Pennsylvania PSSA 2016 Grade 6 Math

Reference Materials
Page 2

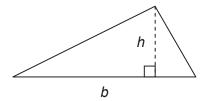
Exam & Answer Key Materials
Pages 3 - 38

Grade 6 Formula Sheet

Formulas that you may need to work questions on this test are found below. You may refer back to this page at any time during the mathematics test.

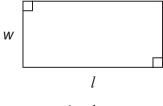
2016 Grade 6

Triangle



$$A = \frac{1}{2} bh$$

Rectangle



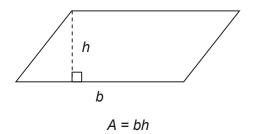
A = lw

Square

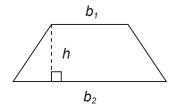


 $A = s^2$

Parallelogram

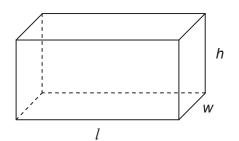


Trapezoid



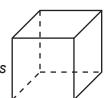
$$A=\frac{1}{2}h(b_1+b_2)$$

Rectangular Prism



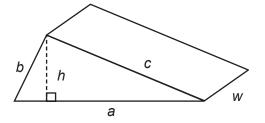
$$V = lwh$$
 $SA = 2lw + 2lh + 2wh$

Cube



$$V = s \cdot s \cdot s$$
 $SA = 6s^2$

Triangular Prism

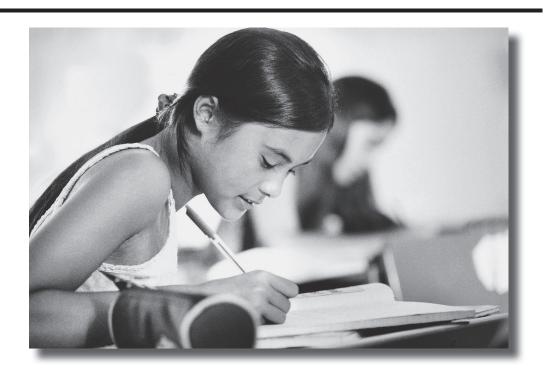


SA = ah + aw + bw + cw



The Pennsylvania System of School Assessment

Mathematics Item and Scoring Sampler



2016-2017 **Grade 6**

Pennsylvania Department of Education Bureau of Curriculum, Assessment and Instruction—September 2016

MATHEMATICS TEST DIRECTIONS

On the following pages are the mathematics questions.

 You may <u>not</u> use a calculator for question 1. You may use a calculator for all other questions on this test.

Directions for Multiple-Choice Questions:

Some questions will ask you to select an answer from among four choices.

For the multiple-choice questions:

- First solve the problem on scratch paper.
- Choose the correct answer and record your choice in the answer booklet.
- If none of the choices matches your answer, go back and check your work for possible errors.
- Only one of the answers provided is the correct response.

Directions for Open-Ended Questions:

Some questions will require you to write your response.

For the open-ended questions:

- These questions have more than one part. Be sure to read the directions carefully.
- You cannot receive the highest score for an open-ended question without completing
 all tasks in the question. For example, if the question asks you to show your work or
 explain your reasoning, be sure to show your work or explain your reasoning in the
 space provided.
- If the question does **not** ask you to show your work or explain your reasoning, you may use the space provided, but only those parts of your response that the question specifically asks for will be scored.
- Write your response in the appropriate location within the response box in the
 answer booklet. Some answers may require graphing, plotting, labeling, drawing, or
 shading. If you use scratch paper, be sure to transfer your final response and any
 needed work or reasoning to the answer booklet.

INFORMATION ABOUT MATHEMATICS

General Description of Scoring Guidelines for Mathematics Open-Ended Questions

4 – The response demonstrates a *thorough* understanding of the mathematical concepts and procedures required by the task.

The response provides correct answer(s) with clear and complete mathematical procedures shown and a correct explanation, as required by the task. Response may contain a minor "blemish" or omission in work or explanation that does not detract from demonstrating a *thorough* understanding.

3 – The response demonstrates a *general* understanding of the mathematical concepts and procedures required by the task.

The response and explanation (as required by the task) are mostly complete and correct. The response may have minor errors or omissions that do not detract from demonstrating a *general* understanding.

2 – The response demonstrates a *partial* understanding of the mathematical concepts and procedures required by the task.

The response is somewhat correct with *partial* understanding of the required mathematical concepts and/or procedures demonstrated and/or explained. The response may contain some work that is incomplete or unclear.

