)	JB	Similarities, recommendations - VK	telecom churn dataset		
Week 8 (10/19-10/25)	Amazon EC2, AWS Datastore, MapReduce - VK	Spark. - RD	Ensemble Methods	HW4	HW3
	<b>Act 3: Bayes, Clustering &amp; Text Analysis</b>				
Week 9 (10/26-11/1)	Bayesian thinking and methods. Prior distributions, likelihood. Naive Bayes. - JB	Advanced Bayesian Thinking. - JB	EC2 and Spark		
Week 10 (11/2-11/8)	Text Analysis. LDA. Topic Modeling. - JB	Interactive Visualizations. Vega. - HP	Bayesian Thinking	HW5	HW4
Week 11 (11/9-11/15)	Clustering. k-means. Mean Shift. Hierarchical Clustering. - VK	Effective Presentations. - HP / JB	Text Analysis: From Naive Bayes to LDA		PROJECT PROPOSALS DUE
Week 12 (11/16-11/22)	Experimental Design. A/B testing. Tirthankar Dasgupta	Deep Learning. - VK	Wrapup: Completely worked example: Chicago Inspections Dataset		HW5, PROJECT REVIEW WEEK
Week 13	<b>No class</b>		<b>No class</b>		

Proposal: final project aspect —> Review is meeting with TF who likes project

# Next Topics

- ML best practices
  - imbalanced data
  - missing values
- Recommender systems
  - collaborative filtering
  - content-based filtering
- Map Reduce

# Cross Validation



training  
data



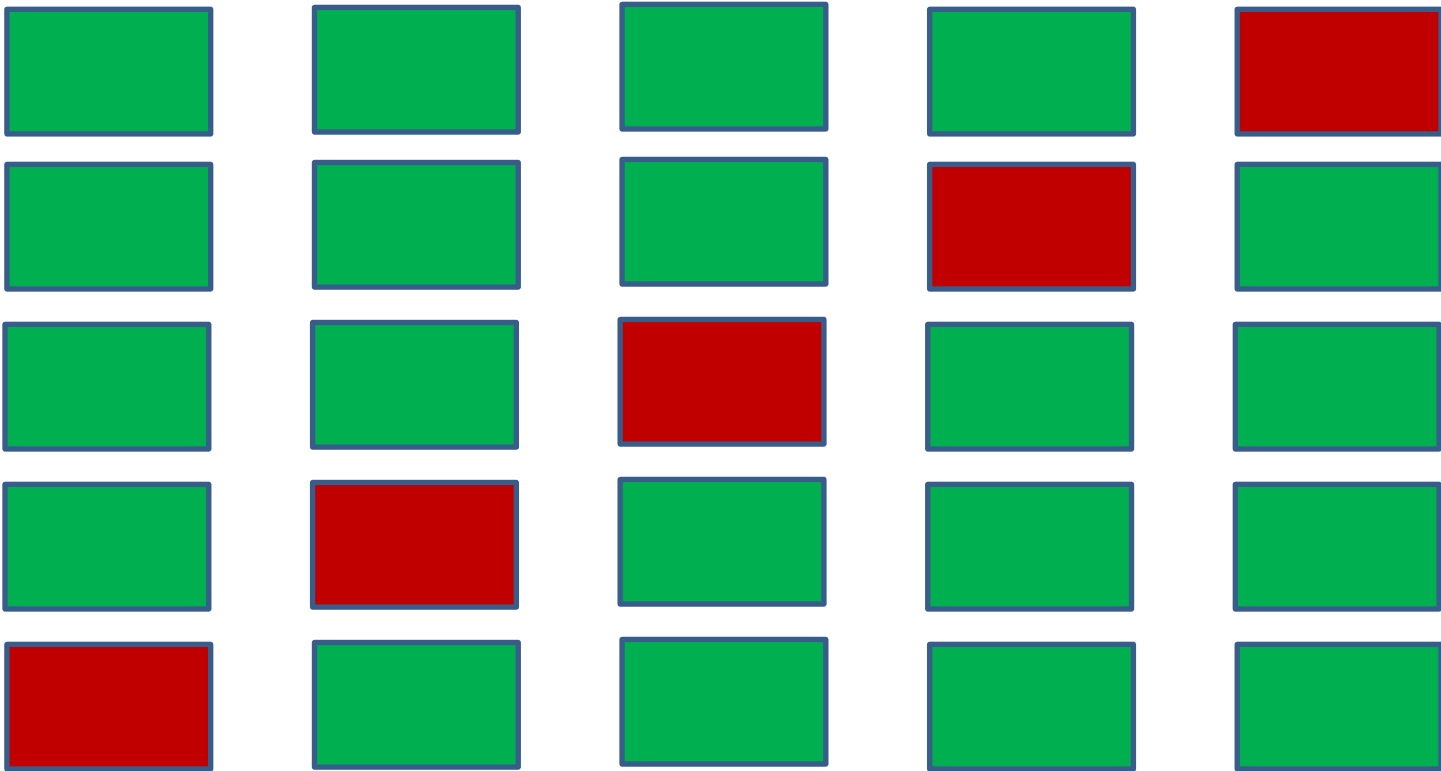
validation  
data



test  
data

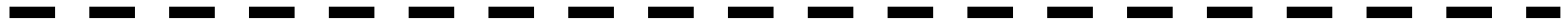
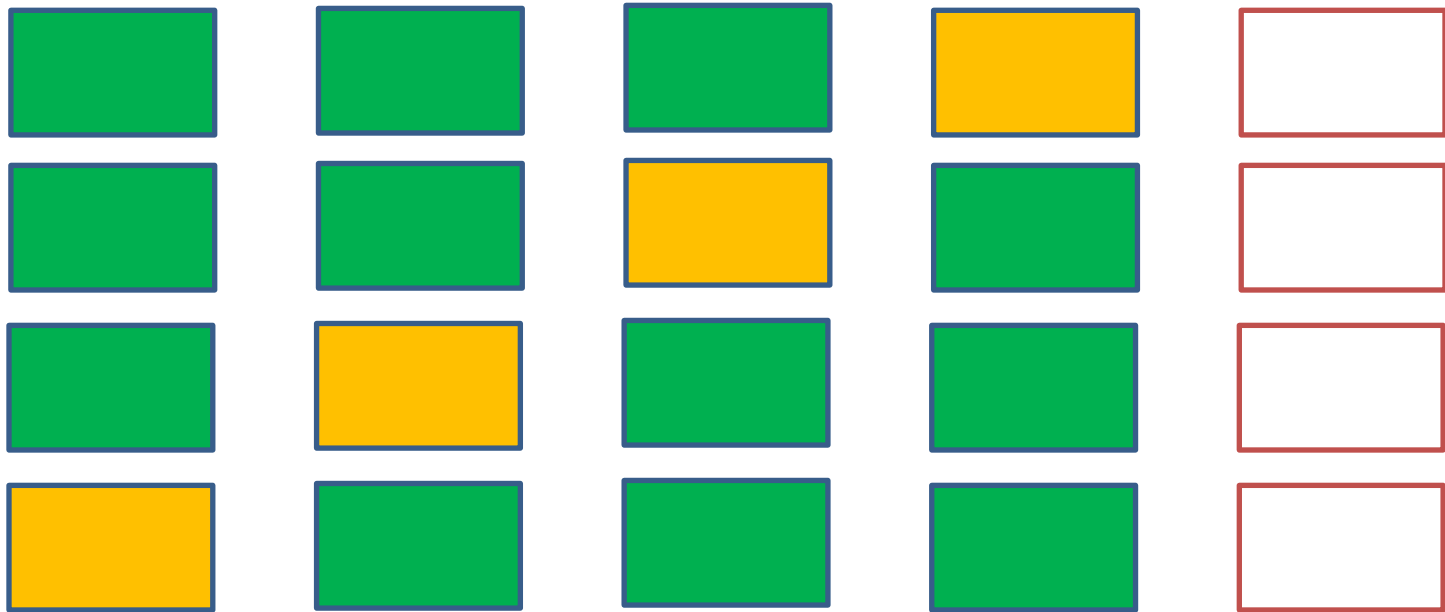
- Training data: train classifier
  - Validation data: estimate hyper parameters
  - Test data: estimate performance
- 
- Be mindful of validation and test set, validation set might refer to test set in some papers.

# 5 – Fold Cross Validation







# 5 – Fold Cross Validation

remove test data



# Last Step of Each Fold

1. Take best parameters  from validation set.
2. Train on training data and validation data together  
3. Test performance on test data   
Only in last step do you touch test data.

This is the **final** result of your method.

# Things to Keep in Mind

- How do you aggregate the parameters?
- What if the hyperparameters are all over the place?
- What if the hyperparameters are at the border of your grid search window?  
Test beyond the window then.



This is wrong: used whole data, including test data for feature selection.

## Scenario - 1

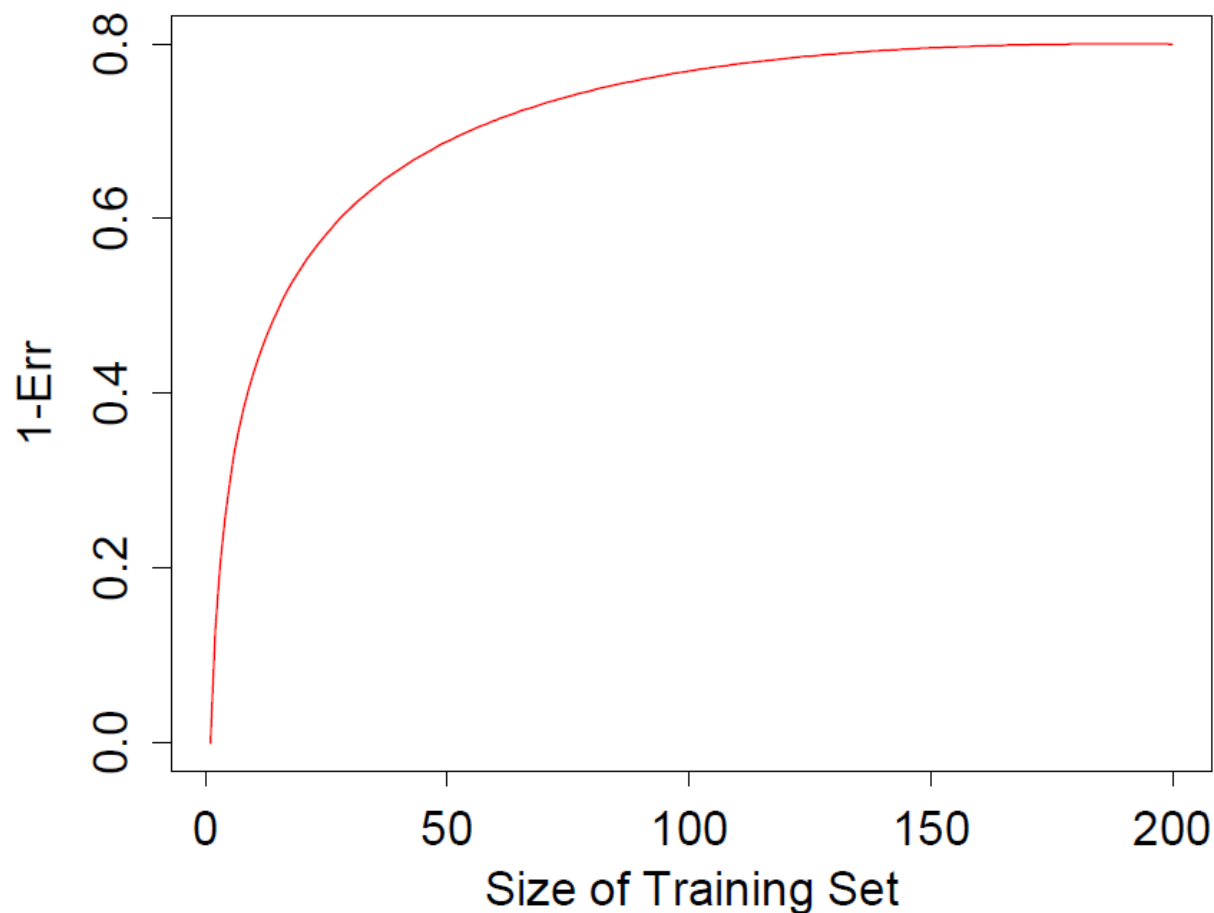
- 1. Screen the predictors: find a subset of “good” predictors that show fairly strong (univariate) correlation with the class labels
- 2. Using just this subset of predictors, build a multivariate classifier.
- 3. Use cross-validation to estimate the unknown tuning parameters and to estimate the prediction error of the final model.

