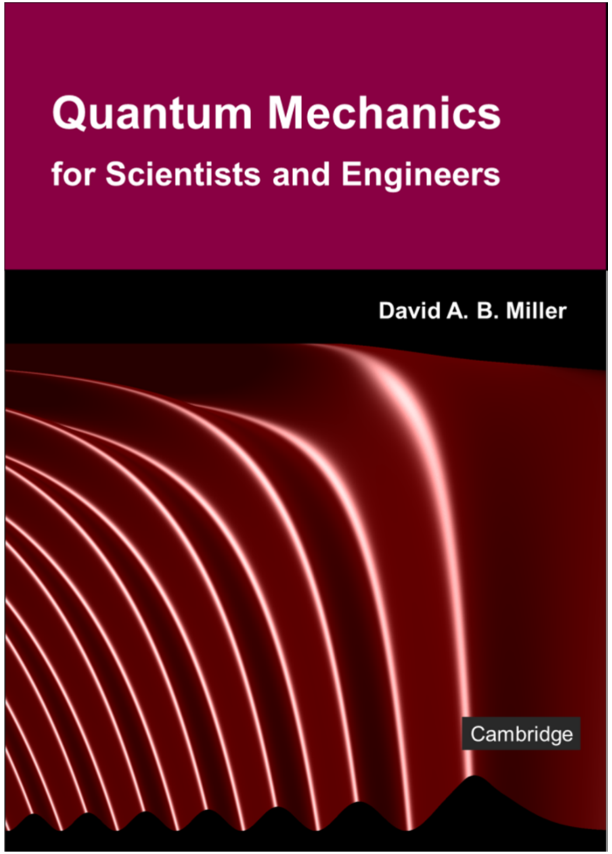
# 7.2 The L squared operator

Slides: Video 7.2.3 Visualizing spherical harmonics

Text reference: Quantum Mechanics for Scientists and Engineers

Section 9.3



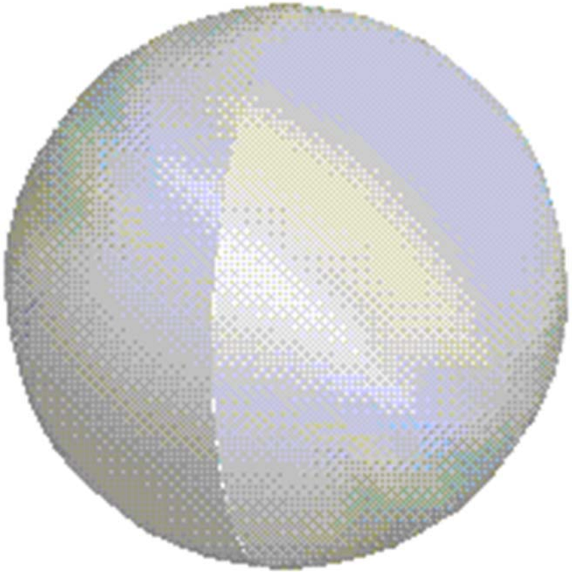
# The L squared operator

## Visualizing spherical harmonics

Quantum mechanics for scientists and engineers

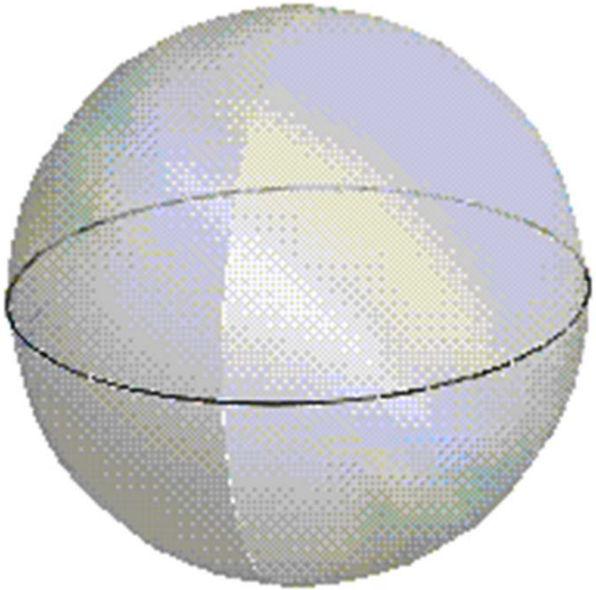
David Miller

# Oscillating modes for spherical shell



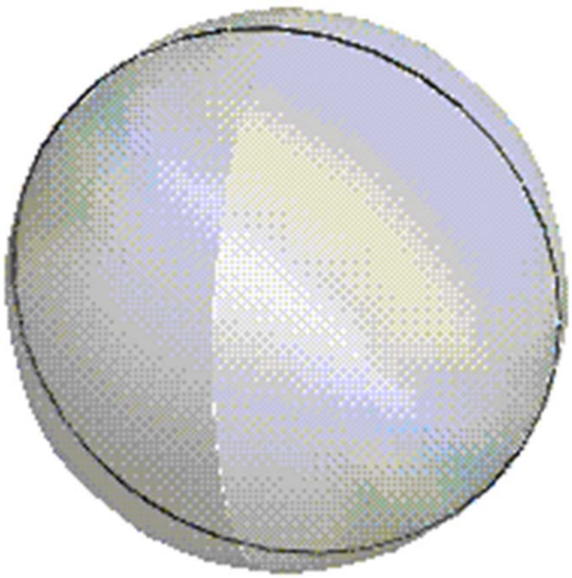
$l = 0$
$m = 0$

# Oscillating modes for spherical shell



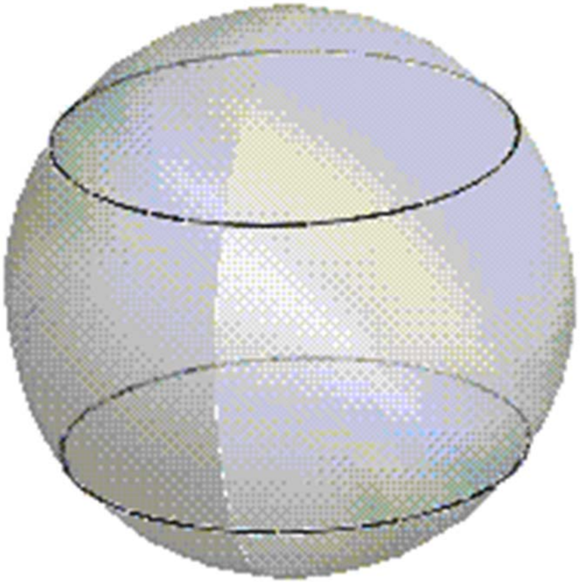
$l = 1$
$m = 0$

# Oscillating modes for spherical shell



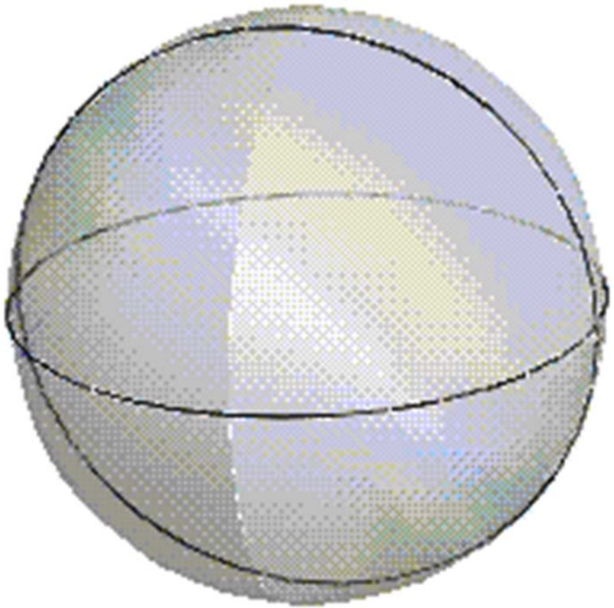
$l = 1$
$m = 1$

# Oscillating modes for spherical shell



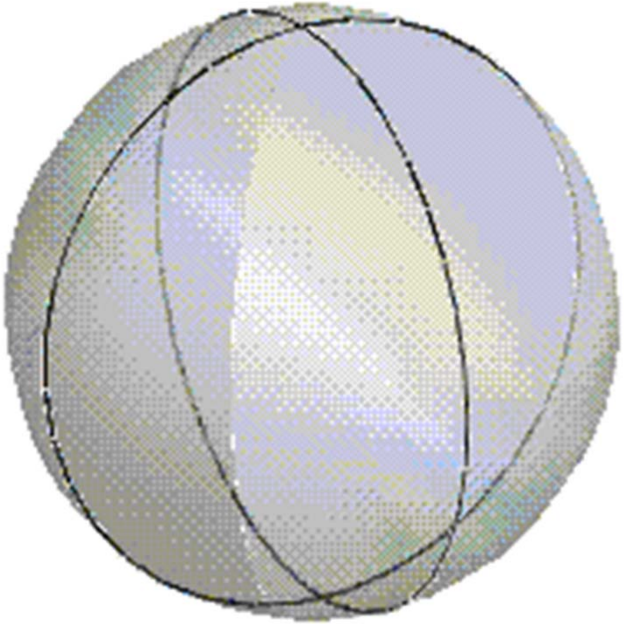
$l = 2$
$m = 0$

# Oscillating modes for spherical shell



$l = 2$
