MATHEMATICS TEST DIRECTIONS

On the following pages are the mathematics questions.

 You may <u>not</u> use a calculator for question 1. You may use a calculator for all other questions on this test.

Directions for Multiple-Choice Questions

Some questions will ask you to select an answer from among four choices.

For the multiple-choice questions:

- First solve the problem on scratch paper.
- Choose the correct answer and record your choice in the answer booklet.
- If none of the choices matches your answer, go back and check your work for possible errors.
- Only one of the answers provided is the correct response.

Directions for Open-Ended Questions

Some questions will require you to write your response.

For the open-ended questions:

- These questions have more than one part. Be sure to read the directions carefully.
- You cannot receive the highest score for an open-ended question without completing all tasks in the question. For example, if the question asks you to show your work or explain your reasoning, be sure to show your work or explain your reasoning in the space provided.
- If the question does **not** ask you to show your work or explain your reasoning, you may use the space provided, but only those parts of your response that the question specifically asks for will be scored.
- Write your response in the appropriate location within the response box in the answer booklet. Some answers may require graphing, plotting, labeling, drawing, or shading. If you use scratch paper, be sure to transfer your final response and any needed work or reasoning to the answer booklet.

Question 1 in this sampler is to be solved without the use of a calculator.

MULTIPLE-CHOICE ITEMS

1. An expression is shown below.

$$^{-}0.31 \bullet 4.2 \div 2$$

What is the value of the expression?

- A. -0.651
- B. -0.093
- C. 0.093
- D. 0.651

Item Information	
Alignment	A-N.1.1.3
Answer Key	A
Depth of Knowledge	1
p-value A	47% (correct answer)
p-value B	22%
p-value C	15%
p-value D	16%
Option Annotations	 A. Correct: first multiplies ~0.31 by 4.2 and then divides that product (~1.302) by 2 B. does not align partial products of 62 and 1,240, adding the partial products as 62 + 124 = 186, and then divides ~0.186 by 2 to determine that the quotient is ~0.093 C. does not align partial products of 62 and 1,240, adding the partial products as 62 + 124 = 186, divides 0.186 by 2 to determine that the quotient is 0.093, and then reasons the quotient is positive since the sign of the "greatest" number is positive D. multiplies ~0.31 by 4.2, then divides that product (~1.302) by 2, but then reasons the quotient is positive since the sign of the "greatest" number is positive

Item Information	
Alignment	A-N.1.1.2
Answer Key	В
Depth of Knowledge	1
p-value A	30%
p-value B	61% (correct answer)
p-value C	5%
p-value D	4%
Option Annotations	A. starts at $1\frac{1}{2}$ and ends at $\frac{1}{4}$ rather than moving $\frac{1}{4}$ units to the left
	B. Correct: starts at $1\frac{1}{2}$ and moves $\frac{1}{4}$ units to the left to show subtracting $\frac{1}{4}$ from $1\frac{1}{2}$
	C. starts at $1\frac{1}{2}$ and moves $\frac{1}{4}$ units to the right rather than moving $\frac{1}{4}$ units to the left
	D. adds the given fractions and then moves $\frac{1}{4}$ units to the left to show subtraction

- 3. Gerry is packing dishes into boxes to be shipped. It takes Gerry $2\frac{3}{4}$ hours to pack $4\frac{1}{2}$ boxes.
 - At this rate, how many boxes of dishes can Gerry pack in 1 hour?
 - A. $\frac{11}{18}$
 - B. $1\frac{3}{4}$
 - C. $1\frac{7}{11}$
 - D. $2\frac{1}{4}$

