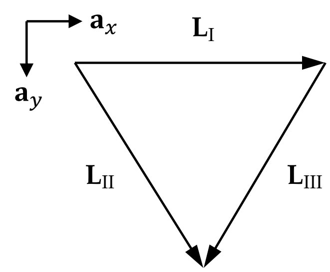
EN580.435/635 APPLIED BIOELECTRICAL ENGINEERING Mar. 1, 2019
The Johns Hopkins University

<u>Solutions for Homework #1</u>

<u>Problem 1 (20 pts)</u>

(a) Using a Cartesian coordinate system, express the lead vectors $\mathbf{L}_{\mathrm{I}}$, $\mathbf{L}_{\mathrm{II}}$ and $\mathbf{L}_{\mathrm{III}}$ in terms of the unit vectors $\mathbf{a}_x$ and $\mathbf{a}_y$.



<u>Answer:</u> Assuming that the magnitude of each lead vector is W, then:

$$\mathbf{L}_{\mathrm{I}} = W(1.0\mathbf{a}_x) \tag{1a}$$

$$\mathbf{L}_{\mathrm{II}} = W(\tfrac{1}{2}\mathbf{a}_x + \tfrac{\sqrt{3}}{2}\mathbf{a}_y) = W(0.5\mathbf{a}_x + 0.866\mathbf{a}_y) \tag{1b}$$

$$\mathbf{L}_{\mathrm{III}} = W(-\tfrac{1}{2}\mathbf{a}_x + \tfrac{\sqrt{3}}{2}\mathbf{a}_y) = W(-0.5\mathbf{a}_x + 0.866\mathbf{a}_y) \tag{1c}$$

(b) Assume that at time t = 20 ms during the QRS complex, the heart vector $\mathbf{H}$ is equal to:

$$\mathbf{H} = (1.5 \text{ mA-cm})\mathbf{a}_x \tag{2a}$$

The lead weight W for all 3 leads can be determined experimentally, but can also be derived theoretically. For the case of a heart dipole in the center of a spherical volume conductor with radius R and conductivity σ (the so-called *centric dipole model*), W is given by:

$$W = \frac{3\sqrt{3}}{4\pi\sigma R^2} \tag{2b}$$

For the purposes of this problem, you may assume that $W = 1$ (but you must specify its units). Use your answer in (a) to determine V_{I}, V_{II} and V_{III}.

<u>Answer:</u> W should have units of Ω/cm (resistance/length). If $W = 1\ \Omega$/cm, then:

$$V_{\mathrm{I}} = \mathbf{H}\cdot\mathbf{L}_{\mathrm{I}} = [(1.5 \text{ mA-cm})\mathbf{a}_x]\cdot[(1\ \Omega/\text{cm})(1.0\mathbf{a}_x)] = 1.5 \text{ mV} \tag{3a}$$

$$V_{\mathrm{II}} = \mathbf{H}\cdot\mathbf{L}_{\mathrm{II}} = [(1.5 \text{ mA-cm})\mathbf{a}_x]\cdot\left[W(0.5\mathbf{a}_x + 0.866\mathbf{a}_y)\right] = 0.75 \text{ mV} \tag{3b}$$

$$V_{\mathrm{III}} = \mathbf{H} \cdot \mathbf{L}_{\mathrm{III}} = [(1.5\text{ mA-cm})\mathbf{a}_x] \cdot \left[W\left(-0.5\mathbf{a}_x + 0.866\mathbf{a}_y\right)\right] = -0.75\text{ mV} \qquad (3c)$$

If you assumed that $W = 1\ \Omega/\text{m}$, then your answers should be 100 times smaller.

(c) At time $t = 40$ ms, **H** is now:

$$\mathbf{H} = (1.5\text{ mA-cm})\mathbf{a}_y \qquad (4)$$

Determine V_{I}, V_{II} and V_{III}.

Answer:

$$V_{\mathrm{I}} = \mathbf{H} \cdot \mathbf{L}_{\mathrm{I}} = \left[(1.5\text{ mA-cm})\mathbf{a}_y\right] \cdot [W(1.0\mathbf{a}_x)] = 0\text{ mV} \qquad (5a)$$

$$V_{\mathrm{II}} = \mathbf{H} \cdot \mathbf{L}_{\mathrm{II}} = \left[(1.5\text{ mA-cm})\mathbf{a}_y\right] \cdot \left[W\left(0.5\mathbf{a}_x + 0.866\mathbf{a}_y\right)\right] = \tfrac{3\sqrt{3}}{4} = 1.30\text{ mV} \qquad (5b)$$

$$V_{\mathrm{III}} = \mathbf{H} \cdot \mathbf{L}_{\mathrm{III}} = \left[(1.5\text{ mA-cm})\mathbf{a}_y\right] \cdot \left[W\left(-0.5\mathbf{a}_x + 0.866\mathbf{a}_y\right)\right] = \tfrac{3\sqrt{3}}{4} = 1.30\text{ mV} \qquad (5c)$$

(d) At time $t = 60$ ms, V_{I} and V_{II} are measured to be -1.0 and 0 mV, respectively. Is this enough information to determine **H**? If so, what is **H**? If not, provide any value you like for V_{III} and determine **H**.