Correct.

## Scenario - 2

- 1. Divide the samples into  $K$  cross-validation folds (groups) at random.
- 2. For each fold  $k = 1, 2, \dots, K$ 
  - Find a subset of “good” predictors that show fairly strong (uni-variate) correlation with the class labels, using all of the samples except those in fold  $k$ .
  - Using just this subset of predictors, build a multivariate classifier, using all of the samples except those in fold  $k$ .
  - Use the classifier to predict the class labels for the samples in fold  $k$ .

# Effect of Sample Size



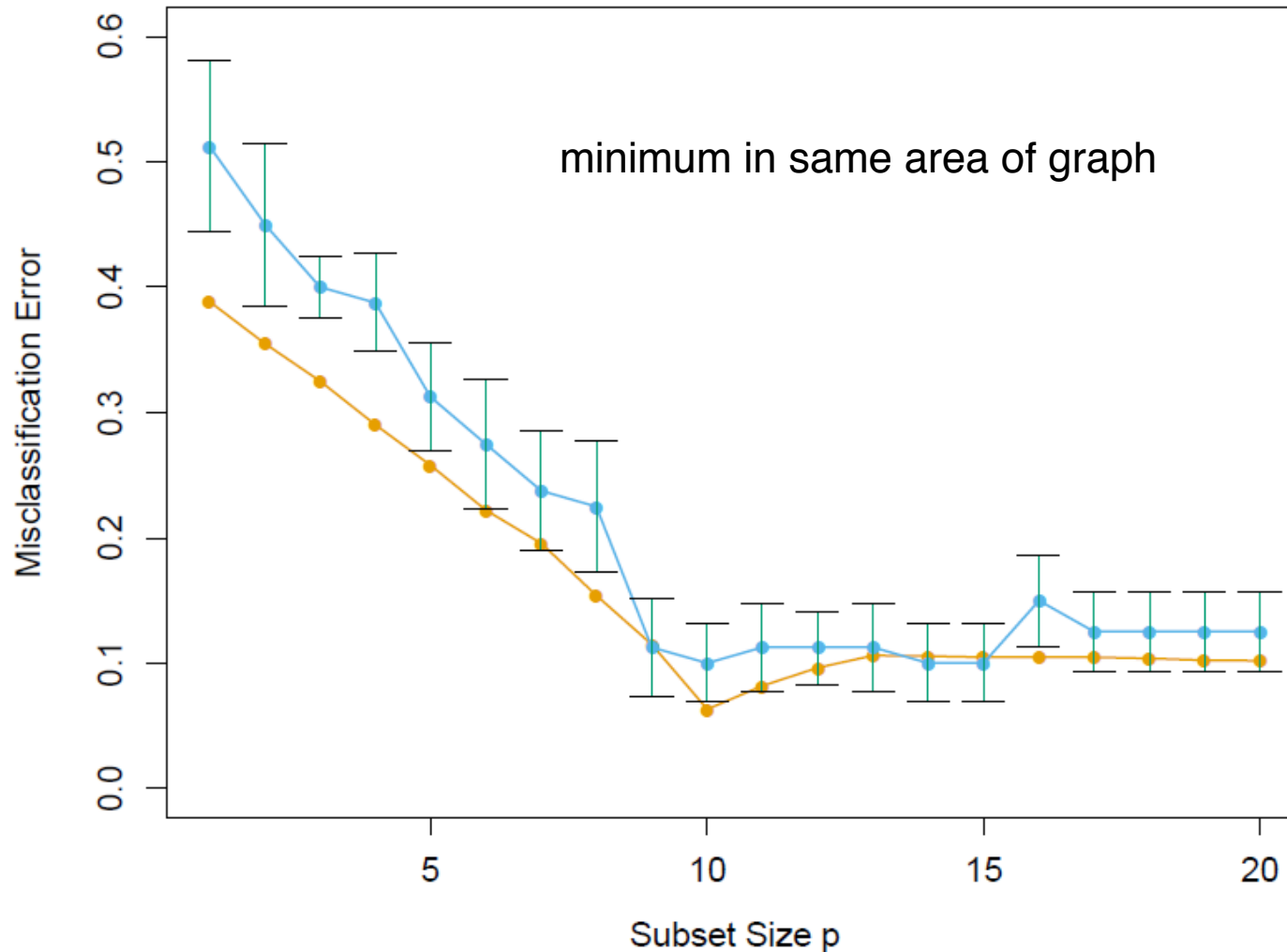
5-fold cross validation:

- $n=200 \Rightarrow 160$  samples
- $n=50 \Rightarrow 40$  samples

small dataset, get big  
diff in error,

This is ok because under-  
promised, but perhaps  
using fewer folds  $\rightarrow$   
set aside less for test and  
you can use more in training

# Cross Validation Over Estimates Error

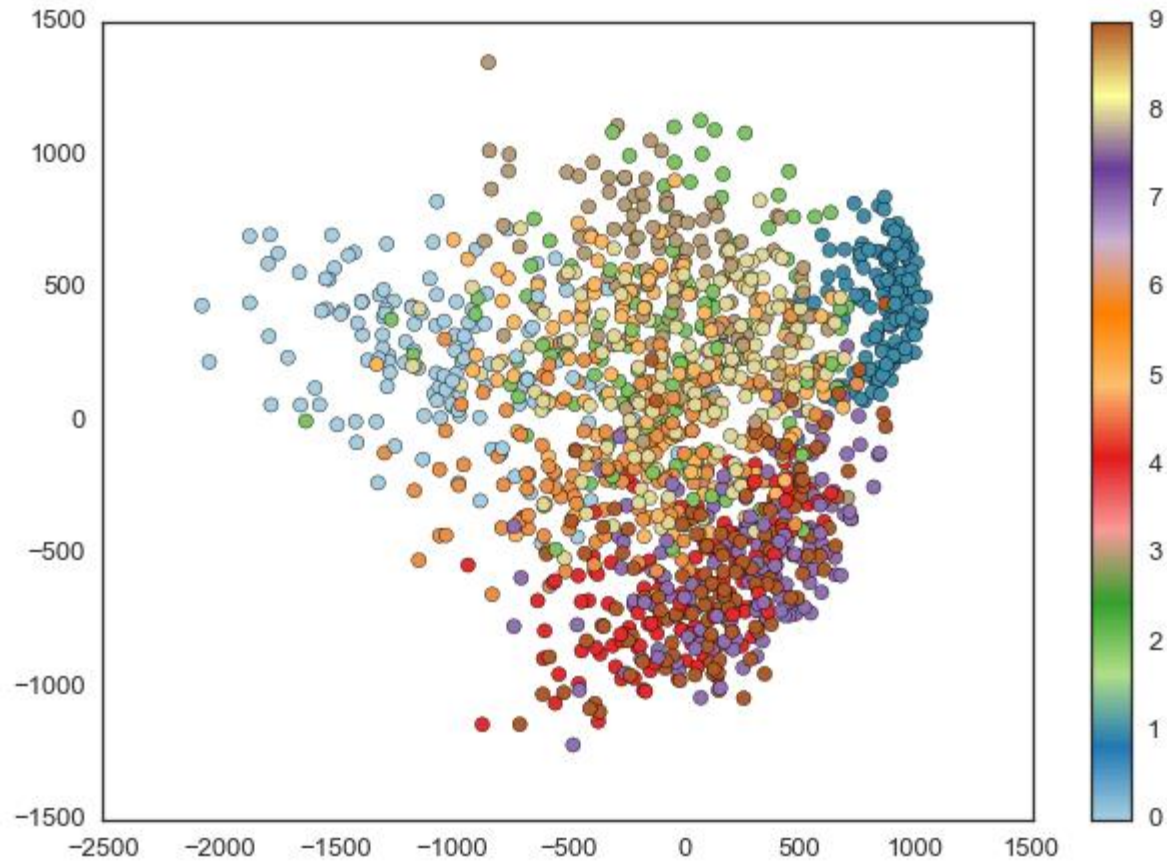


# Normalization

- Be very careful.
- Do not leak into the test data.
- Think about what is useful.  
Units, standard deviation, range of data?

Already normalized: features go (0,1)

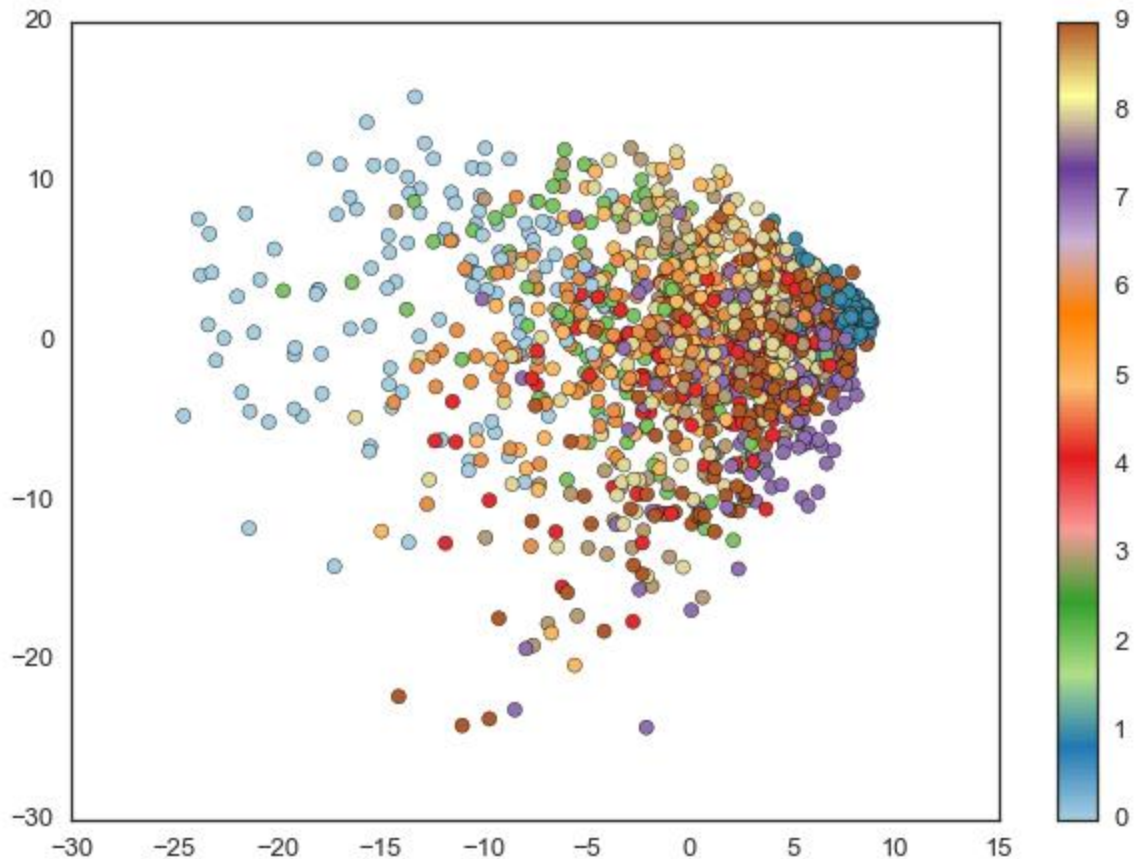
# Example PCA on MNIST



standard PCA

Worse than above: visualization ruined.

