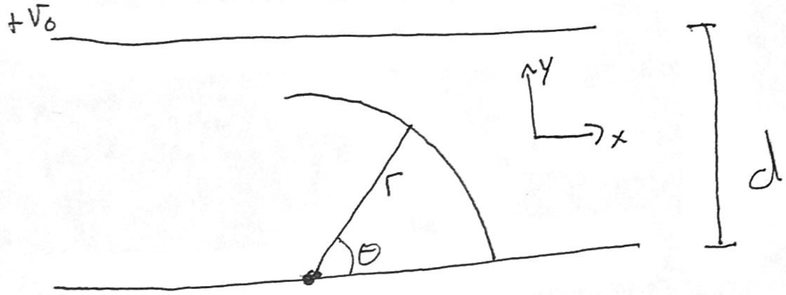
Problem 1

a) The potential along the fiber varies linearly with distance along the fiber so its second derivative will be ϕ.

$$A=0$$

b)



$$x = r\cos\theta$$
$$y = r\sin\theta$$

ϕ_e is independent of x and varies as:

$\phi_e = \frac{V_0}{d} y$ along the fiber. There is also a constant term, but since we're going to take a derivative we need not compute it.

$$\frac{\partial \phi_e}{\partial \theta} = \frac{V_0}{d} r\cos\theta \Rightarrow \frac{\partial^2 \phi_e}{\partial \theta^2} = \frac{-V_0}{d} r\sin\theta = \boxed{\frac{-V_0}{d} y = A}$$

c)

My lab partner placed the point electrode at P, equidistant from all points on the fiber. Since we know:

$\phi = \frac{I_0}{4\pi\sigma r}$ the potential will be constant along the fiber and $\boxed{A=0}$

This can be corrected by moving the electrode anywhere else in the chamber.

Problem 2

a) We can re write as:

$$V_{th} = IR\left(1-e^{-d/\tau}\right)$$

So: $I_1 R\left(1-e^{-d_1/\tau}\right) = I_2 R\left(1-e^{-d_2/\tau}\right)$

The R terms cancel so: $I_1\left(1-e^{-d_1/\tau}\right) = I_2\left(1-e^{-d_2/\tau}\right)$

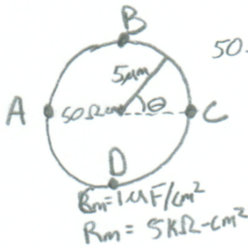
We're now left with an equation where all terms except τ are known!

We can solve it:

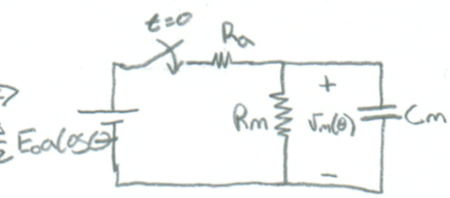
b) The pulses should be square and brief to limit Sodium channel activation. Ideally the pulses would be hyperpolarizing so that the activation could be avoided altogether. This would ensure that the assumptions of constant R and V_{th} remains valid.

Problem 3:

A)



⟹



$R_a = a(\rho_i + \rho_e/2) = 5\times10^{-4}(75\,\Omega\text{-}cm) = 0.0375\ \Omega\text{-}cm$

$tau = \left[\frac{1}{R_m C_m} + \frac{1}{R_a C_m}\right]^{-1} = 3.75\times10^{-8}\,s$

Small time constant ⟹ Small time step in MATLAB.

$$V_m = \frac{R_m}{R_m+R_a}\left(\frac{3}{2}E_0 a\cos\theta\right)\left(1-e^{-t/tau}\right)\Bigg|_{\theta=0,\ \pi/2,\ \pi,\ 3\pi/2}$$

Code for plots attached.