- 1 The response demonstrates a *minimal* understanding of the mathematical concepts and procedures required by the task.
- 0 The response has no correct answer and *insufficient* evidence to demonstrate any understanding of the mathematical concepts and procedures required by the task for that grade level.

Response may show only information copied from the question.

Special Categories within zero reported separately:

BLK (blank)	Blank, entirely erased, or written refusal to respond
OT	Off task
LOE	Response in a language other than English
IL	Illegible

Question 1 in this sampler is to be solved without the use of a calculator.

MULTIPLE-CHOICE ITEMS

- 1. Divide: $\frac{3}{4} \div \frac{2}{3}$
 - A. $\frac{1}{2}$
 - B. $\frac{8}{9}$
 - C. $\frac{9}{8}$
 - D. 2

	Item Information			Option Annotations
	Alignment A-N.1.1.1		.1.1	A. multiplies 3/4 × 2/3
	Answer Key C			B. inverts 3/4 instead of 2/3 prior to multiplying C. correct
Depth of	epth of Knowledge 1			D. inverts both fractions prior to multiplying
	p-values			
Α	В	С	D	
28%	12%	52%	8%	

A calculator is permitted for use in solving questions 2–17 in this sampler.

2. Greg's social studies grade is based on two quizzes, two tests, and one project. The table below shows Greg's scores on these items, as well as each item's weight in determining his final grade.

Greg's Social Studies Scores

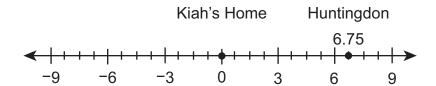
Item	Score	Weight
quiz 1	78	1
quiz 2	90	1
test 1	85	2
test 2	89	2
project	92	4

To determine his final grade, Greg's teacher multiplies each score by its weight, adds the products together, and then divides the sum by 10. What is Greg's final grade?

- A. 79.9
- B. 86.8
- C. 88.4
- D. 92.0

	Item Information			Option Annotations
	Alignmen	t A-N.2	.1.1	A. enters $(78 + 90 \times 1) + (85 + 89 \times 2) + (92 \times 4)$; order of
	Answer Ke	y C		operations error B. finds the average of the scores
Depth of	Knowledge	e 2		C. correct
				D. thinks project grade is final grade because it has greatest weight
	<i>p</i> -values			
Α	В	С	D	
8%	13%	69%	10%	

3. Kiah plotted the locations of her home and the city of Huntingdon on the number line shown below.



Williamsburg is the same distance from Kiah's home as Huntingdon, but it is in the opposite direction. Which statement best describes how to find the location of Williamsburg on the number line?

- A. The opposite of 6.75 is -6.75, so Williamsburg is at -6.75.
- B. The sum of 6.75 and 6.75 is 13.5, so Williamsburg is at 13.5.
- C. The numbers 6.75 and -6.75 are the same, so Williamsburg is at 6.75.
- D. Opposites, such as 6.75 and -6.75, sum to zero, so Williamsburg is at 0.

	Item Information			Option Annotations
	Alignme	nt A-N.3	3.1	A. correct
	Answer K	ey A		B. sums the measurements together C. thinks of absolute values
Depth of	of Knowledge 2			D. wrong property, fails to identify Kiah's home as 0
	p-values			
Α	В	С	D	
76%	8%	8%	8%	

- 4. James measures the water level from the top of a dock twice a day. The water level in the morning is -2 feet. The water level in the afternoon is -6.5 feet. Which statement about the relationship between the two measurements is true?
 - A. A water level of -6.5 feet is higher than a water level of -2 feet, as -6.5 > -2.
 - B. A water level of -2 feet is lower than a water level of -6.5 feet, as -2 > -6.5.
 - C. A water level of -6.5 feet is the same as a water level of -2 feet, as -6.5 = -2.
 - D. A water level of -2 feet is higher than a water level of -6.5 feet, as -2 > -6.5.

	Item Information			Option Annotations
	Alignmen	t A-N.3	3.2.1	A. thinks -6.5 feet is higher than -2 feet because -6.5 is "more"
	Answer Ke	y D		negative than -2 B. thinks -2 is lower than -6.5 in terms of water level, but inequality
Depth of	Knowledg	e 1		is correct
		•		C. thinks measurements are the same since they are both negative
	p-values			D. correct
Α	В	С	D	
15%	11%	6%	68%	
				<u>1</u>

- **5.** The ratio of the number of boys to the number of girls in the cast of a school play is 1:5. Which statement **must** describe the cast of the play?
 - A. There are exactly 6 students in the cast.
 - B. There is 1 boy for every 5 girls in the cast.
 - C. There are 4 more girls than boys in the cast.
 - D. There is 1 boy out of the 5 students in the cast.

