

# The Pennsylvania System of School Assessment

# Mathematics Item and Scoring Sampler



2024-2025 Grade 8

#### **Grade 8 Formula Sheet**

Formulas that you may need on this test are found below. You may refer back to this page at any time during the mathematics test. You may use calculator  $\pi$  or the number 3.14 as an approximation of  $\pi$ .

2024 Grade 8

#### **Exponential Properties**

$$a^m \cdot a^n = a^{m+n}$$

$$(a^m)^n = a^{m \cdot n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$a^{-n} = \frac{1}{a^n}$$

#### **Algebraic Equations**

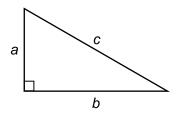
**Slope:** 
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

**Slope-Intercept Form:** 
$$y = mx + b$$

**Point-Slope Form:** 
$$y - y_1 = m(x - x_1)$$

**Standard Form:** 
$$Ax + By = C$$

#### **Pythagorean Theorem**

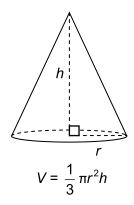


$$a^2 + b^2 = c^2$$

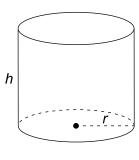
#### **Distance Formula**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

#### Cone

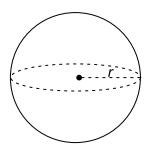


#### Cylinder



 $V = \pi r^2 h$ 

#### **Sphere**



$$V = \frac{4}{3} \pi r^3$$

Question 1 in this Item and Scoring Sampler is to be solved without the use of a calculator.

# **Multiple-Choice Items**

- 1. The speed of sound is approximately  $3 \times 10^2$  meters per second. The speed of light is approximately  $3 \times 10^8$  meters per second. The speed of light is approximately how many times as fast as the speed of sound?
  - (A) 1,000,000
  - ® 3,000,000
  - © 10,000,000,000

Category	Item-Specific Information	
Alignment	B-E.1.1.3	
Answer Key	A	
Depth of Knowledge	1	
p-value A	42% (correct answer)	
p-value B	28%	
p-value C	15%	
p-value D	15%	
Option Annotations	A. Correct: divides $3 \times 10^8$ by $3 \times 10^2$ by dividing the coefficients	
	as $\frac{3}{3}$ = 1 and the powers of 10 as $\frac{10^8}{10^2}$ = $10^{8-2}$ = $10^6$ , which	
	results in a quotient of $1\times10^6$ , and then converts the quotient	
	to standard form by moving the decimal point in 1.0 to the right	
	6 places	
	B. divides the powers of 10 as $\frac{10^8}{10^2} = 10^{8-2} = 10^6$ but then uses	
	the common coefficient (3), resulting in a quotient of $3 \times 10^6$ ,	
	and then converts to standard form	
	C. divides the coefficients as $\frac{3}{3}$ = 1 but then divides the powers of 10 by adding the exponents $\left(\frac{10^8}{10^2} = 10^{8+2} = 10^{10}\right)$ , resulting in a quotient of 1×10 <sup>10</sup> , and then converts to standard form	
	D. uses the common coefficient (3) and divides the powers of 10 by adding the exponents $\left(\frac{10^8}{10^2} = 10^{8+2} = 10^{10}\right)$ , resulting in a quotient of $3 \times 10^{10}$ , and then converts to standard form	

A calculator is permitted for use in solving questions 2-17 in this Item and Scoring Sampler.

- 2. Which pair has one rational number and one irrational number?
  - (a)  $\sqrt{0}$   $\sqrt{9}$

  - ©  $\sqrt{3}$   $\sqrt{6}$
  - (a)  $\sqrt{4}$   $\sqrt{9}$

Category	Item-Specific Information
Alignment	A-N.1.1.1
Answer Key	В
Depth of Knowledge	1
p-value A	20%
p-value B	57% (correct answer)
p-value C	12%
p-value D	11%
Option Annotations	A. considers $\sqrt{0}$ to be irrational (may have confused $\sqrt{0}$ with "dividing by 0")
	B. Correct: recognizes that $\sqrt{4}$ is a rational number since $\sqrt{4}$ simplifies to 2 because $4=2^2$ and recognizes that $\sqrt{5}$ is an irrational number since $\sqrt{5}$ cannot be simplified to a rational number because there is no whole number that when squared equals 5
	C. considers $\sqrt{6}$ to be rational by identifying 6 as a composite number but does not recognize that the prime factors of 6 are 2 and 3 (i.e., $6 = 2 \cdot 3$ ), which means $\sqrt{6}$ cannot be simplified to a rational number since the prime factors do not have even exponents (2 and 3 have exponents of 1)
	D. does not recognize that $\sqrt{4}$ simplifies to 2 and that $\sqrt{9}$ simplifies to 3, which makes both numbers rational

**3.** To determine the width, in units, of a rectangle, Jamal uses a calculator to divide the area of the rectangle by its length. The result on the calculator is shown below.

3.18888889

Which value is most likely the exact width, in units, of the rectangle?

- (A)  $3\frac{17}{99}$
- (B)  $3\frac{17}{90}$
- ©  $3\frac{19}{99}$
- ©  $3\frac{18}{90}$

Category	Item-Specific Information	
Alignment	A-N.1.1.2	
Answer Key	В	
Depth of Knowledge	1	
p-value A	11%	
p-value B	54% (correct answer)	
p-value C	17%	
p-value D	18%	
Option Annotations	A. begins with the fraction $\frac{18}{100}$ and then subtracts 1 from the numerator and from the denominator	
	B. Correct: recognizes that 3.188888889 on a calculator	
	represents the repeating decimal number $3.1\overline{8}$ and then	
	either represents $3.1\overline{8}$ as $3 + 0.1 + 0.0\overline{8}$ , converts $0.1$ to $\frac{1}{10}$ ,	
	and converts $0.0\overline{8}$ as $0.0\overline{8} = 0.1 \cdot 0.\overline{8} = \frac{1}{10} \cdot \frac{8}{9} = \frac{8}{90}$ ,	
	resulting in $3.1\overline{8} = 3 + 0.1 + 0.0\overline{8} = 3 + \frac{1}{10} + \frac{8}{90} =$	
	$3 + \frac{9}{90} + \frac{8}{90} = 3 + \frac{17}{90} = 3\frac{17}{90}$ OR writes the equation	
	$x = 3.1\overline{8}$ , $10x = 31.8\overline{8}$ , and $100x = 318.8\overline{8}$ , and subtracts	
	$10x = 31.8\overline{8}$ from $100x = 318.8\overline{8}$ , resulting in a difference of	
	90x = 287, and then solves for x by dividing both sides by 90,	
	resulting in $x = \frac{287}{90} = 3\frac{17}{90}$	
	C. uses the non-repeating decimal digits (1 and 9) to create a fraction using 99 as the denominator	
	D. uses the 18 as if it were the repeating decimal digits and uses 90 rather than 99 as the denominator	