# Example PCA on MNIST



PCA with normalized std dev

normalize all parts of data separately. —> this is fine

assumes all data from same distribution: and it's ok to have small validation and test sets

# Normalization - 1



training



Estimate  
mean  
values and  
normalize.



validation



Estimate  
mean  
values and  
normalize.



test

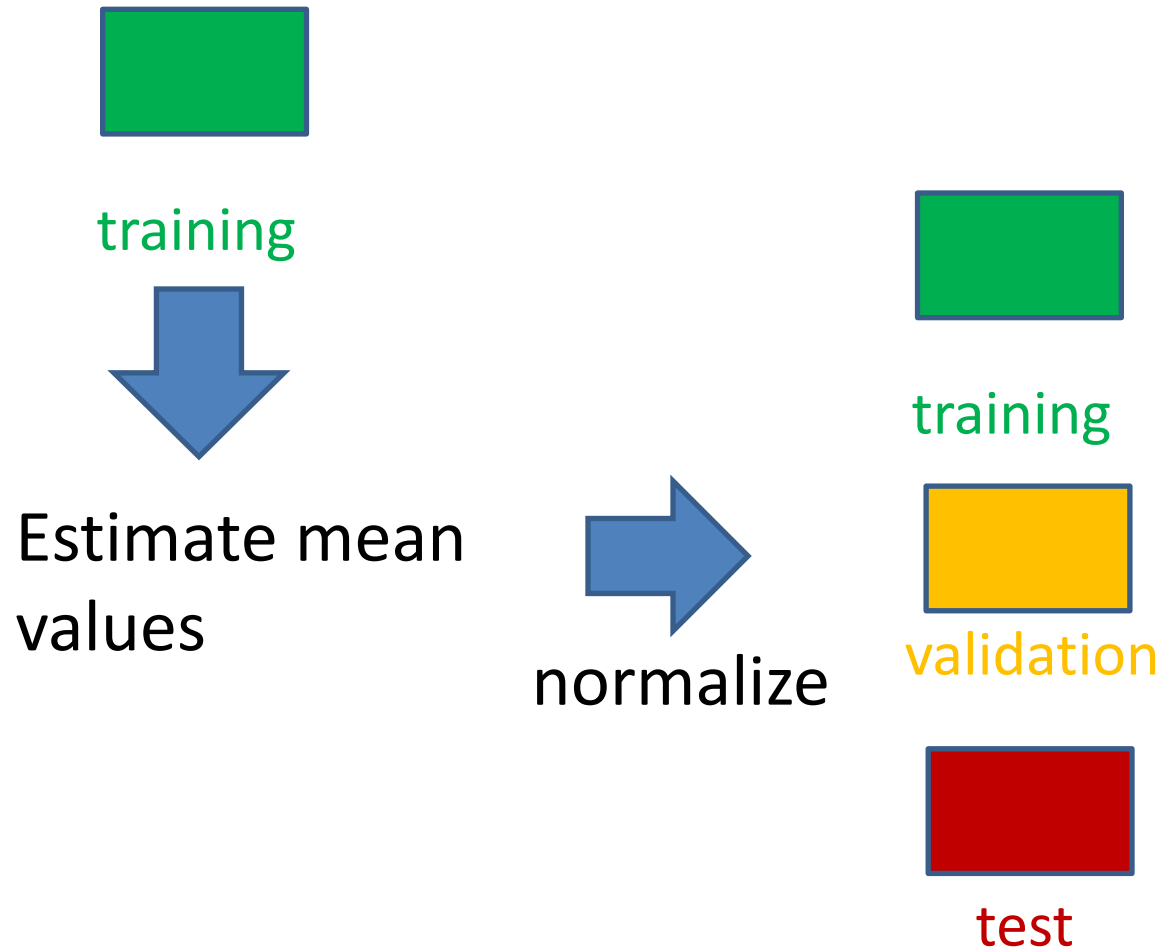


Estimate  
mean  
values and  
normalize.



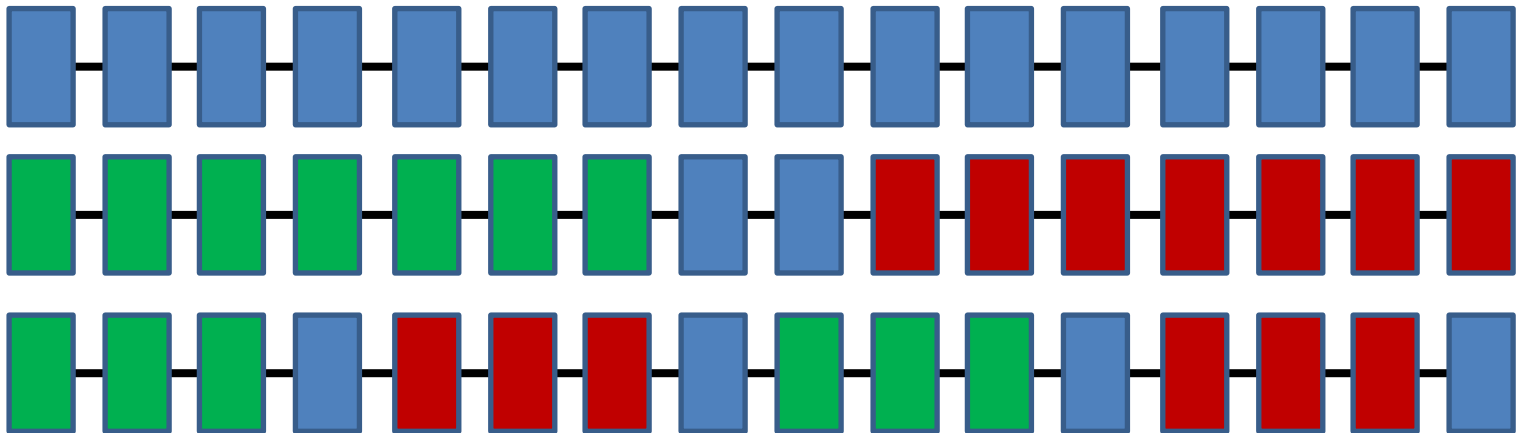
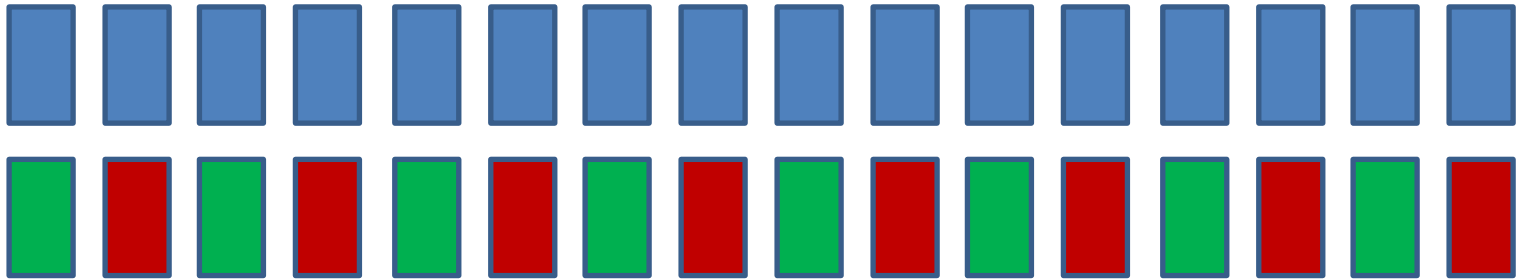
Use largest chunk of data: training to get out mean, then normalize separately  
Now we don't assume same distribution.

## Normalization - 2



This cross validation scheme meant for independent samples.  
What if the data are correlated to each other, even if randomly selected out?

# Know Your Data



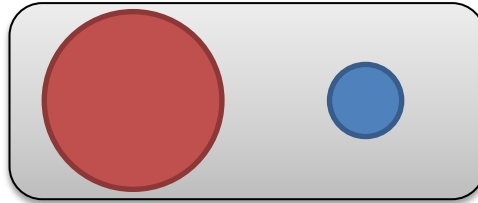
Correlated test and train: well your test data isn't helpful now?

# Imbalanced Data

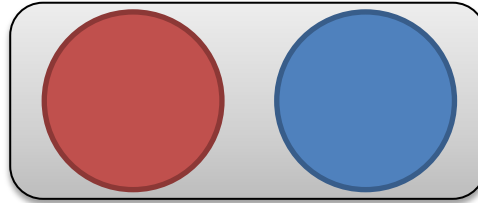
- subsample
- oversample
- re-weight sample points
- use clustering to reduce majority class
  
- re-calibrate classifier output  
ie, change threshold.
  
- Beware the easy true negatives

# Imbalanced Classes

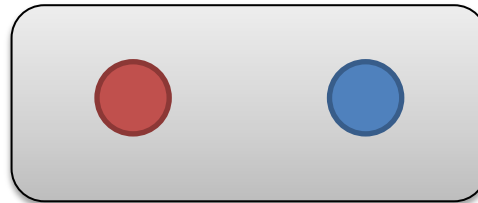
- The Problem:



- Oversample:

