Pennsylvania PSSA 2023 Grade 4 Math

Reference Materials
Page 2

Exam & Answer Key Materials Pages 3 - 41

INFORMATION ABOUT MATHEMATICS

Grade 4 Formula Sheet

Formulas and conversions that you may need on this test are found below. You may refer back to this page at any time during the mathematics test.

2023 Grade 4

Standard Conversions

$$1 \text{ yard (yd)} = 3 \text{ feet (ft)}$$

1 pound (lb) =
$$16$$
 ounces (oz.)

$$1 \text{ pint} = 2 \text{ cups (c)}$$

Metric Conversions

$$1 \text{ kilogram (kg)} = 1,000 \text{ grams (g)}$$

1 liter (L) =
$$1,000$$
 milliliters (mL)

Time Conversions

1 year (yr) = 12 months (mo)

1 year = 52 weeks (wk)

1 year = 365 days

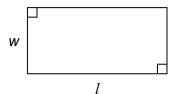
1 week = 7 days

1 day = 24 hours (hr)

1 hour = 60 minutes (min)

1 minute = 60 seconds (sec)

Rectangle



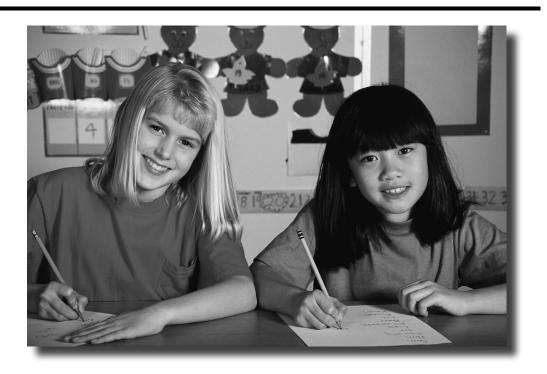
$$A = l \times w$$

$$P = l + l + w + w$$



The Pennsylvania System of School Assessment

Mathematics Item and Scoring Sampler



2023-2024 Grade 4

MATHEMATICS TEST DIRECTIONS

On the following pages are the mathematics questions.

- You may <u>not</u> use a calculator for question 1. You may use a calculator for all other questions on this test.
- You may need a protractor for questions on this test.

Directions for Multiple-Choice Questions

Some questions will ask you to select an answer from among four choices.

For the multiple-choice questions:

- First solve the problem on scratch paper.
- Choose the correct answer and record your choice in the answer booklet.
- If none of the choices matches your answer, go back and check your work for possible errors.
- Only one of the answers provided is the correct response.

Directions for Open-Ended Questions

Some questions will require you to write your response.

For the open-ended questions:

- These questions have more than one part. Be sure to read the directions carefully.
- You cannot receive the highest score for an open-ended question without completing all tasks in the question. For example, if the question asks you to show your work or explain your reasoning, be sure to show your work or explain your reasoning in the space provided.
- If the question does **not** ask you to show your work or explain your reasoning, you may use the space provided, but only those parts of your response that the question specifically asks for will be scored.
- Write your response in the appropriate location within the response box in the answer booklet. Some answers may require graphing, plotting, labeling, drawing, or shading. If you use scratch paper, be sure to transfer your final response and any needed work or reasoning to the answer booklet.

Question 1 in this sampler is to be solved without the use of a calculator.

MULTIPLE-CHOICE ITEMS

1. Divide: $5,113 \div 3$

A. 171

B. 174 R1

C. 1,701

D. 1,704 R1

| Item Information | |
|--------------------|---|
| Alignment | A-T.2.1.3 |
| Answer Key | D |
| Depth of Knowledge | 1 |
| p-value A | 9% |
| p-value B | 17% |
| p-value C | 19% |
| p-value D | 55% (correct answer) |
| Option Annotations | A. omits the 1 in the tens place of the dividend since 3 cannot divide evenly into 1 and does not use a 0 as a placeholder in the tens place of the quotient B. does not use a 0 as a placeholder in the tens place of the quotient C. omits the 1 in the tens place of the dividend since 3 cannot divide evenly into 1 D. Correct: recognizes that 3 divides into 5 (thousands) 1 time with 2 remaining, that 3 divides into 21 (hundreds) 7 times with 0 remaining, that 3 divides into 1 (tens) 0 times with 1 remaining, and that 3 divides into 13 (ones) 4 times with 1 remaining |

A calculator is permitted for use in solving questions 2–17 in this sampler.

2. In 2010, the population of Pittsburgh was 305,704. Which expression shows the population of Pittsburgh in expanded form?

A.
$$300 + 5 + 700 + 4$$

B.
$$30,000 + 5,000 + 700 + 4$$

C.
$$300,000 + 5,000 + 70 + 4$$

D.
$$300,000 + 5,000 + 700 + 4$$

| Item Information | |
|--------------------|--|
| Alignment | A-T.1.1.2 |
| Answer Key | D |
| Depth of Knowledge | 1 |
| p-value A | 7% |
| p-value B | 10% |
| p-value C | 8% |
| p-value D | 75% (correct answer) |
| Option Annotations | A. expands within periods rather than by place value B. splits 305 into two parts: 30 and 5 C. splits 704 into two parts: 70 and 4 D. Correct: recognizes that the 3 is in the hundred-thousands place and represents 300,000, the 5 is in the thousands place and represents 5,000, the 7 is in the hundreds place and represents 700, and the 4 is in the ones place, and then uses these values to write 300,000 + 5,000 + 700 + 4 |

3. The weights, in pounds, of four elephants are listed below.

12,495 12,954 12,599 12,763

Which number sentence correctly compares the weights, in pounds, of two of the elephants?

