**Assignment-3**

**Q.1 Explain the concept of uncertainty?**

* **Uncertainty** is A situation in which something is not known, or something that is not known.
* So to represent uncertain knowledge, where we are not sure about the predicates, we need uncertain reasoning or probabilistic reasoning.
* **Sources of uncertainty:**

1. Implication may be weak.

Doorbell(0.8) →Wake (Mohan)

1. Imprecise language like often, rarely, sometimes

-Need to quantify these terms of frequencies.

1. Precise information may be too complex

- Too many antecedents or consequents

- AtDoor(x) V ShortCkt V Wind …..→ Doorbell

1. Incomplete knowledge.

- We may not know or guess all the possible antecedents or consequents

- ‘The bell rang due to some spooky reason’

1. Conflicting information

- Experts often provide conflicting information.

* **Acting under uncertainty:** Agents may need to handle uncertainty, whether due to partial observability, non-determinism, or a combination of the two.
* A agent may never know for certain what state it is in or where it will end up after a sequence of actions.
* Probability theory provides the basis for our treatment of systems that reasons under uncertainty.
* Four types of uncertainty in the decision-making process: data, prediction, judgment, and action uncertainty.
* **Summarizing uncertainty:** Dealing with a degree of belief is what is done through the probability theory, which assigns a numerical degree of belief between 0 and 1 to sentences.
* Probability provides a way of summarizing the uncertainty that comes from our laziness and ignorance.
* To make such choices, an agent must first have a Preference between the different possible outcomes of various plans.
* We use the Utility theory to represent and reason with preference.
* Utility theory says that every state has a degree of usefulness, to an agent and that agents will prefer states with higher utility.
* **Preference:** Options, choices, what is more preferred.
* **Outcome:** completely specified state
* **Decision theory = Probability Theory + Utility Theory**

**Q.2 What are the sources/causes of uncertainty?**

* **Sources of uncertainty:**
* **Implication may be weak.**

Doorbell(0.8) →Wake (Mohan)

* **Imprecise language like often, rarely, sometimes**

-Need to quantify these terms of frequencies

* **Precise information may be too complex**

- Too many antecedents or consequents

- AtDoor(x) V ShortCkt V Wind …..→ Doorbell

* **Incomplete knowledge.**

- We may not know or guess all the possible antecedents or consequents

- ‘The bell rang due to some spooky reason’

* **Conflicting information**

- Experts often provide conflicting information

* **Causes of uncertainty:**

**1. Laziness: -** It is too much work to list the complete set of antecedents or consequents needed to ensure an exceptionless rule.

**2. Theoretical ignorance: -** Expertise in the area may not be sufficient to have a complete theory for the domain.

**3. Practical ignorance: -** Even if we know all the rules, we might be uncertain about a particular case because not all the necessary tests have been or can be run.

• The agent’s knowledge can at best provide only a degree of belief in the relevant sentence

**Q.3 What are the axioms of probability?**

* All probabilities are between 0 and 1 **0 < P(A) < 1**

1. Necessarily true (i.e., valid) propositions have probability 1, and necessarily false (i.e., unsatisfiable) propositions have probability 0. **P (True) = 1 ; P (False) = 0**
2. The probability of disjunction is given by **𝑃 𝐴 ∨ 𝐵 = 𝑃 𝐴 + 𝑃 𝐵 − 𝑃(𝐴 ∧ 𝐵)**

* From these three axioms, we can derive all other properties of probabilities.

For example, the Probability of negation

𝑃 (𝐴 ∨ ¬𝐴) = 𝑃 (𝐴) + 𝑃 (¬𝐴) − 𝑃(𝐴 ∧ ¬𝐴)

𝑃 (𝑇𝑟𝑢𝑒) = 𝑃 (𝐴) + 𝑃 (¬𝐴) − 𝑃 (𝐹𝑎𝑙𝑠𝑒)

1 = 𝑃 (𝐴) + 𝑃 (¬𝐴)

𝑃 (¬𝐴) = 1 − 𝑃 (A)