Answer:

$$-1.0 = \mathbf{H} \cdot \mathbf{L}_{\mathrm{I}} = (H_x\mathbf{a}_x + H_y\mathbf{a}_y) \cdot (1.0\mathbf{a}_x) \qquad (6a)$$

$$0 = \mathbf{H} \cdot \mathbf{L}_{\mathrm{II}} = (H_x\mathbf{a}_x + H_y\mathbf{a}_y) \cdot \left(0.5\mathbf{a}_x + 0.866\mathbf{a}_y\right) \qquad (6b)$$

Solving (6) leads to:

$$\mathbf{H} = -1.0\mathbf{a}_x + \tfrac{1}{\sqrt{3}}\mathbf{a}_y = -1.0\mathbf{a}_x + \tfrac{1}{\sqrt{3}}\mathbf{a}_y = -1.0\mathbf{a}_x + 0.577\mathbf{a}_y \quad (\text{mA-cm}) \qquad (7)$$

Problem 2 (20 pts)

An action potential is propagating along a cylindrical fiber with radius of 0.1 mm and intracellular resistivity of 80 Ω-cm that is lying in a bath with extracellular resistivity of 50 Ω -cm. It has the following approximate triangular shape:

$$V_m(x,t) = \begin{cases} 40u - 60 & 0 \le u < 2 \\ -20(u-2) + 20 & 2 \le u < 6 \\ -60 & u < 0 \text{ and } u \ge 6 \end{cases} \qquad (1)$$

where

$$u = t - x/\theta$$

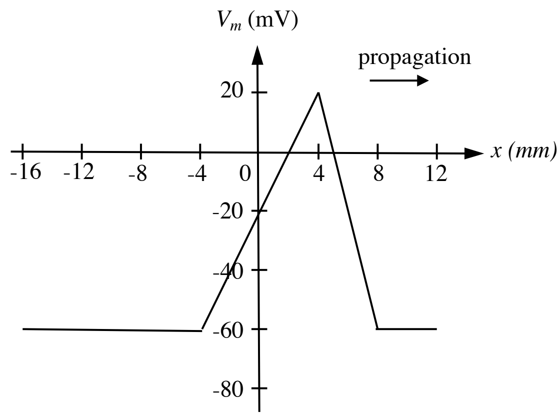
and conduction velocity θ = 2 m/s.~~[provide units for 40].~~ Note that t is in units of ms, x is in units of mm, θ is in units of m/s, and V_m is in units of mV.

The fiber axis is coincident with the x-axis. $V_m(x,t)$ is in units of mV, x is in units of mm, and t is in units of ms. The extracellular resistivity is 50 Ω-cm.

(a) Determine the *distributed dipole line* source τ_l for this waveform at time t = 4 ms. Indicate magnitude(s) and location(s). Be sure to specify units.

Answer: The first thing to do is to determine $V_m(x)$ at t = 4 ms.

$$V_m(x,4) = \begin{cases} 40\left(4-\frac{x}{2}\right)-60 = -20x+100 & 4 < x \leq 8 \\ -20\left(2-\frac{x}{2}\right)+20 = 10x-20 & -4 < x \leq 4 \\ -60 & x > 8 \text{ and } x \leq -4 \end{cases} \quad (2)$$

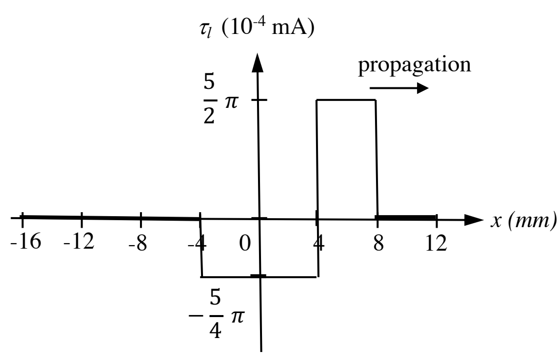


The axial dipole density function for 4 ≤ x ≤ 8 mm is given by,

$$\tau_{l1} = -\pi a^2 \sigma_i \frac{\partial V_m}{\partial x} = -\pi(0.01\text{cm})^2 \frac{1}{80\Omega\text{-cm}} \frac{-80\text{mV}}{0.4\text{cm}} = 0.00025\pi = 7.854\text{x}10^{-4} \text{ mA} \quad (3a)$$

and for -4 ≤ x ≤ 4 mm is given by,

$$\tau_{l2} = -\pi a^2 \sigma_i \frac{\partial V_m}{\partial x} = -\frac{1}{2}(0.00025\pi) = -3.927\text{x}10^{-4} \text{ mA} \quad (3b)$$



(b) Determine the two *lumped dipole* sources *D* for this waveform at time *t* = 4 ms. Indicate magnitudes and location(s). Be sure to specify units.

