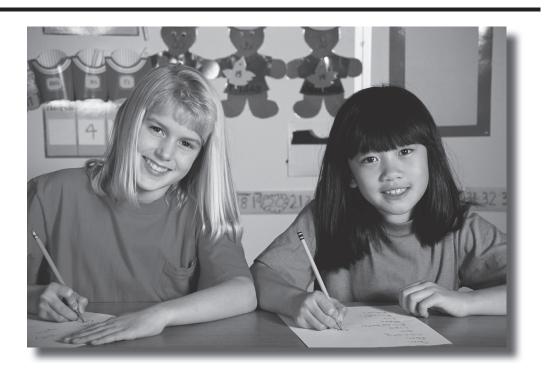


## The Pennsylvania System of School Assessment

### Mathematics Item and Scoring Sampler



2016-2017 **Grade 4** 

Pennsylvania Department of Education Bureau of Curriculum, Assessment and Instruction—September 2016

### MATHEMATICS TEST DIRECTIONS

On the following pages are the mathematics questions.

- You may <u>not</u> use a calculator for question 1. You may use a calculator for all other questions on this test.
- You may need a protractor for questions on this test.

### **Directions for Multiple-Choice Questions:**

Some questions will ask you to select an answer from among four choices.

For the multiple-choice questions:

- First solve the problem on scratch paper.
- Choose the correct answer and record your choice in the answer booklet.
- If none of the choices matches your answer, go back and check your work for possible errors.
- Only one of the answers provided is the correct response.

### **Directions for Open-Ended Questions:**

Some questions will require you to write your response.

For the open-ended questions:

- These questions have more than one part. Be sure to read the directions carefully.
- You cannot receive the highest score for an open-ended question without completing
  all tasks in the question. For example, if the question asks you to show your work or
  explain your reasoning, be sure to show your work or explain your reasoning in the
  space provided.
- If the question does **not** ask you to show your work or explain your reasoning, you may use the space provided, but only those parts of your response that the question specifically asks for will be scored.
- Write your response in the appropriate location within the response box in the
  answer booklet. Some answers may require graphing, plotting, labeling, drawing, or
  shading. If you use scratch paper, be sure to transfer your final response and any
  needed work or reasoning to the answer booklet.

### **INFORMATION ABOUT MATHEMATICS**

### **General Description of Scoring Guidelines for Mathematics Open-Ended Questions**

4 – The response demonstrates a *thorough* understanding of the mathematical concepts and procedures required by the task.

The response provides correct answer(s) with clear and complete mathematical procedures shown and a correct explanation, as required by the task. Response may contain a minor "blemish" or omission in work or explanation that does not detract from demonstrating a *thorough* understanding.

3 – The response demonstrates a *general* understanding of the mathematical concepts and procedures required by the task.

The response and explanation (as required by the task) are mostly complete and correct. The response may have minor errors or omissions that do not detract from demonstrating a *general* understanding.

2 – The response demonstrates a *partial* understanding of the mathematical concepts and procedures required by the task.

The response is somewhat correct with *partial* understanding of the required mathematical concepts and/or procedures demonstrated and/or explained. The response may contain some work that is incomplete or unclear.

- 1 The response demonstrates a *minimal* understanding of the mathematical concepts and procedures required by the task.
- 0 The response has no correct answer and *insufficient* evidence to demonstrate any understanding of the mathematical concepts and procedures required by the task for that grade level.

Response may show only information copied from the question.

Special Categories within zero reported separately:

BLK (blank)......Blank, entirely erased, or written refusal to respond OT.....Off task

LOE .....Response in a language other than English

IL .....Illegible

### **Grade 4 Formula Sheet**

Formulas and conversions that you may need to work questions on this test are 2016 found below. You may refer back to this page at any time during the mathematics test. Grade 4

### **Standard Conversions**

$$1 \text{ yard (yd)} = 3 \text{ feet (ft)}$$

$$1 pint = 2 cups (c)$$

### **Metric Conversions**

$$1 \text{ kilogram (kg)} = 1,000 \text{ grams (g)}$$

1 liter (L) = 
$$1,000$$
 milliliters (mL)

### **Time Conversions**

1 year = 52 weeks (wk)

1 year = 365 days

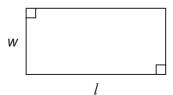
1 week = 7 days

1 day = 24 hours (hr)

1 hour = 60 minutes (min)

1 minute = 60 seconds (sec)

### Rectangle



$$A = l \times w$$

Perimeter = length + length + width + width 
$$P = l + l + w + w$$

Question 1 in this sampler is to be solved without the use of a calculator.