Item Information	
Alignment	A-R.1.1.1
Answer Key	C
Depth of Knowledge	2
p-value A	24%
p-value B	21%
p-value C	36% (correct answer)
p-value D	19%
Option Annotations	A. divides $2\frac{3}{4}$ by $4\frac{1}{2}$
	B. subtracts $2\frac{3}{4}$ from $4\frac{1}{2}$ rather than using division
	C. Correct: recognizes that division is needed to find the unit rate and
	divides the number of boxes $\left(4\frac{1}{2}\right)$ by the number of hours $\left(2\frac{3}{4}\right)$
	by converting both mixed numbers to improper fractions
	$\left(4\frac{1}{2} = \frac{9}{2} \text{ and } 2\frac{3}{4} = \frac{11}{4}\right)$, inverting the divisor $\left(\frac{11}{4} \text{ becomes } \frac{4}{11}\right)$ before
	multiplying $\left(\frac{9}{2} \cdot \frac{4}{11} = \frac{36}{22} = \frac{18}{11}\right)$, and then converting the quotient back
	to a mixed number $\left(\frac{18}{11} = 1 \frac{7}{11}\right)$
	D. subtracts $4-2$ and $\frac{3}{4}-\frac{1}{2}$

Item Information	
Alignment	A-R.1.1.2
	A-R.1.1.4
Answer Key	В
Depth of Knowledge	2
p-value A	12%
p-value B	64% (correct answer)
p-value C	12%
p-value D	12%
Option Annotations	 A. finds the rate of change for table Q but does not consider the constant term (+ 1), writes the equation as y = 2x rather than as y = 2x + 1, and then considers the relationship in table Q to be proportional because the incorrect equation represents a proportional relationship B. Correct: recognizes the relationship in table R to be proportional since the ratio between each y-value and its corresponding x-value is consistent and then finds the equation that represents the relationship by using the unit rate between the y-values and the x-values (3) as the rate of change C. considers that table Q is proportional since the rate of change is consistent (i.e., thinks any linear relationship is proportional) and then sets up the equation as y + 2 = x + 1 since the y-values increase by 2 as the x-values increase by 1 (i.e., confuses the rate of change as the constants) D. recognizes that table R is proportional but then sets up the equation as y + 3 = x + 1 since the y-values increase by 3 as the x-values increase by 1 (i.e., confuses the rate of change as the constants)

- 5. A restaurant buys the eggs it uses from a farmer.
 - The cost to buy 50 dozen eggs from the farmer is \$109.50.
 - The restaurant buys 8 dozen eggs each day from the farmer.
 - The restaurant buys the eggs 7 days each week.

Which equation can be used to determine the total amount (y), in dollars, the restaurant spends on the eggs it buys from the farmer for x weeks?

- A. y = 109.50x
- B. y = 122.64x
- C. y = 109.50x + 56
- D. y = 122.64x + 56

Item Information	
Alignment	A-R.1.1.4 A-R.1.1.3
Answer Key	В
Depth of Knowledge	2
p-value A	25%
p-value B	34% (correct answer)
p-value C	29%
p-value D	12%
Option Annotations	 A. does not consider that the restaurant buys 56 dozen eggs per week and uses \$109.50, the cost of 50 dozen eggs, as the rate of change B. Correct: determines the cost per dozen eggs by dividing the cost \$109.50 by 50 dozen eggs, determines the number of eggs bought per week by multiplying the 8 dozen eggs bought each day by 7 days, and then finds the rate of change by multiplying the 56 dozen eggs bought per week by the cost per dozen (\$2.19) C. does not consider that the restaurant buys 56 dozen eggs per week and uses \$109.50, the cost of 50 dozen eggs, as the rate of change and the product of 7 and 8 as the constant term of equation D. uses the product of 7 and 8 as the constant term of equation

6. The set of ordered pairs below represents the relationship between the number of steps (x) Mr. Davino takes and the distance (y), in miles, he walks.

$$\{(528, 0.25), (1584, 0.75), (2640, 1.25), (3696, 1.75)\}$$

Based on the ordered pairs, which distance is **closest** to the distance Mr. Davino walks by taking 8,900 steps?

- A. 2.4 miles
- B. 2.8 miles
- C. 3.0 miles
- D. 4.2 miles

Item Information	
Alignment	A-R.1.1.6
Answer Key	D
Depth of Knowledge	2
p-value A	16%
p-value B	18%
p-value C	21%
p-value D	45% (correct answer)
Option Annotations	 A. divides 8,900 by 3,696 B. calculates 8,900 ÷ (3,696 – 528) C. determines the factor of increase either from only the first <i>x</i>-value to the next <i>x</i>-value (1584 ÷ 528) OR from only the first <i>y</i>-value to the next <i>y</i>-value (0.75 ÷ 0.25)
	D. Correct: calculates the rate of change as 2,112 by using the ratio $\frac{x}{y}$ (e.g., $\frac{3696}{1.75}$) and then divides 8,900 by 2,112

7. Michael enrolls customers for a food delivery service. For each customer he enrolls, he earns \$20 plus 15% of the value of the customer's first order. The expression below represents Michael's earnings, in dollars, based on the value of a customer's first order (x).

$$20 + 0.15x$$

How would the expression change if Michael earned 10% of the value of each customer's first order instead of 15% of the value?

