Name: ______________________________

(Practice) Exam

1. (10 points) State the law of contraposition and (one of) DeMorgan's laws and prove them using a truth table.

$p \to q \equiv \neg q \to \neg p$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q	$p \to q$	$p \vee q$	$\neg q$	$\neg p$	$\neg q \to \neg p$
T	T	T	T	F	F	T
T	F	F	T	T	F	F
F	T	T	T	F	T	T
F	F	T	F	T	T	T

2. (10 points) Determine the truth values of the following statements if the domain consists of all real numbers, **along with a brief justification of your answer**

1. $\forall x\ 2x > 0$

$\forall x\ x > 0$

False e.g $x=0$ (or $x=-1$) is a counterexample

(any negative or zero number too)

2. $\exists x\ 2x > 0$

True $x=1$

3. $\forall x\ 2x^2 > x^2$

$\forall x\ x^2 > 0$

False $x=0$ is (only) counterexample.

$\forall x\ x^2 \geq 0$

True the square of real number is nonnegative.

3. (15 points) Determine the truth values of the following statements if the domain consists of all real numbers, **along with a brief justification of your answer**

   1. $\forall x \forall y \; x > y$

      False   x=0 y=0
      x=1 y=2   are counterexamples
      ⋮

   2. $\forall x \exists y \; x > y$

      True. take y = x−1

   3. $\exists x \forall y \; x > y$

      False. No such such x exists.
      Given any x, we can find a y making x > y false.
      (If x=2, then y=3)

   4. $\exists x \exists y \; x > y$ True. take x=3 y=2

4. (15 points) Make up a conditional statement, and give examples of converse error, and inverse error applied to your conditional.

   p→q   q→p   ¬p→¬q

   For class discussion today, use the conditional I came up with: *If Patrick wins Super Bowl MVP, then KC won the Super Bowl*

   Converse error (affirming the conclusion)
   If Pat won Super Bowl MVP, then KC won SuperBowl
   KC won Super Bowl
   ∴ Pat won SB MVP.

   Converse error
   p→q
   q
   ∴ p

   Inverse error
   If Pat won Super Bowl MVP, then KC won SuperBow
   Pat did not win SB MVP
   ∴ KC did not win Super Bowl.

   Inverse error
   p→q
   ¬p
   ∴ ¬q

5. (10 points) Consider the argument:
   Quincy likes all action movies. Quincy likes the movie Eight Men Out. Therefore, Eight Men Out is an action movie.

Explain whether the argument is logically valid or invalid, using some of the logic terms we have discussed (not all of these will be needed): *modus ponens, modus tollens, conditional, converse, inverse, affirming the conclusion, denying the hypothesis*

rewrite as a conditional

If a movie is an action movie, then Quincy likes it. ($p \to q$)

Quincy likes EMO. q

$\therefore$ EMO is action $\therefore p$

Converse error / affirm the conclusion

6. (10 points) Prove that the ______ (pick one: sum/product/difference) of an ______ integer (pick one: even/odd) and an ______ (pick one: even/odd) integer is an ______ (pick the CORRECT one: even/odd) integer.

Prove difference of even & odd is odd.

Prove: If x is even & y is odd, $x-y$ is odd.

Direct proof. Assume x is even & y is odd.

Goal: $x-y$ is odd.

$x = 2k$ for some int k.

$y = 2l+1$ for some int l.

7. (30 points) Prove the following statement via the three types of proofs we have discussed (direct proof, proof by contraposition, proof by contradiction): *If n is even, then $n+4$ is even*

$$\begin{aligned} x-y &= 2k-(2l+1) \\ &= 2k-2l-1 \\ &= 2(k-l-1)+1 \end{aligned}$$

$$\begin{aligned} 2k-2l-1 &= 2m+1 \\ 2k-2l-1-1 &= 2m \\ k-l-1 &= m \end{aligned}$$

where $m = k-l-1$ is an integer.

goal: $x-y = 2m+1$ ~~for some int m.~~

7. If $\underbrace{n \text{ is even}}_{p}$, then $\underbrace{n+4 \text{ is even}}_{q}$.

$p \rightarrow q$

$\forall$ integers n

~~$n=6$ even $\rightarrow$ $6+4=10$ is even.~~

Direct proof

Assume p

Goal: show q

Assume n is even

Goal: show $n+4$ is even. $n+4 \stackrel{?}{=} 2\ell$

$n = 2k$ for some integer k.

$n+4 = (2k)+4$

~~$n+4 = 2\ell$~~

$n+4 = 2(k+2)$

$\vdots$

$n+4 = 2\ell$ where $\ell = k+2$ is an integer. $\square$

$$\frac{2k+4}{2} = \frac{2\ell}{2}$$

$k+2 = \ell$

Proof by contraposition

Contrapositive of $p \rightarrow q$ is $\neg q \rightarrow \neg p$

Assume $\neg q$

Goal: show $\neg p$

Assume $n+4$ is odd. (not even)

Goal: show n is odd (not even)

$n+4 = 2k+1$ for some integer k.

$n = 2k+1-4$

$n = 2k-3$

$n = 2(k-2)+1$

$n = 2\ell+1$ (goal)

where $\ell = k-2$, which is an integer $\square$

want $2k-3 = 2\ell+1$

what should ℓ be?

Solve this equation for ℓ.

$2k-3-1 = 2\ell$

$2k-4 = 2\ell$

$k-2 = \ell$

Proof by contradiction

Assume

$\neg(p \rightarrow q) \equiv p \wedge \neg q$

Assume p AND $\neg q$

Goal: get a contradiction.

Assume n is even AND $n+4$ is odd.

$n = 2k$ for some integer k.

$n+4 = 2\cancel{k}+1$

$n+4 = 2\ell+1$ for some integer ℓ.

$\downarrow$

$(2k)+4 = 2\ell+1$

$3 = 2\ell - 2k$

$3 = 2(\ell - k)$

LHS is odd

RHS is even.

contradiction $\square$