- **4.** Which set of statements correctly describes how to compare the values of  $5\sqrt{7}$  and  $7\sqrt{5}$ ?
  - ⓐ  $5\sqrt{7} \approx 17.5$   $7\sqrt{5} \approx 5.9$  $5\sqrt{7} > 7\sqrt{5}$
  - (a)  $5\sqrt{7} \approx 7.6$  $7\sqrt{5} \approx 5.9$  $5\sqrt{7} > 7\sqrt{5}$
  - ©  $5\sqrt{7} \approx 13.2$   $7\sqrt{5} \approx 17.5$  $5\sqrt{7} < 7\sqrt{5}$
  - ⓐ  $5\sqrt{7} \approx 13.2$   $7\sqrt{5} \approx 15.7$  $5\sqrt{7} < 7\sqrt{5}$

Category	Item-Specific Information	
Alignment	A-N.1.1.4	
Answer Key	D	
Depth of Knowledge	2	
p-value A	12%	
<i>p</i> -value B	12%	
p-value C	14%	
p-value D	62% (correct answer)	
Option Annotations	A. calculates $5\sqrt{7}$ as $5\cdot\frac{7}{2}$ and $7\sqrt{5}$ as $\sqrt{7\cdot5}$ B. calculates $5\sqrt{7}$ as $5+\sqrt{7}$ and $7\sqrt{5}$ as $\sqrt{7\cdot5}$ C. calculates $5\sqrt{7}$ as $5\cdot\sqrt{7}$ but then calculates $7\sqrt{5}$ as $7\cdot\frac{5}{2}$ D. Correct: calculates $5\sqrt{7}$ as $5\cdot\sqrt{7}$ , resulting in the product 13.22875, calculates $7\sqrt{5}$ as $7\cdot\sqrt{5}$ , resulting in the product 15.65247, and then uses these values to compare the two expressions	

**5.** A computer monitor uses square pixels to display an image. The expression shown below represents the approximate area, in square inches, of one pixel on the computer monitor.

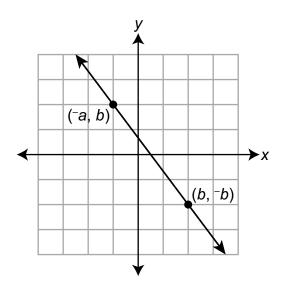
$$(1.04 \times 10^{-2})^2$$

What is the approximate area, in square inches, of one pixel on the computer monitor?

- $\triangle$  1.0816 × 10<sup>-4</sup>
- $\blacksquare$  1.0816 × 10<sup>-2</sup>
- ©  $1.0816 \times 10^{0}$
- 1.0816 × 10<sup>4</sup>

Category	Item-Specific Information
Alignment	B-E.1.1.4
Answer Key	A
Depth of Knowledge	1
<i>p</i> -value A	60% (correct answer)
<i>p</i> -value B	12%
<i>p</i> -value C	17%
<i>p</i> -value D	11%
Option Annotations	A. Correct: recognizes that the expression $(1.04 \times 10^{-2})^2$ can be represented as $(1.04 \times 10^{-2})(1.04 \times 10^{-2}) = 1.04^2 \times (10^{-2})^2$ , squares the 1.04, resulting in 1.0816, and then applies the exponential property $(a^m)^n = a^{m \cdot n}$ to simplify $(10^{-2})^2$ as $10^{-2 \cdot 2}$ , resulting in $10^{-4}$ B. squares only the 1.04  C. squares the 1.04 but adds the exponents when simplifying $(10^{-2})^2$ D. squares the 1.04 but squares the exponent $-2$ when simplifying $(10^{-2})^2$

6. A line on a coordinate grid is shown below.



Which equation can be used to determine the slope (m) of the line?

Category	Item-Specific Information	
Alignment	B-E.2.1	
Answer Key	D	
Depth of Knowledge	1	
p-value A	27%	
p-value B	17%	
p-value C	16%	
p-value D	40% (correct answer)	
Option Annotations	A. uses the reciprocal of the slope and reverses the order of the difference in the numerator	
	B. reverses the order of the difference in the denominator	
	C. uses the reciprocal of the slope	
	D. Correct: applies the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ , using $\begin{pmatrix} -a, b \end{pmatrix}$	
	for $(x_1, y_1)$ and $(b, b)$ for $(x_2, y_2)$ by substituting $b$ for $y_2$ ,	
	$b$ for $y_1$ , $b$ for $x_2$ , and $a$ for $x_1$	

- 7. Information about the costs of apples at an orchard is listed below.
  - The cost of apples is \$1.00 per pound when picked by an employee.
  - The equation t = 0.85n describes the relationship between the total cost (t), in dollars, and the number of pounds of apples (n) when picked by a customer.

An employee and a customer both pick some apples. The employee and the customer each picked \$34.00 worth of apples. How many more pounds of apples did the customer pick than the employee picked?

- A 5.1
- B 6.0
- © 40.0
- 62.9

Category	Item-Specific Information	
Alignment	B-E.2.1.1	
Answer Key	В	
Depth of Knowledge	2	
p-value A	16%	
p-value B	45% (correct answer)	
p-value C	32%	
<i>p</i> -value D	7%	
Option Annotations	A. determines that the employee picked 34 pounds of apples since the cost is \$1.00 per pound of apples but then determines that the customer picked 28.9 pounds of apples by substituting 34.00 for $n$ (rather than for $t$ ) into the equation $t = 0.85n$ , resulting in $t = 0.85(34.00) = 28.90$ , and then finds the difference between the two amounts picked $(34 - 28.9 = 5.1)$	
	B. Correct: determines that the employee picked 34 pounds of apples since the cost is \$1.00 per pound of apples, determines that the customer picked 40 pounds of apples by substituting 34.00 for $t$ into the equation $t = 0.85n$ , resulting in $34.00 = 0.85n$ , and dividing both sides of the equation by $0.85$ resulting in $40 = n$ , and then finds the difference between the two amounts picked $(40 - 34 = 6)$	
	C. substitutes 34.00 for $t$ into the equation $t = 0.85n$ , resulting in $34.00 = 0.85n$ , and then solves the equation for $n$ by dividing both sides of the equation by 0.85, resulting in 40 pounds of apples picked by the customer, but does not consider the pounds of apples picked by the employee	
	D. adds the two costs, resulting in 1 + 0.85 = 1.85, and then multiplies the sum by 34.00	

**8.** Mr. Klavier rents out two different studio spaces to bands. The equations shown below represent the total cost (*y*), in dollars, to rent each studio for *x* hours.

large studio: 
$$y = 80x + 40$$

small studio: 
$$y = 60x + 60$$

Based on the system of equations, which statement about the rental costs is correct?