- A. 12,495 > 12,954
- B. 12,599 > 12,954
- C. 12,495 = 12,954
- D. 12,763 < 12,954

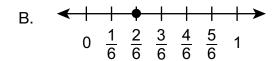
| Item Information | |
|--------------------|--|
| Alignment | A-T.1.1.3 |
| Answer Key | D |
| Depth of Knowledge | 1 |
| p-value A | 7% |
| p-value B | 6% |
| p-value C | 5% |
| p-value D | 82% (correct answer) |
| Option Annotations | A. looks only at the ones place B. sees the two 9s in 12,599 and thinks the number is automatically larger C. sees that both numbers have the same digits and thinks that the numbers must be equal D. Correct: compares the numbers in the ten-thousands place (10,000 = 10,000), then the numbers in the thousands place (2,000 = 2,000), and then the numbers in the hundreds place (700 < 900) |

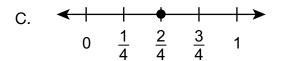
- **4.** A total of 4,896 people went into a football stadium. The stadium had 8 gates. The same number of people went through each gate. Which sentence describes the **closest estimate** of the number of people who went through each gate?
 - A. Since $8 \times 6 = 48$, a little less than 600 people went through each gate.
 - B. Since $8 \times 6 = 48$, a little more than 600 people went through each gate.
 - C. Since $8 \times 6 = 48$, a little less than 800 people went through each gate.
 - D. Since $8 \times 6 = 48$, a little more than 800 people went through each gate.

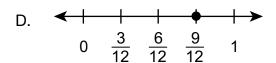
| Item Information | |
|--------------------|--|
| Alignment | A-T.2.1 |
| Answer Key | В |
| Depth of Knowledge | 2 |
| p-value A | 18% |
| p-value B | 47% (correct answer) |
| p-value C | 16% |
| p-value D | 19% |
| Option Annotations | A. does not recognize that the actual number of people should round down to 600 and not round up to 600 B. Correct: recognizes that since 8 × 6 = 48, then 8 × 600 = 4,800, and since 4,800 is less than 4,896, the number of people would be a little more than 600 C. confuses the meaning of the two multiplicands (interprets the meaning as 800 × 6 = 4,800) and does not recognize that the actual number of people should round down and not round up D. confuses the meaning of the two multiplicands (interprets the meaning as 800 × 6 = 4,800) |

5. Which number line shows a point that is graphed at a fraction that is equivalent to $\frac{2}{3}$?









| 11 1 4 11 | |
|--------------------|---|
| Item Information | |
| Alignment | A-F.1.1.1 |
| Answer Key | A |
| Depth of Knowledge | 1 |
| p-value A | 52% (correct answer) |
| p-value B | 18% |
| p-value C | 20% |
| p-value D | 10% |
| Option Annotations | A. Correct: recognizes that $\frac{2}{3} = \frac{4}{6}$ and selects a graph with a point plotted at $\frac{4}{6}$ |
| | B. selects a point plotted at a fraction with the same numerator |
| | C. selects a point plotted on the second of three tick marks between 0 and 1 |
| | D. either does not multiply the numerator and denominator of $\frac{2}{3}$ by the |
| | same amount when finding an equivalent fraction OR finds a fraction |
| | equivalent to $\frac{3}{4} \left(\frac{9}{12} \right)$ rather than one that is equivalent to $\frac{2}{3} \left(\frac{8}{12} \right)$ |

- 6. Angela practiced piano and guitar each day for 5 days.
 - She practiced piano for $\frac{3}{6}$ hour each day.
 - She practiced guitar for $\frac{1}{6}$ hour each day.

What is the total amount of time, in hours, Angela practiced piano and guitar for the 5 days?

- A. $\frac{20}{60}$
- B. $\frac{20}{30}$
- C. $\frac{20}{12}$
- D. $\frac{20}{6}$

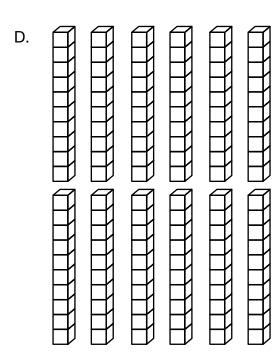
| Item Information | |
|--------------------|---|
| Alignment | A-F.2.1 |
| Answer Key | D |
| Depth of Knowledge | 2 |
| p-value A | 14% |
| p-value B | 18% |
| p-value C | 12% |
| p-value D | 56% (correct answer) |
| Option Annotations | A. adds numerators (3 + 1) and denominators (6 + 6) to get $\frac{4}{12}$ and then |
| | multiplies both the numerator and the denominator of $\frac{4}{12}$ by 5 |
| | B. adds $\frac{3}{6}$ to $\frac{1}{6}$ and determines the sum is $\frac{4}{6}$ but then multiplies both |
| | the numerator and the denominator of $\frac{4}{6}$ by 5 |
| | C. adds numerators (3 + 1) and denominators (6 + 6) to get $\frac{4}{12}$ before |
| | multiplying $\frac{4}{12}$ by 5 |
| | D. Correct: adds $\frac{3}{6}$ to $\frac{1}{6}$ and determines the sum is $\frac{4}{6}$, since $3 + 1 = 4$ |
| | and the common denominator does not change, and then |
| | multiplies $\frac{4}{6}$ by 5 and determines the product is $\frac{20}{6}$, since $4 \times 5 = 20$ |
| | and the denominator does not change when multiplying a fraction |
| | by a whole number |