- A. The 20 would decrease by 5.
- B. The 0.15 would change to 0.10.
- C. The entire expression would be multiplied by $\frac{2}{3}$.
- D. The entire expression would be multiplied by 0.10.

Item Information	
Alignment	B-E.1
Answer Key	В
Depth of Knowledge	2
p-value A	14%
p-value B	63% (correct answer)
p-value C	10%
p-value D	13%
Option Annotations	 A. decreases the fixed amount by 5 rather than decreasing the percentage B. Correct: recognizes that the 15% is represented by 0.15 in the original equation and changes 0.15 to 0.10 since 0.10 is the decimal equivalent of 10%
	C. considers that the whole expression should be multiplied by $\frac{10\%}{15\%} = \frac{0.10}{0.15} = \frac{2}{3}$
	D. recognizes that 0.10 is the decimal equivalent of 10% but thinks the entire expression should be multiplied by 0.10, which would result in 10% of Michael's total earnings rather than a change from 15% to 10% of the first order

8. A rectangular prism has a length of *x* units, a width of *y* units, and a height of 3 units. The expression shown below represents the surface area, in square units, of the prism.

$$2(xy + 3x + 3y)$$

Which statement describes how to create an equivalent expression to represent the surface area, in square units, of the prism?

- A. Combine 3x and 3y because they are like terms.
- B. Apply the distributive property and multiply only the *xy* term by 2.
- C. Combine the three terms in the parentheses because they are all like terms.
- D. Apply the distributive property and multiply each of the three terms in the parentheses by 2.

Item Information	
Alignment	B-E.1.1
Answer Key	D
Depth of Knowledge	1
p-value A	14%
p-value B	17%
p-value C	16%
p-value D	53% (correct answer)
Option Annotations	 A. considers terms with the same coefficient (3) to be like terms B. recognizes that the distributive property can be used to create an equivalent expression but applies the distributive property to only the first term rather than to all three terms C. considers all three terms in the parentheses to be like terms because they all have at least one variable (an x, a y, or both an x and a y) D. Correct: recognizes that the distributive property can be used to create an equivalent expression and understands that each term in the parentheses (not just the first term) needs to be multiplied by the factor 2

9. Ms. Carter makes shirts. She charges \$12 per shirt. She also charges \$70 per order regardless of the number of shirts ordered. Which equation could be used to calculate the total cost (*c*), in dollars, of an order of *n* shirts?

A.
$$c = n + 82$$

B.
$$c = 12n + 70$$

C.
$$c = 840n$$

D.
$$c = \frac{n}{12} + 70$$

Item Information	
Alignment	B-E.2.2
Answer Key	В
Depth of Knowledge	2
p-value A	12%
p-value B	70% (correct answer)
p-value C	8%
p-value D	10%
Option Annotations	 A. uses the sum \$12 + \$70 = \$82 as the initial fee B. Correct: recognizes that \$12 per shirt represents the rate of change, which is multiplied by the number of shirts (n), and recognizes that \$70 is the constant term since this initial fee is added to the total cost regardless of the number of shirts ordered C. uses the product \$12 • \$70 = \$840 as the rate of change D. divides the number of shirts by \$12 rather than multiplying by \$12

10. The equation shown below can be used to determine the total income (*m*), in dollars, a salesperson makes in a month based on the total sales amount (*s*), in dollars, the salesperson generates that month.

$$m = 4,250 + 0.06s$$

Last month, the salesperson generated a total of \$21,400 in sales. What was the salesperson's total income last month?