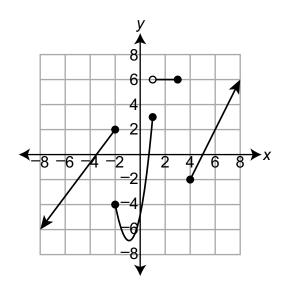
- There is no length of time at which the rental cost for the large studio is equal to the rental cost for the small studio since the graph consists of two curves that never intersect.
- ® There is no length of time at which the rental cost for the large studio is equal to the rental cost for the small studio since the graph consists of two straight lines that never intersect.
- © There is a length of time at which the rental cost for the large studio is equal to the rental cost for the small studio since the graph consists of two curves that intersect at two points.
- There is a length of time at which the rental cost for the large studio is equal to the rental cost for the small studio since the graph consists of two straight lines that intersect at a single point.

Category	Item-Specific Information	
Alignment	B-E.3.1.3	
Answer Key	D	
Depth of Knowledge	2	
p-value A	13%	
<i>p</i> -value B	23%	
p-value C	19%	
p-value D	45% (correct answer)	
Option Annotations	A. recognizes that the equations are not equivalent because the <i>y</i> -intercepts are not equal (40 ≠ 60) but thinks that each equation can be represented by a curve rather than a line and that the curves are "parallel" since 80 + 40 = 60 + 60	
	<ul> <li>B. recognizes that each equation can be represented by a line and that the lines are not collinear since the <i>y</i>-intercepts are not equal (40 ≠ 60) but thinks that the lines are parallel since 80 + 40 = 60 + 60</li> <li>C. thinks that each equation can be represented by a curve rather than a line, which would make two points of intersection possible</li> </ul>	
	D. Correct: recognizes that each equation can be represented by a line since $80x$ and $60x$ both represent constant rates of change (80 and 60 are the rates of change, and the $x$ indicates the rate of change is constant because the exponent for the $x$ variable is 1) and recognizes that the lines will have a single point of intersection since the rates of change are not equal ( $80 \neq 60$ )	

- 9. The Rodriguez family buys 4 student tickets and 2 adult tickets to the school play for a total of \$22. The Delgado family buys 2 student tickets and 3 adult tickets for a total of \$21. What is the price of an adult ticket to the school play?
  - A \$3
  - ® \$5
  - © \$7
  - \$8

Category	Item-Specific Information	
Alignment	B-E.3.1.5	
Answer Key	В	
Depth of Knowledge	2	
<i>p</i> -value A	15%	
<i>p</i> -value B	58% (correct answer)	
p-value C	18%	
p-value D	9%	
Option Annotations	A. solves the system of equations $4x + 2y = 22$ and $2x + 3y = 21$ for $x$ , resulting in $x = 3$ , but misinterprets this solution as the price of an adult ticket rather than the price of a student ticket (i.e., did not define the variables $x$ and $y$ prior to writing the system of equations)	
	B. Correct: writes the equation for the Rodriguez family as	
	4s + 2a = 22, writes the equation for the Delgado family as	
	2s + 3a = 21, and then solves the system of two equations	
	either by multiplying both sides of the second equation by 2,	
	resulting in $4s + 6a = 42$ , subtracting the first equation from	
	the second equation, resulting in $4a = 20$ , and then dividing	
	each side of the equation by 4, resulting in <i>a</i> = 5 OR by	
	multiplying both sides of the first equation by $\frac{1}{2}$ , resulting in	
	2s + 1a = 11, subtracting the first equation from the second	
	equation, resulting in $2a = 10$ , and then dividing each side of	
	the equation by 2, resulting in $a = 5$	
	C. writes the system of equations as $4s + 2a = 22$ and $2s + 3a = 21$ , multiplies both sides of the second equation by 2, resulting in $4s + 6a = 42$ , but then eliminates the $4s$ terms and uses only the remaining part of the second equation, resulting in $6a = 42$ , and then divides each side of the equation by 6, resulting in $a = 7$	
	D. writes the system of equations as $4s + 2a = 22$ and $2s + 3a = 21$ , multiplies both sides of the second equation by 2, resulting in $4s + 6a = 42$ , but then combines the two equations by eliminating the $4s$ terms and adding the remaining terms of the equations, resulting in $8a = 64$ , and then divides each side of the equation by 8, resulting in $a = 8$	

**10.** A relation is shown on the graph below.



Which statement explains why this relation is **not** a function?

- (a) At x = -2, the relation has two closed points.
- ® Between x = -2 and x = 1, the relation is not a straight line.
- © At x = 1, the relation has a closed point and an open point.
- ① Between x = 3 and x = 4, the relation has no values.

Category	Item-Specific Information	
Alignment	B-F.1.1.1	
Answer Key	A	
Depth of Knowledge	1	
p-value A	31% (correct answer)	
p-value B	30%	
p-value C	21%	
p-value D	18%	
Option Annotations	A. Correct: either recognizes that a relation cannot be a function if there are two or more distinct <i>y</i> -values for any given <i>x</i> -value	
	and that the two closed points at $x = -2$ represent two distinct	
	y-values (2 and ⁻4) OR uses the vertical line test and identifies	
	<ul> <li>two distinct points for the vertical line at x = -2, resulting in a failure of the vertical line test</li> <li>B. considers that functions can be represented only by straight lines and notices that the relation is represented by a curved segment between x = -2 and x = 1</li> </ul>	
	C. recognizes that a relation cannot be a function if there are two or more distinct <i>y</i> -values for any given <i>x</i> -value but does not realize that the open point at (1, 6) does not represent an actual value for <i>x</i> = 1 and that the open point only indicates that the line segment extends to the right of (1, 6)	
	D. considers that functions must be continuous for all $x$ -values and notices a gap in the relation between $x = 3$ and $x = 4$	

- 11. Which equation is a linear function?

  - (B)  $y = \frac{1.2}{x} + 65$
  - © y = 65 + 1.2x
  - ①  $y = 1.2x^2 + 65$

Category	Item-Specific Information	
Alignment	B-F.1.1.3	
Answer Key	С	
Depth of Knowledge	2	
p-value A	15%	
<i>p</i> -value B	15%	
p-value C	50% (correct answer)	
p-value D	20%	
Option Annotations	A. considers any equation with no <i>y</i> -intercept to represent a proportional relationship and is therefore a linear function	
	B. selects an equation that appears to be in the form $y = mx + b$ but does not consider that the $x$ -variable is in the denominator	
	C. Correct: recognizes that the equation can be written in the form $y = mx + b$ , with $m = 1.2$ and $b = 65$	
	D. selects an equation that appears to be in the form $y = mx + b$ but does not consider the exponent of the $x$ -variable	

**12.** A library charges a flat fee of \$0.50 for keeping a book past its due date. The library also charges \$0.15 for each day the book is overdue. The table shown below represents some of the library's fees.

**Library Overdue Fees** 

Days Overdue	Total Fee
1	\$0.65
2	?
3	\$0.95

What is the total fee when a library book is 2 days overdue?