**Q.4 Discuss the concept of Independence?**

* **Independent event:** An example of an independent event is the probability of getting heads on two coin tosses. The probability of getting heads on the first coin toss does not have an impact on the probability of getting heads on the second coin toss.
* Independence between propositions a and b can be written as, **P(a|b) =p(a) or P(b|a) =p(b)**

**Or P(a ∧ b) = P(a) P(b)**

* Example, add Weather variable in our previous example**, P(Toothache,Catch,Cavity,Weather)**
* To find the **P(toothache, catch, cavity, cloudy)** and **P(Toothache,Catch,Cavity)** related? we Can use the Product rule

**P(toothache, catch, cavity, cloudy)** = **P(cloudy | toothache, catch, cavity) P(toothache, catch, cavity).**

* It seems safe to say that the weather does not influence the dental variables. Therefore the following assertion seems reasonable.

**P(cloudy | toothache, catch, cavity) = P(cloudy)** From this we can deduce

**P(toothache, catch, cavity, cloudy) = P(cloudy) P(toothache, catch, cavity)**

* **P(toothache, catch, cavity, Weather) = P(toothache, catch, cavity) P(Weather)**

**Q.5 What are the basic probability notations?**

* Notation for describing the degree of belief.
* Probability can be defined as a chance that an uncertain event will occur. It is the numerical measure of the likelihood that an event will occur. The value of probability always remains between 0 and 1 which represents ideal uncertainties.
* 0 ≤ P(A) ≤ 1, where P(A) is the probability of an event A.
* P(A) = 0, indicates total uncertainty in an event A.
* P(A) =1, indicates total certainty in an event A.
* The dependence on experience is reflected in the syntactic distinction between : -

**-Prior probability** statements which apply before any evidence is obtained

**-Conditional probability** statement which includes the evidence explicitly.

* **Sample space:** The set of all possible worlds i.e., all possible outcomes is referred to as sample space.
* **Notation:**
* **Ω** ∶ 𝑆𝑎𝑚𝑝𝑙𝑒 𝑆𝑝𝑎𝑐𝑒
* **𝜔:** 𝐴𝑛 𝑒𝑙𝑒𝑚𝑒𝑛𝑡 𝑖𝑛 𝑠𝑎𝑚𝑝𝑙𝑒 𝑠𝑝𝑎𝑐𝑒
* **𝜑:** 𝐴𝑛 𝑒𝑣𝑒𝑛𝑡, An event 𝜑 is a subset of sample space Ω: 𝜑 ⊆ Ω
* **Example** : Two Dice adding up to 11 is an event 𝜑 = { (5,6) , (6,5) }

**Q.6 What are the different types of probabilities?**

* **Unconditional probability** is when you don’t consider any other information except for the object in the question.
* **Example:** Two Dice – red and blue; consider only one – red.
* In **Conditional probability** we have evidence i.e., extra information already revealed.
* **Example:** Rolling two dice – one is 6; the sum can not be 5!
* **Prior Probability:** Use the notation P(A) for the unconditional or prior probability that proposition A true.
* **For example**, if Fever denotes the proposition that a particular patient has a fever, P(Fever) = 0.1 This means that in the absence of any other information, the agent will assign a probability of 0.1 (a 10% chance) to the event of the patient having a fever.
* P(A) can only be used when there is no other information. As soon as some new information B is known, we have to reason with the conditional probability of A given B instead of P(A)
* **Joint Probability:** Probability that the intersection of two events occurs.
* **Experiment 1:** Rolling a die
* Event A1: Rolling an even number
* Event A2: Rolling an odd number
* **Experiment 2:** Flipping a coin
* Event B1: Flipping heads
* Event B2: Flipping tails
* **Four possible intersections:** **1.** Rolling even and flipping heads, **2.** Rolling even and flipping tails, **3.** Rolling odd and flipping heads, **4.** Rolling odd and flipping tails.
* **Notation:** Either “P(A and B)” or “P(A Ⴖ B)”
* **Calculating joint probabilities:**

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* **Marginal Probability:** Probability that an individual event from one experiment occurs, regardless of the outcomes from another experiment.
* Computed by adding the probabilities across the row (or down the column) of the desired event.
* Always involve only one experiment.
* Get their names from the fact that they are written in the margins of the table
* **Notation:** P(A1)
* **Calculating marginal probabilities:**