	Item Information			Option Annotations
	Alignment	A-R.1	.1.1	A. thinks ratio shows total number
	Answer Key	В		B. correct C. thinks 5 – 1
Depth of	f Knowledge	1		D. mistakes ratio of boys to girls for ratio of boys to all students in
				the cast
	<i>p</i> -values			
Α	В	С	D	
15%	64%	8%	13%	

- **6.** For a recipe, Harris uses 2 cups of white sugar for each cup of brown sugar. How many cups of brown sugar does Harris use when he uses 1 cup of white sugar?
 - A. $\frac{1}{2}$
 - B. 1
 - C. $1\frac{1}{2}$
 - D. 2

	Item Info	rmation		Option Annotations
	Alignment A-R.1.1.2		.1.2	A. correct
	Answer K	ey A		B. thinks the cups of brown sugar is a constant C. subtracts 1/2 from 2
Depth of	Depth of Knowledge 2			D. reverses the original ratio
	p-values			
Α	В	С	D	
71%	10%	10%	9%	

- **7.** Jasmine earns \$36 for 4 hours of baby-sitting. She charges a constant hourly rate. Which table correctly shows the amount Jasmine earns baby-sitting for different numbers of hours?
 - A. Jasmine's Baby-Sitting Earnings

Number of Hours	Amount Earned (\$)
2	18
4	36
6	54
8	72

B. Jasmine's Baby-Sitting Earnings

Number of Hours	Amount Earned (\$)
2	34
4	36
6	38
8	40

C. Jasmine's Baby-Sitting Earnings

Number of Hours	Amount Earned (\$)
2	27
4	36
6	45
8	54

D. Jasmine's Baby-Sitting Earnings

Number of Hours	Amount Earned (\$)
2	32
4	36
6	42
8	50

	Item Infor	mation		Option Annotations
	Alignme	A-R.1 A-R.1		A. correct B. adds/subtracts change in hours from earnings
	Answer Key A			C. finds rate is \$9 per hour, but assumes hours in table are in increments of 1 D. adds/subtracts amount in hours from earnings
Depth of	Depth of Knowledge 2			
	<i>p</i> -valu	es		
Α	A B C D			
78%	78% 8%		5%	

- **8.** Alon started in 60% of his team's basketball games this season. He started a total of 12 games. How many games did Alon's team play this season?
 - A. 6
 - B. 7
 - C. 18
 - D. 20

	Item Inform	nation		Option Annotations
	Alignment A-R.1.1.5			A. subtracts 12 – 6
	Answer Key D			B. finds 60% of 12, rounding down to 7 C. adds 12 + 6 D. correct
Depth of	Depth of Knowledge 2			
	p-value	es		
Α	A B C D			
13%	13% 23% 18% 46%		46%	

9. An inequality is shown below.

$$x + 2.5 < 20$$

What is the **greatest** value of x from the set {10.5, 12.5, 17.5, 19.5} that makes the inequality true?

- A. 10.5
- B. 12.5
- C. 17.5
- D. 19.5

	Item Infor	rmation		Option Annotations
	Alignme	nt B-E.2	.1.1	A. $10.5 + 2.5 = 13$; makes the inequality true but is not the greatest
	Answer Key B			value from the set to do so B. correct
Depth of	Knowled	ge 1		C. 17.5 + 2.5 = 20; makes the inequality not true D. 19.5 + 2.5 = 22; though 19.5 is the greatest value in the set that is less than 20, adding the value to 2.5 makes the inequality not true
	<i>p</i> -valu	ues		
Α	АВ		D	
11%	11% 55% 19% 159		15%	

- **10.** There are 150 children playing in a park.
 - The number of boys (x) playing in the park is **greater** than 50.
 - The number of girls (y) playing in the park is **less** than 100.

Which values of x and y could be the numbers of boys and girls playing in the park?

A.
$$x = 35$$

 $y = 115$

B.
$$x = 50$$

 $y = 100$

C.
$$x = 55$$

 $y = 85$

D.
$$x = 60$$

 $y = 90$

	Item Inforn	nation		Option Annotations
	Alignmen	B-E.2 B-E.2		A. reverses inequality statements B. uses numbers given, does not consider inequality statements
	Answer Key D			C. does not consider total number of boys and girls D. correct
Depth of	Depth of Knowledge 2			
	p-value	26		
	p-value			
Α	A B		D	
3%	3% 9% 1		77%	

11. Gary saves \$2.50 each day. Which equation describes the relationship between the number of days (*d*) Gary saves money and the total amount of money (*m*), in dollars, that he saves?