- \$0.70
- ® \$0.75
- © \$0.80
- \$0.90

Category	Item-Specific Information		
Alignment	B-F.2.1		
Answer Key	С		
Depth of Knowledge	2		
p-value A	4%		
p-value B	14%		
p-value C	79% (correct answer)		
p-value D	3%		
Option Annotations	A. uses \$0.05 rather than \$0.15 and adds \$0.05 to the 1-day overdue fee (\$0.65)		
	B. adds \$0.10 to the 1-day overdue fee (\$0.65) since the change in the number of days was 1		
	C. Correct: uses the description "charges a flat fee of \$0.50 [and] \$0.15 for each day" to write the equation $y = 0.50 + 0.15x$ and then substitutes 2 for $x$ into the equation, resulting in $y = 0.50 + 0.15(2) = 0.50 + 0.30 = 0.80$ OR uses the table and finds the midpoint between \$0.65 and \$0.95 (\$0.65 + \$0.95 = \$1.60, \$1.60 $\div$ 2 = \$0.80) since 2 days overdue is the midpoint between 1 day overdue and 3 days overdue OR uses the description "charges \$0.15 for each day" along with the table and adds \$0.15 to the 1-day overdue fee (\$0.65)		
	D. uses \$0.05 rather than \$0.15 and subtracts \$0.05 from the 3-day overdue fee (\$0.95)		

**13.** A science experiment requires students to find the mass of a bucket with different numbers of pennies in it. The table below shows the total masses, in grams, of the bucket with different numbers of pennies in the bucket.

**Penny Experiment** 

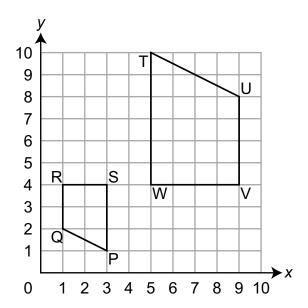
Number of Pennies	Total Mass (grams)
1	252.5
3	257.5
4	260
6	265
9	272.5

Which equation represents the relationship between the total mass (y), in grams, of the bucket and the number of pennies (x) in the bucket?

- (A) y = 2.5x
- y = 2.5x + 250
- © y = 250x
- ① y = 250x + 2.5

B-F.2.1.1 B 2		
2		
2		
18%		
54% (correct answer)		
11%		
17%		
A. calculates the rate of change by using the change in     total mass for any two rows divided by the change		
in the number of pennies for the same two rows $\left(e.g.,\ \frac{257.5-252.5}{3-1}=\frac{5}{2}=2.5\right) \text{ but does not include the } y\text{-intercept}$		
<ul> <li>B. Correct: calculates the rate of change by using the change in total mass for any two rows divided by the change in the number of pennies for the same two rows \[ \begin{align*} \left( \text{e.g.}, \frac{260 - 257.5}{4 - 3} = \frac{2.5}{1} = 2.5 \end{align*} \] and then determines the y-intercept by subtracting the rate of change from the total mass for 1 penny (252.5 - 2.5 = 250)</li> <li>C. uses the y-intercept as the rate of change and does not include a y-intercept</li> <li>D. switches the y-intercept and rate of change</li> </ul>		

14. Trapezoid PQRS and trapezoid TUVW are graphed on the coordinate grid shown below.



Which series of two transformations can be used to map trapezoid PQRS onto trapezoid TUVW?

- a rotation of 180° centered at (4, 4)
   a dilation with a scale factor of 2 centered at (5, 4)
- a rotation of 180° centered at (4, 4)
   a dilation with a scale factor of 2 centered at the origin
- © a rotation of 180° centered at the origin a dilation with a scale factor of 2 centered at (5, 4)
- a rotation of 180° centered at the origin
   a dilation with a scale factor of 2 centered at the origin

Category	Item-Specific Information		
Alignment	C-G.1.1.4		
Answer Key	A		
Depth of Knowledge	2		
p-value A	41% (correct answer)		
p-value B	20%		
p-value C	25%		
<i>p</i> -value D	14%		
Option Annotations	A. Correct: recognizes that to orient trapezoid PQRS in the same direction as trapezoid TUVW, a rotation of 180° must occur and that the rotation should be centered at (4, 4) to move point S from (3, 4) to (5, 4), which is the location of point W, and this would result in point P rotating from (3, 1) to (5, 7), point Q rotating from (1, 2) to (7, 6), and point R rotating from (1, 4) to (7, 4), and then recognizes that, to increase the size of trapezoid PQRS to match the size of trapezoid TUVW, a dilation with a scale factor of 2 must occur and that the dilation should be centered at (5, 4) to keep the rotated point S on point W, and this would result in moving the rotated point P from (5, 7) to (5, 10), which is the location of point T, moving the rotated point Q from (7, 6) to (9, 8), which is the location of point U, and moving the rotated point R from (7, 4) to (9, 4), which is the location of point V		
	B. considers that all dilations must be centered at the origin		
	C. considers that all rotations must be centered at the origin		
	D. considers that all rotations and all dilations must be centered at the origin		
	Note: On future assessments, centers of dilation will be limited to the origin and centers of rotation will be limited to the origin or a vertex on the original figure.		

**15.** The lengths of the two legs of a right triangle are 4 centimeters and 8 centimeters. The length (*h*), in centimeters, of the hypotenuse of the right triangle can be determined using the equation shown below.

$$4^2 + 8^2 = h^2$$

What is the length, in centimeters, of the hypotenuse of the right triangle?

- ©  $\sqrt{40}$

Category	Item-Specific Information		
Alignment	C-G.2.1.2		
Answer Key	D		
Depth of Knowledge	1		
<i>p</i> -value A	11%		
p-value B	11%		
p-value C	9%		
p-value D	69% (correct answer)		
Option Annotations	<ul> <li>A. adds 4 and 8 without applying the exponents, resulting in the equation 12 = h², and then takes the square root of both sides of the equation to remove the exponent of 2 from h²</li> <li>B. multiplies 4 and 8 without applying the exponents, resulting in the equation 32 = h², and then takes the square root of both sides of the equation to remove the exponent of 2 from h</li> </ul>		
	C. determines that $4^2 = 16$ and $8^2 = 64$ , adds 16 and 64, resulting in the equation $80 = h^2$ , but then divides 80 by 2 before taking the square root of both sides of the equation to remove the exponent of 2 from $h^2$		
	D. Correct: determines that $4^2 = 16$ and $8^2 = 64$ , adds $16$ and $64$ , resulting in the equation $80 = h^2$ , and then takes the square root of both sides of the equation to remove the exponent of 2 from $h^2$		

- **16.** An artist has a piece of wood in the shape of a cylinder. It has a height of 36 inches and a diameter of 8 inches. The artist cuts the piece of wood into 6 equal-sized pieces. Which measurement is **closest** to the volume of each piece?