15

- 7. Which expression represents $1\frac{1}{3}$ written as a sum of unit fractions?
 - A. $\frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1}$
 - B. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$
 - C. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$
 - D. $\frac{1}{6} + \frac{1}{6}$

| Item Information | |
|--------------------|---|
| Alignment | A-F.2.1.2 |
| Answer Key | В |
| Depth of Knowledge | 1 |
| p-value A | 11% |
| p-value B | 78% (correct answer) |
| p-value C | 6% |
| p-value D | 5% |
| Option Annotations | A. recognizes that the first $\frac{1}{1}$ represents the 1 whole but then |
| | adds $\frac{1}{1} + \frac{1}{1} + \frac{1}{1}$ and determines the sum is $\frac{1}{3}$ by adding the |
| | denominators rather than the numerators |
| | B. Correct: either adds numerators to get $\frac{4}{3}$, which is equal to $1\frac{1}{3}$, OR recognizes that $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1$ and that $1 + \frac{1}{3} = 1\frac{1}{3}$ |
| | C. confuses $\frac{4}{3}$ with $\frac{3}{4}$ |
| | D. ignores the whole number 1 |

8. Saul builds a model that represents the decimal 0.12. He uses a tens rod to represent one tenth and a ones cube to represent one hundredth. Which set of blocks could be the model Saul builds?



| Item Information | |
|--------------------|---|
| Alignment | A-F.3 |
| Answer Key | A |
| Depth of Knowledge | 2 |
| p-value A | 67% (correct answer) |
| p-value B | 9% |
| p-value C | 10% |
| p-value D | 14% |
| Option Annotations | A. Correct: either identifies that 1 tens rod = 1/10 (1 tenth) and 2 ones cubes = 2/100 (2 hundredths) and then adds 1/10 to 2/100 to get 1/100, which is equal to 0.12, OR recognizes that a tens rod is the same as 10 ones cubes and identifies that 12 ones cubes = 0.12 (12 hundredths) B. switches the tenths and the hundredths C. either models the correct number of tenths but includes an additional 12 hundredths OR correctly models 12 hundredths but also includes an extra tenth D. models 12 tenths rather than 12 hundredths |

- **9.** A farmer owns four times as many sheep as cows. She owns two more cows than horses. When the farmer places her sheep in 3 pens, each pen holds 8 sheep. How many horses does the farmer own?
 - A. 4
 - B. 8
 - C. 94
 - D. 98

| Item Information | |
|--------------------|--|
| Alignment | B-O.1.1.3 |
| Answer Key | A |
| Depth of Knowledge | 2 |
| p-value A | 42% (correct answer) |
| p-value B | 29% |
| p-value C | 19% |
| p-value D | 10% |
| Option Annotations | A. Correct: multiplies 3 by 8 to get 24 sheep, divides 24 by 4 to get 6 cows, and subtracts 2 from 6 to get 4 horses B. adds 2 to the number of cows C. multiplies the number of sheep by 4 and subtracts 2 from the number of cows D. multiplies the number of sheep by 4 and adds 2 to the number of cows |

- **10.** Mr. Larson has 57 tomato plants. He wants to put them in rows with the same number of tomato plants in each row. How many rows could Mr. Larson have using all 57 tomato plants?
 - A. 3
 - B. 5
 - C. 7
 - D. 12

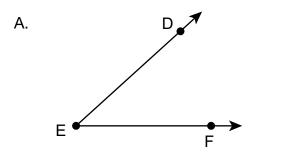
| Item Information | |
|--------------------|--|
| Alignment | B-O.2.1.1 |
| Answer Key | A |
| Depth of Knowledge | 2 |
| p-value A | 53% (correct answer) |
| p-value B | 9% |
| p-value C | 21% |
| p-value D | 17% |
| Option Annotations | A. Correct: recognizes that 3 and 19 is a factor pair of 57 B. uses the digit in the tens place as a factor C. uses the digit in the ones place as a factor D. uses 5 + 7 as a factor |

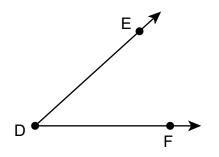
- **11.** Hannah makes a dog-walking area in the shape of a triangle. She uses 8 feet of fencing on each side of the dog-walking area. Which term describes each angle of Hannah's dog-walking area?
 - A. acute
 - B. obtuse
 - C. right
 - D. straight

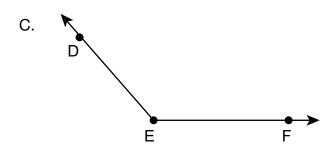
| Item Information | |
|--------------------|--|
| Alignment | C-G.1.1 |
| Answer Key | A |
| Depth of Knowledge | 1 |
| p-value A | 60% (correct answer) |
| p-value B | 15% |
| p-value C | 15% |
| p-value D | 10% |
| Option Annotations | A. Correct: recognizes that a triangle with three equal side lengths is an equilateral triangle and that each angle of an equilateral triangle is an acute angle B. thinks an obtuse triangle can have three equal side lengths C. thinks a right triangle can have three equal side lengths D. does not realize that a straight angle cannot be part of a triangle |

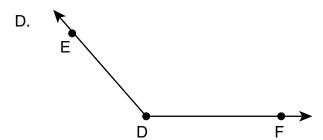
В.

