Bioelectric Dipole Sources

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580.435/635 Applied Bioelectrical Engineering
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What you can expect to learn today

- Concept of duality
- Concept of current dipole source
- Transitioning from cable theory to volume conductors
- Distributed and lumped dipole model of activation wave
- Example of a triangularized action potential

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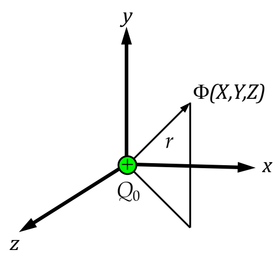
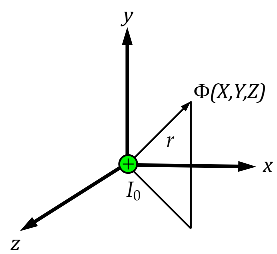
Concept of Duality

Dielectric medium



Conductive medium



electric field → $\mathbf{E} = -\nabla\Phi$ ← electrical potential

displacement field → $\mathbf{D} = \varepsilon\mathbf{E}$

$\nabla \cdot \mathbf{D} = \rho$ ← charge density

$$\Rightarrow \nabla^2\, \Phi = -\frac{\rho}{\varepsilon}$$

permittivity

electric field → $\mathbf{E} = -\nabla\Phi$ ← electrical potential

current density → $\mathbf{J} = \sigma\mathbf{E}$

$\nabla \cdot \mathbf{J} = I_v$ ← volumetric current density

$$\Rightarrow \nabla^2\Phi = -\frac{I_v}{\sigma}$$

conductivity

If $\rho(X,Y,Z) = Q_0\, \delta(r)$

$$\Rightarrow \Phi(X,Y,Z) = \frac{Q_0}{4\pi\varepsilon r}$$

If $I_v(X,Y,Z) = I_0\, \delta(r)$

$$\Rightarrow \Phi(X,Y,Z) = \frac{I_0}{4\pi\sigma r}$$

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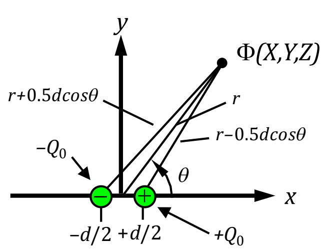
Current Dipole Source



$$\Phi(X,Y,Z) = \frac{I_0 d}{4\pi\sigma r^2}\cos\theta$$

$$= \frac{p}{4\pi\sigma r^2}\cos\theta$$

$p = I_0\, d$ is the current dipole moment

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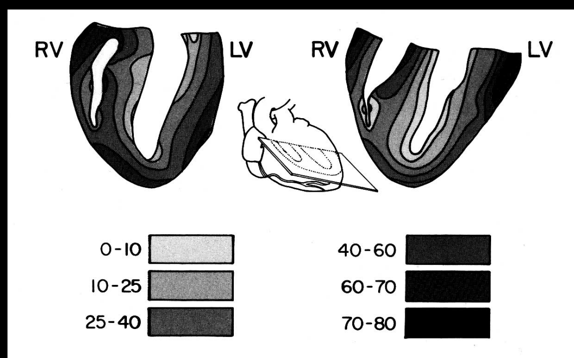
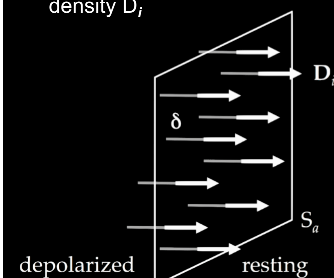
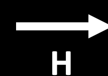
Transmural Activation in the Ventricles Occurs from Endocardium to Epicardium



The activation surface is a <u>dipole sheet</u> with dipole density D_i



The heart vector **H** is the vectorial sum of the dipoles in the sheet.



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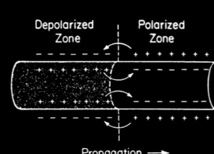
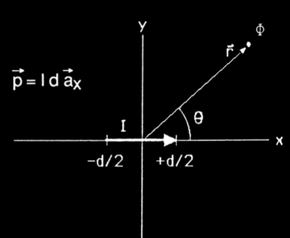
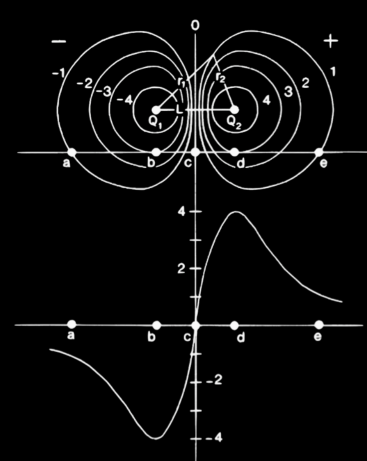
Concept of an Excitation Dipole in a Volume Conductor