B)

$\perp$ $V_m = V_i - V_e \Rightarrow V_e = V_i - V_m\Big|_{V_i=0} \Rightarrow V_e = -V_m$

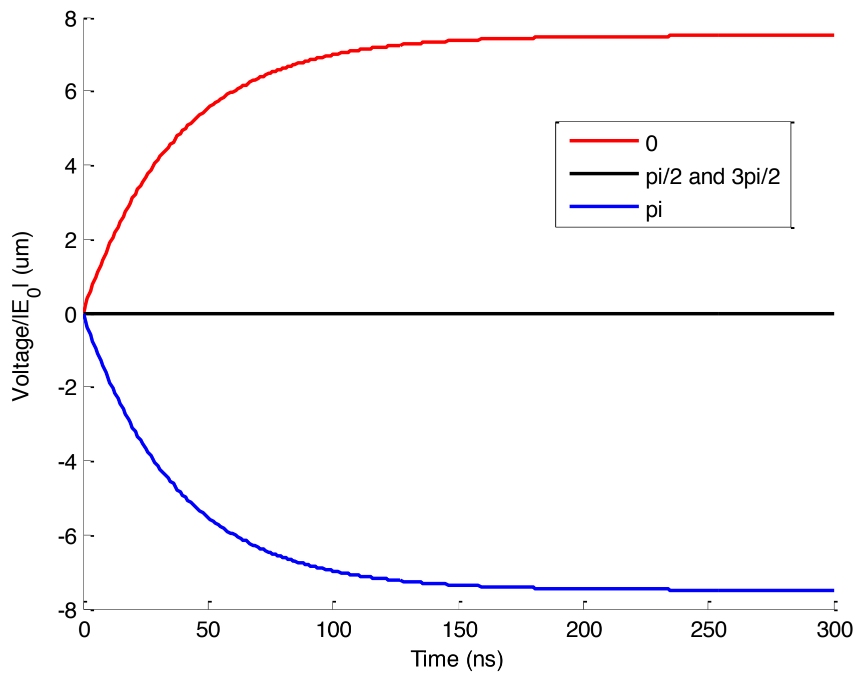
$$\vec{E} = -\nabla\Phi \Rightarrow \vec{E} = \nabla V_m = \frac{\partial V_m}{\partial r}\vec{a}_r + \frac{1}{r}\frac{\partial V_m}{\partial\theta}\vec{a}_\theta = -\frac{3}{2}E_0\frac{a}{r}\sin\theta\left(1-e^{-t/\tau}\right)\frac{R_m}{R_m+R_a}\vec{a}$$

$$\vec{E}\Big|_{r=a} = \frac{-R_m}{R_m+R_a}\left(\frac{3}{2}E_0\sin\theta\right)\left(1-e^{-t/\tau}\right)\vec{a}_\theta$$

$\vec{a}_\theta = \cos\phi\cos\theta\,\vec{a}_x + \sin\phi\cos\theta\,\vec{a}_y - \sin\theta\,\vec{a}_z$

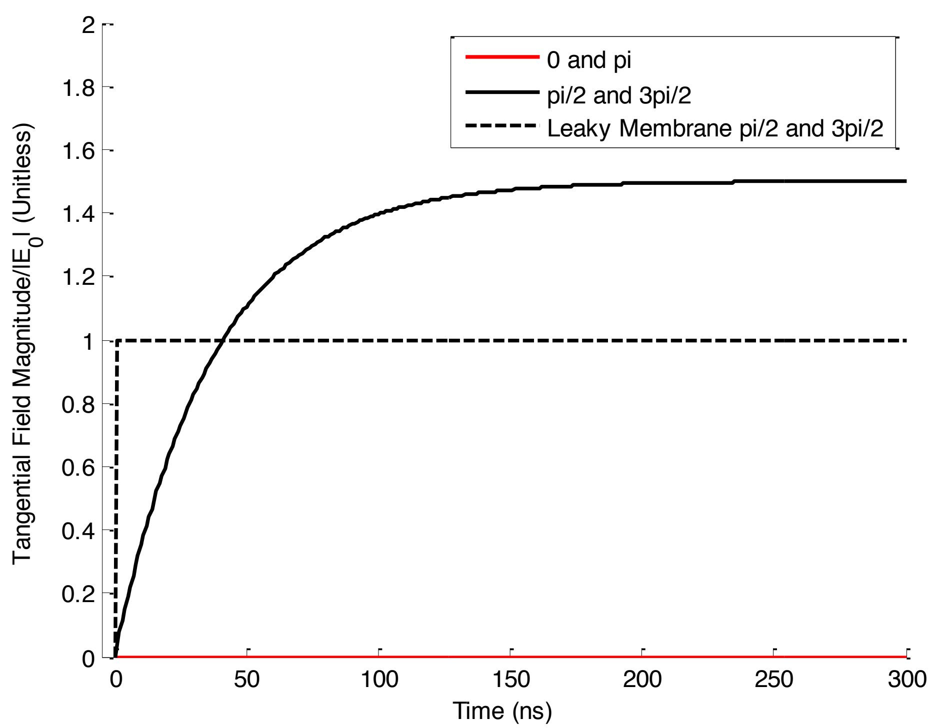
C) With a leaky membrane the field will not be perturbed (field lines will flow directly through w/o bending). $A=C=\phi$ $B=D=E_0$