12. Which angle is obtuse and can be identified as angle DEF?





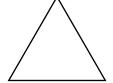




| Item Information | |
|--------------------|--|
| Alignment | C-G.1.1.1 |
| Answer Key | С |
| Depth of Knowledge | 1 |
| p-value A | 11% |
| p-value B | 7% |
| p-value C | 71% (correct answer) |
| p-value D | 11% |
| Option Annotations | A. selects an angle with a measure that is smaller than 90° B. selects an angle with a measure that is smaller than 90° and has point D (the first letter of DEF) as its vertex C. Correct: selects an angle with a measure that is larger than 90° and has point E (the middle letter of DEF) as its vertex D. selects an angle with a measure that is larger than 90° but has point D (the first letter of DEF) as its vertex |

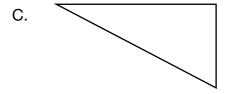
13. Which triangle most likely has both a right angle and a line of symmetry?

A.

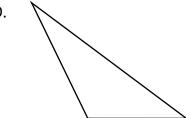


В.





D.

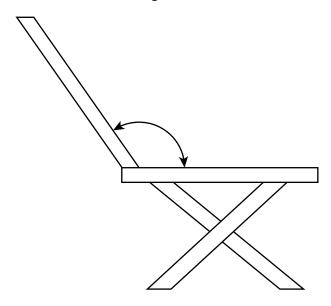


| Item Information | | | | | |
|--------------------|--|--|--|--|--|
| Alignment | C-G.1.1.3 C-G.1.1.2 | | | | |
| Answer Key | В | | | | |
| Depth of Knowledge | 2 | | | | |
| p-value A | 22% | | | | |
| p-value B | 58% (correct answer) | | | | |
| p-value C | 12% | | | | |
| p-value D | 8% | | | | |
| Option Annotations | A. recognizes that a triangle with 2 equal sides (or 2 equal angles) has a line of symmetry but does not recognize that all the angles are acute (no right angle) B. Correct: identifies the lower-left angle as a right angle and that a triangle with 2 equal sides (or 2 equal angles) has a line of symmetry C. identifies the upper-right angle as a right angle but does not recognize that a triangle with no equal sides (or no equal angles) does not have a line of symmetry | | | | |
| | D. does not recognize that the largest angle is obtuse (no right angle) or that a triangle with no equal sides (or no equal angles) does not have a line of symmetry | | | | |

- 14. The length of a fence is 12 yards. What is the length, in feet, of the fence?
 - A. 4 feet
 - B. 36 feet
 - C. 120 feet
 - D. 144 feet

| Item Information | | | | |
|--------------------|---|--|--|--|
| Alignment | D-M.1.1.1 | | | |
| Answer Key | В | | | |
| Depth of Knowledge | 1 | | | |
| p-value A | 15% | | | |
| p-value B | 66% (correct answer) | | | |
| p-value C | 11% | | | |
| p-value D | 8% | | | |
| Option Annotations | A. divides 12 by 3 rather than multiplying 12 by 3 B. Correct: multiplies 12 x 3 since there are 3 feet in each yard C. multiplies 12 by 10 (i.e., uses a metric conversion factor) D. multiplies 12 by 12 (i.e., converts using 1 foot = 12 inches rather than 1 yard = 3 feet) | | | |

15. The seat and the back of a chair form an angle as shown below.

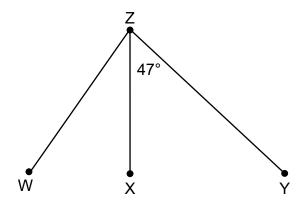


Using your protractor, what is the measure of the angle formed by the seat and the back of the chair?

- A. 55°
- B. 65°
- C. 125°
- D. 135°

| Item Information | | | | |
|--------------------|---|--|--|--|
| Alignment | D-M.3.1.1 | | | |
| Answer Key | С | | | |
| Depth of Knowledge | 1 | | | |
| p-value A | 20% | | | |
| p-value B | 15% | | | |
| p-value C | 46% (correct answer) | | | |
| p-value D | 19% | | | |
| Option Annotations | A. uses the outer scale on the protractor (but sees that the back of the chair passes halfway between 50° and 60°) B. uses the outer scale on the protractor, starts at 60° (rather than 50°), and then counts "up" 5 tick marks | | | |
| | C. Correct: uses the inner scale on the protractor and sees that the back of the chair passes halfway between 120° and 130° D. uses the inner scale on the protractor but counts up from 130° rather than from 120° | | | |

16. In the picture shown below, the measure of angle WZY is 82°.



What is the measure, in degrees, of angle WZX?

- A. 35°
- B. 43°
- C. 129°
- D. 164°

| Item Information | | | | | |
|--------------------|---|--|--|--|--|
| Alignment | D-M.3.1.2 | | | | |
| Answer Key | A | | | | |
| Depth of Knowledge | 2 | | | | |
| p-value A | 50% (correct answer) | | | | |
| p-value B | 21% | | | | |
| p-value C | 20% | | | | |
| p-value D | 9% | | | | |
| Option Annotations | A. Correct: subtracts 47° from 82° and determines the difference is 35° B. subtracts 47° from 90°, without considering that angle WZY is 82° and not 90° C. adds the two given angle measures (82° + 47°) D. subtracts 47° from 82° and determines the difference is 35° but then adds all three angle measures together (82° + 47° + 35°) | | | | |

OPEN-ENDED QUESTION

- 17. Three number patterns are described below.
 - pattern A: Start at 10 and add 4.
 - pattern B: Start at 30 and subtract 3.
 - pattern C: Start at 6 and add 5.

| A. | What are the first four terms of pattern A? |
|----|---|
| | |
| | |
| | |
| | |

B. What numbers are in both pattern B and pattern C? Show or explain all your work.

Go to the next page to finish question 17.