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* **Conditional probability: t**he probability that a second event (B) will occur given that we know that the first event (A) has already occurred,
* A and B come from two different experiments.
* **Notation:** P(B|A) → vertical bar “|” means “given”
* **Calculating conditional probabilities:**

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**Q.7 What is Baye's rule and its use?**

* **Baye's rule:**

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**Q.8 Demonstrate the wumpus world revisted problem ?**

* **The Wumpus World Revisited:** **Aim –** To calculate the probability that each of the three squares contains a pit. .
* **Variables:**  Boolean variable Pi,j for each square, which is true if square [i,j] actually contains a pit Boolean variable Bi,j that are true if square [i,j] is breezy; we include this variable only for the observed squares – in this case, [1,1], [1,2], and [2,1].
* **Properties of the world**:  **1)** A pit causes breezes in all neighboring squares, **2)** Each square other than [1,1] contains a pit with a probability 0.2

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**Q.9 What do you mean by Bayesian network and semantics of Bayesian network?**

* **Bayesian network:**
* The full joint probability distribution can answer any question about the domain, but can become intractably large as the number of variables grows. This section introduces the Bayesian network to represent the dependencies among variables.
* Bayesian network is also known as belief network, probabilistic network, causal network, and knowledge map.
* A Bayesian network is a directed graph in which each node is annotated with quantitative probability information.
* **The full specification is as follows:**

1. Each node corresponds to a random variable, which may be discrete or continuous.
2. A set of directed links or arrows connect pairs of nodes. If there is an arrow from node X to node Y, X is said to be a parent of Y. The graph has no directed cycles, hence it is a directed acyclic graph or DAG.
3. Each node Xi has a conditional probability distribution P(Xi |parents(Xi )) that quantifies the effect of the parents on the node.

* The intuitive meaning of an arrow is typically that X has a direct influence on Y.
* It is usually easy for a domain expert to decide what direct influences exist in the domain.
* Once the topology of the Bayesian network is laid out, we need only specify a conditional probability distribution for each variable, given its parents.
* **Bayesian Belief Network –Example:**  You have a new burglar alarm installed at home. It is fairly reliable at detecting burglary, but also sometimes responds to minor earthquakes.
* You have two neighbors, John and Merry, who promised to call you at work when they hear the alarm.
* Jhon always calls when he hears the alarm, but sometimes confuses telephone ringing with alarm and calls too.
* Merry likes loud music and sometimes misses the alarm.
* Given the evidence of who has or has not called, we would like to estimate the probability of a burglary.
* **The semantics of Bayesian Network:**
* A Bayesian network is a directed acyclic graph with some numeric parameters attached to each node.
* The network semantics – is to define the way in which it represents a specific joint distribution over all the variables.
* We know that P(Xi |parents(Xi )).
* Joint distribution is the probability of a conjunction of particular assignments to each variable , such as **𝑃 (𝑋1 = 𝑥1 ∧ ⋯ ∧ 𝑋𝑛 = 𝑥n)**

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**Q.10 Explain the exact inference in the Bayesian network?**

* The graphical independence representation

- yields an efficient inference scheme

* We generally want to compute

- Marginal probability Pr(Z),

- Pr(Z|E) where E is (conjunctive) evidence.

* Z : query variable(s),
* E : evidence variable(s),
* everything else: hidden variable.
* **Variable elimination:** P(B,E,A, J,M ) = P(B)× P(E)× P(A | B,E)× P(J | A)× P(M | A)

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**Q.11 The Bayesian network is given to answer the following questions:**

**1. What is the probability that an alarm has sounded but neither a burglary nor an earthquake has occurred, and both Jhon and Mery called?**

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* **Solution:** 𝑃 (𝑗 ∧ 𝑚 ∧ 𝑎 ∧ ¬𝑏 ∧ ¬𝑒) = 𝑃 (𝑗/ 𝑎) 𝑃 (𝑚/𝑎) 𝑃 (𝑎/ ¬𝑏), ¬𝑒) 𝑃 (¬𝑏) 𝑃( ¬𝑒)

= 0.90 × 0.70 × 0.001 × 0.999 × 0.998

= **0.00062**