  - 301 cubic inches
  - © 1,206 cubic inches
  - 1,356 cubic inches

Category	Item-Specific Information		
Alignment	C-G.3.1.1		
Answer Key	В		
Depth of Knowledge	2		
p-value A	15%		
<i>p</i> -value B	58% (correct answer)		
p-value C	16%		
p-value D	11%		
Option Annotations	A. divides the diameter by 6 rather than 2, resulting in a quotient of $\frac{4}{3}$ , and applies the volume formula for a cylinder as $V = \pi \cdot \left(\frac{4}{3}\right)^2 \cdot 36 = 64\pi$ , which can be approximated using $64 \cdot 3.14 = 200.96$		
	B. Correct: divides the diameter (8) by 2 to determine the		
	radius, resulting in a radius of 4, applies the volume formula		
	for a cylinder $(V = \pi r^2 h)$ , using $r = 4$ and $h = 36$ to write the		
	equation $V = \pi \cdot 4^2 \cdot 36$ , which can be simplified as		
	$V = \pi \cdot 16 \cdot 36 = 576\pi$ , and then divides the volume by 6,		
	resulting in a quotient of $96\pi$ , which can be approximated		
	using 96 · 3.14 = 301.44		
	C. uses $d^2$ rather than $r^2$ to apply the volume formula for a		
	cylinder as $V = \pi \cdot (8)^2 \cdot 36 = 2,304\pi$ , and then divides the		
	volume by 6, resulting in a quotient of $384\pi$ , which can be		
	approximated using 384 · 3.14 = 1,205.76		
	D. uses $\left(\frac{h}{2}\right)^2 \cdot d$ rather than $\left(\frac{d}{2}\right)^2 \cdot h$ to apply the volume formula for a cylinder as $V = \pi \cdot \left(\frac{36}{2}\right)^2 \cdot 8 = 2,592\pi$ ,		
	and then divides the volume by 6, resulting in a quotient of		
	$432\pi$ , which can be approximated using $432 \cdot 3.14 = 1,356.48$		

#### **Open-Ended Item**

17. A quiz competition was held for grade 7 and grade 8 students. In the first round, each of the 150 contestants was given a quiz. Contestants advanced to the second round if they answered enough questions correctly. The tables below show some information about the 150 contestants in the competition as well as some information about the grade 8 students who competed.

#### **Quiz Competition Contestants**

	Grade 7	Grade 8
Advanced to Second Round	44	44
Did Not Advance to Second Round	33	29

#### **Grade 8 Quiz Competition Contestants**

	Male	Female
Advanced to Second Round	20	24
Did Not Advance to Second Round	?	12

Α.	How many grade 8	male students	did <b>not</b> advance to	the second round?

A grade 8 female student who competed is randomly selected.

**B.** What is the probability that the randomly selected student advanced to the second round?

Go to the next page to finish question 17.

GOON



One of the 150 students who competed is randomly selected.

**C.** Explain why the randomly selected student **most likely** advanced to the second round.

**D.** Explain why a grade 7 student is less likely to have advanced to the second round than a grade 8 student even though the same number of grade 7 students advanced to the second round as grade 8 students.

After you have finished your work, close this booklet so your teacher will know you are finished.



# **Item-Specific Scoring Guideline**

### #17 Item Information

Category	Item-Specific Information
Alignment	D-S.1.2.1
Depth of Knowledge	2
Mean Score	1.45

#### Assessment Anchor this item will be reported under:

M08.D-S.1 Investigate patterns of association in bivariate data.

### **Specific Anchor Descriptor addressed by this item:**

**M08.D-S.1.2** Understand that patterns of association can be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

### **Item-Specific Scoring Guideline**

Score	In this item, the student
4	Demonstrates a thorough understanding of how to investigate patterns of association in bivariate data by correctly solving problems and clearly explaining procedures.
3	Demonstrates a general understanding of how to investigate patterns of association in bivariate data by correctly solving problems and clearly explaining procedures with only minor errors or omissions.
2	Demonstrates a partial understanding of how to investigate patterns of association in bivariate data by correctly performing a significant portion of the required task.
1	Demonstrates minimal understanding of how to investigate patterns of association in bivariate data.
0	The response has no correct answer and insufficient evidence to demonstrate any understanding of the mathematical concepts and procedures as required by the task. Response may show only information copied from the question.

# **Top-Scoring Student Response and Training Notes**

Score	Description
4	Student earns 4 points.
3	Student earns 3.0–3.5 points.
2	Student earns 2.0–2.5 points.
1	Student earns 0.5–1.5 points.  OR  Student demonstrates minimal understanding of how to investigate patterns of association in bivariate data.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.

### **Top-Scoring Response**

## Part A (1 point):

1 point for correct answer

#### What?

17 (male students)

### Part B (1 point):

1 point for correct answer

#### What?

24/36 **OR** 2/3

**OR** equivalent

# Part C (1 point):

1 point for correct and complete explanation

**OR** 1/2 point for correct but incomplete explanation

### Why?

### **Sample Explanation:**

The student most likely advanced to the second round because more than half the students who competed advanced to the second round (88 out of 150).

**OR** equivalent

### Part D (1 point):

1 point for correct and complete explanation

**OR** 1/2 point for correct but incomplete explanation

### Why?

### Sample Explanations:

Even though the same number of grade 7 students advanced to the second round as grade 8 students, a grade 7 student is less likely to advance to the second round than a grade 8 student because more grade 7 students participated in the competition than grade 8 students.

#### OR

The probability a grade 7 student advanced to the second round is 44/77. The probability a grade 8 student advanced to the second round is 44/73. Since 44/77 is less than 44/73, a grade 7 student is less likely to advance to the second round than a grade 8 student.

#### **OR**

When comparing the probabilities, you have to use both the numerator and the denominator. Since the numerators are the same, the probability with the larger denominator represents a student who is less likely to advance to the second round. So a grade 7 student is less likely to advance to the second round than a grade 8 student.

### **OR** equivalent

#### STUDENT RESPONSE

### **Response Score: 4 points**

17. A quiz competition was held for grade 7 and grade 8 students. In the first round, each of the 150 contestants was given a quiz. Contestants advanced to the second round if they answered enough questions correctly. The tables below show some information about the 150 contestants in the competition as well as some information about the grade 8 students who competed.

**Quiz Competition Contestants** 

	Grade 7	Grade 8
Advanced to Second Round	44	44
Did Not Advance to Second Round	33	29

### **Grade 8 Quiz Competition Contestants**

	Male	Female
Advanced to Second Round	20	24
Did Not Advance to Second Round	?	12

**A.** How many grade 8 male students did **not** advance to the second round?

17 males

A grade 8 female student who competed is randomly selected.

**B.** What is the probability that the randomly selected student advanced to the second round?

3

Go to the next page to finish question 17.



Part A: The student provided the correct answer (17 males). While support is not required for Part A, the student likely calculated the total number of grade 8 students (44 + 29 = 73), found the total number of grade 8 students already accounted for in the second table (20 + 24 + 12 = 56), and then subtracted the sums (73 – 56 = 17). [1 point]

**Part B:** The student provided the correct answer  $\left(\frac{2}{3}\right)$ . While support is not required for Part B, the student likely calculated the total number of grade 8 female students from the second table (24 + 12 = 36), identified the number of grade 8 female students who advanced to the second round (24), and then wrote the probability as  $\frac{24}{36}$ , and then simplified the fraction to  $\frac{2}{3}$ . [1 point]

**17.** *Continued.* Please refer to the previous page for task explanation.

One of the 150 students who competed is randomly selected.