GO ON

17. Continued. Please refer to the previous page for task explanation.

Another number pattern is shown below.

16, 14, 15, 13, 14, 12, 13, ____

C. Explain how to determine whether the next number in the pattern is even or odd.

After you have finished your work, close this booklet so your teacher will know you are finished.



Item-Specific Scoring Guideline

#17 Item Information

| Alignment | B-O.3 | Depth of Knowledge | 2 | Mean Score | 1.38 |
|-----------|-------|-----------------------|---|------------|------|
|-----------|-------|-----------------------|---|------------|------|

Assessment Anchor this item will be reported under:

M04.B-O.3—Generate and analyze patterns.

Specific Anchor Descriptor addressed by this item:

M04.B-O.3.1 — Recognize, describe, extend, create, and replicate a variety of patterns.

Scoring Guide

| Score | In this item, the student |
|-------|---|
| 4 | Demonstrates a thorough understanding of how to generate and analyze patterns by correctly solving problems and clearly explaining procedures. |
| 3 | Demonstrates a general understanding of how to generate and analyze patterns by correctly solving problems and clearly explaining procedures with only minor errors or omissions. |
| 2 | Demonstrates a partial understanding of how to generate and analyze patterns by correctly performing a significant portion of the required task. |
| 1 | Demonstrates minimal understanding of how to generate and analyze patterns. |
| 0 | The response has no correct answer and insufficient evidence to demonstrate any understanding of the mathematical concepts and procedures as required by the task. Response may show only information copied from the question. |

Top-Scoring Student Response and Training Notes

| Score | Description |
|-------|--|
| 4 | Student earns 4 points. |
| 3 | Student earns 3.0–3.5 points. |
| 2 | Student earns 2.0–2.5 points. |
| 1 | Student earns 0.5–1.5 points. OR Student demonstrates minimal understanding of how to generate and analyze patterns. |
| 0 | Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured. |

Top-Scoring Response

Part A (1 point):

1 point for correct answer

| What? | Why? |
|----------------|------|
| 10, 14, 18, 22 | |

Part B (2 points):

 $\frac{1}{2}$ point for <u>each</u> correct answer

OR 0 points for any incorrect answers

1 point for correct and complete explanation

OR $\frac{1}{2}$ point for correct but incomplete explanation

| What? | Why? |
|----------|---|
| 6 and 21 | Sample Work: pattern B: 30, 27, 24, <u>21</u> , 18, 15, 12, 9, <u>6</u> , 3, 0 |
| | pattern C: <u>6</u> , 11, 16, <u>21</u> , 26, 31, |
| | OR |
| | Sample Explanation: Pattern B represents all the multiples of 3 up to 30, so I wrote out pattern C and circled the numbers that are multiples of 3 up to 30. OR equivalent |

Part C (1 point):

1 point for correct and complete explanation

 $OR \frac{1}{2}$ point for correct but incomplete explanation

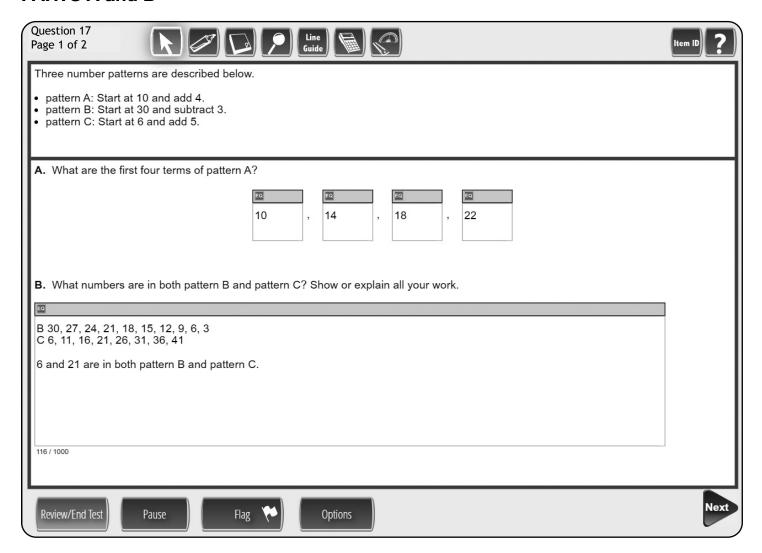
| What? | Why? |
|-------|---|
| | Sample Explanations: The rule for the pattern is subtract 2, then add 1. The last number is 13. This comes from $12 + 1 = 13$. So, the next number is $13 - 2 = 11$, which is an odd number. |
| | OR |
| | The pattern is two even numbers followed by two odd numbers. Since the pattern shown ends with one odd number, the next number is also odd. |
| | OR equivalent |