A.
$$m = 2.50d$$

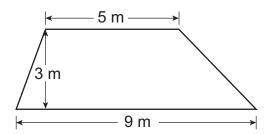
B.
$$m = d - 2.50$$

C.
$$d = 2.50m$$

D.
$$d = \frac{2.50}{m}$$

	Item Infor	mation		Option Annotations
	Alignment B-E.3.1.1			A. correct
	Answer Key A			B. equation creates incorrect relationship C. equation switches <i>d</i> and <i>m</i>
Depth of	Depth of Knowledge 2			D. equation creates incorrect relationship
	n voli	100		
	p-valı	25		
Α	A B C		D	
55%	5% 10% 2		11%	
			•	

12. A trapezoid is shown below.

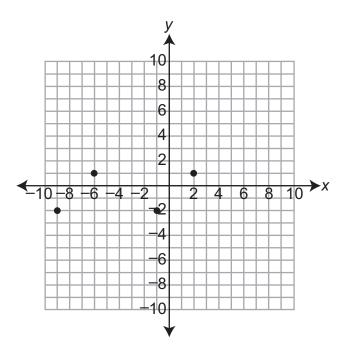


What is the area of the trapezoid?

- A. 21 m²
- B. 27 m²
- C. 34 m²
- D. 42 m²

	Item Info	rmation		Option Annotations
	Alignment C-G.1.1.1			A. correct
	Answer Key A			B. 3 × 9 C. (5 + 9 + 3) × 2; incorrect formula
Depth of	Depth of Knowledge 1			D. (5 + 9) × 3; forgot to multiply by 1/2
	p-val	ues		
Α	В	С	D	
60%	60% 19% 9%		12%	

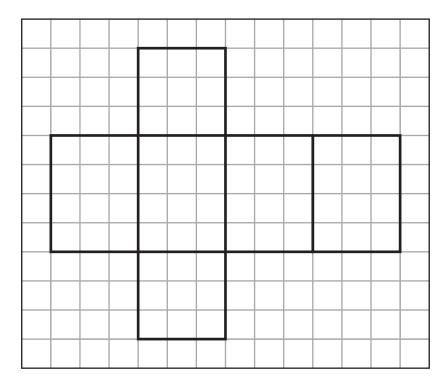
13. The vertices of a parallelogram are plotted on the coordinate plane shown below.



- What is the area, in square units, of the parallelogram?
- A. 8
- B. 9
- C. 12
- D. 24

	Item Info	rmation		Option Annotations
	Alignme	ent C-G.1	.1.4	A. uses (-9, 2) and (-1, 2) instead of (-9, -2) and (-1, -2)
	Answer Key D			B. uses the vertical and horizontal distances between (-6, 1) and (-9, -2) C. thinks the formula is (1/2) × bh
Depth of	Depth of Knowledge 2			
		•		D. correct
	<i>p</i> -val	ues		
Α	A B C D		D	
11%	11% 7% 14		68%	

14. Rosa built a jewelry box. She first cut out all the pieces she would need by using the pattern shown below.

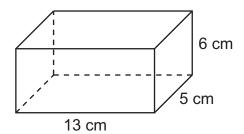


Based on the pattern, which phrase best describes the shape of the completed jewelry box?

- A. a cube with a box top
- B. a cube with no box top
- C. a rectangular prism with a box top
- D. a rectangular prism with no box top

Item Information				Option Annotations
	Alignment C-G.1.1.5			A. thinks any six-sided solid is a cube
	Answer Key C			B. thinks any six-sided solid is a cube and does not realize that the top is included
Depth of	Depth of Knowledge 2			C. correct
	<i>p</i> -valu	ues		D. does not see that the row of four sides eventually folds around to make the top
Α	A B C D		D	
21%	21% 4% 64% 11%		11%	

15. A rectangular prism is pictured below.



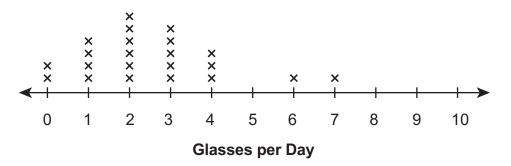
What is the surface area of the rectangular prism?