- A. \$1,284
- B. \$1,539
- C. \$5,534
- D. \$5,789

Item Information			
Alignment	B-E.2.2.1		
Answer Key	С		
Depth of Knowledge	2		
p-value A	13%		
p-value B	16%		
p-value C	60% (correct answer)		
p-value D	11%		
Option Annotations	 A. multiplies 0.06 by 21,400 but does not add the constant term B. adds 4,250 to 21,400 before multiplying the sum (25,650) by 0.06 C. Correct: recognizes that the total sales (\$21,400) should be multiplied by 0.06 before adding 4,250 to the product (\$1,284) D. adds 4,250 to 21,400 before multiplying the sum (25,650) by 0.06 and then adds 4,250 again 		

- 13. A circle has a circumference of 36π inches. What is the area, in square inches, of the circle?
 - Α. 12π
 - B. 72π
 - C. 324π
 - D. 1,296π

Item Information	
Alignment	C-G.2.2.1
Answer Key	С
Depth of Knowledge	2
p-value A	19%
p-value B	27%
p-value C	39% (correct answer)
p-value D	15%
Option Annotations	 A. uses the area formula (A = πr²) rather than the circumference formula (C = 2πr), substitutes 36π in for A (36π = πr²) to determine that r = 6, and then calculates the circumference using C = 2π(6) = 12π (i.e., determines the circumference of a circle with an area of 36π square inches) B. thinks that the radius is 36 inches, substitutes r = 36 into the circumference formula, and calculates C = 2π(36) = 72π (i.e., doubles the radius of 36 inches) C. Correct: uses the circumference formula to solve for r (C = 2πr → 36π = 2πr → 18 = r) and then finds the area using the area formula for a circle [A = πr² = π(18)² = 324π] D. uses C = πr as the circumference formula rather than C = 2πr, substitutes 36π in for C (36π = πr) to determine that r = 36, and then calculates the area using A = π(36)² = 1,296π

14. A sample of 8 pitchers of skim milk and 8 pitchers of whole milk are in a refrigerator. The table below lists the amount of calcium, in milligrams, in each pitcher of milk.

Amount of Calcium (milligrams)

Skim Milk	853	854	856	857	865	886	904	916
Whole Milk	818	836	841	870	874	879	881	938

Based on the information shown in the table, which statement about the distribution of calcium in skim milk and the distribution of calcium in whole milk is true?

- A. Both distributions have exactly one mode.
- B. Both distributions have a median that is less than 870 milligrams.
- C. The range of the distribution of calcium in whole milk is almost twice the range of the distribution of calcium in skim milk.
- D. The range of the distribution of calcium in whole milk is 85 milligrams more than the range of the distribution of calcium in skim milk.

Harra Information					
Item Information					
Alignment	D-S.2.1.1				
Answer Key	C				
Depth of Knowledge	2				
p-value A	17%				
p-value B	18%				
p-value C	44% (correct answer)				
p-value D	21%				
Option Annotations	 A. does not recognize that neither distribution has a mode B. either does not recognize that the whole milk distribution has a median of 872 OR uses the median value of both distributions combined (867.5) rather than the median values of each distribution C. Correct: determines the range of each distribution by finding the difference between the maximum and minimum values (916 − 853 = 63 and 938 − 818 = 120) and then compares the two differences (63 • 2 = 126, which is close to 120) D. subtracts the minimum value for the skim milk (853) from the maximum value for the whole milk (938) and considers this to be the difference between the range of each distribution 				

15. A crate contains green, red, and yellow apples. Information about the number of apples of each color in the crate is listed below.

green: 24red: 15yellow: ?

One apple is randomly selected from the crate. The probability of the apple being red is $\frac{1}{3}$. How many yellow apples are in the crate?