**C.** Explain why the randomly selected student **most likely** advanced to the second round.

A student, who is condomly sclected, is most likely to advance to the second round because there are more students who it to the scrond round than students that didn't make it to the second round.

**D.** Explain why a grade 7 student is less likely to have advanced to the second round than a grade 8 student even though the same number of grade 7 students advanced to the second round as grade 8 students.

Even though both grades have the same amount of students Advancing grade 7 has more students not advancing meaning it is less likely for a grade 7 student to advance to the second round. Then a grade 8 student.

After you have finished your work, close this booklet so your teacher will know you are finished.



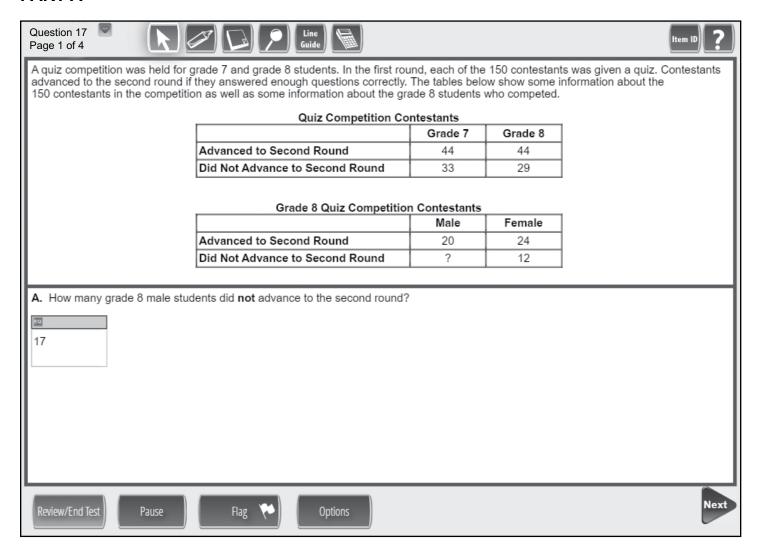
**Part C:** The student provided a correct and complete explanation as to why a randomly selected student most likely advanced to the second round (there are more students who [made] it to the second round than students that didn't make it to the second round). [1 point]

**Part D:** The student provided a correct and complete explanation as to why a grade 7 student is less likely to have advanced to the second round than a grade 8 student (though both grades have the same amount of students Advancing grade 7 has more students not advoncing). [1 point]

### STUDENT RESPONSE

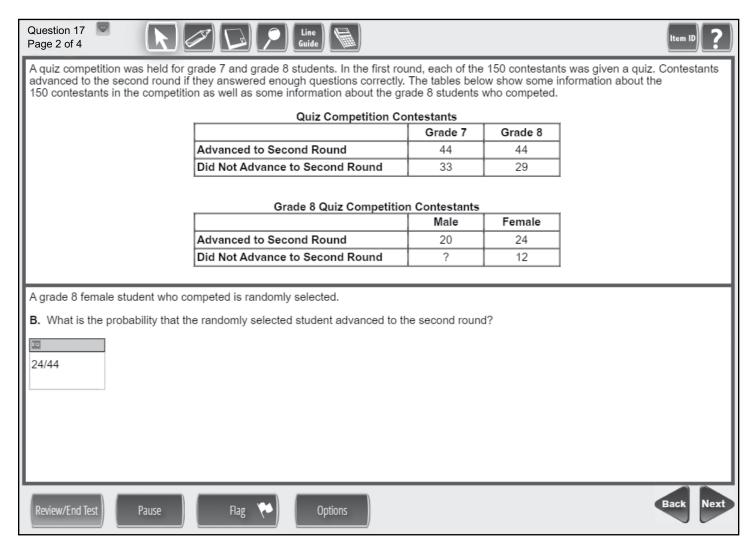
Computer Response Score: 3 points

#### **PART A**



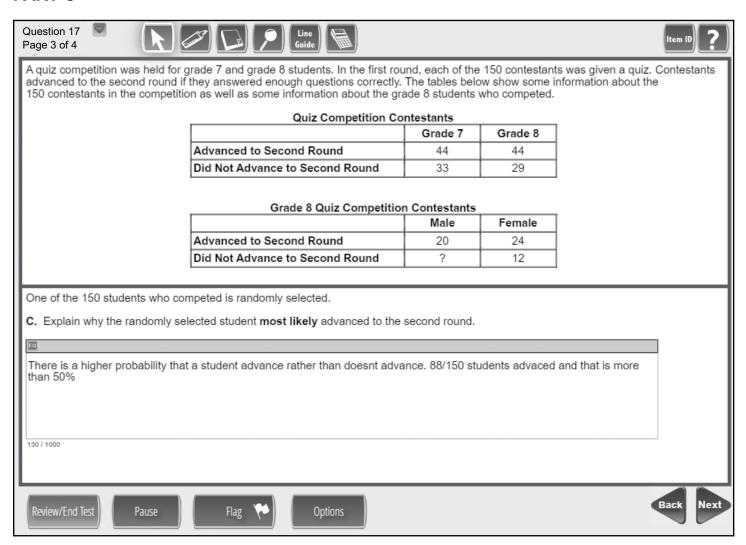
Part A: The student provided the correct answer (17). While support is not required for Part A, the student may have looked at the first table and noted that 29 grade 8 students did not advance to the second round, looked at the second table and noted that 12 female students did advance, and then subtracted these values (29 – 12 = 17). [1 point]





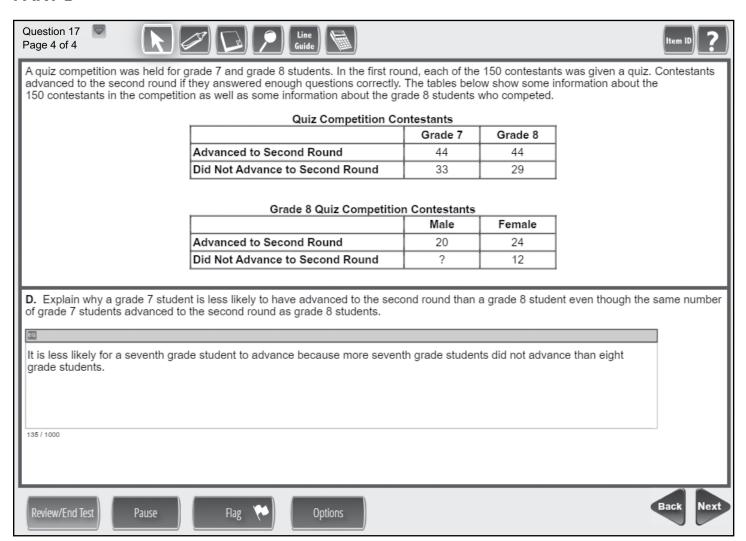
**Part B:** The student provided an incorrect answer (24/44). While the numerator (24) is correct, the denominator (44) is incorrect. No support (work or explanation) is required, so it is unclear where an error was made. The student may have looked at the first table and used the number of grade 8 students who advanced to the second round rather than calculating the total number of female grade 8 students from the second table. [0 points]

#### **PART C**



**Part C:** The student provided a correct and complete explanation as to why a randomly selected student likely advanced to the second round (88/150 students advaced and that is more than 50%). [1 point]





**Part D:** The student provided a correct and complete explanation as to why a grade 7 student is less likely to have advanced to the second round than a grade 8 student (*more seventh grade students did not advance than eight grade students*). [1 point]

#### STUDENT RESPONSE

### **Response Score: 2 points**

17. A quiz competition was held for grade 7 and grade 8 students. In the first round, each of the 150 contestants was given a quiz. Contestants advanced to the second round if they answered enough questions correctly. The tables below show some information about the 150 contestants in the competition as well as some information about the grade 8 students who competed.