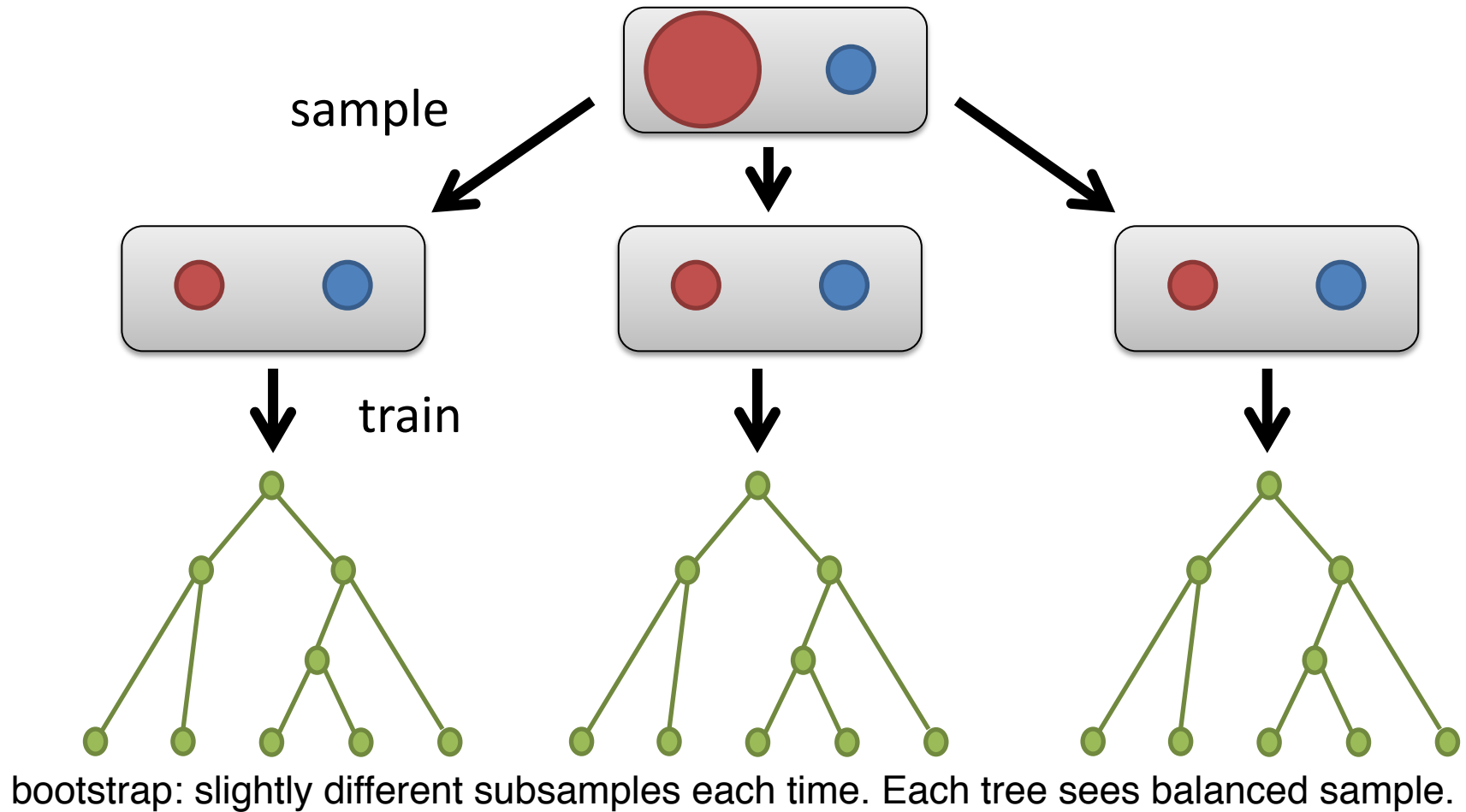


- Subsample:



- Subsample for each tree in a random forest

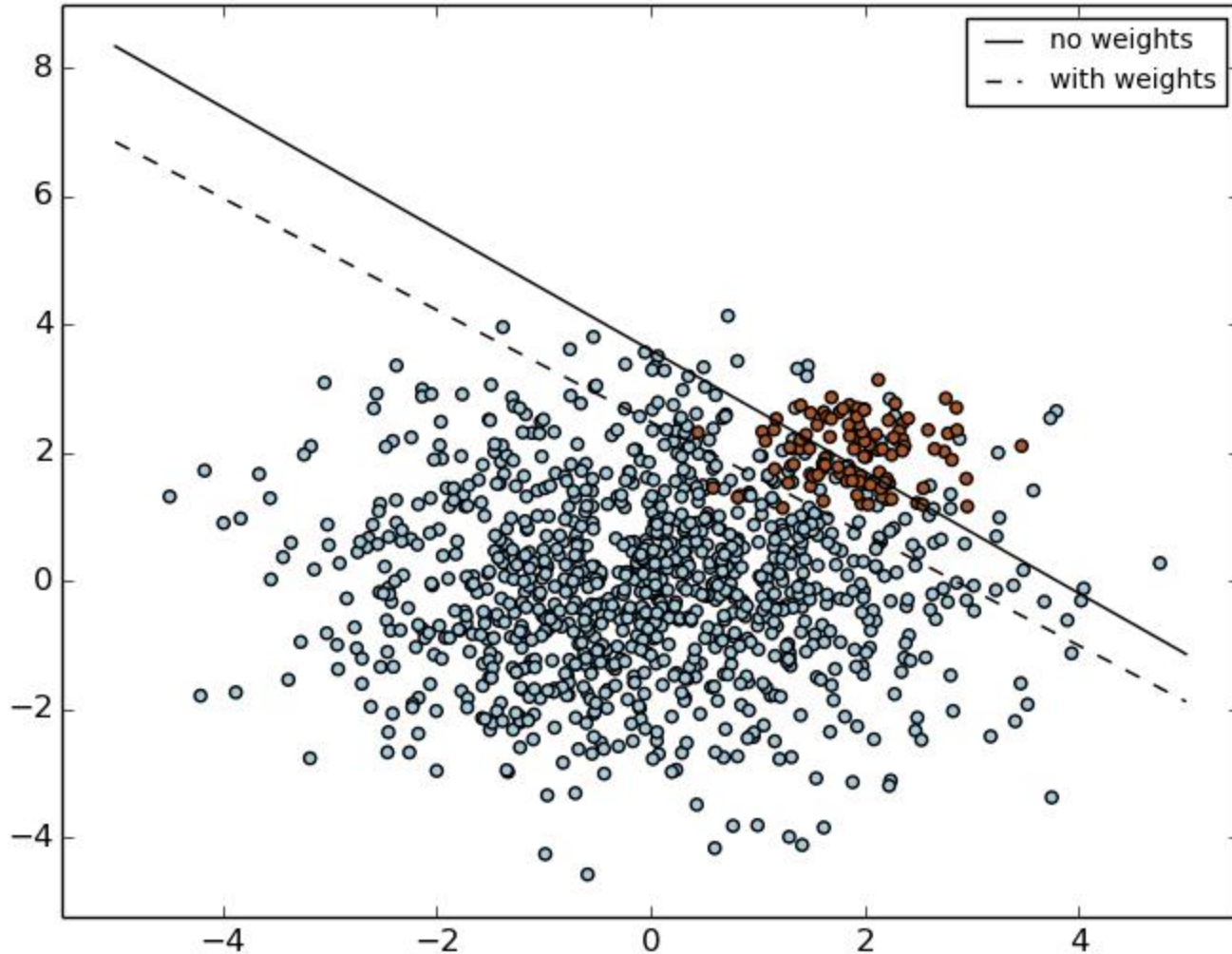
# Example: Random Forest Subsampling



Reweight points. I.e., boosting.

Similar to counting parts of bootstrap sample more.

# Class Weights



[http://scikit-learn.org/stable/\\_images/plot\\_separating\\_hyperplane\\_unbalanced\\_0011.png](http://scikit-learn.org/stable/_images/plot_separating_hyperplane_unbalanced_0011.png)

# Cross Validation with Imbalanced Classes

- Think about using stratified sampling to generate the folds
- The goal is to have the same class ratio in training, validation and test set.

# Missing data

- Delete data points
  - Can cause sample size to be way too small
- Use the mean of the feature
  - Does not change the sample mean, but is independent of the other features.  
destroyed correlation within one observation and other features.
- Use regression to estimate the value
  - Values will be deterministic  
interpolating



I think you'd like to buy X given you've bought Y before.

# Recommender Systems

- We are already surrounded by them



# Good Resources (also for this lecture)

Survey on recommender systems by Michael D. Ekstrand et al.

- <http://files.grouplens.org/papers/FnT%20CF%20Recsys%20Survey.pdf>

Good slides from Stanford lecture by Lester Mackey

- <http://web.stanford.edu/~lmackey/papers/cf-slides-pml09.pdf>


























recommendation system: want to fill in blanks based on other:

want to use users similar to the one predicting on based on past preferences.

# Rating Matrix Completion Problem

features

observations  
(users)

# Collaborative Filtering

Insight: Personal preferences are correlated

- If Jack loves A and B, and Jill loves A, B, and C, then Jack is more likely to love C
- Does not rely on item or user attributes (e.g. demographic info, author, genre)

May recommend diff category (ie movie genre) based on preferences of other  
—> move outside comfort zone.

Netflix: genre, actors, screen writer, etc. Recommend movies that are similar along these features if user has liked something similar before.

# Content-based Filtering

- Each item is described by a set of features
- Measure similarity between items
- Recommend items that are similar to the items the User liked

Keeps user in comfort zone: similar items.

# Comparison

- Collaborative filtering:
  - Items entirely described by user ratings
  - Good for new discoveries
  - People who like SciFi maybe also like Fantasy
- Content-based filtering:
  - Predictions are in users comfort zone
  - Can start with a single item
- Can do a hybrid approach

# User Based Collaborative Filtering

Intuition:

- I like what people similar to me like
- Users give ratings
- People with similar ratings in the past  
assumed to have similar ratings in the future

cold start problem: new user, no information.

# Item-based Collaborative Filtering

- Similar to user-based, but looks at the items instead of the users
- Useful if the user base is way larger than the number of items.
- More useful: Items are relatively stable in their rating, users vary more.



# We Could Use Missing Data Strategies

All that we talked about earlier:

- Omitting samples
- Using the mean rating of an item
- Doing regression



# CF as Regression


























- Choose favorite regression algorithm
- Train a predictor for each item
- Each user who rated that item provides one sample
- To predict rating of an item  $A$ , apply predictor for  $A$  to the user's incomplete ratings vector.

# Recommendation by Regression

- **Pros:**
  - Reduces recommendations to a well-studied problem
  - Many good prediction algorithms available
- **Cons:**
  - Have to handle tons of missing data
  - Training  $M$  predictors is expensive








Very fast to train → closets neighbors: ppl similar

# KNN

# KNN for Collaborative Filtering

- Widely used
- Item-based and User-based focus
- Represent each user as incomplete vector of item ratings
- Compute similarity between query user and all other users similarity = proximity
- Find K most similar users who rated the query item
- Predict weighted average of ratings

# Similarity Measures

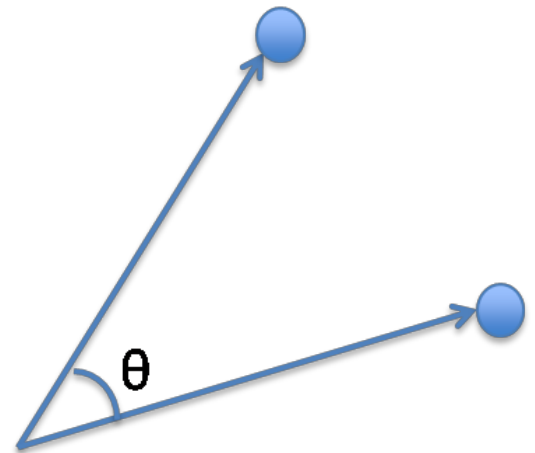
- Pearson Correlation Coefficient
  - bound between 1 and -1
  - suffers from computing high similarity between users with few ratings in common If too many missing values
  - set threshold for minimum number of co-rated items suffers from computing high similarity between users with few ratings in common  
threshold: at least n common ratings.

$$s(u, v) = \frac{\sum_{i \in I_u \cap I_v} (r_{u,i} - \bar{r}_u)(r_{v,i} - \bar{r}_v)}{\sqrt{\sum_{i \in I_u \cap I_v} (r_{u,i} - \bar{r}_u)^2} \sqrt{\sum_{i \in I_u \cap I_v} (r_{v,i} - \bar{r}_v)^2}}$$

# Similarity Measures

- Cosine similarity
  - vector-space approach based on linear algebra
  - Unknown ratings are considered to be 0
  - this causes them to effectively drop out of the numerator

$$\text{sim}(A, B) = \cos(\theta) = \frac{A \cdot B}{\|A\| \|B\|}$$



# Netflix Prize

- Remember when we saw the Netflix prize video they mentioned SVD
- SimonFunk did this publicly on his blog with the title “Try this at home”
- <http://sifter.org/~simon/journal/20061027.2.html>