### **MULTIPLE-CHOICE ITEMS**

- 1. Which value is **closest** to 79 × 4?
  - A. 280
  - B. 320
  - C. 350
  - D. 400

	Item Infor	mation		Option Annotations
	Alignmer	t A-T.2	.1.4	A. rounds 79 down to 70
	Answer Ke	у В		B. correct C. rounds 79 down to 70 and 4 up to 5
Depth of	Depth of Knowledge 1			D. rounds both multiplicands up 1
	p-values			
Α	В	С	D	
13%	73%	7%	7%	

### A calculator is permitted for use in solving questions 2-17 in this sampler.

- 2. Asia covers about  $\frac{3}{10}$  of the land on Earth. South America covers about  $\frac{12}{100}$  of the land on Earth. Which statement correctly compares the land sizes of Asia and South America?
  - A. Since  $\frac{12}{100}$  is equivalent to  $\frac{12}{10}$  and  $\frac{3}{10} < \frac{12}{10}$ , Asia covers less land than South America.
  - B. Since  $\frac{3}{10}$  is equivalent to  $\frac{3}{100}$  and  $\frac{3}{100} < \frac{12}{100}$ , Asia covers less land than South America.
  - C. Since  $\frac{3}{10}$  is equivalent to  $\frac{30}{100}$  and  $\frac{30}{100} > \frac{12}{100}$ , Asia covers more land than South America.
  - D. Since  $\frac{3}{10}$  is equivalent to  $\frac{93}{100}$  and  $\frac{93}{100} > \frac{12}{100}$ , Asia covers more land than South America.

	Item Inform	mation		Option Annotations
	3			A. does not convert denominator     B. does not convert denominator     C. correct     D. converts 3/10 to hundredths by adding 90 to both numerator and denominator
	Answer Key			
Depth of	Depth of Knowledge 2			
	p-values			
	p valu			
Α	В	С	D	
21%	19%	52%	8%	

- 3. In a box of 24 chocolate pieces,  $\frac{2}{3}$  of the pieces have peanuts in them. How many of the chocolate pieces have peanuts in them?
  - A. 8
  - B. 12
  - C. 16
  - D. 19

	Item Infor	mation		Option Annotations
	Alignmer	nt A-F.2.	1.6	A. 24/3
	Answer Ke	уС		B. 24/2 C. correct
Depth of	Depth of Knowledge			D. 24 – 2 – 3
	p-values			
Α	В	С	D	
24%	22%	42%	12%	

- **4.** Miguel went to a baseball stadium and a football stadium.
  - The baseball stadium has thirty-seven thousand, four hundred ninety-five seats.
  - The football stadium has sixty-nine thousand, one hundred forty-three seats.

How many times greater is the value of the digit 3 in the number of seats at the baseball stadium than the value of the digit 3 in the number of seats at the football stadium?

- A. 10 times
- B. 100 times
- C. 1,000 times
- D. 10,000 times

	Item Inforn	nation			Option Annotations
	Alignment A-T.1.1.2 A-T.1.1.1		A. states mathematical rule rather than applying it (each place to the left is 10 times greater)		
	Answer Key	, D	D		B. incorrectly counts number of places C. counts 3 places between the ones and tens thousands place,
Depth of	Depth of Knowledge 2			thus 10 × 10 × 10	
	p-values			D. correct	
Α	В	С	D		
13%	13%	17%	57%		
	·				

- **5.** Rounded to the nearest ten, 8,300 books were read by the students at Matilda's school during a read-a-thon. Which value could be the actual total number of books read?
  - A. 8,289
  - B. 8,296
  - C. 8,307
  - D. 8,312

