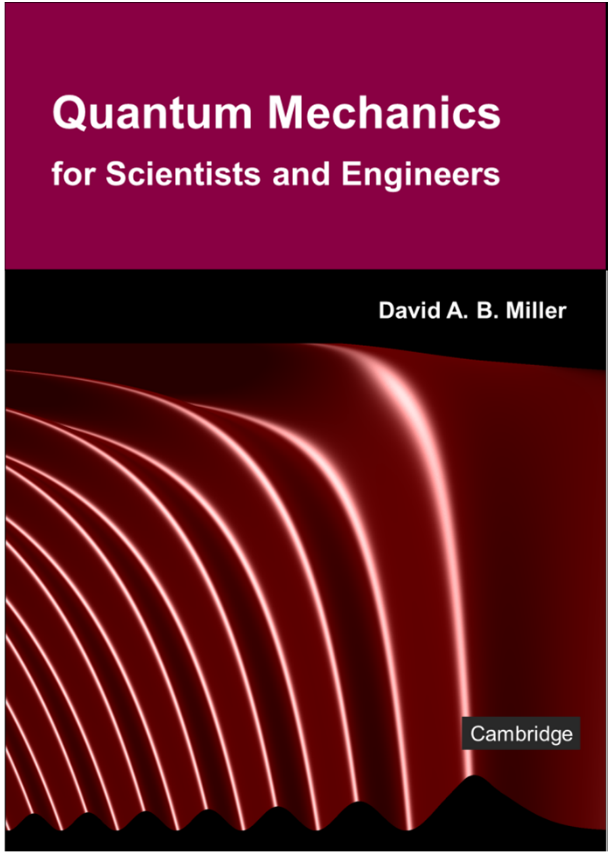
6.3 Operators and quantum mechanics

Slides: Video 6.3.5 Specific uncertainty principles

Text reference: Quantum Mechanics for Scientists and Engineers

Section 5.2 (starting from “Position-momentum uncertainty principle”)



Operators and quantum mechanics

Specific uncertainty principles

Quantum mechanics for scientists and engineers

David Miller

Position-momentum uncertainty principle

We now formally derive the position-momentum relation

Consider the commutator of $\hat{p}_x$ and x

(We treat the function x as the operator for position)

To be sure we are taking derivatives correctly

we have the commutator operate on an arbitrary function

$$\left[\hat{p}_x, x\right]|f\rangle = -i\hbar\left(\frac{d}{dx}x - x\frac{d}{dx}\right)|f\rangle = -i\hbar\left\{\frac{d}{dx}\left(x|f\rangle\right) - x\frac{d}{dx}|f\rangle\right\}$$

$$= -i\hbar\left\{|f\rangle + x\frac{d}{dx}|f\rangle - x\frac{d}{dx}|f\rangle\right\} = -i\hbar|f\rangle$$

Position-momentum uncertainty principle

In $\left[\hat{p}_x, x\right] \left| f \right\rangle = -i\hbar \left| f \right\rangle$

since $\left| f \right\rangle$ is arbitrary

we can write $\left[\hat{p}_x, x\right] = -i\hbar$

and the commutation rest operator $\hat{C}$

is simply the number $\hat{C} = -\hbar$

Hence $\bar{C} = -\hbar$

and so, from $\Delta A \Delta B \geq \left|\bar{C}\right| / 2$

we have $\Delta p_x \Delta x \geq \dfrac{\hbar}{2}$

Energy-time uncertainty principle

The energy operator is the Hamiltonian $\hat{H}$

and from Schrödinger's equation $\hat{H}|\psi\rangle = i\hbar \frac{\partial}{\partial t}|\psi\rangle$

so we use $\hat{H} \equiv i\hbar\partial / \partial t$

If we take the time operator to be just t

then using essentially identical algebra as used for the momentum-position uncertainty principle

$$\left[\hat{H}, t\right] = i\hbar\left(\frac{\partial}{\partial t}t - t\frac{\partial}{\partial t}\right) = i\hbar$$

so, similarly we have

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Frequency-time uncertainty principle

We can relate this result mathematically to

the frequency-time uncertainty principle

that occurs in Fourier analysis

Noting that $E = \hbar\omega$ in quantum mechanics

we have

$$\Delta\omega\Delta t \geq \frac{1}{2}$$