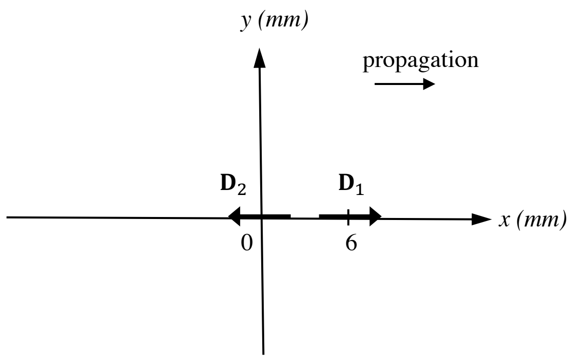
Answer:

$$\mathbf{D}_1 = D_1 \mathbf{a}_x \quad (4a)$$

$$D_1 = -\pi a^2 \sigma_i \left[V_{rest} - V_{peak}\right] = -\pi (0.01\text{cm})^2 \frac{1}{80\Omega\text{-cm}} (-80\text{mV}) = 10^{-4}\pi \text{ (mA-cm)} \quad (4b)$$

$$\mathbf{D}_2 = D_2 \mathbf{a}_x \quad (5a)$$

$$D_2 = -D_1 = -10^{-4}\pi \text{ (mA-cm)} = -10^{-3}\pi \text{ (mA-mm)} \quad (5b)$$



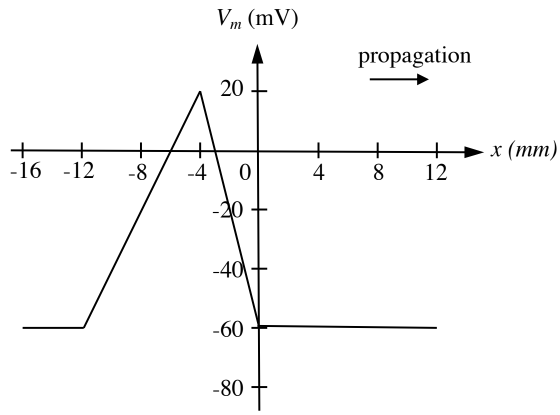
Note that according the definition of (5a), $\mathbf{D}_2$ should be pointing in the +*x*-direction (with negative amplitude). However, it is useful to draw $\mathbf{D}_2$ as in the diagram above (i.e., $\mathbf{D}_2 = -D_2 \mathbf{a}_x$) so that regions of positive and negative potential are easily visualized, in which case D_2 should no longer carry the negative sign.

$\mathbf{D}_1$ and $\mathbf{D}_2$ have been placed at the centers of the two groups of distributed dipoles so that their potential fields best approximate those of the distributed dipoles. This would not be a good approximation at a distance *r* close to the dipole sources, but works better farther away (i.e., where *r* >> width of distributed dipole source, which is either 4 or 8 mm and where *r* >> radius of the fiber, which is 0.1mm).

(c) Assuming that the fiber lies in an infinite volume conductor, calculate the extracellular potentials Φ_e at the *x*,*y*,*z* coordinates of (0,4,0) and (0,0,4) at time *t* = 0 ms. Be sure to specify units.

Answer: At *t* = 0 ms,

$$V_m(x,0) = \begin{cases} 40\left(-\frac{x}{2}\right) - 60 = -20x - 60 & -4 < x \le 0 \\ -20\left(-2 - \frac{x}{2}\right) + 20 = 10x + 60 & -12 < x \le -4 \\ -60 & x > 0 \text{ and } x \le -12 \end{cases} \quad (6)$$



This problem was intended to utilize the lumped dipole approximation, but turned out to be a poor example, since as noted above, the approximation works only at distances sufficiently away from the fiber. Since the measurement point is only 4 mm from the dipoles, the more exact solution should be based on the distributed dipole distribution, but this is more complicated than what was originally intended. The solution looks like,

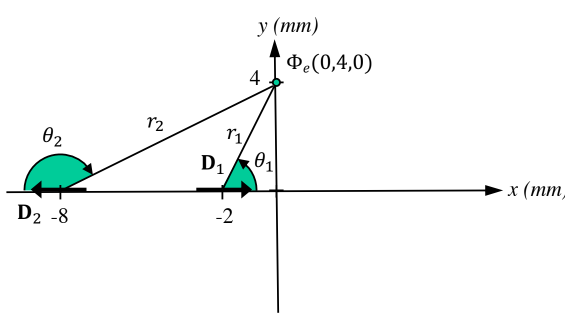
$$\Phi_e(0,4,0) = \frac{1}{4\pi\sigma_e}\int_L \frac{1}{r^2}\tau_l \cos\theta\, dx = \frac{500\Omega\text{-mm}}{4\pi}\int_{-12}^{-4}\frac{1}{r^2}\left(-\frac{5}{4}\pi\cdot 10^{-4}\text{mA}\right)\cos\theta\, dx$$

$$+\frac{500\Omega\text{-mm}}{4\pi}\int_{-4}^{0}\frac{1}{r^2}\left(\frac{5}{2}\pi\cdot 10^{-4}\text{mA}\right)\cos\theta\, dx$$

$$= -\frac{0.25\text{mV-mm}}{16}\int_{-12}^{-4}\frac{1}{(x^2+4^2)}\left(\frac{-x}{(x^2+4^2)^{1/2}}\right)dx + \frac{0.25\text{mV-mm}}{8}\int_{-4}^{0}\frac{1}{(x^2+4^2)}\left(\frac{-x}{(x^2+4^2)^{1/2}}\right)dx$$

$$= -\frac{0.25\text{mV-mm}}{16}\left[\frac{1}{(x^2+16)^{1/2}\text{mm}}\right]_{-12}^{-4} + \frac{0.25\text{mV-mm}}{8}\left[\frac{1}{(x^2+16)^{1/2}\text{mm}}\right]_{-4}^{0} = 0.761\ \mu\text{V} \qquad (7)$$