A.



8
6
4
2
0
-2
-4
-6
-8
Voltage/|E$_0$| (um)
0
50
100
150
200
250
300
Time (ns)
0
pi/2 and 3pi/2
pi

B+C.



2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Tangential Field Magnitude/|E$_0$| (Unitless)
0
50
100
150
200
250
300
Time (ns)
0 and pi
pi/2 and 3pi/2
Leaky Membrane pi/2 and 3pi/2

Problem 4:

a) Color in theta =60 degrees (see below)

b) $V_m = \frac{R_m}{R_m + R_a}\left(\frac{3}{2} E_0 a \cos\theta\right)\left(1 - e^{-t/\tau}\right)$ We'll ignore this at steady state (rheobase is after a long time)

$R_m = 5\,k\Omega\text{-}cm^2$

$R_a = (20\times10^{-4}\,cm)(2\,k\Omega\text{-}cm + 0.5\,k\Omega\text{-}cm) = 5\,\Omega\text{-}cm^2$

$\Rightarrow \frac{R_m}{R_m+R_a} \sim 1 \quad \Rightarrow V_m \cong \frac{3}{2} E_0 (20\times10^{-4}\,cm)\cos\theta$

Liminal area $= \pi a^2 \big|_{a=1 \text{ for unit sphere}} = \pi = 2\pi(1-\cos\theta) \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$ at farthest point

$\Rightarrow V_m = \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)(20\times10^{-4}\,cm)\,E_0$ and V_m is the change in potential, so we set $V_m = 20\,mV$

$\Rightarrow E_0 = \frac{20\,mV\left(\frac{2}{3}\right)(2)}{20\times10^{-4}\,cm} = 1.333\times10^{4}\,mV/cm = \boxed{13.33\,V/cm}$

I used the fact that area of a spherical cap for a given angle is A=2*pi*r^2*(1-cos(theta))

Problem 5:

Answer:

(a)
For a monopolar *cathodal* current source with strength -I_e at position x = 0 a distance z_0 above a fiber (see Fig. 9 in Handout #5):

$$\Phi_e = -\frac{I_e}{4\pi\sigma_e r} = -\frac{I_e}{4\pi\sigma_e\sqrt{x^2+z_0^2}} \tag{1}$$

$$A = \frac{\partial^2\Phi_e}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{-I_e}{4\pi\sigma_e\left(x^2+z_0^2\right)^{3/2}}\left(\frac{-1}{2}\right)2x\right) = \frac{\partial}{\partial x}\left(\frac{I_e x}{4\pi\sigma_e\left(x^2+z_0^2\right)^{3/2}}\right) \tag{2a}$$

$$= \frac{I_e}{4\pi\sigma_e\left(x^2+z_0^2\right)^{3/2}} + \frac{I_e x}{4\pi\sigma_e}\frac{1}{\left(x^2+z_0^2\right)^{5/2}}\left(\frac{-3}{2}\right)2x \tag{2b}$$