# Singular Value Decomposition

The diagram illustrates the Singular Value Decomposition (SVD) of a matrix  $A$ . Matrix  $A$  is represented by a large rectangle with dimensions  $|U|$  (height) and  $|I|$  (width). It is equal to the product of three matrices:  $U$ ,  $\Sigma$ , and  $T^T$ . Matrix  $U$  is a tall, narrow rectangle. Matrix  $\Sigma$  is a small square with dimension  $k$  indicated above it, and is labeled "diagonal matrix" below it. Matrix  $T^T$  is a wide, short rectangle.

$$\begin{matrix} |I| \\ \boxed{A} \\ |U| \end{matrix} = \boxed{U} \begin{matrix} k \\ \boxed{\Sigma} \\ \text{diagonal matrix} \end{matrix} \boxed{T^T}$$

- If we know the SVD, we could compute the missing values in  $R$ .
- Try to infer SVD from matrix with missing data, and reconstruct full matrix  $R$

# Best SVD Explanation I have seen!

- Leskovec, Rajaraman, Ullman
- <https://www.youtube.com/watch?v=YKmkAolUxkU>

# ■ $A = U \Sigma V^T$ - example: Users to Movies

Matrix

	Matrix	Alien	Serenity	Casablanca	Amelie
1	1	1	0	0	
3	3	3	0	0	
4	4	4	0	0	
5	5	5	0	0	
0	2	0	4	4	
0	0	0	5	5	
0	1	0	2	2	

U

$\Sigma$

$V^T$

$n$

$m$

$=$

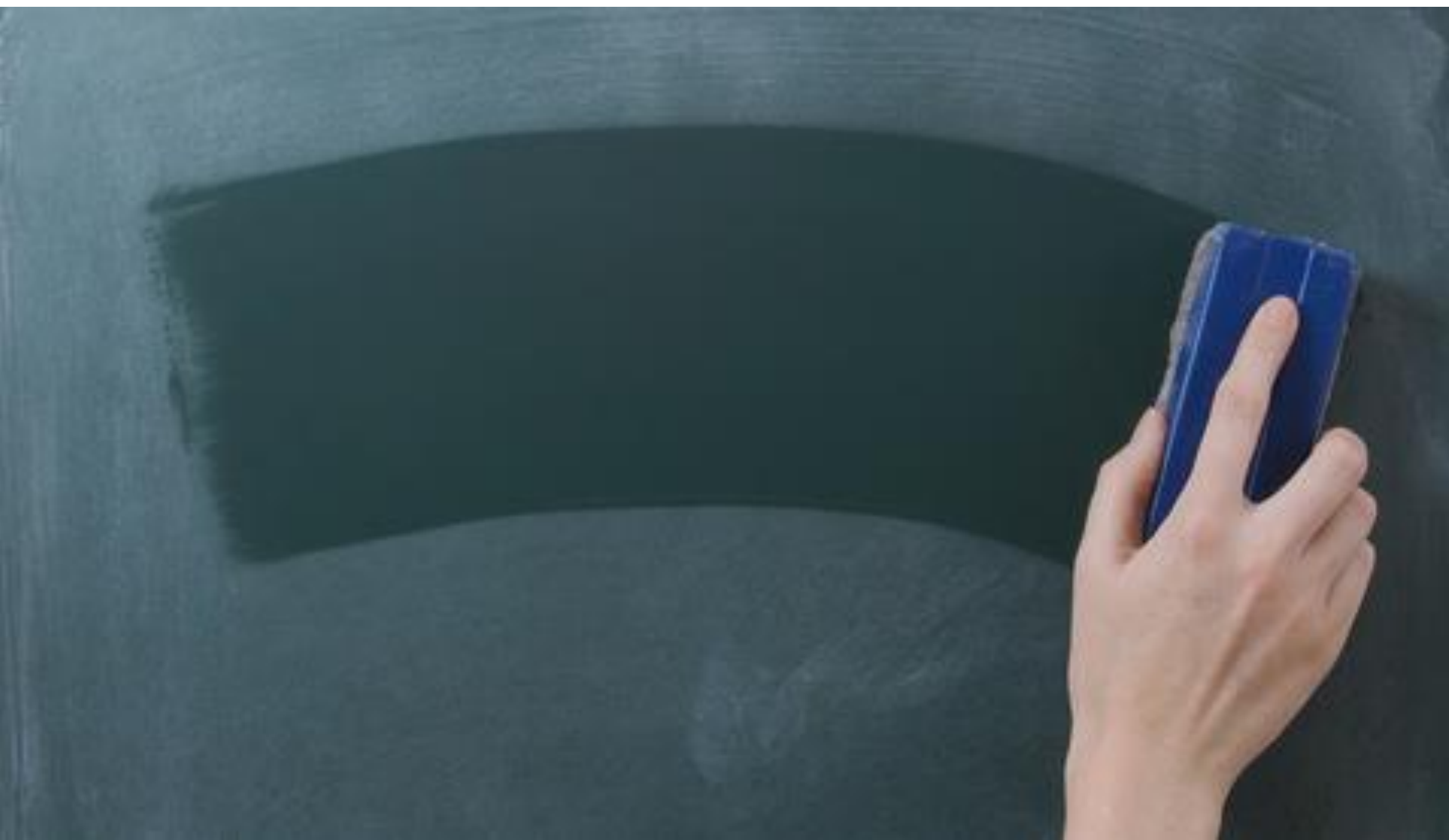
# ■ $A = U \Sigma V^T$ - example: Users to Movies

$$\begin{array}{c} \text{Matrix} \\ \text{Alien} \\ \text{Serenity} \\ \text{Casablanca} \\ \text{Amelie} \end{array} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 \\ 4 & 4 & 4 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 2 & 0 & 4 & 4 \\ 0 & 0 & 0 & 5 & 5 \\ 0 & 1 & 0 & 2 & 2 \end{bmatrix} = \begin{array}{c} \text{U: Users x Topics} \\ \text{scifi} \quad \text{romance} \quad \text{romance} \end{array} \begin{bmatrix} 0.13 & 0.02 & -0.01 \\ 0.41 & 0.07 & -0.03 \\ 0.55 & 0.09 & -0.04 \\ 0.68 & 0.11 & -0.05 \\ 0.15 & -0.59 & 0.65 \\ 0.07 & -0.73 & -0.67 \\ 0.07 & -0.29 & 0.32 \end{bmatrix} \times \begin{array}{c} \text{eigenvalue} \\ \Sigma: \text{Topics x Topics} \\ \text{eigenvalue diagonal} \\ \text{if mean center: these are sqrt(eigenvalue)} \\ \text{can see 3rd topic low weight/is redundant.} \end{array} \begin{bmatrix} 12.4 & 0 & 0 \\ 0 & 9.5 & 0 \\ 0 & 0 & 1.3 \end{bmatrix} \times \begin{array}{c} \text{V}^T: \text{Topics x Movies} \end{array} \begin{bmatrix} 0.56 & 0.59 & 0.56 & 0.09 & 0.09 \\ 0.12 & -0.02 & 0.12 & -0.69 & -0.69 \\ 0.40 & -0.80 & 0.40 & 0.09 & 0.09 \end{bmatrix}$$

# SVD for Recommender Systems

- Not only good for estimating missing data
- We might actually care about the topics more

Find decomposition for incomplete  $A$ , then reconstruct  $A$ .



# What is Map Reduce

- programming model
  - addressing large data sets very large: pentabytes of data.
  - parallel and distributed algorithms
  - cluster framework
- 
- It also is a way of thinking!

# Map Reduce Background

- Originally developed by Google
- Apache Hadoop is open source implementation in Java
- MrJob is a Python interface to Hadoop  
Does not run from an iPython notebook.

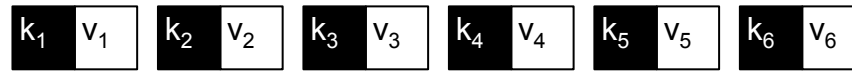




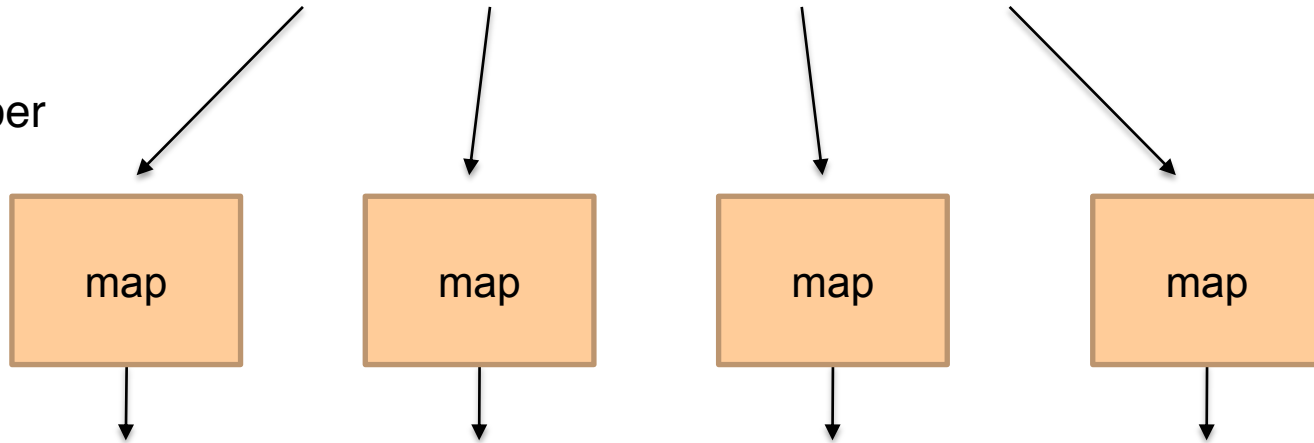
# The Map and the Reduce

- Map:
  - performs filtering and sorting
- Reduce:
  - summary operation

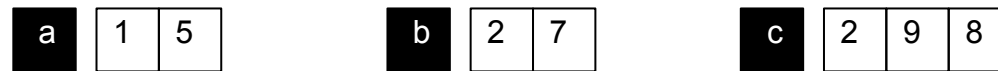
key, value pair.



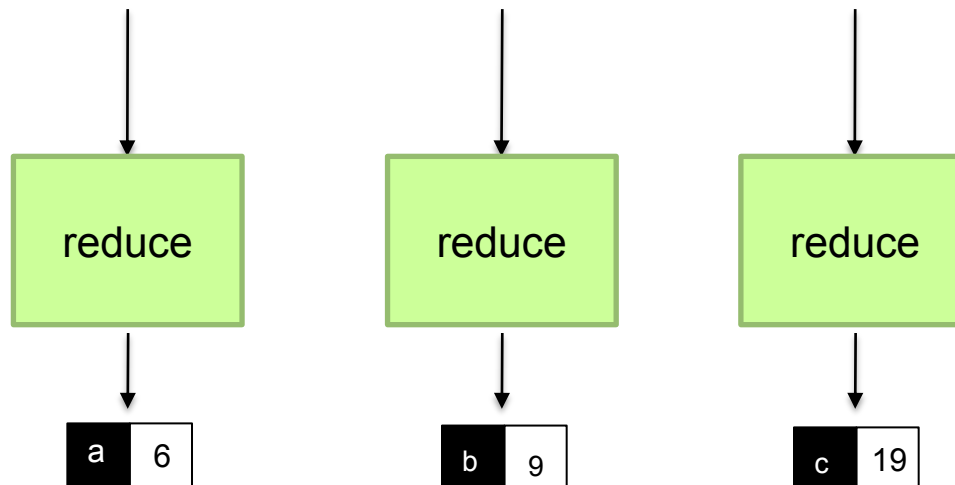
program mapper



shuffle done automatically **Shuffle and Sort:** aggregate values by keys



we implement reduce  
ie, reduce is sum  
of the values.