	Item Info	mation		Option Annotations
	Alignme	nt A-T.1	1.4	A. rounds to nearest hundred
	Answer Key			B. correct C. rounds down to the nearest ten D. rounds to nearest hundred
Depth of	pth of Knowledge 1			
	p-values			
Α	В	С	D	
11%	53%	20%	16%	

- **6.** Each team in a football league has 53 players on it. There are 32 teams in the league. How many total players are in the league?
  - A. 256
  - B. 265
  - C. 1,506
  - D. 1,696

	Item Infor	mation		Option Annotations
	Alignmer	t A-T.2	.1.2	A. does $32 \times 53$ and when multiplying $2 \times 5$ does not put down the
	Answer Ke	y D		0 since there is already a (placeholder) 0 there  B. does 53 × 32 and when multiplying 3 × 3 does not put in a
Depth of	Depth of Knowledge 1			placeholder 0
				C. multiplies 5 × 3 and 2 × 3 D. correct
	<i>p</i> -valu	es		
Α	В	С	D	
5%	5%	5%	85%	
	<u>'</u>			

- **7.** Albert has been a chef for *y* years. Maria has been a chef for 3 years more than 2 times as many years as Albert. Which expression shows how many years Maria has been a chef?
  - A. 3 + 2 + y
  - B.  $3 + 2 \times y$
  - C.  $3 \times y + 2$
  - D.  $3 \times y \times 2$

	Item Info	rmation		Option Annotations
	Alignme	nt B-O.	1.1	A. adds all the values
	Answer K	еу В		B. correct C. does 3 times as many plus 2
Depth of	Depth of Knowledge 2			D. multiplies all the values
	p-values			
Α	В	С	D	
14%	31%	27%	28%	

- **8.** Kara, Lynn, and Molly each play on a basketball team. The points scored in their last game are listed below.
  - Kara scored 3 points.
  - Lynn scored 5 times as many points as Kara.
  - Molly scored 7 times as many points as Kara.

How many more points did Molly score than Lynn in their last game?

- A. 2
- B. 6
- C. 8
- D. 16

	Item Infor	nation		Option Annotations
	9			A. 7 – 5 B. correct
	Answer Ke	у В		C. (3 × 5) – 7 D. (3 × 7) – 5
Depth of	Depth of Knowledge			
	p-values			
	<del></del> .			
Α	В	С	D	
24%	49%	10%	17%	
			•	

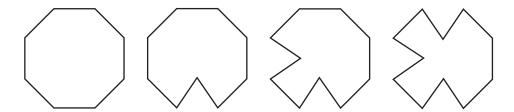
- **9.** Rosie, Stella, and Tiffany are all on a bowling team. In a recent game, Stella scored twice as many points as Rosie. Tiffany scored 2 more points than Rosie. Rosie scored 70 points. What is the total number of points scored by all three women?
  - A. 177
  - B. 216
  - C. 282
  - D. 350

	Item Inforn	nation		Option Annotations
	<b>Alignment</b> B-O.1.1.2 B-O.1.1.3			A. thinks Stella scored half as many points (35) not twice as many B. thinks Tiffany, Stella, and Rosie all scored 72
	Answer Key	, C		C. correct D. thinks Tiffany and Stella both scored 140
Depth of	Depth of Knowledge			2. Timing Thany and Groing Sour Source 140
	p-values			
Α	В	С	D	
14%	16%	64%	6%	
			•	

- **10.** Frank made a pattern starting with the number 7. He used the rule "add 7." What is true about every number in Frank's pattern?
  - A. Every number is a multiple of 7.
  - B. The last digit of every number is a 7.
  - C. The first digit of every number is a 7.
  - D. The digits in every number add up to 7.

	Item Info	mation		Option Annotations
	B-O.3.1.1			A. correct B. only true of the 1st term, 11th term, 21st term, 31st term, etc. C. is true for some but not all D. only true for some of the terms but not all the terms
	Answer Key			
Depth of	Depth of Knowledge			
	p-values			
Α	В	С	D	
70%	4%	6%	20%	
			•	

**11.** Starting with an octagon, Hillary used the rule "Replace one side with two new sides" to create the pattern shown below.



How many sides will the next shape in Hillary's pattern have?