STUDENT RESPONSE

Response Score: 4 points

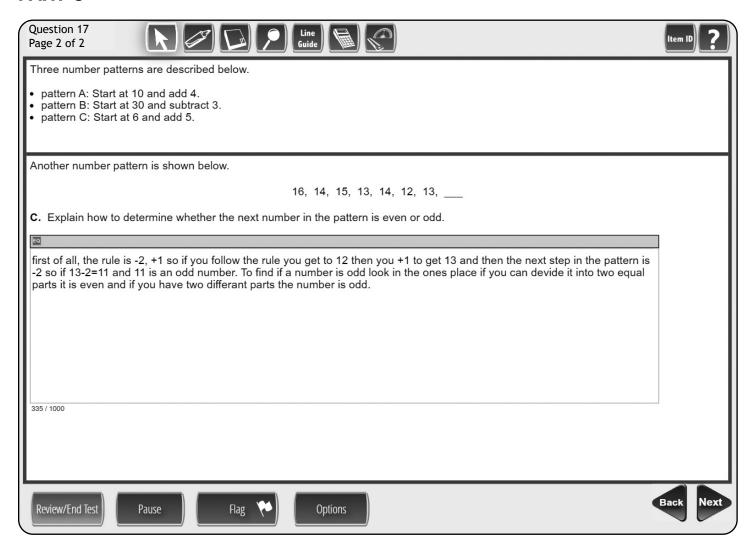


PARTS A and B



- Part A. The student provided the correct terms for pattern A by starting at 10 and adding 4 each time (10, 14, 18, 22). [1 point]
- Part B. The student provided the correct answer (6 and 21 are in both pattern B and pattern C) with correct and complete support. The student wrote out both patterns correctly: pattern B started at 30 and subtracted 3 each time, while pattern C started at 6 and added 5 each time. The student continued each pattern until pattern B reached a term less than any term in pattern C and pattern C reached multiple terms that were greater than any term in pattern B. The numbers 6 and 21 appeared in both patterns. [2 points]

PART C



Part C. The student provided a correct and complete explanation of how to determine whether the next number in the pattern is even or odd. First, the student correctly identified the pattern as –2, +1. Then, the student correctly explained how to use the pattern to find the next number and identified the next number as odd (if you follow the rule you get to 12 then you +1 to get 13 and then the next step in the pattern is –2 so if 13 – 2 = 11 and 11 is an odd number). [1 point]

STUDENT RESPONSE

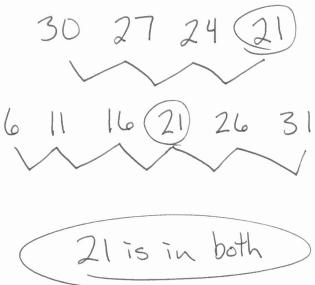
Response Score: 3 points

- 17. Three number patterns are described below.
 - pattern A: Start at 10 and add 4.
 - pattern B: Start at 30 and subtract 3.
 - pattern C: Start at 6 and add 5.
 - A. What are the first four terms of pattern A?

10 14 18 22

Part A. The student provided the correct terms for pattern A by starting at 10 and adding 4 each time (10, 14, 18, 22). [1 point]

B. What numbers are in both pattern B and pattern C? Show or explain all your work.



Part B. The student provided one of the two correct answers (21 is in both) with correct but incomplete support. The student started both patterns correctly: pattern B started at 30 and subtracted 3 each time, while pattern C started at 6 and added 5 each time. The student continued pattern B until a term that was also in pattern C was found (21). The student continued pattern C until a term that was greater than any term in pattern B was found (31). To earn full credit, the student needed to continue pattern B until a term that was less than or equal to the smallest term in pattern C was found. Extending pattern B further, the student would have seen a second number that is in both patterns (6). [1 point]

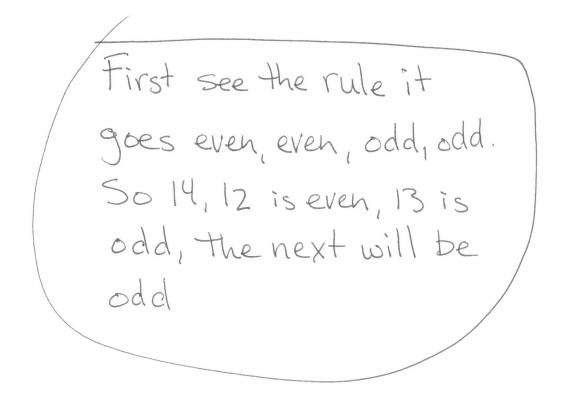
Go to the next page to finish question 17.



17. *Continued.* Please refer to the previous page for task explanation.

Another number pattern is shown below.

C. Explain how to determine whether the next number in the pattern is even or odd.



Part C. The student provided a correct and complete explanation of how to determine whether the next number in the pattern is even or odd. First, the student correctly identified the pattern as even, even, odd, odd. Then, the student correctly explained how to use the pattern to find whether the next number is even or odd and identified the next number as odd (So 14, 12 is even, 13 is odd, the next will be odd). [1 point]

After you have finished your work, close this booklet so your teacher will know you are finished.