- A. 2
- B. 6
- C. 39
- D. 45

Item Information	
Alignment	D-S.3
Answer Key	В
Depth of Knowledge	2
p-value A	8%
p-value B	51% (correct answer)
p-value C	29%
p-value D	12%
Option Annotations	A. uses the two known values to determine $\frac{15}{24 + 15} = \frac{15}{39} = \frac{5}{13}$ but then
	uses $\frac{1}{3} = \frac{5}{15}$ to set up the equation $\frac{5}{13 + ?} = \frac{5}{15}$, resulting in ? = 2 as
	the number of yellow apples
	B. Correct: calculates the total number of apples in the crate by solving
	the proportion $\frac{1}{3} = \frac{15}{x}$, determining that $x = 45$, and then determines
	the number of yellow apples by subtracting the number of green
	apples and the number of red apples from $45 (45 - 24 - 15 = 6)$
	C. adds 24 and 15 (i.e., thinks the number of yellow apples is equal to the number of green apples and red apples combined)
	D. calculates the total number of apples in the crate by solving the
	proportion $\frac{1}{3} = \frac{15}{x}$, determining that $x = 45$, but then considers that x
	represents the number of yellow apples rather than the total number
	of apples in the crate

- **16.** Dorian and Sarah are the only two students running for class president. There are 311 votes in the election. Every vote is for either Dorian or Sarah. Which outcome is **certain** to happen?
 - A. Either Dorian or Sarah will receive exactly 156 votes.
 - B. Neither Dorian nor Sarah will receive exactly 156 votes.
 - C. Either Dorian or Sarah will receive at least 156 votes.
 - D. Neither Dorian nor Sarah will receive at least 156 votes.

Item Information				
Alignment	D-S.3.1.1			
Answer Key	С			
Depth of Knowledge	2			
p-value A	13%			
p-value B	23%			
p-value C	54% (correct answer)			
p-value D	10%			
Option Annotations	A. assumes that one student will receive exactly 1 more vote than the other student			
	 B. either recognizes that a vote of 156-to-155 is unlikely but confuses unlikely for impossible OR divides 311 by 2 and thinks each student will receive 155.5 votes and does not consider that the number of votes needs to be a whole number C. Correct: recognizes that one student must receive more than half the 			
	votes since half of 311 is 155.5			
	D. confuses certain with impossible			

Item-Specific Scoring Guideline

#17 Item Information

Alignment	A-R.1.1.1 A-R.1.1.5 A-R.1.1.6	Depth of Knowledge	2	Mean Score	1.03
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Assessment Anchor this item will be reported under:

M07.A-R.1 — Demonstrate an understanding of proportional relationships.

Specific Anchor Descriptor addressed by this item:

M07.A-R.1.1—Analyze, recognize, and represent proportional relationships and use them to solve real-world and mathematical problems.

Scoring Guide

Score	In this item, the student
4	Demonstrates a thorough understanding of proportional relationships by correctly solving problems and clearly explaining procedures.
3	Demonstrates a general understanding of proportional relationships by correctly solving problems and clearly explaining procedures with only minor errors or omissions.
2	Demonstrates a partial understanding of proportional relationships by correctly performing a significant portion of the required task.
1	Demonstrates minimal understanding of proportional relationships.
0	The response has no correct answer and insufficient evidence to demonstrate any understanding of the mathematical concepts and procedures as required by the task. Response may show only information copied from the question.

Top-Scoring Student Response and Training Notes

Score	Description
4	Student earns 4 points.
3	Student earns 3.0–3.5 points.
2	Student earns 2.0–2.5 points.
1	Student earns 0.5–1.5 points. OR Student demonstrates minimal understanding of proportional relationships.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.

Top-Scoring Response

Part A (1 point):

1 point for correct answer

What?	Why?
Answers may vary. Accept any fraction equivalent to $\frac{2}{23}$ or any decimal from 0.08 to 0.09.	
Sample Responses:	
$\frac{2}{23}$	
0.0869	

Part B (1 point):

1 point for correct and complete response

OR $\frac{1}{2}$ point for correct but incomplete response

What?	Why?
Sample Responses: The <i>y</i> -coordinate of this point represents the number of pages Sara types each minute.	
OR	
Sara can type 0.5 pages per minute.	
OR equivalent	

Part C (2 points):