- A. 173 cm²
- B. 320 cm²
- C. 346 cm²
- D. 390 cm²

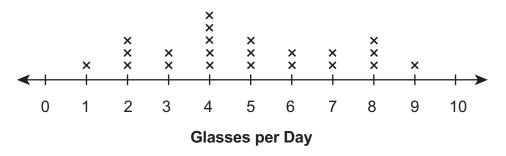
Item Information				Option Annotations
	Alignment C-G.1.1.6			 A. does not multiply by 2 B. uses 5 × 13 face 4 times instead of twice, forgetting that 2 faces are 6 × 13 C. correct D. calculates volume instead of surface area
	Answer Key C			
Depth of	Depth of Knowledge 1			
	<i>p</i> -valu	es		D. Calculates volume instead of surface area
Α	A B C D		D	
10%	10% 7%		37%	

16. Franco asked his soccer team how many glasses of milk and how many glasses of water each player drinks per day. The line plots below show his data.

Amount of Milk Each Player Drinks



Amount of Water Each Player Drinks



Which statement correctly describes the number of glasses of milk and the number of glasses of water each player drinks per day?

- A. The mean would be a better measure of center than the median for the number of glasses of milk the players drink.
- B. There is less variability in the number of glasses of milk the players drink than the number of glasses of water they drink.
- C. The median number of glasses of milk the players drink is greater than the mean number of glasses of milk the players drink.
- D. The range for the number of glasses of milk and the range for the number of glasses of water the players drink are the same.

	Item Infor	mation		Option Annotations
	Alignment			A. confuses mean and median
	Answer Key			B. correct C. thinks extreme values affect the median more than the mean
Depth of	Depth of Knowledge 3			D. forgets the × on the 9 for glasses of water
	<i>p</i> -valu	ies		
Α	В	С	D	
21%	50%	19%	10%	

OPEN-ENDED QUESTION

A.	Write an expression to represent how many kilometers Annika rides her bike to school.
Saı	m rides his bike at most 2 kilometers to school.
B.	What is the greatest distance, in kilometers, Annika could ride her bike to school? Show or explain all your work.

Go to the next page to finish question 17.

17. Continued. Please refer to the previous page for task explanation.

C. Explain why Sam must ride his bike more than $\frac{1}{2}$ kilometer to school.

Item-Specific Scoring Guideline

#17 Item Information

Alignment	B-E.1	Depth of Knowledge	3	Mean Score	1.40
-----------	-------	--------------------	---	------------	------

Assessment Anchor this item will be reported under:

M06.B-E.1—Apply and extend previous understandings of arithmetic to numerical and algebraic expressions.

Specific Anchor Descriptor addressed by this item:

M06.B-E.1.1—Identify, write, and evaluate numerical and algebraic expressions.

Scoring Guide

Score	In this item, the student
4	Demonstrates a thorough understanding of identifying, writing, and evaluating numerical and algebraic expressions by correctly solving problems and clearly explaining procedures.
3	Demonstrates a general understanding of identifying, writing, and evaluating numerical and algebraic expressions by correctly solving problems and clearly explaining procedures with only minor errors or omissions.
2	Demonstrates a partial understanding of identifying, writing, and evaluating numerical and algebraic expressions by correctly performing a significant portion of the required task.
1	Demonstrates minimal understanding of identifying, writing, and evaluating numerical and algebraic expressions.
0	The response has no correct answer and insufficient evidence to demonstrate any understanding of the mathematical concepts and procedures as required by the task. Response may show only information copied from the question.

Top-Scoring Student Response and Training Notes

Score	Description				
4	Student earns 4 points.				
3	Student earns 3.0–3.5 points.				
2	Student earns 2.0–2.5 points.				
1	Student earns 0.5–1.5 points. OR Student demonstrates minimal understanding of identifying, writing, and evaluating numerical and algebraic expressions.				
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.				

Top-Scoring Response

Part A (1 point):

1 point for correct answer

What?	Why?
4k – 2	
OR equivalent	

Part B (2 points):

1 point for correct answer

1 point for complete explanation

OR $\frac{1}{2}$ point for correct but incomplete explanation

What?	Why?		
6 (kilometers)	Sample Work:		
[Note: Carry over any error from	4(2) - 2 = 8 - 2 = 6		
Part A unless it is blank.]	OR		
	Sample Explanation:		
	Since Annika rides her bicycle 2 less than 4 times as many kilometers as Sam, she rides her bike 2 less than $4(2) = 8$ kilometers, which is $8 - 2 = 6$ kilometers.		