- A. 8
- B. 11
- C. 12
- D. 16

	Item Infor	mation		Option Annotations
	Alignme	<b>nt</b> B-0.3	.1.1	A. assumes each shape has 8 sides because it starts with an
	Answer Key C			octagon  B. gives the number of sides of the last shape given
Depth of	Depth of Knowledge 1			C. correct
	p-values			D. thinks the number of sides is increasing by 2 with each iteration and not 1
Α	В	С	D	
21%	13%	58%	8%	

**12.** In a video game, players can collect a special item that is worth different points based on what level of the game they are on, as shown in the table below.

**Special Item Points** 

Level	Number of Points
1	10
2	20
3	40
4	80

The pattern for the number of points for collecting the special item continues. Which statement explains how to find the correct number of points for collecting the special item while on level 5?

- A. Add 10 to 80 to get 90 points.
- B. Add 40 to 80 to get 120 points.
- C. Multiply 80 by 2 to get 160 points.
- D. Multiply 80 by 5 to get 400 points.

	Item Inforn	nation		Option Annotations			
Alignment B-0.3.1.3 B-0.3.1.2				A. returns to idea of adding 10, which only works going from level 1 to 2			
	Answer Key C			B. adds the previous value C. correct			
Depth of	Depth of Knowledge 2			D. multiplies by the new level			
	<i>p</i> -value	es		1			
Α	A B C D		D				
22%	22% 18% 53% 7%		7%	]			
				1			

**13.** As part of a map, Lewis drew a ray with three points labeled A, B, and C on it. The ray Lewis drew started at point A. Which could be the part of the map Lewis drew?



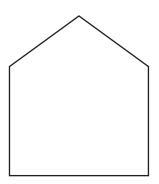






	Item Info	rmation		Option Annotations
	Alignme	nt C-G.1	1.1.1	A. confuses a ray and a line
	Answer Key B			B. correct C. confuses a ray with a line segment
Depth of	Depth of Knowledge 1			D. confuses a ray with a line segment and chooses a starting point
				that is neither at the left-most nor right-most point
	p-val	ues		
Α	A B C		D	
16% 75% 6		6%	3%	
			•	

**14.** Eric is making a design for a school flag. He draws the pentagon shown below.

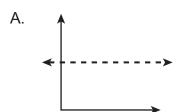


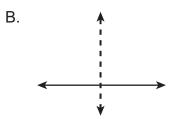
Eric will cut the figure along a line of symmetry of the shape. What are the two shapes Eric will make?

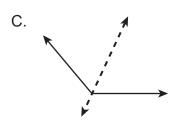
- A. two trapezoids
- B. two parallelograms
- C. a rectangle and a triangle
- D. a rectangle and a pentagon

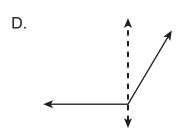
	Item Infor	mation		Option Annotations
			l.1.2 l.1.3	A. correct B. sees a set of parallel lines
	Answer Key			<ul><li>C. draws a horizontal line through 2 vertices</li><li>D. draws a horizontal line halfway through the shape</li></ul>
Depth of	Depth of Knowledge 2			an analysis and analysis and analysis
	<i>p</i> -valı	ıes		
Α	В	С	D	
51% 20% 19%		19%	10%	

**15.** A designer drew a line of symmetry in an angle to create two acute angles. Which figure could the designer have drawn?



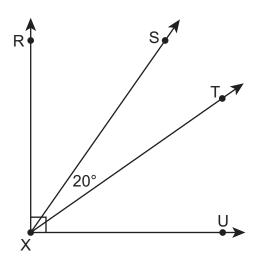






	Item Infor	mation		Option Annotations
	•			A. does not create a line of symmetry     B. creates two right angles, not acute angles
	Answer Key C			C. correct D. does not create a line of symmetry
Depth of	Depth of Knowledge 2			
	p-valu	IAS		
	p vaic		ı	
Α	В	С	D	
6%	6% 12% 69% 1		13%	
	•		_	

**16.** A 90° angle is divided into three smaller angles, as shown below.



The middle angle is 20°. The other two angles have the same measure. What is the measure of one of the other two angles?