Alternatively, the simpler although less accurate solution (but acceptable for this homework) uses the lumped dipole approximation:



Thus, $\Phi_e(0,4,0)$ is the superposition of potentials from the two dipoles.

$$\Phi_e(0,4,0) = \frac{D_1}{4\pi\sigma_e r_1{}^2}\cos\theta_1 + \frac{D_2}{4\pi\sigma_e\, r_2{}^2}\cos\theta_2$$

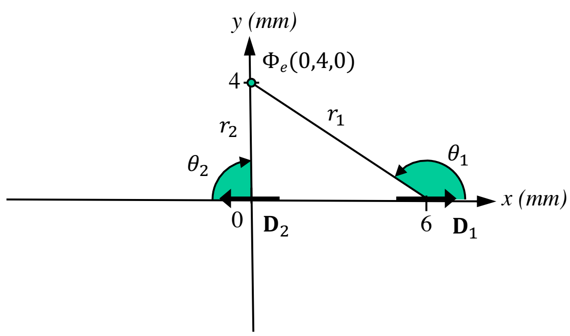
$$= \frac{(500\Omega\text{-mm})(10^{-3}\pi\ \text{mA-mm})}{4\pi(2^2+4^2)\text{mm}^2}\frac{2}{(2^2+4^2)^{1/2}} + \frac{(500\Omega\text{-cm})(10^{-3}\pi\ \text{mA-mm})}{4\pi(8^2+4^2)\text{mm}^2}\frac{-8}{(8^2+4^2)^{1/2}} = 1.398\ \mu\text{V} \qquad (8a)$$

Because of axial symmetry along the *x*-axis,

$$\Phi_e(0,0,4) = \Phi_e(0,4,0) = 1.398\ \mu\text{V} \qquad (8b)$$

(d) Calculate Φ_e at $(x,y,z) = (0,4,0)$ and $(0,0,4)$ at time $t = 4$ ms.

Answer: Using the lumped dipole approximation, at $t = 4$ ms,



$$\Phi_e(0,4,0) = \frac{D_1}{4\pi\sigma_e r_1{}^2}\cos\theta_1 + \frac{D_2}{4\pi\sigma_e r_2{}^2}\cos\theta_2$$

$$= \frac{(500\Omega\text{-mm})(10^{-3}\pi\ \text{mA-mm})}{4\pi(6^2+4^2)\text{mm}^2}\frac{-6}{(6^2+4^2)^{1/2}} + 0 = -2.000\ \mu\text{V} \qquad (9a)$$

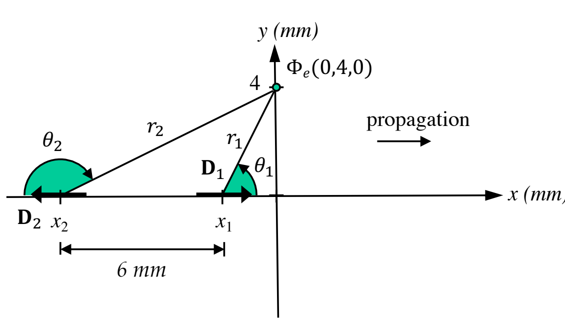
By axial symmetry,

$$\Phi_e(0,0,4) = \Phi_e(0,4,0) = -2.000\ \mu\text{V} \qquad (9b)$$

For extra credit (10 pts):

(e) Calculate and plot Φ_e at $(x,y,z) = (0,4,0)$ over the time interval t = -10 to 10 ms in 0.1 ms (or smaller) time steps.

Answer: The calculations above at times $t = 0$ and $t = 4$ms can be generalized to any time:



$$\Phi_e(0,4,0) = \frac{D_1}{4\pi\sigma_e {r_1}^2}\frac{-x_1}{r_1} + \frac{D_2}{4\pi\sigma_e\, {r_2}^2}\frac{x_2}{r_2} \tag{10}$$

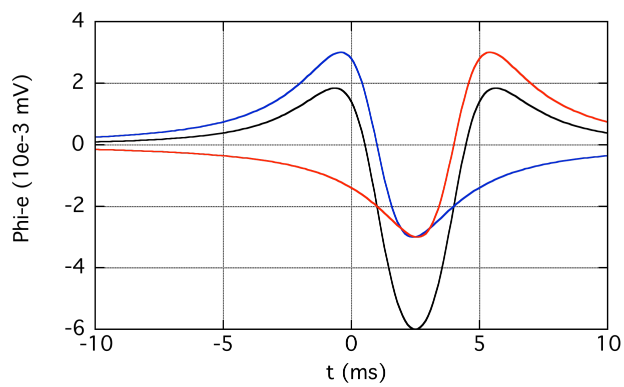
where $$x_1 = \theta t - 2;\; x_2 = x_1 - 6 = \theta t - 8 \tag{11a,b}$$