1 point for correct answer

1 point for correct and complete support

OR $\frac{1}{2}$ point for correct but incomplete support

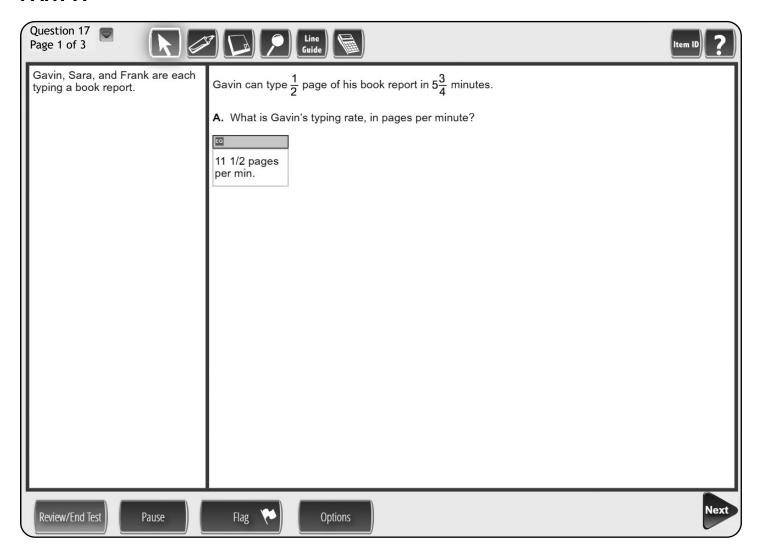
What?	Why?
14.4 (minutes) OR 14. $\frac{2}{5}$ (minutes) OR 14 minutes 24 seconds	Sample Work: $0.4(3) = 1.2$ pages $3 - 1.2 = 1.8$ pages $1.8 \div \frac{1}{8} = 14.4$ minutes OR Sample Explanation: Since Frank has 40% of his report typed and his final report will be 3 pages long, that means he has typed $0.4(3) = 1.2$ pages. So, he has $3 - 1.2 = 1.8$ pages remaining to type. Since he types at a rate of $\frac{1}{8}$ page per minute, Frank needs $1.8 \div \frac{1}{8} = 14.4$ more minutes to finish typing his book report. OR equivalent