- A. 35°
- B. 45°
- C. 30°
- D. 70°

	Item Info	rmation		Option Annotations
	Alignment D-M.3.1.2			A. correct
	Answer Key			B. 90 divided by 2 (does not subtract the 20) C. 90 divided by 3 (thinks all three angles are equal)
Depth of	Depth of Knowledge 2			D. 90 minus 20 (does not divide the difference)
		·		
	p-val	ues		
Α	В	С	D	
47%	47% 20% 14		19%	

### **OPEN-ENDED QUESTION**

17. Curt has two pieces of rope. The first piece of rope is  $\frac{4}{10}$  meter long. The second piece of rope is  $\frac{42}{100}$  meter long.

**A.** Write the length, in meters as a decimal, of the first piece of rope.

**B.** Write the total length, in meters as a fraction, of both pieces of rope. Show or explain all your work.

Go to the next page to finish question 17.

17. Continued. Please refer to the previous page for task explanation. The length of a third piece of rope is between the lengths of the first and second pieces of rope. The length of the third piece is expressed as a decimal to the hundredths place. **C.** Explain why there is only one possible length for the third piece of rope. As part of the explanation, find the length, in meters, of the third piece of rope and express it as a decimal to the hundredths place.

### **Item-Specific Scoring Guideline**

### **#17 Item Information**

Alignment	A-F.3	Depth of Knowledge	3	Mean Score	1.75
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### Assessment Anchor this item will be reported under:

M04.A-F.3—Understand decimal notation for fractions, and compare decimal fractions.

### Specific Anchor Descriptor addressed by this item:

M04.A-F.3.1—Use operations to solve problems involving decimals, including converting between fractions and decimals (may include word problems).

### **Scoring Guide**

Score	In this item, the student
4	Demonstrates a thorough understanding of decimal notation for fractions and comparing decimal fractions by correctly solving problems and clearly explaining procedures.
3	Demonstrates a general understanding of decimal notation for fractions and comparing decimal fractions by correctly solving problems and clearly explaining procedures with only minor errors or omissions.
2	Demonstrates a partial understanding of decimal notation for fractions and comparing decimal fractions by correctly performing a significant portion of the required task.
1	Demonstrates minimal understanding of decimal notation for fractions and comparing decimal fractions.
0	The response has no correct answer and insufficient evidence to demonstrate any understanding of the mathematical concepts and procedures as required by the task. Response may show only information copied from the question.

### **Top-Scoring Student Response and Training Notes**

Score	Description
4	Student earns 4 points.
3	Student earns 3.0–3.5 points.
2	Student earns 2.0–2.5 points.
1	Student earns 0.5–1.5 points.  OR  Student demonstrates minimal understanding of decimal notation for fractions and comparing decimal fractions.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.

### **Top-Scoring Response**

### Part A (1 point):

1 point for correct answer

What?	Why?
0.4 (meter)	

### Part B (2 points):

- 1 point for correct answer
- 1 point for complete support

 $OR\frac{1}{2}$  point for correct but incomplete support

What?	Why?
$\frac{82}{100}$ (meter)	Sample Work:
OR equivalent	$\frac{4}{10} = \frac{40}{100}$ $\frac{40}{100} + \frac{42}{100} = \frac{82}{100}$
	OR
	Sample Explanation:
	First, I changed $\frac{4}{10}$ to $\frac{40}{100}$ by multiplying the numerator and denominator by 10. Then I added $\frac{40}{100} + \frac{42}{100}$ to get $\frac{82}{100}$ .  OR equivalent

27

### Part C (1 point):

1 point for complete explanation

 $OR\frac{1}{2}$  point for correct but incomplete explanation

What?	Why?
	Sample Explanation:
	The first piece of rope is equal to $\frac{40}{100}$ meter and the second
	piece of rope is $\frac{42}{100}$ meter. There is only one possible length
	between these two lengths since 41 is the only whole
	number between 40 and 42. So the length of the third piece
	of rope must be $\frac{41}{100}$ meter, which can also be written as
	0.41 meter.
	OR equivalent

### STUDENT RESPONSE

**Response Score: 4 points** 

- 17. Curt has two pieces of rope. The first piece of rope is  $\frac{4}{10}$  meter long. The second piece of rope is  $\frac{42}{100}$  meter long.
  - A. Write the length, in meters as a decimal, of the first piece of rope.