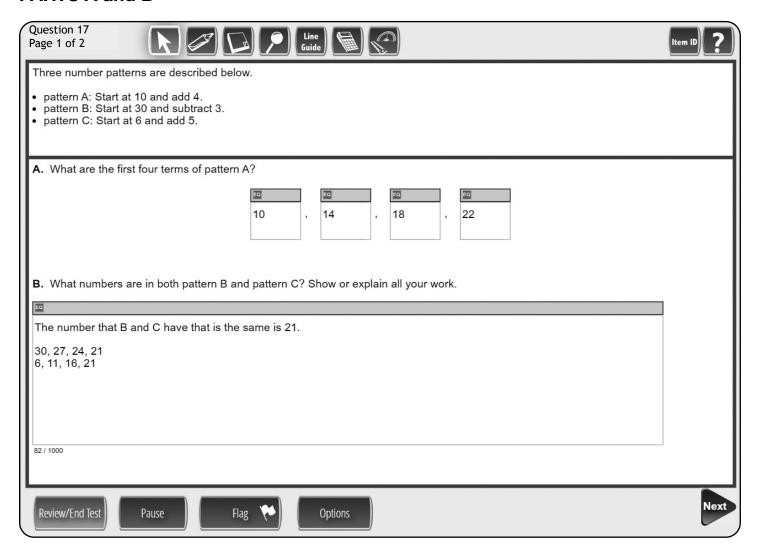


STUDENT RESPONSE

Response Score: 2 points

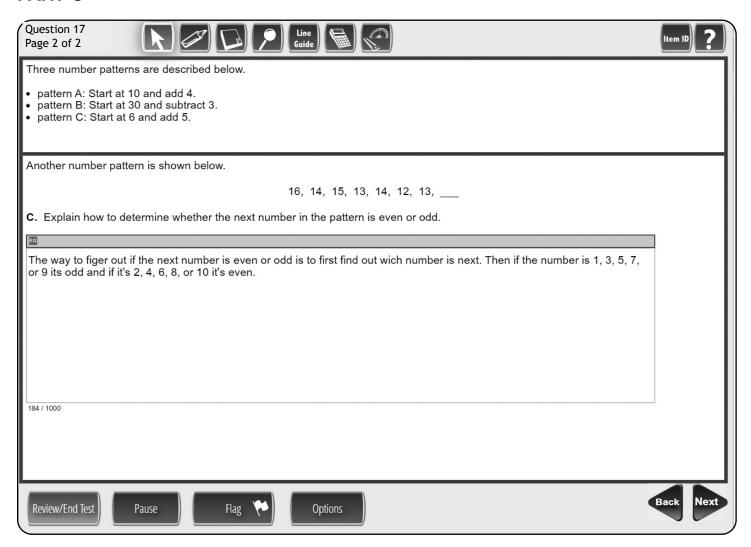


PARTS A and B



- Part A. The student provided the correct terms for pattern A by starting at 10 and adding 4 each time (10, 14, 18, 22). [1 point]
- Part B. The student provided one of the two correct answers (*The number that B and C have that is the same is 21*.) with correct but incomplete support. The student started both patterns correctly: pattern B started at 30 and subtracted 3 each time, while pattern C started at 6 and added 5 each time. The student continued each pattern until a term that was also in the other pattern was found (21). To earn full credit, the student needed to continue pattern B until a term that was less than or equal to the smallest term in pattern C was found and continue pattern C until a term that was greater than or equal to the largest term in pattern B was found. If both patterns had been extended further, the student would have seen a second number that is in both patterns (6). [1 point]

PART C



Part C. The student provided an incorrect explanation. Rather than explaining how to determine whether the next number in the pattern is even or odd, the student attempted to give a definition of whether a number is even or odd (if the number is 1, 3, 5, 7, or 9 its odd and if it's 2, 4, 6, 8, or 10 it's even); however, the student's definition only works for numbers less than or equal to 10. [0 points]

STUDENT RESPONSE

Response Score: 1 point

- 17. Three number patterns are described below.
 - pattern A: Start at 10 and add 4.
 - pattern B: Start at 30 and subtract 3.
 - pattern C: Start at 6 and add 5.
 - A. What are the first four terms of pattern A?

14 18 22 26

- Part A. The student provided an incorrect pattern (14, 18, 22, 26) by starting at 14 rather than starting at 10. The student did correctly add 4 each time, resulting in terms 2 through 5 of pattern A rather than terms 1 through 4. [0 points]
- **B.** What numbers are in both pattern B and pattern C? Show or explain all your work.

Partern B: 27, 24, 21, 18

Pattern C: 11, 16, 21, 26

Part B. The student provided no correct answers and incorrect support (work or explanation). For pattern B, the student began the pattern with the second term (27) rather than the first term (30) and then subtracted 3 each time. For pattern C, the student began the pattern with the second term (11) rather than the first term (6) and then added 5 each time. Although the student continued both patterns to one term past a common number in each pattern (21), the student did not identify that this number is in both patterns. [0 points]

Go to the next page to finish question 17.

17. Continued. Please refer to the previous page for task explanation.

Another number pattern is shown below.

C. Explain how to determine whether the next number in the pattern is even or odd.

The next number is odd because the pattern is $-\lambda + 1$. $14-\lambda = |\lambda| + |a| = |3|$. Is starts the pattern again. $|3-\lambda| = |1|$ is an odd number.

Part C. The student provided a correct and complete explanation of how to determine whether the next number in the pattern is even or odd. First, the student correctly identified the pattern as -2 + 1. Then, the student explained how to use the pattern to find the next number (14 - 2 = 12 + 1 = 13. 13 starts the pattern again. 13 - 2 = 11) and identified the next number as odd (11 is an odd number). The student's explanation contains a run-on equation, which is mathematically incorrect and would not allow a student to earn all 4 points for the item; however, this was ignored since this part of the explanation was extraneous. [1 point]

After you have finished your work, close this booklet so your teacher will know you are finished.