27

Part C (1 point):

1 point for complete explanation

 $OR\frac{1}{2}$ point for correct but incomplete explanation

What?	Why?
	Sample Explanation:
	If Sam were to ride his bike $\frac{1}{2}$ kilometer, that means that Annika would
	ride her bike $4\left(\frac{1}{2}\right) - 2 = 2 - 2 = 0$ kilometers. Then she would not be
	riding her bike to school at all. So Sam must ride a distance greater
	than $\frac{1}{2}$ kilometer from his home to school.
	[Note: Carry over any error from Part A unless it is blank.]

STUDENT RESPONSE

Response Score: 4 points

- Sam and Annika ride their bikes to school. Sam rides his bike k kilometers, Annika rides her bike 2 less than 4 times as many kilometers as Sam rides.
 - A. Write an expression to represent how many kilometers Annika rides her bike to school.

K4-2

The student has given a correct expression.

Sam rides his bike at most 2 kilometers to school.

B. What is the **greatest** distance, in kilometers, Annika could ride her bike to school? Show or explain all your work.

the gretest distance is 6 Kilometers because if you insert 2 into the Epression it would be 2.4-2= A So 2 times 4 is 8 and 2 subtracted

from 8 is Six



The student has given a correct answer and complete support.

- **17.** *Continued.* Please refer to the previous page for task explanation.
 - **C.** Explain why Sam must ride his bike more than $\frac{1}{2}$ kilometer to school.

if San doesn't ride his like More than & Kilometer then annika wont be able to ride her bike at all. So if you insert half in the

1.4-2=A will get this,

So $\pm .4 = 2$ and 2-2 = 0So anniha couldn't go anywhere Unless he rides more than 1 Kilometer.

> The student has given a complete explanation.

STUDENT RESPONSE

Response Score: 3 points

Next

₽
D
S
\triangleright
\triangleright
Z
D
B

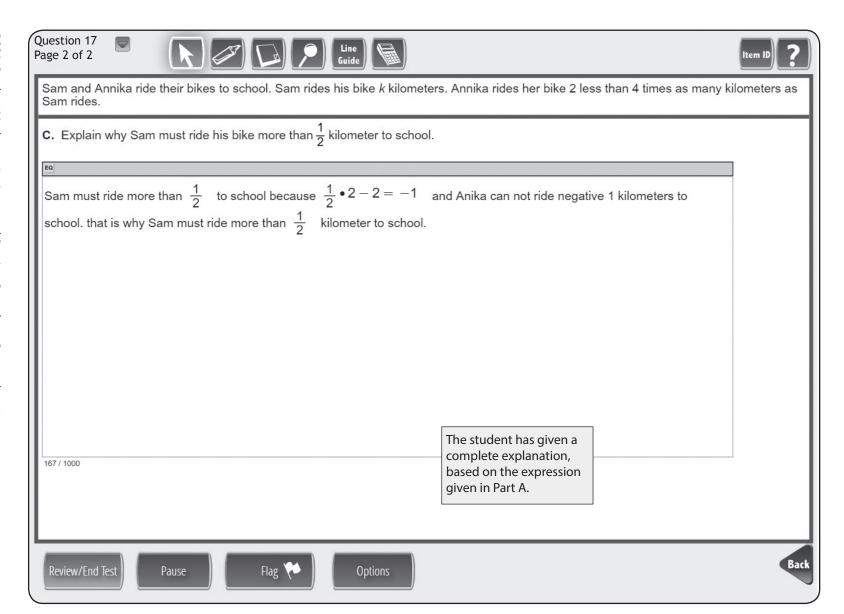
P

Question 17 Page 1 of 2 Sam and Annika ride their bikes to school. Sam rides his bike k kilometers. Annika rides her bike 2 less than 4 times as many kilometers as Sam rides. A. Write an expression to represent how many kilometers Annika rides her bike to school. $k^2 - 2 =$ The student has given an incorrect expression. Sam rides his bike at most 2 kilometers to school. B. What is the greatest distance, in kilometers, Annika could ride her bike to school? Show or explain all your work. The greatest distance she could ride is 2 kilometers. I used the expression k2 - 2 = 2. First, I plugged 2 in for k, $2 \cdot 2 = 4$ and 4 - 2 = 2. That is how I got my answer. The student has given a correct answer and complete 158 / 1000 support, based on the expression given in Part A.