0.4 meters long

The student has given a correct answer.

B. Write the total length, in meters as a fraction, of both pieces of rope. Show or explain all your work.

$$\frac{40}{100} + \frac{42}{100} = \frac{82}{100}$$

The student has given a correct answer and complete support.

Go to the next page to finish question 17.

17. Continued. Please refer to the previous page for task explanation.

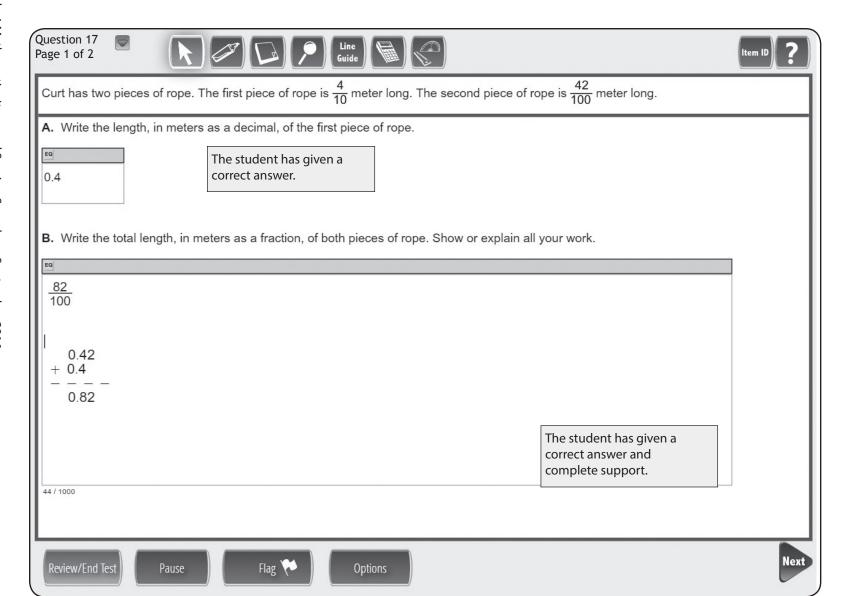
The length of a third piece of rope is between the lengths of the first and second pieces of rope. The length of the third piece is expressed as a decimal to the hundredths place.

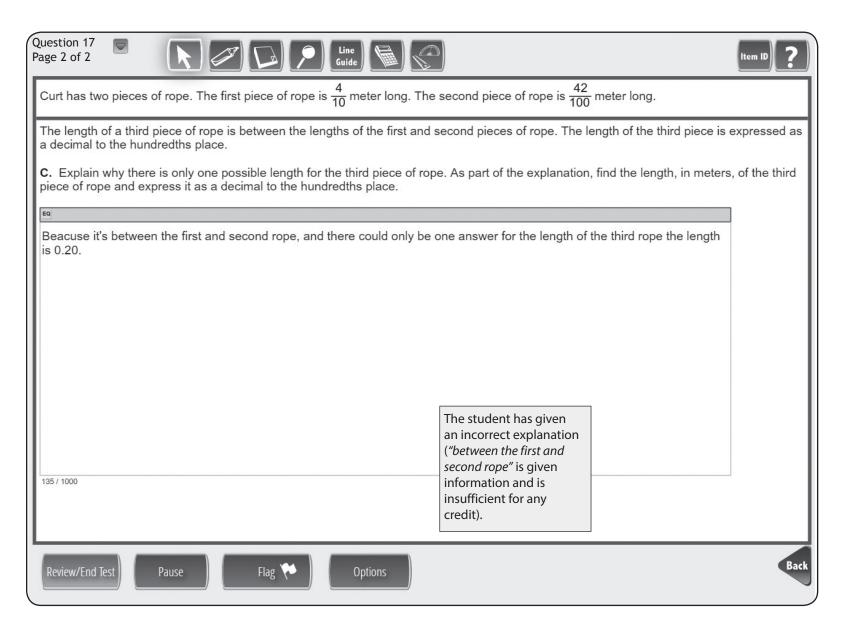
**C.** Explain why there is only one possible length for the third piece of rope. As part of the explanation, find the length, in meters, of the third piece of rope and express it as a decimal to the hundredths place.

