EN580.435/635 APPLIED BIOELECTRICAL ENGINEERING revised Feb. 11, 2019
The Johns Hopkins University (corrections in red)

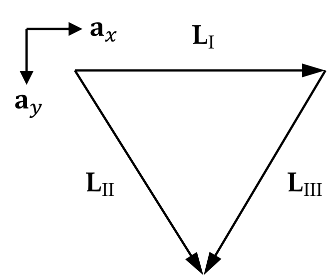
The homework policy for this course is that you are expected to work on the problems on your own. You may collaborate on homework assignments only insofar as group discussion and peer learning. Assignments must be written up individually in each individual's words and NOT COPIED. It is highly recommended that the actual solution of the problem be done by yourself, much like you would do on an exam since the exam will be modelled after the homework questions. If you need further help to understand the problem, you should contact or meet with either Dr. Hunter or Dr. Tung. Homework turned in after the start of class on the due date will receive only 50% credit until noon the next day and only if it is not discussed in class, and 0% after that.

Furthermore, some homework questions given throughout the course may require that you do some research or make some assumptions regarding relevant parameters or other information. Conversely, extra information may sometimes be given that is not needed to solve the problem. You will need to recognize these situations when they come up.

Homework #1 (due beginning of class on Feb. 12)

Problem 1 (20 pts)

(a) Using a Cartesian coordinate system, express the lead vectors $\mathbf{L}_{\mathrm{I}}$, $\mathbf{L}_{\mathrm{II}}$ and $\mathbf{L}_{\mathrm{III}}$ in terms of the unit vectors $\mathbf{a}_x$ and $\mathbf{a}_y$.



(b) Assume that at time t = 20 ms during the QRS complex, the heart vector $\mathbf{H}$ is equal to:

$$\mathbf{H} = (1.5\ \text{mA-cm})\mathbf{a}_x \qquad (1a)$$

The lead weight W for all 3 leads can be determined experimentally, but can also be derived theoretically. For the case of a heart dipole in the center of a spherical volume conductor with radius R and conductivity σ (the so-called *centric dipole model*), W is given by:

$$W = \frac{3\sqrt{3}}{4\pi\sigma R^2} \qquad (1b)$$

For the purposes of this problem, you may assume that W = 1 (but you must specify its units). Use your answer in (a) to determine V_{I}, V_{II} and V_{III}.

(c) At time t = 40 ms, **H** is now:

$$\mathbf{H} = (1.5 \text{ mA-cm})\mathbf{a}_y \tag{2}$$

Determine V_{I}, V_{II} and V_{III}.

(d) At time t = 60 ms, V_{I} and V_{II} are measured to be -1.0 and 0 mV, respectively. Is this enough information to determine **H**? If so, what is **H**? If not, provide any value you like for V_{III} and determine **H**.

Problem 2 (20 pts)

An action potential is propagating along a cylindrical fiber with radius of 0.1 mm and intracellular resistivity of 80 Ω-cm that is lying in a bath with extracellular resistivity of 50 Ω -cm. It has the following approximate triangular shape:

$$V_m(x,t) = \begin{cases} 40u - 60 & 0 \le u < 2 \\ -20(u-2) + 20 & 2 \le u < 6 \\ -60 & u < 0 \text{ and } u \ge 6 \end{cases} \tag{3}$$

where

$$u = t - x/\theta$$

and conduction velocity θ = 2 m/s.[provide units for 40]. Note that t is in units of ms, x is in units of mm, θ is in units of m/s, and V_m is in units of mV.

The fiber axis is coincident with the x-axis. $V_m(x,t)$ is in units of mV, x is in units of mm, and t is in units of ms. The extracellular resistivity is 50 Ω-cm.

(a) Determine the *distributed dipole line* source τ_l for this waveform at time t = 4 ms. Indicate magnitude(s) and location(s). Be sure to specify units.

(b) Determine the two *lumped dipole* sources D for this waveform at time t = 4 ms. Indicate magnitudes and location(s). Be sure to specify units.