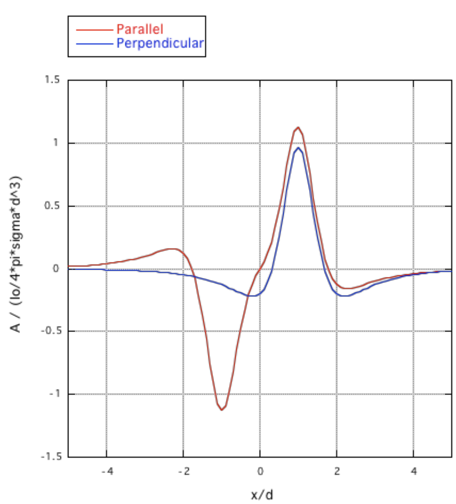
$$= \frac{I_e}{4\pi\sigma_e}\frac{\left(x^2+z_0^2\right)-3x^2}{\left(x^2+z_0^2\right)^{5/2}} = \frac{I_e}{4\pi\sigma_e}\frac{z_0^2-2x^2}{\left(x^2+z_0^2\right)^{5/2}} \tag{2c}$$

Superimposing the activating functions for a cathodal source with strength -I_0 at position (d,d) and an anodal source with strength I_0 at position (d,-d), yields:

$$A = \frac{I_0}{4\pi\sigma_e}\frac{d^2-2(x-d)^2}{((x-d)^2+d^2)^{5/2}} - \frac{I_0}{4\pi\sigma_e}\frac{d^2-2(x+d)^2}{((x+d)^2+d^2)^{5/2}} \tag{3}$$

for the parallel case. For the perpendicular case, a cathodal source with strength -I_0 is at position (d,d) and an anodal source with strength I_0 is at position (d,3d),

$$A = \frac{I_0}{4\pi\sigma_e}\frac{d^2-2(x-d)^2}{((x-d)^2+d^2)^{5/2}} - \frac{I_0}{4\pi\sigma_e}\frac{9d^2-2(x-d)^2}{((x-d)^2+9d^2)^{5/2}} \tag{4}$$



Note that $X = \frac{x}{d}$ and A in the graph is normalized to $\frac{I_0}{4\pi\sigma_e d^3}$.

$$A = \frac{I_0}{4\pi\sigma_e d^3}\left(\frac{1-2(X-1)^2}{((X-1)^2+1)^{5/2}} - \frac{1-2(X+1)^2}{((X+1)^2+1)^{5/2}}\right) \quad \text{(parallel)} \qquad (5)$$

$$A = \frac{I_0}{4\pi\sigma_e d^3}\left(\frac{1-2(X-1)^2}{((X-1)^2+1)^{5/2}} - \frac{9-2(X-1)^2}{((X-1)^2+9)^{5/2}}\right) \quad \text{(perpendicular)} \qquad (6)$$

(b) Take the ratio of peak amplitudes of A in (a). At X = 1 where the cathodal amplitudes are maximal (note: this is only approximate for the parallel case; the amplitude is maximal at X = 0.988).

$$A = \frac{I_0}{4\pi\sigma_e d^3}\left(1 - \frac{-7}{5^{5/2}}\right) \quad \text{(parallel)} \qquad (7)$$

$$A = \frac{I_0}{4\pi\sigma_e d^3}\left(1 - \frac{9}{9^{5/2}}\right) \quad \text{(perpendicular)} \qquad (8)$$

Parallel amplitude/perpendicular amplitude $=\left(1 + \frac{7}{5^{5/2}}\right) / \left(1 - \frac{9}{9^{5/2}}\right)$ = 1.1252/0.9630 = 1.16849 [or for the exact answer, = 1.125865/0.962963 = 1.16917].

Thus, the parallel orientation is 16.8% more effective than the perpendicular orientation for the same amplitude current.