Options

Review/End Test

Pause

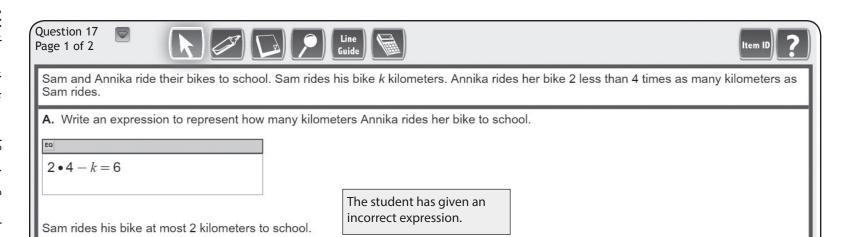


STUDENT RESPONSE

PARTS A AND B

Response Score: 2 points





The student has given a correct answer and complete

Next

support.

B. What is the greatest distance, in kilometers, Annika could ride her bike to school? Show or explain all your work.

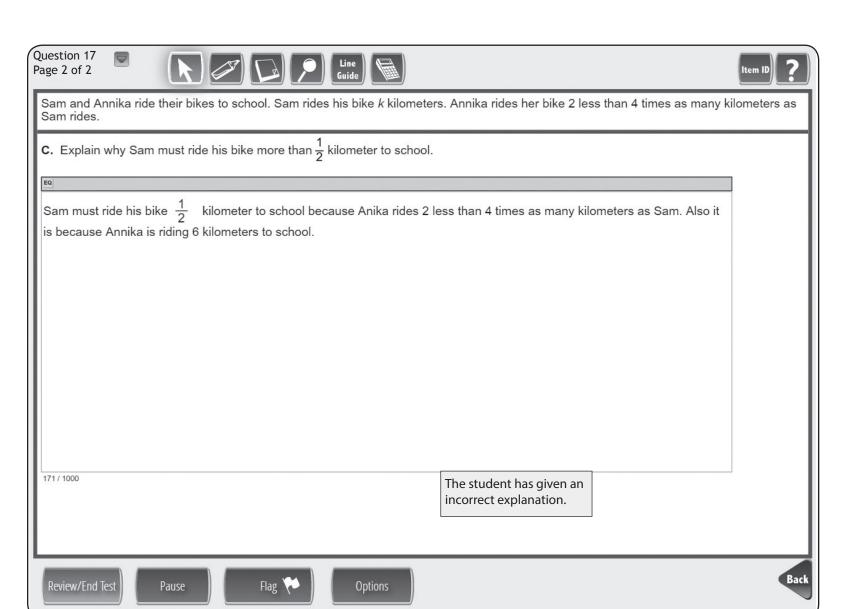
Annika could ride her bike 6 kilometers to school.

8 - 2 = 6

62 / 1000

Review/End Test Pause Options

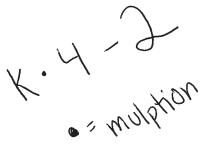
PART C



STUDENT RESPONSE

Response Score: 1 point

- **17.** Sam and Annika ride their bikes to school. Sam rides his bike *k* kilometers. Annika rides her bike 2 less than 4 times as many kilometers as Sam rides.
 - **A.** Write an expression to represent how many kilometers Annika rides her bike to school.



The student has given a correct expression.

Sam rides his bike at most 2 kilometers to school.

B. What is the **greatest** distance, in kilometers, Annika could ride her bike to school? Show or explain all your work.

the greatest distence Annika could ride her bike to achool is Ok. because dam rides Zk. and Annika rides ZK less than sam, making it Ok.

The student has given an incorrect answer and incorrect support.

Go to the next page to finish question 17.

- 17. Continued. Please refer to the previous page for task explanation.
 - **C.** Explain why Sam must ride his bike more than $\frac{1}{2}$ kilometer to school.

because his house is further than 1/2 kilo. to school? ? ? school???