(c) Assuming that the fiber lies in an infinite volume conductor, calculate the extracellular potentials Φ_e at the x,y,z coordinates of (0,4,0) and (0,0,4) at time t = 0 ms. Be sure to specify units.

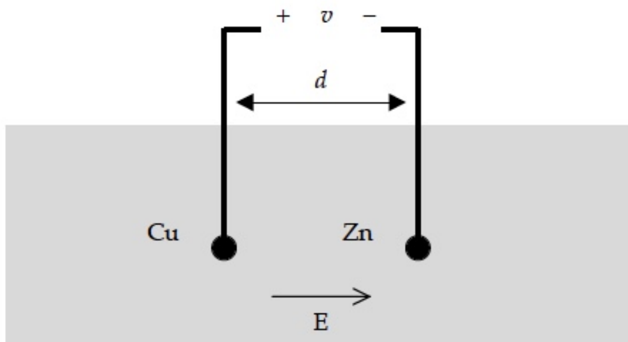
(d) Calculate Φ_e at (x,y,z) = (0,4,0) and (0,0,4) at time t = 4 ms.

For extra credit (10 pts):

(e) Calculate and plot Φ_e at (x,y,z) = (0,4,0) over the time interval t = -10 to 10 ms in 0.1 ms (or smaller) time steps.

Problem 3 (15 pts)

A copper electrode and zinc electrode are placed in a bath as shown below to measure the local bioelectric field.



The field is estimated as the voltage difference *v* divided by the interelectrode distance *d*.

$$E = \frac{v}{d} \tag{4}$$

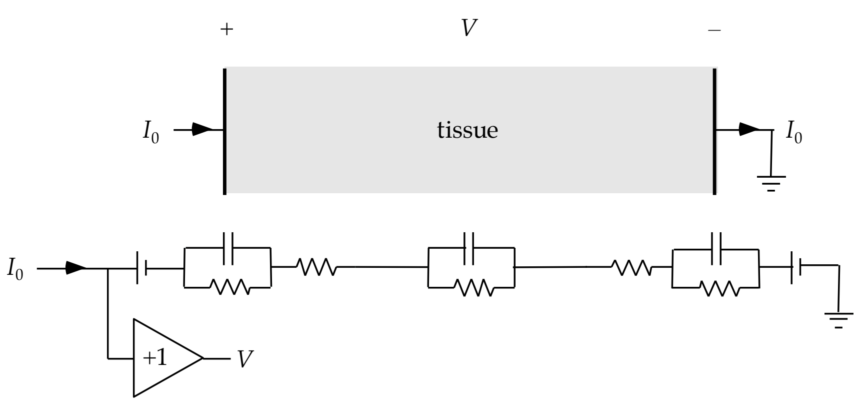
(a) Under standard measurement conditions (1 molar solute concentration, 1 atm pressure and 25°C), (4) is off by a constant value. What is the constant value? [Hint: if the field is turned off, what is the value of *v*?]

(b) The copper electrode is now replaced by nickel, and the zinc electrode by iron. Is there still an offset potential? If so, what is it?

(c) Both electrodes are now replaced by silver. Is there still an offset potential? If so, what is it?

Problem 4 (30 pts)

Consider the electrode system below, in which a rectangular slab of tissue is placed between two plate electrodes:



A steady current of 1mA is is injected into one electrode and returns to ground at the other electrode. The tissue slab has length L = 1 cm and cross-sectional area A = 0.1 cm^2. The tissue has a resistivity of ρ = 50 Ω-cm.

(a) Assume that the electrodes are ideal (no half-cell potential and zero impedance). What is V?

(b) What are the magnitudes of **E** (the electric field in the tissue) and **J** (the current density in the tissue)?

(c) Suppose the electrode on the left is now silver, having a half cell potential of 0.223 V, a specific capacitance (=C/A) of 10 μF/cm^2, and series and parallel specific resistances (= R A) of 0.5 kΩ-cm^2 and 15 kΩ-cm^2 respectively. The right electrode is still assumed to be ideal. Now what is V?