0.41

The one possible length is 0.41. It is because in hundred this of 0.4 is also 0.40. Then the other one is 0.42. Then I thought it was 0.41. That's how I got my answer.

The student has given a complete explanation.





**PART C** 

### **STUDENT RESPONSE**

**Response Score: 2 points** 

- 17. Curt has two pieces of rope. The first piece of rope is  $\frac{4}{10}$  meter long. The second piece of rope is  $\frac{42}{100}$  meter long.
  - A. Write the length, in meters as a decimal, of the first piece of rope.



The student has given an incorrect answer.

**B.** Write the total length, in meters as a fraction, of both pieces of rope. Show or explain all your work.



The student has given a correct answer and complete support.

Go to the next page to finish question 17.

17. Continued. Please refer to the previous page for task explanation.

The length of a third piece of rope is between the lengths of the first and second pieces of rope. The length of the third piece is expressed as a decimal to the hundredths place.

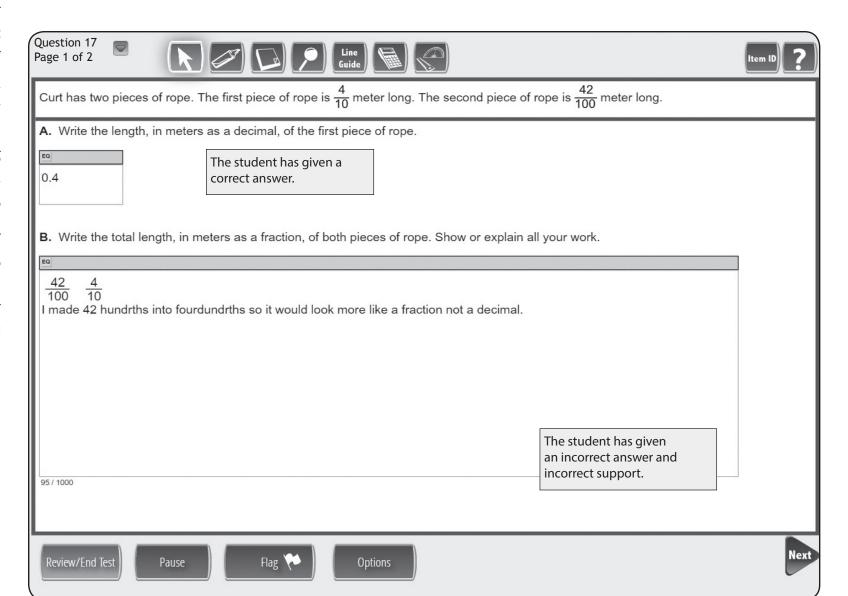
**C.** Explain why there is only one possible length for the third piece of rope. As part of the explanation, find the length, in meters, of the third piece of rope and express it as a decimal to the hundredths place.

there is Gnly one possible length For the rope to Be Because I changed 4 to an equivelent Fraction so I could add them. I multiplied By 10 to get 40 also Have 42 100. I also Have 42 100. I also Have 42 100. Therefore the only possible answer could Be 41