# The Famous Word Count Example

```
from mrjob.job import MRJob

class mrWordCount(MRJob):

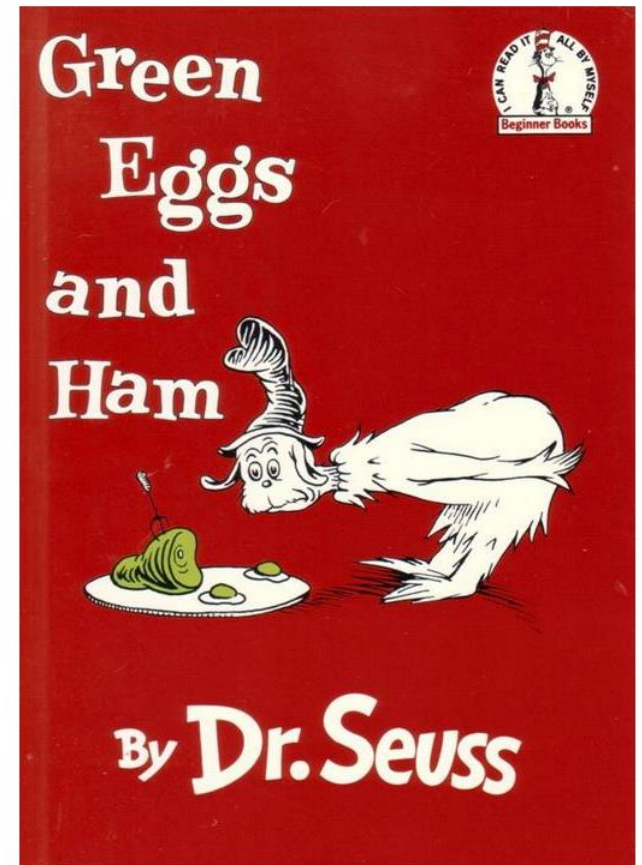
    def mapper(self, key, line):
        for word in line.split(' '):
            yield word.lower(), 1

    def reducer(self, word, occurrences):
        yield word, sum(occurrences)

if __name__ == '__main__':
    mrWordCount.run()
```

# Green Eggs and Ham

- Result of a bet:
- Can Dr. Seuss write a book using only 50 words?
- Bennett Cerf (Dr. Seuss's publisher) lost.
- It is the fourth best selling English-language children's hardcover book of all time.



# Example Input File

I am Sam

I am Sam  
Sam I am

That Sam I am  
That Sam I am  
I do not like  
that Sam I am

Do you like  
green eggs and ham

I do not like them  
Sam I am  
I do not like  
green eggs and ham

```
from mrjob.job import MRJob
```

```
class mrWordCount(MRJob):
```

```
    def mapper(self, key, line):  
        for word in line.split(' '):  
            yield word.lower(), 1
```

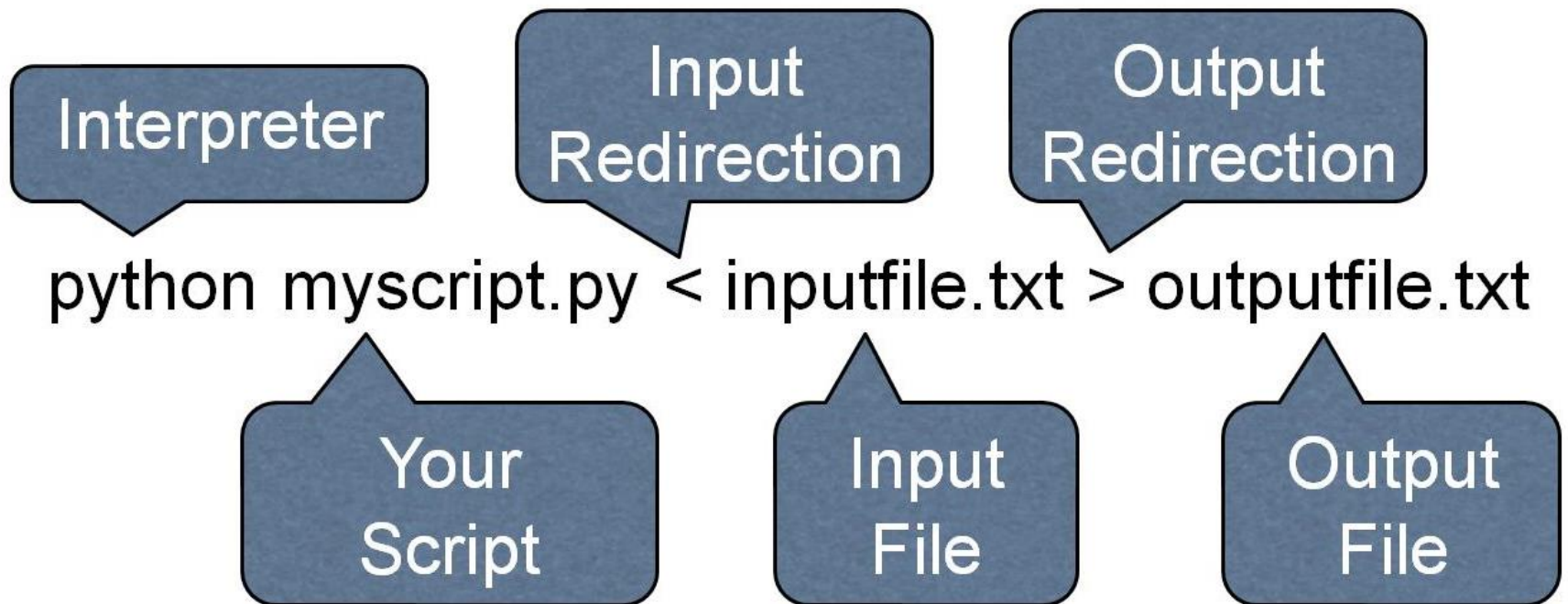
```
    def reducer(self, word, occurrences):  
        yield word, sum(occurrences)
```

```
if __name__ == '__main__':  
    mrWordCount.run()
```

Test files go in: mapper sees row by row.

Shuffle sort handles sorting and aggregating: reduced only sums.

# Launching the Job



# Output File

8

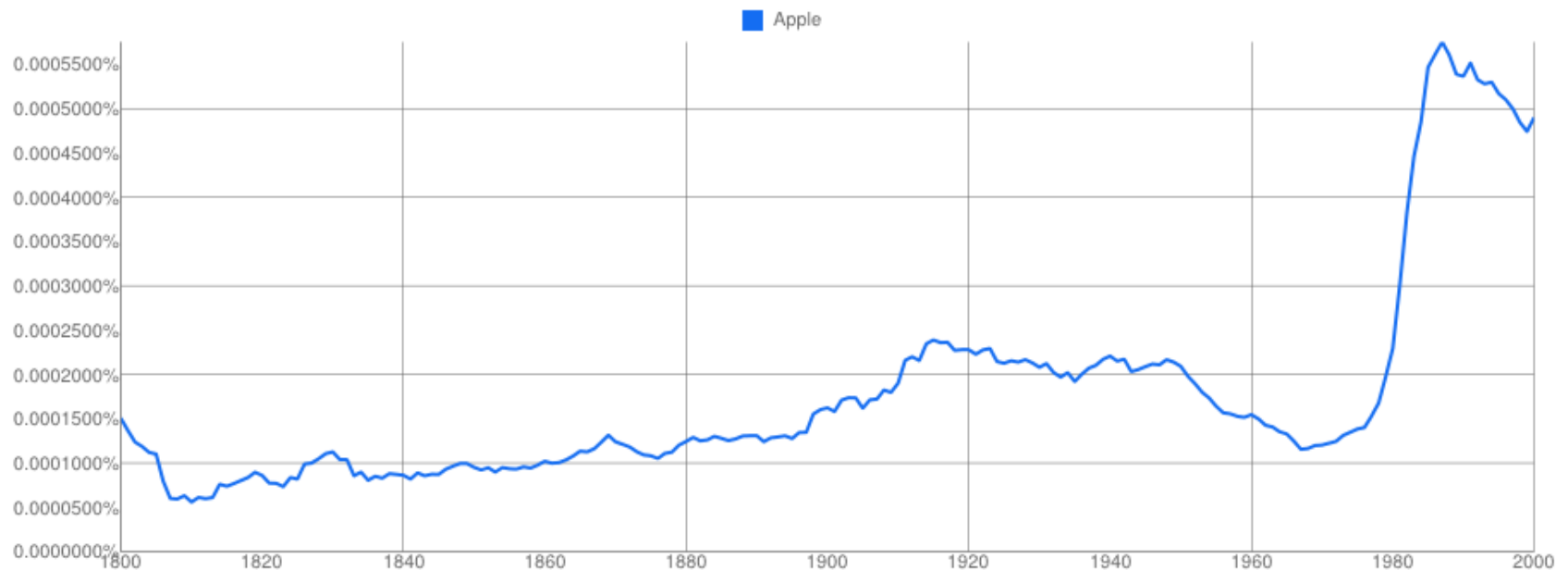
"	36
"a"	60
"am"	16
"and"	24
"anywhere"	
"are"	2
"be"	4
"boat"	3
"box"	7
"car"	7
"could"	14
"dark"	7
"do"	36
"eat"	24
"eggs"	10
"fox"	7
"goat"	4
"good"	2
"green"	10
"ham"	10
"here"	11
"house"	8
"i"	84
"if"	1
"in"	41
"let"	4
"like"	11

This word count on all of google books.

# Culturomics

Google books Ngram Viewer

Graph these **case-sensitive** comma-separated phrases:   
between  and  from the corpus  with smoothing of .



Search in Google Books:

<a href="#">1800 - 1839</a>	<a href="#">1840 - 1978</a>	<a href="#">1979 - 1987</a>	<a href="#">1988 - 1993</a>	<a href="#">1994 - 2000</a>	<a href="#">Apple (English)</a>
-----------------------------	-----------------------------	-----------------------------	-----------------------------	-----------------------------	---------------------------------

Run your own experiment! Raw data is available for download [here](#).