**Quiz Competition Contestants** 

	Grade 7	Grade 8
Advanced to Second Round	44	44
Did Not Advance to Second Round	33	29

### **Grade 8 Quiz Competition Contestants**

	Male	Female
Advanced to Second Round	20	24
Did Not Advance to Second Round	?	12

**A.** How many grade 8 male students did **not** advance to the second round?

10 male students

A grade 8 female student who competed is randomly selected.

**B.** What is the probability that the randomly selected student advanced to the second round?

she has a 50% chance of advancing to the second round 12 is 1/2 of 24 (50 is 1/2 of 100)

Go to the next page to finish question 17.

GO ON

Part A: The student provided an incorrect answer (10 male students). No support (work or explanation) is required for Part A, so it is unclear where an error was made. The student may have looked at the second table and noticed that, for the female column, those who did not advance happened to be half the number of those who did advance (12 did not advance compared to 24 who did advance), and then assumed the same would hold true for the male column (20 ÷ 2 = 10). [0 points]

**Part B:** The student provided an incorrect answer (50%). The explanation provided, though not required for Part B, shows that the student incorrectly compared the 24 and the 12 from the second table (12 is  $\frac{1}{2}$  of 24). The student used the ratio of grade 8 female students who did not advance (12) to those who did advance (24) rather than the ratio of grade 8 female students who advanced (24) to the total number of grade 8 female students (12 + 24 = 36). [0 points]

**17.** *Continued.* Please refer to the previous page for task explanation.

One of the 150 students who competed is randomly selected.

**C.** Explain why the randomly selected student **most likely** advanced to the second round.

they most likely advanced to the second round because more kids advanced than did not advance.

**D.** Explain why a grade 7 student is less likely to have advanced to the second round than a grade 8 student even though the same number of grade 7 students advanced to the second round as grade 8 students.

Because there was more 7th grade students who did not advance to the second round than there were 8th grade students who didn't advance.

After you have finished your work, close this booklet so your teacher will know you are finished.



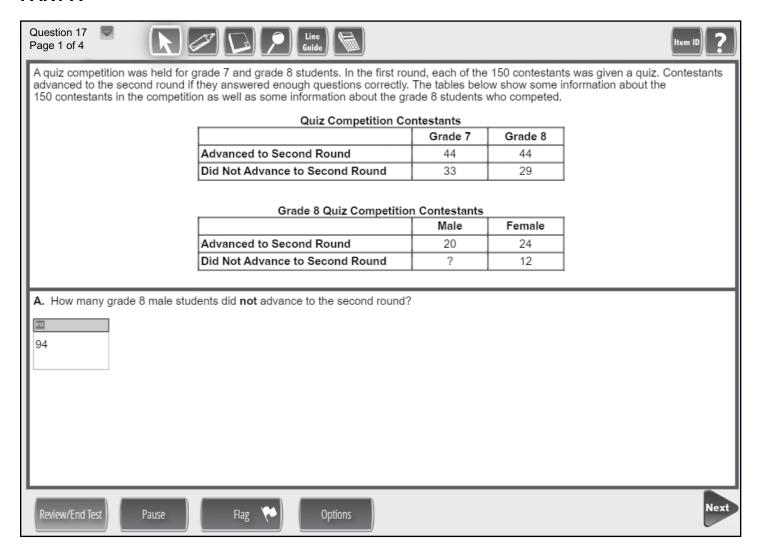
**Part C:** The student provided a correct and complete explanation as to why a randomly selected student likely advanced to the second round (*more kids advanced than did not advance*). [1 point]

**Part D:** The student provided a correct and complete explanation as to why a grade 7 student is less likely to have advanced to the second round than a grade 8 student (there was more 7th grade students who <u>did not</u> advance . . . than there were 8th grade students who didn't advance). [1 point]

### STUDENT RESPONSE

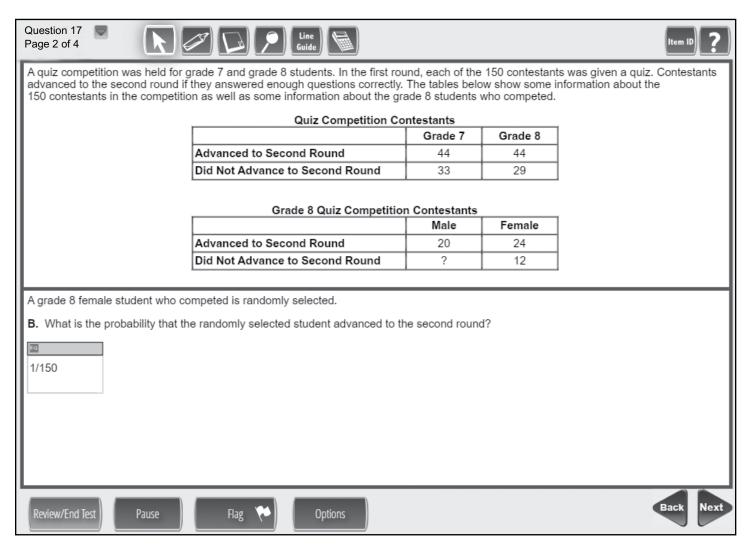
Computer Response Score: 1 point

#### **PART A**



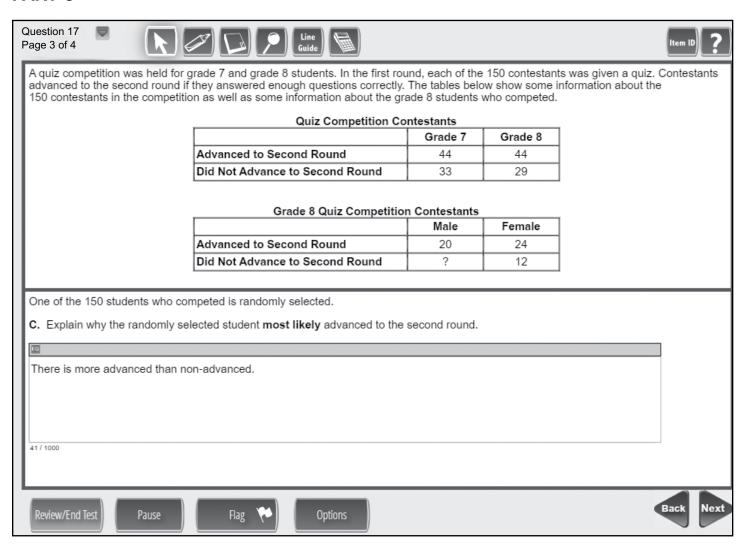
**Part A:** The student provided an incorrect answer (94). No support (work or explanation) is required, so it is unclear where an error was made. The student likely started with 150 (the total number of grade 7 and grade 8 students) and subtracted the values from the second table (150 - 20 - 24 - 12 = 94). [0 points]