and $$r_1 = \sqrt{{x_1}^2 + 4^2};\; r_2 = \sqrt{{x_2}^2 + 4^2} \tag{12a,b}$$

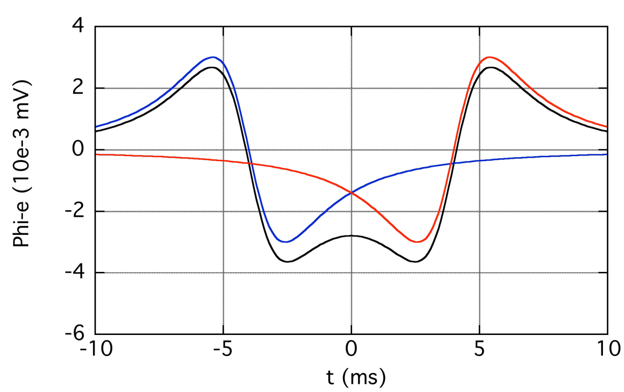
$$\Phi_e(0,4,0) = \frac{D_1}{4\pi\sigma_e {r_1}^2}\cos\theta_1 + \frac{D_2}{4\pi\sigma_e\, {r_2}^2}\cos\theta_2$$

$$= \frac{(500\Omega\text{-mm})(10^{-3}\pi\ \text{mA-mm})}{4\pi}\frac{-(2t-2)}{((2t-2)^2+4^2)^{3/2}\text{mm}^2} + \frac{(500\Omega\text{-mm})(10^{-3}\pi\ \text{mA-mm})}{4\pi}\frac{(2t-8)}{((2t-8)^2+4^2)^{3/2}\text{mm}^2}$$

$$= \frac{0.5}{4}\frac{-(2t-2)}{((2t-2)^2+16)^{3/2}} + \frac{0.5}{4}\frac{-(2t-8)}{((2t-8)^2+16)^{3/2}}\ \text{mV} \tag{13}$$



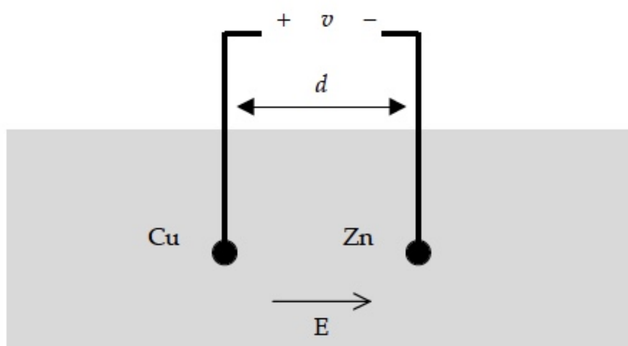
In the plots above, the blue and red curves are Φ_e resulting from $\mathbf{D}_1$ and $\mathbf{D}_2$, respectively. Each is biphasic in nature. The black curve is Φ_e for the sum of the two responses, and is triphasic in nature. If the distance between the two dipoles were greater, say 16 mm, then the negative portion of the waveform resolves into two peaks as shown below.



This discussion may help in your understanding of Problem 6 of this homework assignment (note, however, that in this problem Φ_e is plotted as a function of time at one point in space, whereas in Problem 6, Φ_e is plotted as a function of position at one instance of time.

Problem 3 (15 pts)

A copper electrode and zinc electrode are placed in a bath as shown below to measure the local bioelectric field.

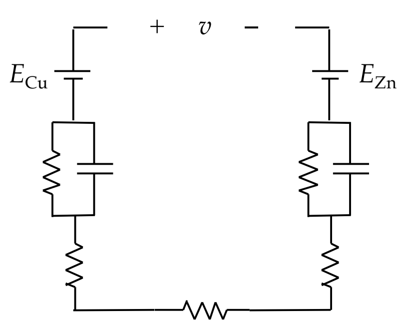


The field is estimated as the voltage difference *v* divided by the interelectrode distance *d*.

$$E = \frac{v}{d} \tag{1}$$

(a) Under standard measurement conditions (1 molar solute concentration, 1 atm pressure and 25°C), (4) is off by a constant value. What is the constant value? [Hint: if the field is turned off, what is the value of *v*?]

Answer:



The offset potential would be the open circuit voltage, equal to the difference in half-cell potentials, $E_{Cu} - E_{Zn} = 0.340 - (-0.763) = 1.103$ V. The half-cell potential of 0.340 is chosen for Cu because this is for the reaction between aqueous copper (Cu^{2+}) and solid copper.

(b) The copper electrode is now replaced by nickel, and the zinc electrode by iron. Is there still an offset potential? If so, what is it?

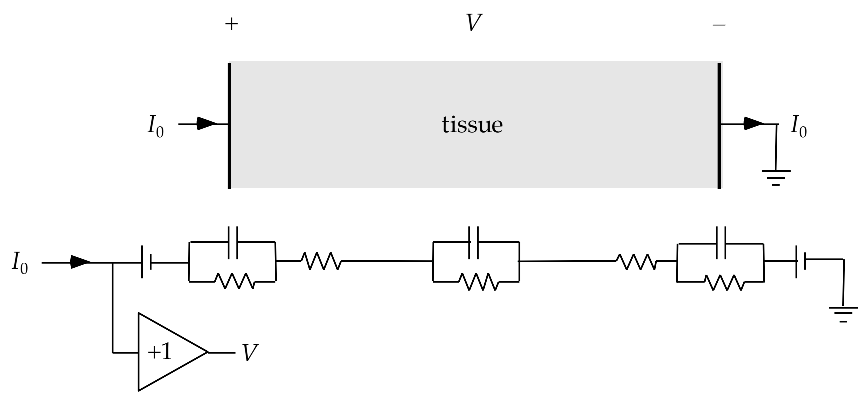
Answer: The offset potential is now the difference in half-cell potentials, $E_{Ni} - E_{Fe}$ = -0.230 – (-0.409) = 0.179 V.