# Anagram Finder

- Anagram: Words or phrases consisting of the same letters
- Examples:
  - Dormitory – Dirty room
  - Astronomer – Moon starer
  - Election results – Lies let's recount
- Verifying anagrams with map reduce
- Input: file with one word per line

```
from mrjob.job import MRJob
```

```
class MRAnagram(MRJob):
```

```
    def mapper(self, _, line):
```

```
        # Convert word into a list of characters, sort them, and convert  
        # back to a string.
```

```
        letters = list(line)
```

```
        letters.sort()
```

```
        # Key is the sorted word, value is the regular word.
```

```
        yield letters, line
```

```
    def reducer(self, _, words):
```

```
        # Get the list of words containing these letters.
```

```
        anagrams = [w for w in words]
```

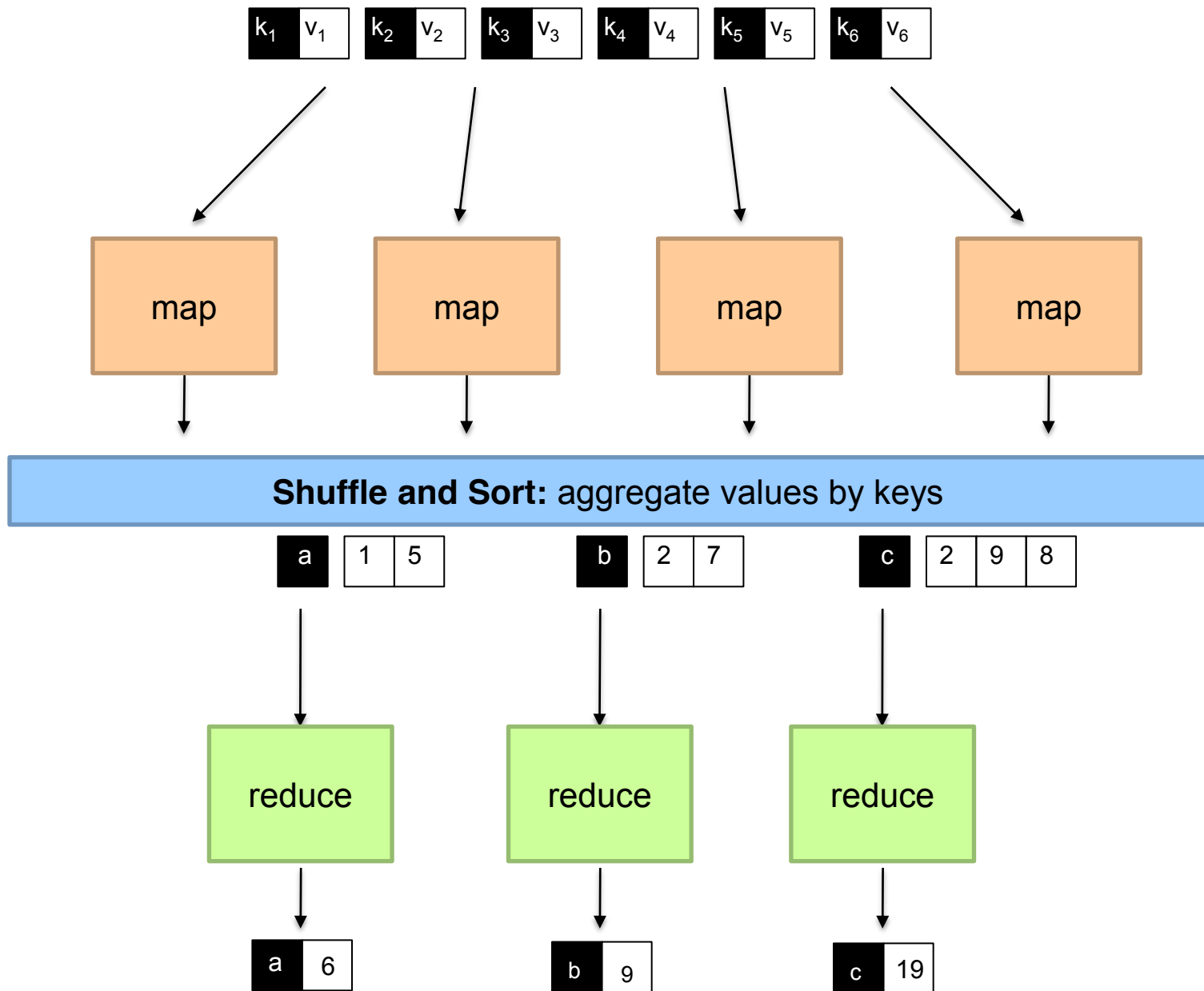
```
        # Only yield results if there are at least two words which are  
        # anagrams of each other.
```

```
        if len(anagrams) > 1: If > 1 then word had anagrams
```

```
            yield len(anagrams), anagrams
```

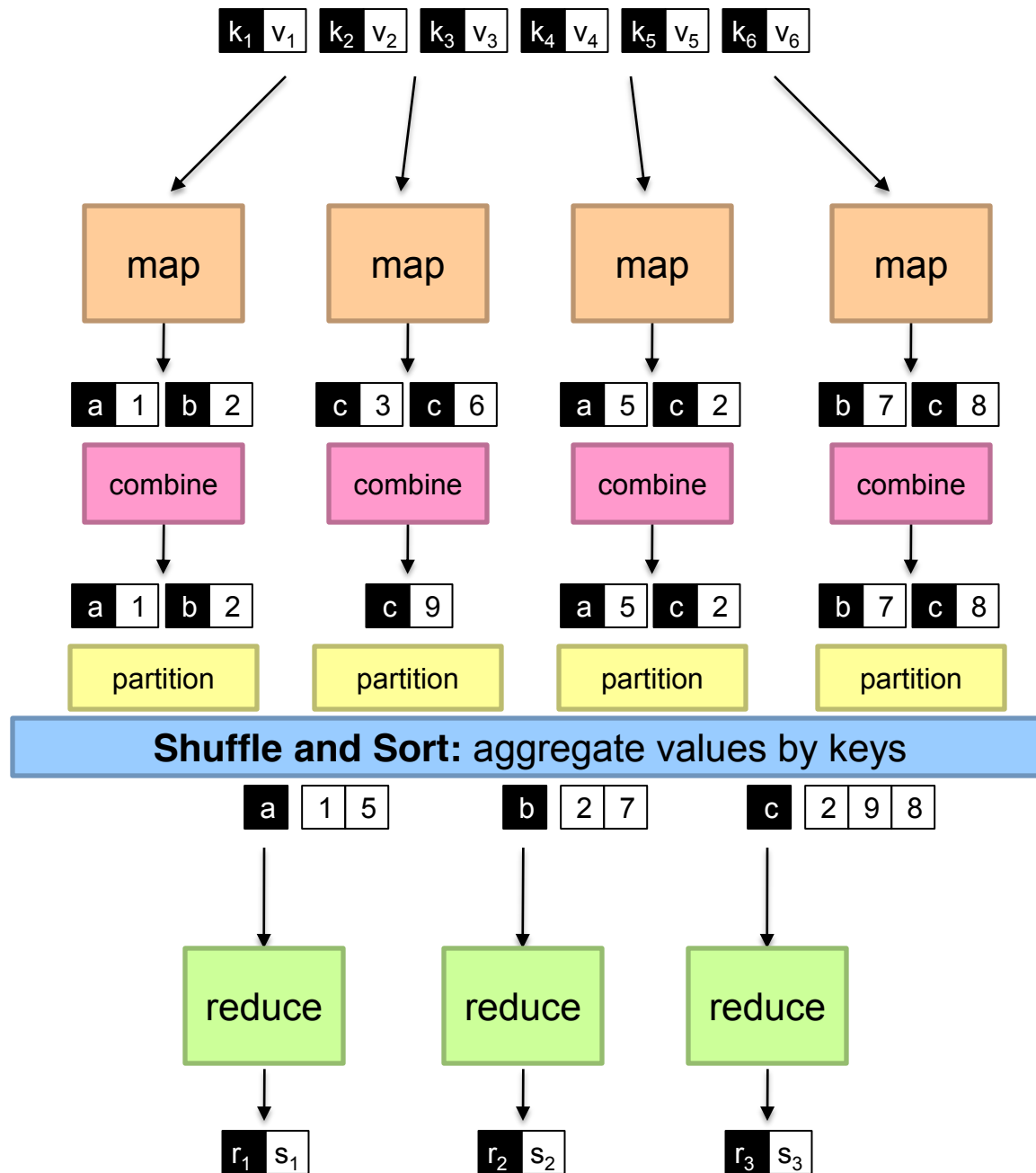
```
if __name__ == "__main__":
```

```
    MRAnagram.run()
```



# Importance of Local Aggregation

- Ideal scaling characteristics:
  - Twice the data, twice the running time
  - Twice the resources, half the running time
- Why can't we achieve this?
  - Synchronization requires communication
  - Communication kills performance
- Thus... avoid communication!
  - Reduce intermediate data via local aggregation
  - Two possibilities:
    - Combiners
    - In-mapper combining



# Combiner

- “mini-reducers”
- Takes mapper output before shuffle and sort
- Can significantly reduce network traffic
- No access to other mappers
- Not guaranteed to get all values for a key
- Not guaranteed to run at all!
- Key and value output must match mapper

Why does the key and value output have to match the mapper output?

# Word Count with Combiner

```
from mrjob.job import MRJob

class mrWordCount(MRJob):

    def mapper(self, key, line):
        for word in line.split(' '):
            yield word.lower(), 1

    def combiner(self, word, occurrences):
        yield word, sum(occurrences)

    def reducer(self, word, occurrences):
        yield word, sum(occurrences)

if __name__ == '__main__':
    mrWordCount.run()
```

# Combiner Design

- Combiners and reducers share same method signature
  - Sometimes, reducers can serve as combiners
  - Often, not...
- Remember: combiners are optional optimizations
  - Should not affect algorithm correctness
  - May be run 0, 1, or multiple times
- Example: find average of all integers associated with the same key



# Computing the Mean: Version 1

```
1: class MAPPER
2:   method MAP(string  $t$ , integer  $r$ )
3:     EMIT(string  $t$ , integer  $r$ )

1: class REDUCER
2:   method REDUCE(string  $t$ , integers  $[r_1, r_2, \dots]$ )
3:      $sum \leftarrow 0$ 
4:      $cnt \leftarrow 0$ 
5:     for all integer  $r \in$  integers  $[r_1, r_2, \dots]$  do
6:        $sum \leftarrow sum + r$ 
7:        $cnt \leftarrow cnt + 1$ 
8:        $r_{avg} \leftarrow sum / cnt$ 
9:       EMIT(string  $t$ , integer  $r_{avg}$ )
```

Why can't we use reducer as combiner?

# Computing the Mean: Version 2

```
1: class MAPPER
2:   method MAP(string  $t$ , integer  $r$ )
3:     EMIT(string  $t$ , integer  $r$ )

1: class COMBINER
2:   method COMBINE(string  $t$ , integers  $[r_1, r_2, \dots]$ )
3:      $sum \leftarrow 0$ 
4:      $cnt \leftarrow 0$ 
5:     for all integer  $r \in$  integers  $[r_1, r_2, \dots]$  do
6:        $sum \leftarrow sum + r$ 
7:        $cnt \leftarrow cnt + 1$ 
8:     EMIT(string  $t$ , pair ( $sum, cnt$ ))           ▷ Separate sum and count

1: class REDUCER
2:   method REDUCE(string  $t$ , pairs  $[(s_1, c_1), (s_2, c_2) \dots]$ )
3:      $sum \leftarrow 0$ 
4:      $cnt \leftarrow 0$ 
5:     for all pair  $(s, c) \in$  pairs  $[(s_1, c_1), (s_2, c_2) \dots]$  do
6:        $sum \leftarrow sum + s$ 
7:        $cnt \leftarrow cnt + c$ 
8:      $r_{avg} \leftarrow sum / cnt$ 
9:     EMIT(string  $t$ , integer  $r_{avg}$ )
```

Why doesn't this work?