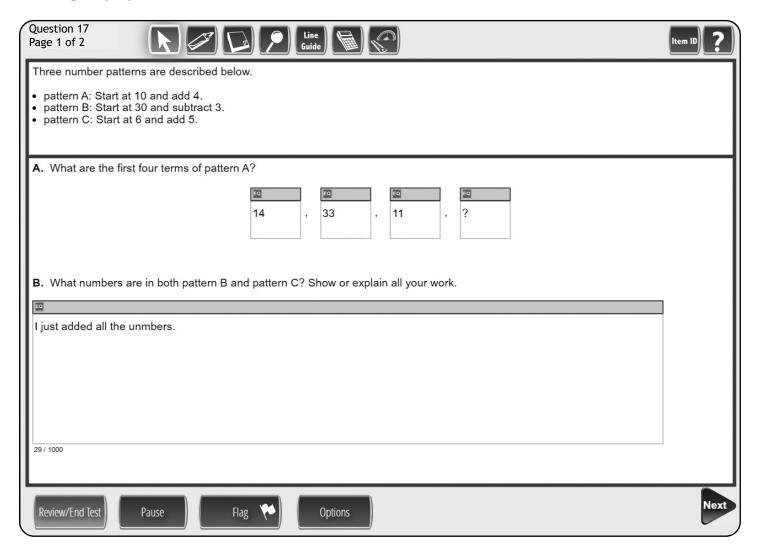


STUDENT RESPONSE

Response Score: 0 points

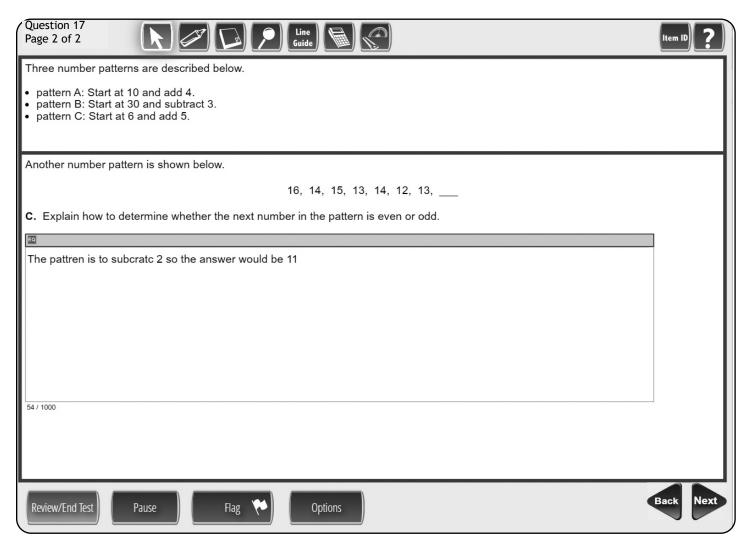


PARTS A and B



- Part A. The student provided an incorrect pattern (14, 33, 11, ?). No support (work or explanation) is required, so it is unclear where an error was made. The student may have added the numbers together from the prompt for the first three numbers (10 + 4 = 14, pattern B: 30 + 3 = 33, pattern C: 6 + 5 = 11). [0 points]
- Part B. The student provided an incorrect explanation that may refer to the numbers provided in Part A (*I just added all the unmbers* [numbers].). The student did not identify which numbers are in both pattern B and pattern C, and the student did not provide any support (work or explanation) showing either pattern B or pattern C. [0 points]

PART C



Part C. The student provided an incorrect explanation (*The pattren is to subcratc* [subtract] 2 so the answer would be 11) of how to determine whether the next number in the pattern is even or odd. The student only identified the first part of the pattern (subcratc [subtract] 2) and did not explain whether 11 is even or odd. Identifying the next number as 11 does not earn any credit. [0 points]

MATHEMATICS—SUMMARY DATA

Multiple-Choice

| Sample Number | Alignment | Answer Key | Depth of Knowledge | <i>p</i> -value A | <i>p</i> -value B | <i>p</i> -value C | <i>p</i> -value D |
|------------------|------------------------|------------|--------------------|-------------------|----------------------|----------------------|----------------------|
| 1 | A-T.2.1.3 | D | 1 | 9% | 17% | 19% | 55% |
| 2 | A-T.1.1.2 | D | 1 | 7% | 10% | 8% | 75% |
| 3 | A-T.1.1.3 | D | 1 | 7% | 6% | 5% | 82% |
| 4 | A-T.2.1 | В | 2 | 18% | 47% | 16% | 19% |
| 5 | A-F.1.1.1 | А | 1 | 52% | 18% | 20% | 10% |
| 6 | A-F.2.1 | D | 2 | 14% | 18% | 12% | 56% |
| 7 | A-F.2.1.2 | В | 1 | 11% | 78% | 6% | 5% |
| 8 | A-F.3 | А | 2 | 67% | 9% | 10% | 14% |
| 9 | B-O.1.1.3 | А | 2 | 42% | 29% | 19% | 10% |
| 10 | B-O.2.1.1 | А | 2 | 53% | 9% | 21% | 17% |
| 11 | C-G.1.1 | А | 1 | 60% | 15% | 15% | 10% |
| 12 | C-G.1.1.1 | С | 1 | 11% | 7% | 71% | 11% |
| 13 | C-G.1.1.3 C-G.1.1.2 | В | 2 | 22% | 58% | 12% | 8% |
| 14 | D-M.1.1.1 | В | 1 | 15% | 66% | 11% | 8% |
| 15 | D-M.3.1.1 | С | 1 | 20% | 15% | 46% | 19% |
| 16 | D-M.3.1.2 | А | 2 | 50% | 21% | 20% | 9% |

Open-Ended

| Sample Number | Alignment | Points | Depth of Knowledge | Mean Score |
|------------------|-----------|--------|-----------------------|------------|
| 17 | B-O.3 | 4 | 2 | 1.38 |