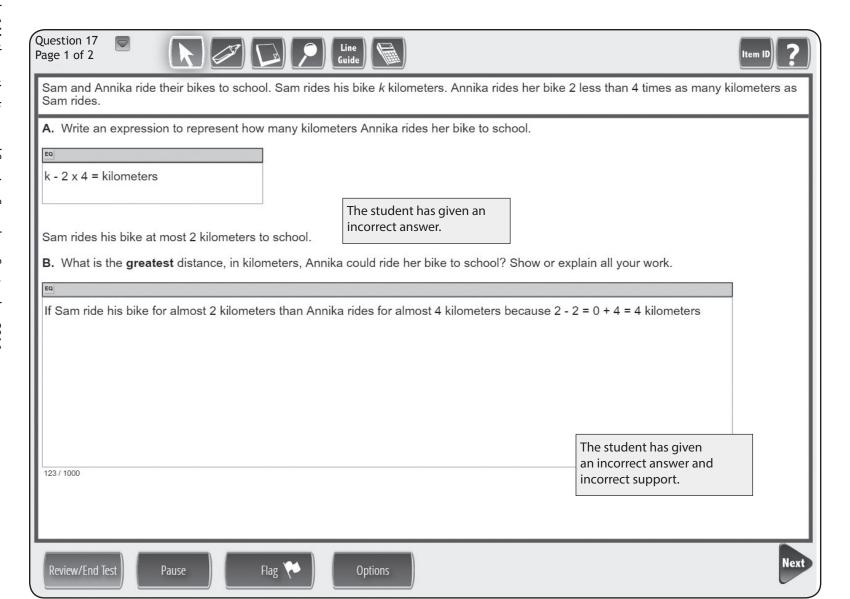
The student has given an incorrect explanation.

STUDENT RESPONSE

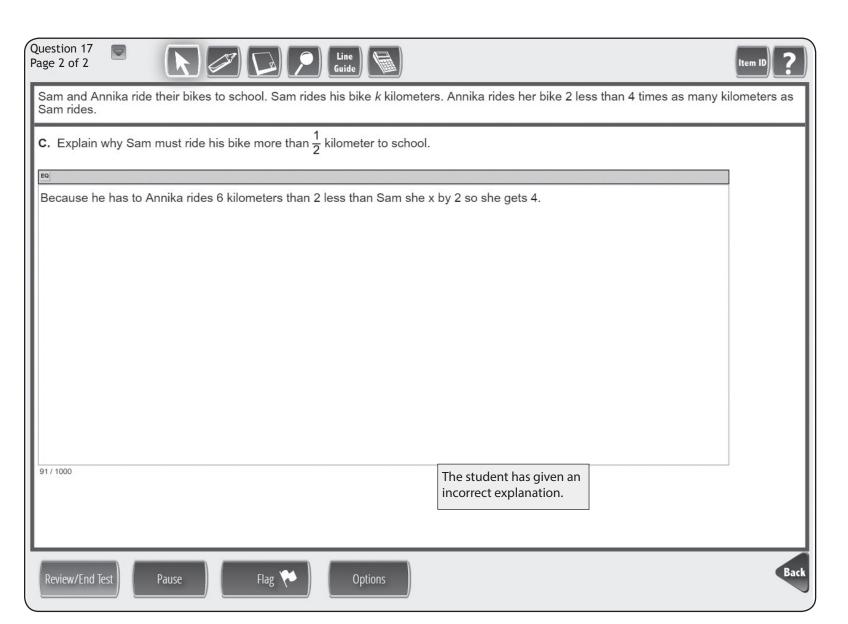
PARTS A AND B

Response Score: 0 points





PART C





MATHEMATICS—SUMMARY DATA

MULTIPLE-CHOICE

Sample			Depth of	p-values			
Number	Alignment	Answer Key	Knowledge	Α	В	С	D
1	A-N.1.1.1	С	1	28%	12%	52%	8%
2	A-N.2.1.1	С	2	8%	13%	69%	10%
3	A-N.3.1	Α	2	76%	8%	8%	8%
4	A-N.3.2.1	D	1	15%	11%	6%	68%
5	A-R.1.1.1	В	1	15%	64%	8%	13%
6	A-R.1.1.2	А	2	71%	10%	10%	9%
7	A-R.1.1.4 A-R.1.1.3	А	2	78%	8%	9%	5%
8	A-R.1.1.5	D	2	13%	23%	18%	46%
9	B-E.2.1.1	В	1	11%	55%	19%	15%
10	B-E.2.1.3 B-E.2.1.4	D	2	3%	9%	11%	77%
11	B-E.3.1.1	А	2	55%	10%	24%	11%
12	C-G.1.1.1	Α	1	60%	19%	9%	12%
13	C-G.1.1.4	D	2	11%	7%	14%	68%
14	C-G.1.1.5	С	2	21%	4%	64%	11%
15	C-G.1.1.6	С	1	10%	7%	46%	37%
16	D-S.1	В	3	21%	50%	19%	10%

OPEN-ENDED

Sample Number	Alignment	Points	Depth of Knowledge	Mean Score
17	B-E.1	4	3	1.40