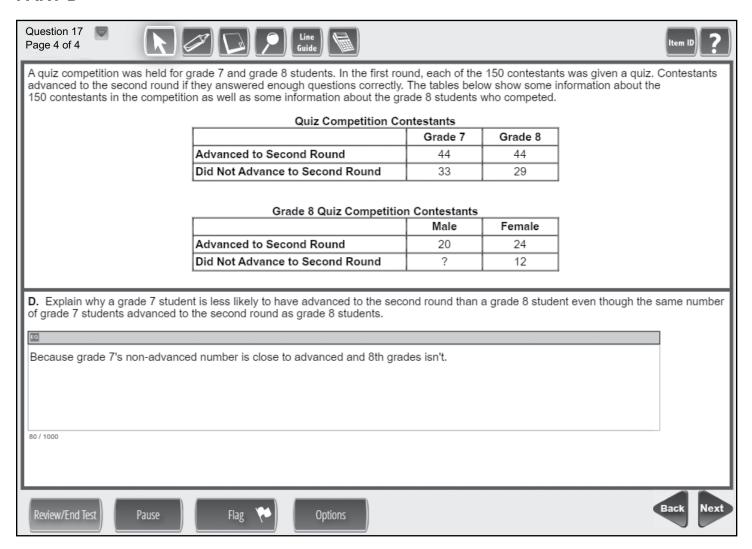
**Part B:** The student provided an incorrect answer (1/150). No support (work or explanation) is required, so it is unclear where an error was made. The student likely wrote the probability for 1 student out of all 150 students who participated in the quiz competition rather than the probability for one of the 24 grade 8 female students who advanced to the second round out of the 36 grade 8 female students who participated in the quiz competition. [0 points]

#### **PART C**



**Part C:** The student provided a correct and complete explanation as to why a randomly selected student likely advanced to the second round (*more advanced than non-advanced*). [1 point]





**Part D:** The student provided an incorrect explanation as to why a grade 7 student is less likely to have advanced to the second round than a grade 8 student (*grade 7's non-advanced number is close to advanced and 8th grades isn't*). The student incorrectly attempted to associate likelihood with the difference between those who advanced and those who did not advance. [0 points]

#### STUDENT RESPONSE

**Response Score: 0 points** 

17. A quiz competition was held for grade 7 and grade 8 students. In the first round, each of the 150 contestants was given a quiz. Contestants advanced to the second round if they answered enough questions correctly. The tables below show some information about the 150 contestants in the competition as well as some information about the grade 8 students who competed.

### **Quiz Competition Contestants**

	Grade 7	Grade 8
Advanced to Second Round	44	44
Did Not Advance to Second Round	33	29

### **Grade 8 Quiz Competition Contestants**

	Male	Female
Advanced to Second Round	20	24
Did Not Advance to Second Round	?	12

A. How many grade 8 male students did not advance to the second round?

9 did not advance

A grade 8 female student who competed is randomly selected.

**B.** What is the probability that the randomly selected student advanced to the second round?

Their is a good Probability thatsh will advance to the second round.

Go to the next page to finish question 17.

GO ON

Part A: The student provided an incorrect answer (9 did not advance). No support (work or explanation) is required for Part A, so it is unclear where an error was made. The student may have looked at the first table and noted that 29 grade 8 students did not advance to the second round, looked at the second table and noted that 20 male students advanced (rather than noting the 12 female students who did not advance), and then subtracted these values (29 – 20 = 9). [0 points]

**Part B:** The student provided an incorrect answer (*Their is a good probability thatsh* [that she] *will advance to the second round*). The student attempted to respond with a likelihood ("good probability" being similar to "more likely") rather than an actual probability. [0 points]

17. Continued. Please refer to the previous page for task explanation.

One of the 150 students who competed is randomly selected.

**C.** Explain why the randomly selected student **most likely** advanced to the second round.

Because their is a even chance of getting it right Just like their is getting it Wrong.

**D.** Explain why a grade 7 student is less likely to have advanced to the second round than a grade 8 student even though the same number of grade 7 students advanced to the second round as grade 8 students.

Because their is less kids in the seventh grade to start with.

After you have finished your work, close this booklet so your teacher will know you are finished.



- Part C: The student provided an incorrect explanation as to why a randomly selected student likely advanced to the second round (their is a even chance of getting it right just like their is getting it wrong). The student's explanation actually attempts to explain why the randomly selected student is equally likely, not most likely, to have advanced to the second round. [0 points]
- **Part D:** The student provided an incorrect explanation as to why a grade 7 student is less likely to have advanced to the second round than a grade 8 student (*their is less kids in the seventh grade to start with*). The student incorrectly attempted to associate likelihood with the size of the population; however, there are more, not fewer, grade 7 students than grade 8 students who participated in the quiz competition. [0 points]

# **Mathematics—Summary Data**

# **Multiple-Choice**

An asterisk (\*) indicates the key.

Sample Number	Alignment	Answer Key	Depth of Knowledge	<i>p</i> -value A	<i>p</i> -value B	<i>p</i> -value C	<i>p</i> -value D
1	B-E.1.1.3	Α	1	42%*	28%	15%	15%
2	A-N.1.1.1	В	1	20%	57%*	12%	11%
3	A-N.1.1.2	В	1	11%	54%*	17%	18%
4	A-N.1.1.4	D	2	12%	12%	14%	62%*
5	B-E.1.1.4	А	1	60%*	12%	17%	11%
6	B-E.2.1	D	1	27%	17%	16%	40%*
7	B-E.2.1.1	В	2	16%	45%*	32%	7%
8	B-E.3.1.3	D	2	13%	23%	19%	45%*
9	B-E.3.1.5	В	2	15%	58%*	18%	9%
10	B-F.1.1.1	А	1	31%*	30%	21%	18%
11	B-F.1.1.3	С	2	15%	15%	50%*	20%
12	B-F.2.1	С	2	4%	14%	79%*	3%
13	B-F.2.1.1	В	2	18%	54%*	11%	17%
14	C-G.1.1.4	А	2	41%*	20%	25%	14%
15	C-G.2.1.2	D	1	11%	11%	9%	69%*
16	C-G.3.1.1	В	2	15%	58%*	16%	11%

# **Open-Ended**

Sample Number	Alignment	Points	Depth of Knowledge	Mean Score
17	D-S.1.2.1	4	2	1.45