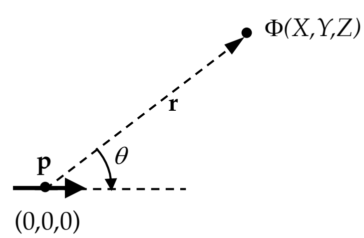
(d) Now what are the magnitudes of **E** and **J** in the tissue?

(e) Using your answer in (c), what would you have concluded the resistivity of the tissue to be had you assumed that both electrodes were ideal? How much error is there compared with the correct value of 50 Ω-cm?

<u>The following is extra credit for 580.435; required for 580.635</u>

<u>Problem 5 (10 pts)</u>

We have seen that the potential field Φ of a current dipole **p** in a volume conductor that is positioned at the origin and oriented along the x-axis:



is given by,

$$\Phi(X, Y, Z) = \frac{p}{4\pi\sigma r^2}\cos\theta \tag{5}$$

Note that we can write the dipole source as **p**,

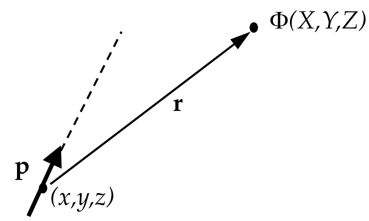
$$\mathbf{p} = p\mathbf{a}_x \tag{6}$$

where p is the dipole moment, $\mathbf{a}_x$ is the unit vector in the x-direction, and **r** is the vector from the origin to the measurement point (X,Y,Z), with magnitude r and unit direction $\mathbf{a}_r$:

$$\mathbf{r} = r\mathbf{a}_r \quad (7)$$

Now consider the case where the dipole source is not constrained to be at the origin or have an orientation along the x-axis.

(a) Determine the expression for the potential field for the current dipole located at position (x,y,z) with orientation along the general vector $\mathbf{p}$:



where

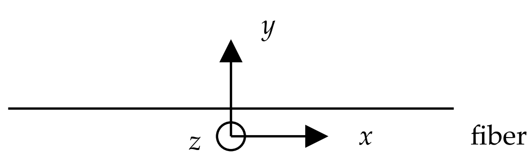
$$\mathbf{p} = p\mathbf{a}_p \quad (8)$$

$\mathbf{a}_p$ is the unit vector along $\mathbf{p}$, p is the dipole moment, and $\mathbf{r}$ is the vector from (x,y,z) to (X,Y,Z), with magnitude r and unit direction $\mathbf{a}_r$. Your answer should be expressed in terms of vectors and σ.

(b) What is r (as a function of the source and measurement coordinates)?

<u>Problem 6 (20 pts)</u>

An action potential is propagating along a cylindrical fiber with radius of 0.1 mm and intracellular resistivity of 80 Ω-cm that is lying in a bath with extracellular resistivity of 50 Ω-cm. The coordinate system is shown below (z-axis is out of the paper).



It has the following approximate triangular shape:

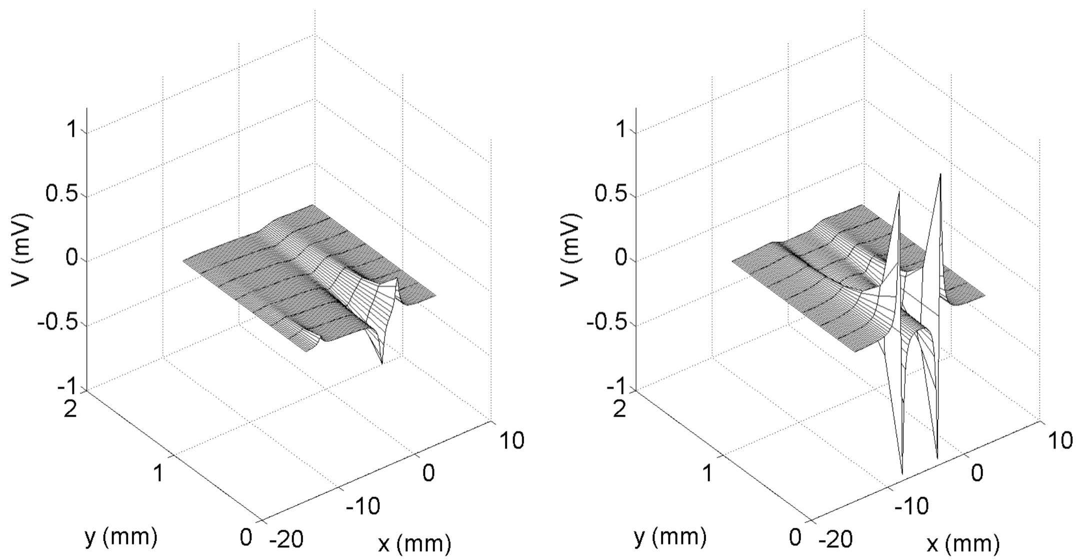
$$V_m(x,t) = \begin{cases} 80u - 60 & 0 \le u < 1 \\ -20(u-1) + 20 & 1 \le u < 5 \\ -60 & u < 0 \text{ and } u \ge 5 \end{cases} \quad (9)$$

where

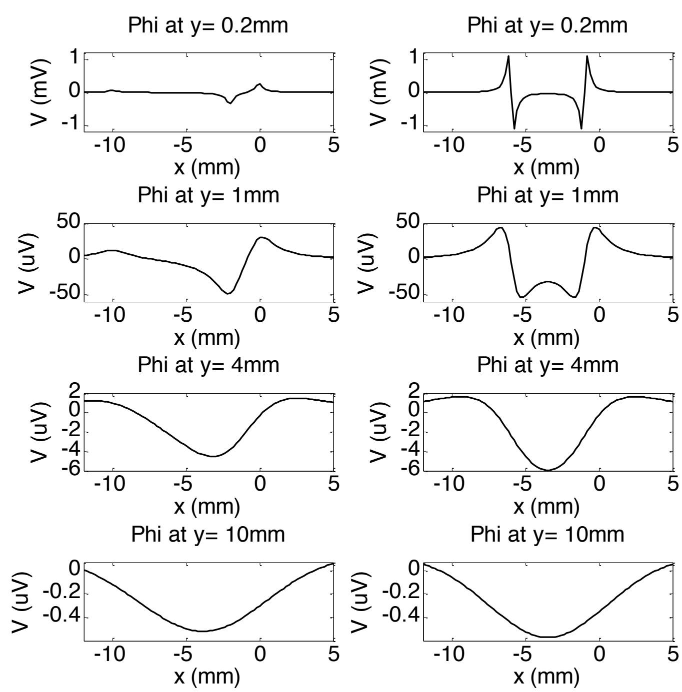
$$u = t - x/\theta$$

and conduction velocity θ = 2 m/s. The fiber axis is coincident with the x-axis. $V_m(x,t)$ is in units of mV, x is in units of mm, and t is in units of ms.

(a) Below are plots of $\Phi_e(x,y,0)$ at time t = 0 for both the distributed and lumped dipole sources associated with the action potential. Which plot is for which dipole source?



(b) Below are pairs of plots of $\Phi_e(x,y,0)$ for the distributed and lumped dipole sources at different values of y: 0.2 mm, 1 mm, 4 mm and 10 mm. Each column has only one source type (distributed or lumped), and each row is a measurement at the different y values. Describe the qualitative changes in the potential profiles for each column as y increases from 0.2 mm to 10 mm. Also describe the qualitative differences in the potential profiles for each row. Explain the basis for the differences between rows and between columns.



(c) What would happen to the plots in (b) if the conduction velocity is increased to 4 m/s? Sketch your expected results on the figure in (b) and turn it in with your homework.

(d) Based on the figure in (b), what is a reasonable distance at which the sources can be represented as lumped dipoles?