(c) Both electrodes are now replaced by silver. Is there still an offset potential? If so, what is it?

Answer: Assuming no motion artifacts and identical environmental conditions for both electrodes, the offset potentially is nominally zero. However, if current were to pass between the electrodes, the half cell potentials would no longer balance, since they are also a function of current (and its direction).

Problem 4 (30 pts)

Consider the electrode system below, in which a rectangular slab of tissue is placed between two plate electrodes:



A steady current of 1mA is is injected into one electrode and returns to ground at the other electrode. The tissue slab has length L = 1 cm and cross-sectional area A = 0.1 cm^2. The tissue has a resistivity of ρ = 50 Ω-cm.

(a) Assume that the electrodes are ideal (no half-cell potential and zero impedance). What is V?

Answer: At dc, the tissue has a resistance of,

$$R_t = \frac{\rho L}{A} = \frac{(50\Omega\text{-cm})(1\text{cm})}{0.1\text{cm}^2} = 500\Omega \qquad (1)$$

Hence,

$$V = IR_t = (1\text{mA})(500\Omega) = 500\text{mV} \qquad (2)$$

(b) What are the magnitudes of **E** (the electric field in the tissue) and **J** (the current density in the tissue)?

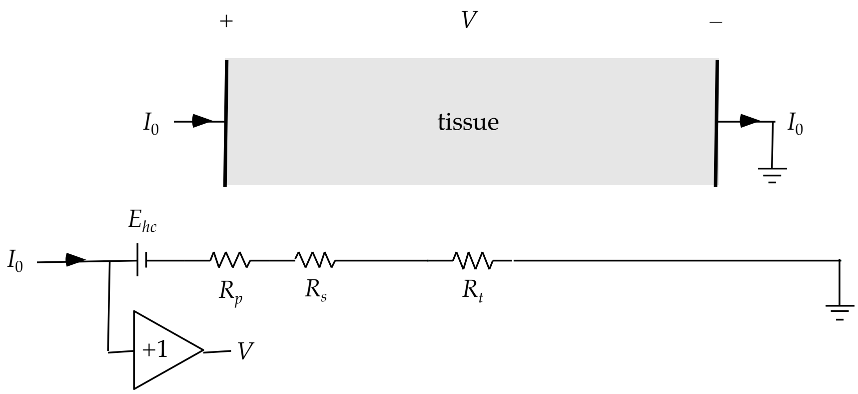
Answer:

$$E = \frac{V}{L} = \frac{0.5\text{V}}{1\text{cm}} = 0.5\text{V/cm} \qquad (3)$$

Hence, $$J = \frac{E}{\rho} = \frac{0.5\text{V/cm}}{50\Omega\text{-cm}} = 10\text{mA/cm}^2 \tag{4}$$

(c) Suppose the electrode on the left is now silver, having a half cell potential of 0.223 V, a specific capacitance (=C/A) of 10 μF/cm², and series and parallel specific resistances (= $R\,A$) of 0.5 kΩ-cm² and 15 kΩ-cm² respectively. The right electrode is still assumed to be ideal. Now what is V?

Answer: The equivalent circuit of the system now looks like:



where

$$R_p = \frac{15\text{k}\Omega\text{-cm}^2}{A} = \frac{15\text{k}\Omega\text{-cm}^2}{0.1\text{cm}^2} = 150\text{k}\Omega \tag{5a}$$

$$R_s = \frac{0.5\text{k}\Omega\text{-cm}^2}{A} = \frac{0.5\text{k}\Omega\text{-cm}^2}{0.1\text{cm}^2} = 5\text{k}\Omega \tag{5b}$$

Hence, $$V = (1\text{mA})(150\text{k}\Omega + 5\text{k}\Omega + 500\Omega) + 0.223\text{V} = 155.723\text{V} \tag{6}$$

(d) Now what are the magnitudes of **E** and **J** in the tissue?

Answer: Note that E is no longer equal to V/L because of the voltage drop across the left electrode. But J is still defined because the current I_0 is injected and held constant:

$$J = \frac{I_0}{A} = \frac{1\text{mA}}{0.1\text{cm}^2} = 10\text{mA/cm}^2 \tag{7}$$

so that $$E = \rho J = (50\Omega\text{-cm})(10\text{mA/cm}^2) = 0.5\text{V/cm} \tag{8}$$

Note that J and E are unchanged from the answer in (b). Most of the applied V occurs across R_p and R_s, leaving only 0.5V across the tissue. This should be evident from the equivalent circuit model.

(e) Using your answer in (c), what would you have concluded the resistivity of the tissue to be had you assumed that both electrodes were ideal? How much error is there compared with the correct value of 50 Ω-cm?