STUDENT RESPONSE

Response Score: 3 points

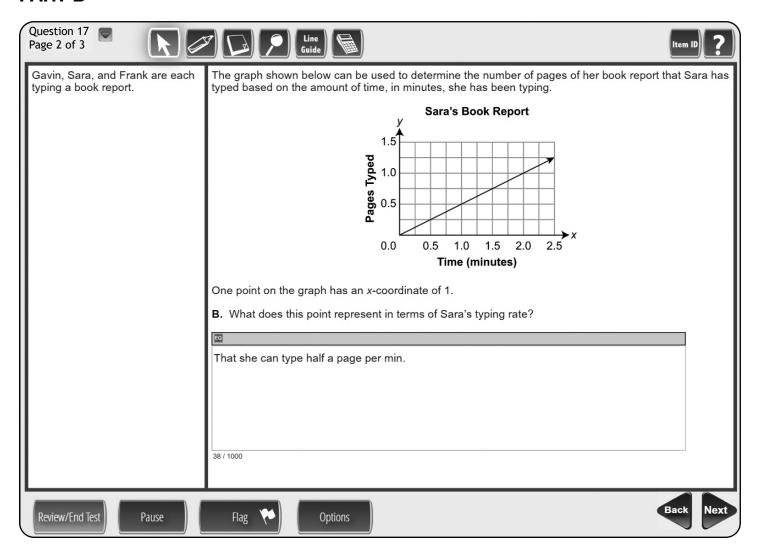


PART A



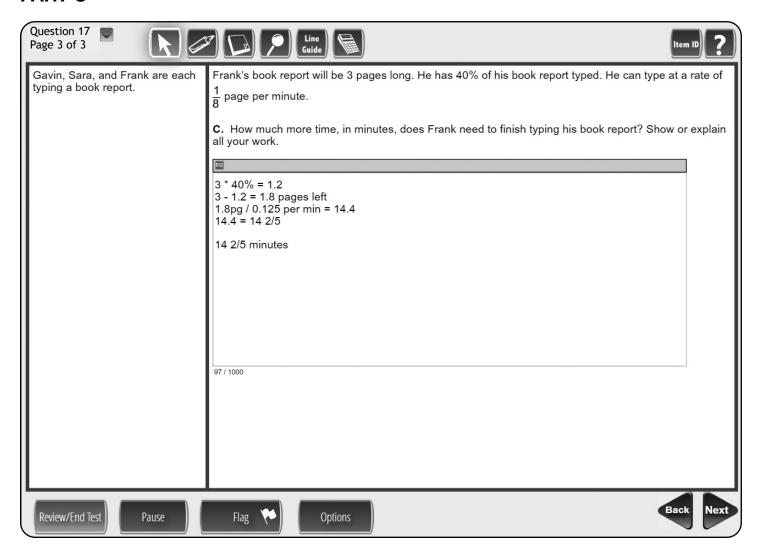
Part A. The student provided an incorrect answer (11 1/2 pages per min.) for Gavin's typing rate. While support is not required for Part A, the student likely set up the division incorrectly, using $5\frac{3}{4} \div \frac{1}{2}$ rather than $\frac{1}{2} \div 5\frac{3}{4}$. [0 points]

PART B



Part B. The student provided the correct response (*half a page per min.*). The student correctly interpreted the graph, observing that the *x*-coordinate of 1 represents 1 minute and corresponds to 0.5 pages typed on the *y*-axis. [1 point]

PART C



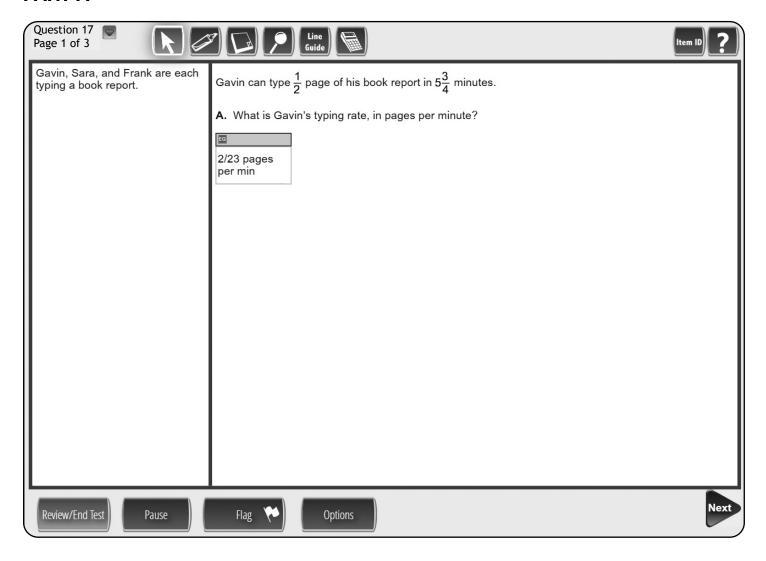
Part C. The student provided the correct answer ($14\ 2/5\ minutes$) with correct and complete support. The work provided shows the student first found how many pages have been typed so far (3*40%=1.2) and then subtracted the 1.2 pages completed from 3 pages total to find there are 1.8 pages left. The student divided the remaining 1.8 pages by the $\frac{1}{8}$ page per minute typing rate (converted to decimal form of 0.125) to find the answer of 14.4. The student then converted the decimal form of the answer to a fraction ($14.4=14\ 2/5$). [2 points]