# Computing the Mean: Version 3

```
1: class MAPPER
2:   method MAP(string  $t$ , integer  $r$ )
3:     EMIT(string  $t$ , pair ( $r$ , 1))

1: class COMBINER
2:   method COMBINE(string  $t$ , pairs  $[(s_1, c_1), (s_2, c_2) \dots]$ )
3:      $sum \leftarrow 0$ 
4:      $cnt \leftarrow 0$ 
5:     for all pair  $(s, c) \in$  pairs  $[(s_1, c_1), (s_2, c_2) \dots]$  do
6:        $sum \leftarrow sum + s$ 
7:        $cnt \leftarrow cnt + c$ 
8:     EMIT(string  $t$ , pair ( $sum$ ,  $cnt$ ))

1: class REDUCER
2:   method REDUCE(string  $t$ , pairs  $[(s_1, c_1), (s_2, c_2) \dots]$ )
3:      $sum \leftarrow 0$ 
4:      $cnt \leftarrow 0$ 
5:     for all pair  $(s, c) \in$  pairs  $[(s_1, c_1), (s_2, c_2) \dots]$  do
6:        $sum \leftarrow sum + s$ 
7:        $cnt \leftarrow cnt + c$ 
8:      $r_{avg} \leftarrow sum / cnt$ 
9:     EMIT(string  $t$ , pair ( $r_{avg}$ ,  $cnt$ ))
```

Fixed? What if combiner does not run?

# In-Mapper Combining

- “Fold the functionality of the combiner into the mapper by preserving state across multiple map calls

```
1: class MAPPER
2:   method INITIALIZE
3:      $S \leftarrow \text{new ASSOCIATIVEARRAY}$ 
4:      $C \leftarrow \text{new ASSOCIATIVEARRAY}$ 
5:   method MAP(string  $t$ , integer  $r$ )
6:      $S\{t\} \leftarrow S\{t\} + r$ 
7:      $C\{t\} \leftarrow C\{t\} + 1$ 
8:   method CLOSE
9:     for all term  $t \in S$  do
10:       EMIT(term  $t$ , pair ( $S\{t\}, C\{t\}$ ))
```

# In-Mapper Combining

- Advantages
  - Speed
  - Why is this faster than actual combiners?
- Disadvantages
  - Explicit memory management required
  - Potential for order-dependent bugs

# Word Count with In-Mapper-Comb.

```
from collections import defaultdict
from mrjob.job import MRJob

class mrWordCount(MRJob):
    def __init__(self, *args, **kwargs):
        super(mrWordCount, self).__init__(*args, **kwargs)
        self.localWordCount = defaultdict(int)

    def mapper(self, key, line):
        if False:
            yield
        for word in line.split(' '):
            self.localWordCount[word.lower()] += 1

    def mapper_final(self):
        for (word, count) in self.localWordCount.iteritems():
            yield word, count

    def reducer(self, word, occurrences):
        yield word, sum(occurrences)

if __name__ == '__main__':
    mrWordCount.run()
```

# Which is better?

- For large dictionaries?
  - Combiner has no memory problems
- For skewed word distributions (“the”)?
  - In-mapper reduces load on reducer

# Word of Caution

```
from mrjob.job import MRJob
import sys

class SimpleTest(MRJob):

    def __init__(self, *args, **kwargs):
        super(SimpleTest, self).__init__(*args, **kwargs)
        self.test = 1

    def mapper(self, key, value):
        self.test = 2
        yield 1, self.test

    def mapper_final(self):
        yield 1, self.test

    def reducer(self, key, value):
        sys.stderr.write(str(self.test))
        yield 1, value

if __name__ == '__main__':
    SimpleTest.run()
```



1!!!



# Pairs and Stripes:

- Term co-occurrence matrix for a text collection
  - $M = N \times N$  matrix ( $N$  = vocabulary size)
  - $M_{ij}$ : number of times  $i$  and  $j$  co-occur in some context
  - Context can be a sentence, sequence of  $m$  words, etc.
  - In this case co-occurrence matrix is symmetric

# MapReduce: Large Counting Problems

- Term co-occurrence matrix for a text collection  
= specific instance of a large counting problem
  - A large event space (number of terms)
  - A large number of observations (the collection itself)
  - Goal: keep track of interesting statistics about the events
- Basic approach
  - Mappers generate partial counts
  - Reducers aggregate partial counts

# First Try: “Pairs”

- Each mapper takes a sentence:
  - Generate all co-occurring term pairs
  - For all pairs, emit (a, b)  $\rightarrow$  count
- Reducers sum up counts associated with these pairs
- Use combiners!

# Pairs: Pseudo-Code

```
1: class MAPPER
2:   method MAP(docid  $a$ , doc  $d$ )
3:     for all term  $w \in \text{doc } d$  do
4:       for all term  $u \in \text{NEIGHBORS}(w)$  do
5:         EMIT(pair ( $w, u$ ), count 1)      ▷ Emit count for each co-occurrence

1: class REDUCER
2:   method REDUCE(pair  $p$ , counts  $[c_1, c_2, \dots]$ )
3:      $s \leftarrow 0$ 
4:     for all count  $c \in \text{counts } [c_1, c_2, \dots]$  do
5:        $s \leftarrow s + c$                   ▷ Sum co-occurrence counts
6:     EMIT(pair  $p$ , count  $s$ )
```

# “Pairs” Analysis

- Advantages
  - Easy to implement, easy to understand
- Disadvantages
  - Lots of pairs to sort and shuffle around
  - Not many opportunities for combiners to work

# Another Try: “Stripes”

- Idea: group together pairs into an associative array

$(a, b) \rightarrow 1$

$(a, c) \rightarrow 2$

$(a, d) \rightarrow 5$

$(a, e) \rightarrow 3$

$(a, f) \rightarrow 2$

$a \rightarrow \{ b: 1, c: 2, d: 5, e: 3, f: 2 \}$

- Each mapper takes a sentence:
  - Generate all co-occurring term pairs
  - For each term, emit  $a \rightarrow \{ b: \text{count}_b, c: \text{count}_c, d: \text{count}_d \dots \}$
- Reducers perform element-wise sum of associative arrays

$$\begin{array}{r} a \rightarrow \{ b: 1, \quad d: 5, e: 3 \} \\ + \quad a \rightarrow \{ b: 1, c: 2, d: 2, \quad f: 2 \} \\ \hline a \rightarrow \{ b: 2, c: 2, d: 7, e: 3, f: 2 \} \end{array}$$

**Key: cleverly-constructed data structure  
brings together partial results**

# Stripes: Pseudo-Code

```
1: class MAPPER
2:   method MAP(docid  $a$ , doc  $d$ )
3:     for all term  $w \in \text{doc } d$  do
4:        $H \leftarrow \text{new ASSOCIATIVEARRAY}$ 
5:       for all term  $u \in \text{NEIGHBORS}(w)$  do
6:          $H\{u\} \leftarrow H\{u\} + 1$  ▷ Tally words co-occurring with  $w$ 
7:       EMIT(Term  $w$ , Stripe  $H$ )

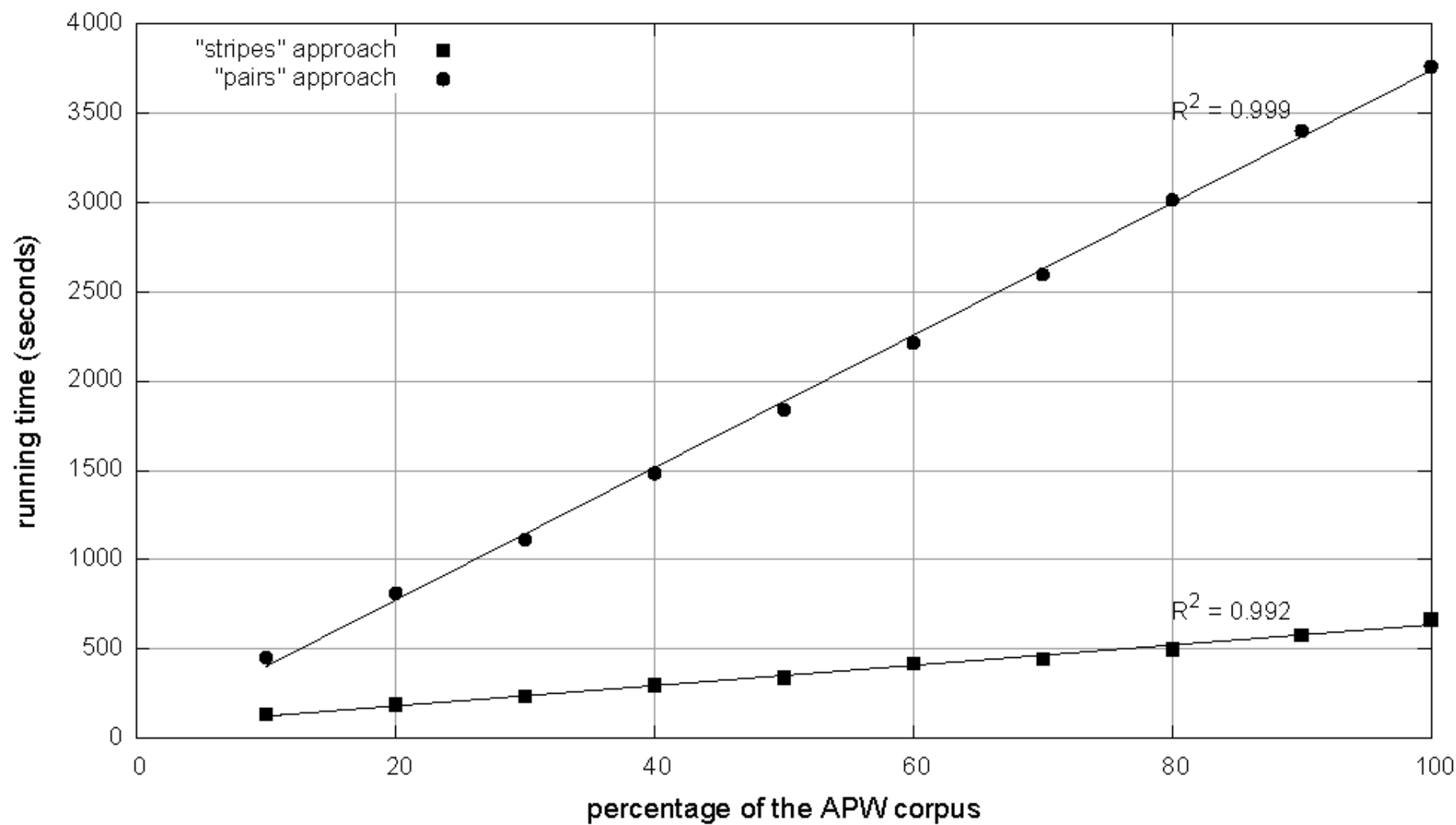
1: class REDUCER
2:   method REDUCE(term  $w$ , stripes  $[H_1, H_2, H_3, \dots]$ )
3:      $H_f \leftarrow \text{new ASSOCIATIVEARRAY}$ 
4:     for all stripe  $H \in \text{stripes } [H_1, H_2, H_3, \dots]$  do
5:       SUM( $H_f, H$ ) ▷ Element-wise sum
6:     EMIT(term  $w$ , stripe  $H_f$ )
```

# “Stripes” Analysis

- Advantages
  - Far less sorting and shuffling of key-value pairs
  - Keys are less unique than in pairs approach
  - Can make better use of combiners
- Disadvantages
  - More difficult to implement
  - Underlying object more heavyweight
  - Fundamental limitation in terms of size of event space



## Comparison of "pairs" vs. "stripes" for computing word co-occurrence matrices



Cluster size: 38 cores

Data Source: Associated Press Worldstream (APW) of the English Gigaword Corpus (v3), which contains 2.27 million documents (1.8 GB compressed, 5.7 GB uncompressed)

# Map Reduce for Machine Learning

- Random Forest?
- SVM?