$$\Phi(\vec{r}) = \frac{Id\cos\theta}{4\pi\sigma r^2}$$



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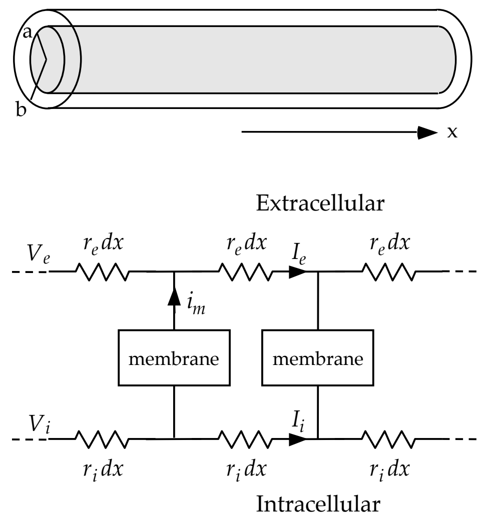
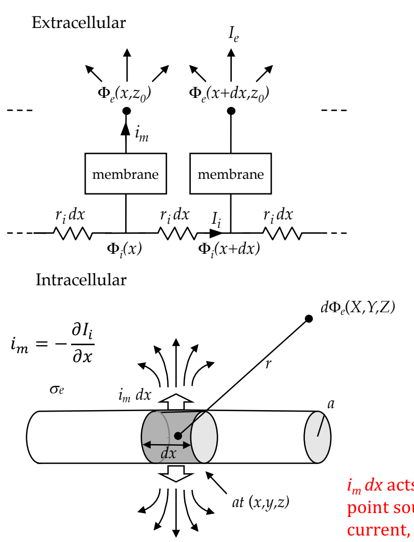
Core conductor model for excitable fiber



Transitioning from a linear cable to a volume conductor model



$i_m\, dx$ acts like a point source of current, $I_0\,\delta(r)$

$$d\Phi_e(X,Y,Z) = \frac{i_m dx}{4\pi\sigma_e r}$$

$$\Phi_e(X,Y,Z) = \frac{1}{4\pi\sigma_e}\int_L \frac{i_m}{r}dx$$

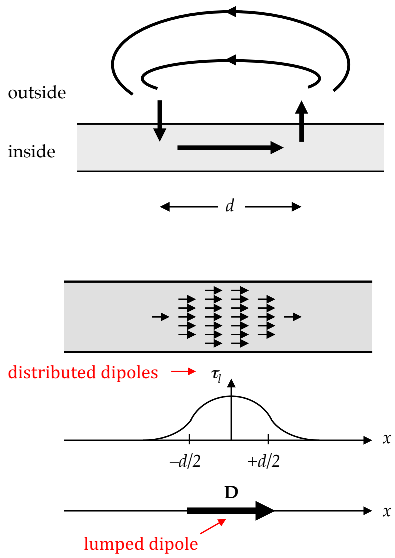
$$i_m = \pi a^2 \sigma_i \frac{\partial^2 \Phi_i}{\partial x^2}$$

When $\sigma_i \approx \sigma_e$

$$i_m \approx \pi a^2 \sigma_i \frac{\partial^2 V_m}{\partial x^2}$$

$$\boxed{\Phi_e(X,Y,Z) = \frac{a^2\sigma_i}{4\sigma_e}\int_L \frac{1}{r}\frac{\partial^2 V_m}{\partial x^2}dx}$$

Current Dipole Source



$$\Phi_e(X,Y,Z) = \frac{1}{4\pi\sigma_e}\int_L \frac{1}{r^2}\tau_l \cos\theta \, dx$$

$$\text{where,} \quad \tau_l = -\pi a^2 \sigma_i \frac{\partial V_m}{\partial x}$$

$$D = \int_{x_1}^{x_2} \tau_l \, dx = -\pi a^2 \sigma_i \int_{x_1}^{x_2} \frac{\partial V_m}{\partial x} dx$$

$$= -\pi a^2 \sigma_i [V_m(x_2) - V_m(x_1)]$$

At the activation wavefront,

$$D = \pi a^2 \sigma_i [V_{peak} - V_{rest}]$$

$$\Phi_e(X,Y,Z) = \frac{D\cos\theta}{4\pi\sigma_e r^2}$$

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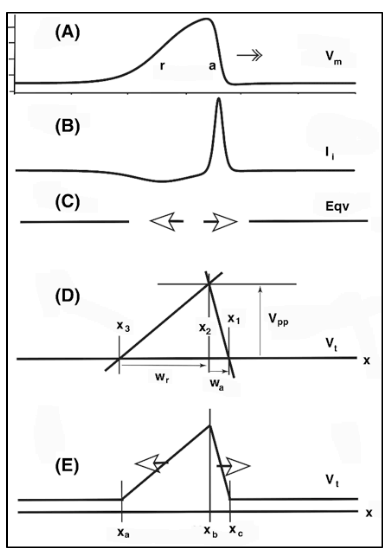
Example – Triangularized Action Potential



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