Answer: You would have assumed the electric field is V/L, i.e.,

$$E = \frac{V}{L} = \frac{155.723\text{V}}{1\text{cm}} = 155.723\text{V/cm} \tag{9}$$

and,

$$\rho = \frac{E}{J} = \frac{155.723\text{V/cm}}{10\text{mA/cm}^2} = 15572.3\Omega\text{-cm} \tag{10}$$

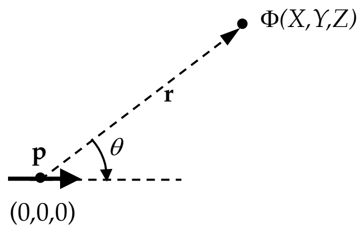
which would be an overestimate of,

$$Error = \frac{15572.3 - 50}{50} = +31045\% \tag{11}$$

The following is extra credit for 580.435; required for 580.635

Problem 5 (10 pts)

We have seen that the potential field Φ of a current dipole **p** in a volume conductor that is positioned at the origin and oriented along the *x*-axis:



is given by,

$$\Phi(X, Y, Z) = \frac{p}{4\pi\sigma r^2}\cos\theta \tag{1}$$

Note that we can write the dipole source as **p**,

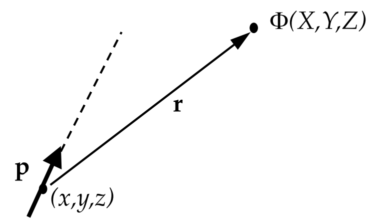
$$\mathbf{p} = p\mathbf{a}_x \tag{2}$$

where p is the dipole moment, $\mathbf{a}_x$ is the unit vector in the x-direction, and **r** is the vector from the origin to the measurement point (X,Y,Z), with magnitude r and unit direction $\mathbf{a}_r$:

$$\mathbf{r} = r\mathbf{a}_r \tag{3}$$

Now consider the case where the dipole source is not constrained to be at the origin or have an orientation along the *x*-axis.

(a) Determine the expression for the potential field for the current dipole located at position (x,y,z) with orientation along the general vector **p**:



where

$$\mathbf{p} = p\mathbf{a}_p \tag{4}$$

$\mathbf{a}_p$ is the unit vector along $\mathbf{p}$, p is the dipole moment, and $\mathbf{r}$ is the vector from (x,y,z) to (X,Y,Z), with magnitude r and unit direction $\mathbf{a}_r$. Your answer should be expressed in terms of vectors and σ.

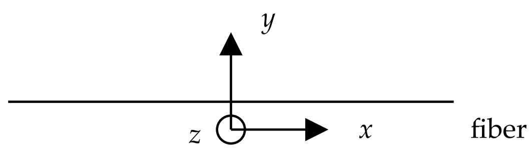
Answer: $$\Phi(X,Y,Z) = \frac{p}{4\pi\sigma r^2}\mathbf{a}_p \cdot \mathbf{a}_r = \frac{1}{4\pi\sigma r^2}\mathbf{p} \cdot \mathbf{a}_r = \frac{1}{4\pi\sigma r^3}\mathbf{p} \cdot \mathbf{r} \tag{5}$$

(b) What is r (as a function of the source and measurement coordinates)?

Answer: $$r = \sqrt{(X-x)^2 + (Y-y)^2 + (Z-z)^2} \tag{6}$$

Problem 6 (20 pts)

An action potential is propagating along a cylindrical fiber with radius of 0.1 mm and intracellular resistivity of 80 Ω-cm that is lying in a bath with extracellular resistivity of 50 Ω-cm. The coordinate system is shown below (z-axis is out of the paper).



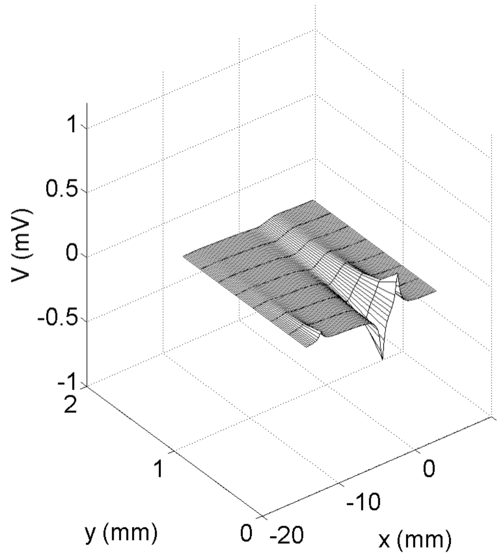
It has the following approximate triangular shape:

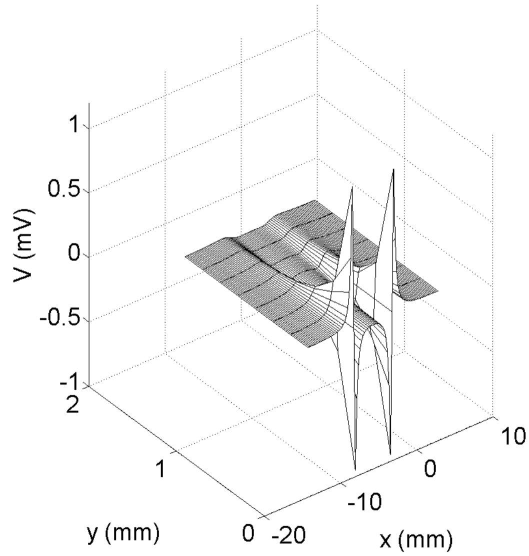
$$V_m(x,t) = \begin{cases} 80u - 60 & 0 \le u < 1 \\ -20(u-1) + 20 & 1 \le u < 5 \\ -60 & u < 0 \text{ and } u \ge 5 \end{cases} \tag{1}$$

where, $$u = t - x/\theta \tag{2}$$

and conduction velocity θ = 2 m/s. The fiber axis is coincident with the x-axis. $V_m(x,t)$ is in units of mV, x is in units of mm, and t is in units of ms.