STUDENT RESPONSE

Response Score: 1 point

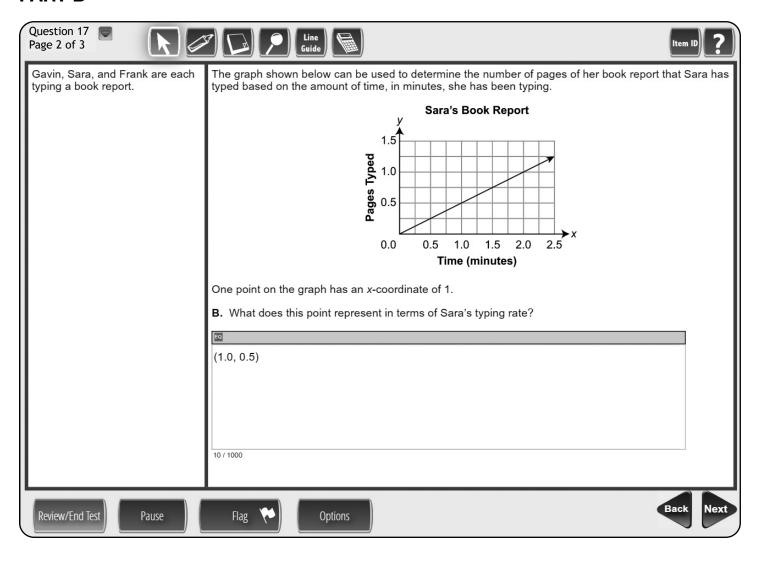


PART A



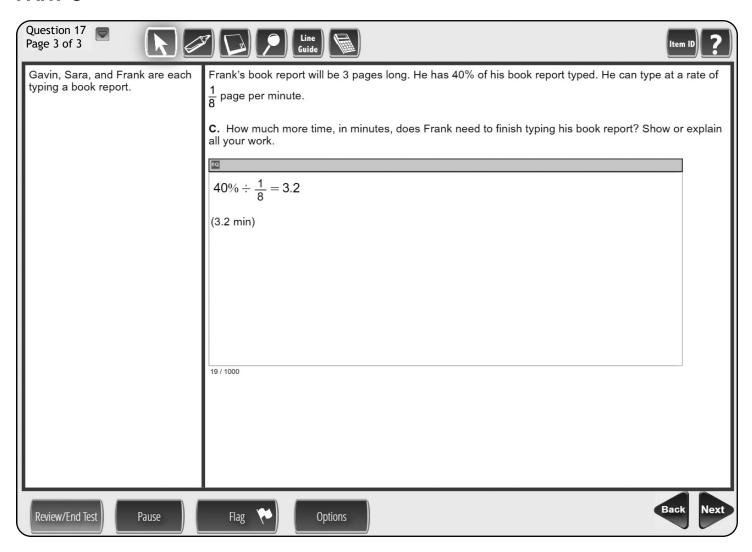
Part A. The student provided the correct answer (2/23 pages per min) for Gavin's typing rate. While support is not required for Part A, the student likely divided $\frac{1}{2}$ by $5\frac{3}{4}$. [1 point]

PART B



Part B. The student provided an incorrect response. Rather than describing what the point represents, the student identified the ordered pair for the point that has an *x*-coordinate of 1 [(1.0, 0.5)]. [0 points]

PART C



Part C. The student provided an incorrect answer (3.2 min) with incorrect and incomplete support. The work shown $\left|40\% \div \frac{1}{8}\right|$ is an incorrect step and does not earn any credit. [0 points]

MATHEMATICS—SUMMARY DATA

Multiple-Choice

Sample Number	Alignment	Answer Key	Depth of Knowledge	<i>p</i> -value A	<i>p</i> -value B	<i>p</i> -value C	<i>p</i> -value D
1	A-N.1.1.3	А	1	47%	22%	15%	16%
2	A-N.1.1.2	В	1	30%	61%	5%	4%
3	A-R.1.1.1	С	2	24%	21%	36%	19%
4	A-R.1.1.2 A-R.1.1.4	В	2	12%	64%	12%	12%
5	A-R.1.1.4 A-R.1.1.3	В	2	25%	34%	29%	12%
6	A-R.1.1.6	D	2	16%	18%	21%	45%
7	B-E.1	В	2	14%	63%	10%	13%
8	B-E.1.1	D	1	14%	17%	16%	53%
9	B-E.2.2	В	2	12%	70%	8%	10%
10	B-E.2.2.1	С	2	13%	16%	60%	11%
11	C-G.1.1	D	2	13%	11%	21%	55%
12	C-G.2.1.2	С	1	17%	19%	38%	26%
13	C-G.2.2.1	С	2	19%	27%	39%	15%
14	D-S.2.1.1	С	2	17%	18%	44%	21%
15	D-S.3	В	2	8%	51%	29%	12%
16	D-S.3.1.1	С	2	13%	23%	54%	10%

Open-Ended

Sample Number	Alignment	Points	Depth of Knowledge	Mean Score
17	A-R.1.1.1 A-R.1.1.5 A-R.1.1.6	4	2	1.03