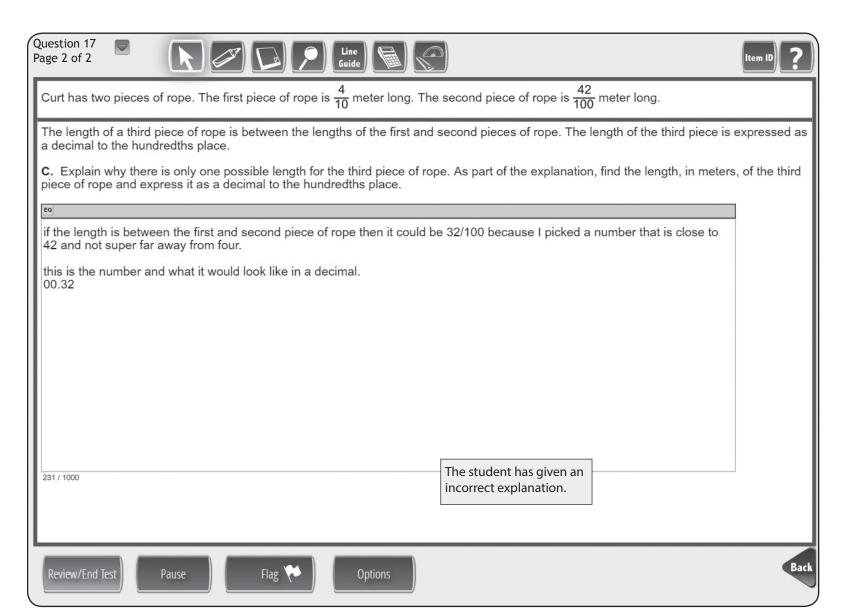
The student has given a correct but incomplete explanation (has not expressed the length as a decimal).

# **STUDENT RESPONSE**

**PARTS A AND B Response Score: 1 point** 



**PART C** 



### **STUDENT RESPONSE**

**Response Score: 0 points** 

- 17. Curt has two pieces of rope. The first piece of rope is  $\frac{4}{10}$  meter long. The second piece of rope is  $\frac{42}{100}$  meter long.
  - **A.** Write the length, in meters as a decimal, of the first piece of rope.

The student has given an incorrect answer.

**B.** Write the total length, in meters as a fraction, of both pieces of rope. Show or explain all your work.

46

I put the answer because I added the 4 to 42 and got 46 then I added 10 to 100 and got 110.

The student has given an incorrect answer and incorrect support.

Go to the next page to finish question 17.

17. Continued. Please refer to the previous page for task explanation.

The length of a third piece of rope is between the lengths of the first and second pieces of rope. The length of the third piece is expressed as a decimal to the hundredths place.

**C.** Explain why there is only one possible length for the third piece of rope. As part of the explanation, find the length, in meters, of the third piece of rope and express it as a decimal to the hundredths place.

 $\frac{4}{10}$   $\frac{28}{100}$   $\frac{42}{100}$  28.0

I put 26.0 because I Cut the 42 in half and got 26 then I Cut 4 in half and got 2 so I added the 26 and the 2 together and got 28 then I put the 28 in meters like this 28 loop because it said hundreds.

When I put the 28 in decimal form to the hundreds it looks lik this.

The student has given an incorrect explanation.

### **MATHEMATICS—SUMMARY DATA**

### **MULTIPLE-CHOICE**

Sample			Depth of	<i>p</i> -values			
Number	Alignment	Answer Key	Knowledge	Α	В	С	D
1	A-T.2.1.4	В	1	13%	73%	7%	7%
2	A-F.1.1.2 A-F.1.1.1	С	2	21%	19%	52%	8%
3	A-F.2.1.6	С	1	24%	22%	42%	12%
4	A-T.1.1.2 A-T.1.1.1	D	2	13%	13%	17%	57%
5	A-T.1.1.4	В	1	11%	53%	20%	16%
6	A-T.2.1.2	D	1	5%	5%	5%	85%
7	B-O.1.1	В	2	14%	31%	27%	28%
8	B-O.1.1.1 B-O.1.1.3	В	2	24%	49%	10%	17%
9	B-O.1.1.2 B-O.1.1.3	С	2	14%	16%	64%	6%
10	B-O.2.1.1 B-O.3.1.1	А	1	70%	4%	6%	20%
11	B-O.3.1.1	С	1	21%	13%	58%	8%
12	B-O.3.1.3 B-O.3.1.2	С	2	22%	18%	53%	7%
13	C-G.1.1.1	В	1	16%	75%	6%	3%
14	C-G.1.1.2 C-G.1.1.3	А	2	51%	20%	19%	10%
15	C-G.1.1.3 C-G.1.1.1	С	2	6%	12%	69%	13%
16	D-M.3.1.2	А	2	47%	20%	14%	19%

### **OPEN-ENDED**

Sample Number	Alignment	Points	Depth of Knowledge	Mean Score
17	A-F.3	4	3	1.75