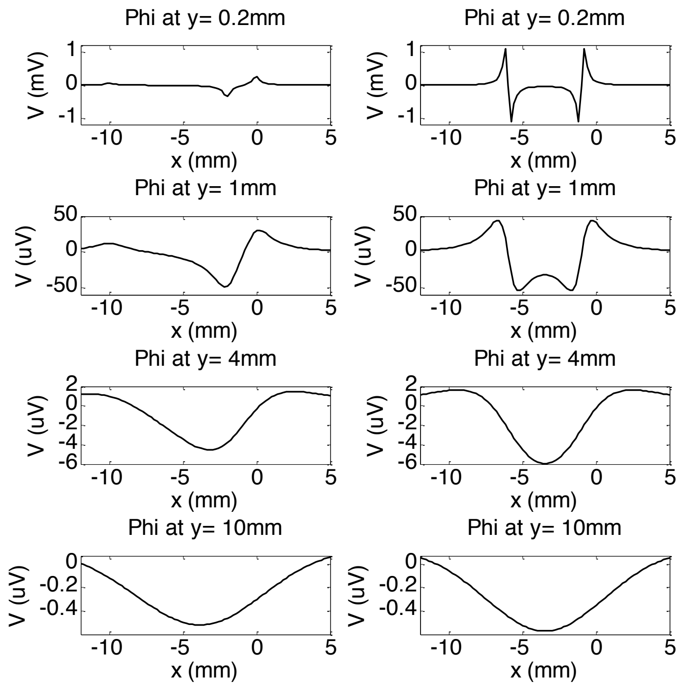
(a) Below are plots of $\Phi_e(x,y,0)$ at time t = 0 for both the distributed and lumped dipole sources associated with the action potential. Which plot is for which dipole source?





Answer: The plot on the left is for the *distributed* source; the one on the right is for the *lumped dipole* source. You can tell because the individual biphasic dipole potential fields are more distinct for the lumped dipole source and are equally large for the depolarizing and repolarizing phases of the action potential. For the distributed dipole source, the potentials for the repolarizing phase are much smaller and spread out than for the depolarizing phase, reflecting the lower amplitude and broader spatial distribution of the sources. Hence, on first glance it looks like only a single dipole is present.

(b) Below are pairs of plots of $\Phi_e(x,y,0)$ for the distributed and lumped dipole sources at different values of y: 0.2 mm, 1 mm, 4 mm and 10 mm. Each column has only one source type (distributed or lumped), and each row is a measurement at the different y values. Describe the qualitative changes in the potential profiles for each column as y increases from 0.2 mm to 10 mm. Also describe the qualitative differences in the potential profiles for each row. Explain the basis for the differences between rows and between columns.



Answer: For both types of sources, as y increases, the peaks of Φ_e decrease in amplitude and become broader in space. The positive and negative phases also start to fuse. Comparing distributed to discrete sources (left to right columns), the distributed sources result in two biphasic responses with the left hand (repolarizing) response very small in amplitude, wherase for the lumped sources the two biphasic responses are much closer together (reflecting the relatively close spacing of the two dipoles) and equal in amplitude. The biphasic responses are also mirrored, left to right, relative to one another, reflecting the opposite directions of the dipole soures. At increasing distances from the fiber, the fine features of the local field that were apparent at 0.2mm become lost, due to a low pass spatial filtering effect, and at y=10 mm the potential field from the lumped dipole sources is nearly identical to that from the distributed dipole sources.

(c) What would happen to the plots in (b) if the conduction velocity is increased to 4 m/s? Sketch your expected results on the figure in (b) and turn it in with your homework.

Answer: If conduction velocity is doubled, the action potential will be stretched over twice the distance. To put it another way, if θ is doubled in Eq. 2, then x must change twice as much to produce the same change in u. Hence, one would expect the plots of Φ_e to stretch out over twice the distance. The amplitude of Φ_e for the distributed dipoles would be halved, since all of the spatial V_m gradients would be halved, whereas the amplitudeof Φ_e for the distributed dipoles would remain the same (since the dipole source strength is related to the net change in V_m which remains unchanged), but spaced twice as far apart.

(d) Based on the figure in (b), what is a reasonable distance at which the sources can be represented as lumped dipoles?

Answer: To clarify, the question intended to ask, "what is a reasonable distance at which the sources can be represented as lumped dipoles instead of distributed dipoles?". Comparing the left and right columns, the two are nearly identical at y = 10 mm and beginning to resemble each other at y = 4 mm. So somewhere between 4 and 10 mm; 10 mm for sure.