$m = 1$

# Oscillating modes for spherical shell



$l = 2$
$m = 2$

# Constructing spherical harmonics for a shell

The lowest solution

$$l = 0,\ m = 0$$

- is the "breathing" mode
  - The spherical shell expands and contracts periodically

For all other solutions

- there are one or more nodal circles on the sphere
  - A nodal circle is one that is unchanged in that particular oscillating mode

# Constructing spherical harmonics for a shell

Note the following rules for the spherical shell modes

- the surfaces on opposite sides of a nodal circle oscillate in opposite directions
- the total number of nodal circles is equal to $l$
- the number of nodal circles passing through the poles is $m$, and they divide the sphere equally in the azimuthal angle $\phi$
- the remaining nodal circles are either equatorial or parallel to the equator

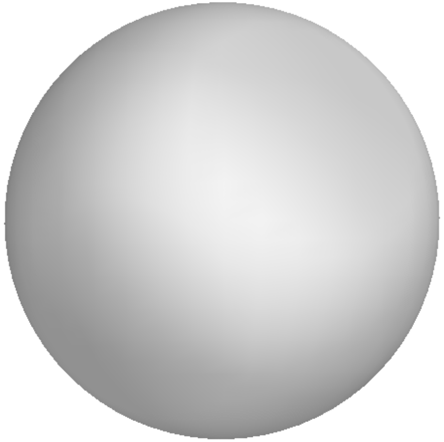
  symmetrically distributed between the top and bottom halves of the sphere

# Spherical harmonics

We can formally also plot the spherical harmonic in a parametric plot

where the distance from the center at a given angle

represents the magnitude of amplitude of the spherical harmonic



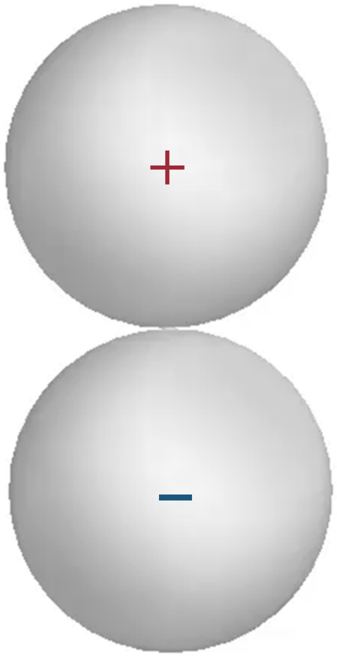
$l = 0$
$m = 0$

# Spherical harmonics

We can formally also plot the spherical harmonic in a parametric plot

- where the distance from the center at a given angle
  - represents the magnitude of amplitude of the spherical harmonic
    - Adjacent "lobes" have opposite signs

$$l = 1$$
$$m = 0$$



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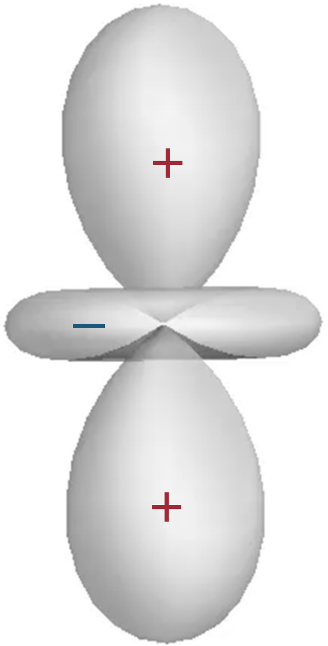
$l = 2$
$m = 0$



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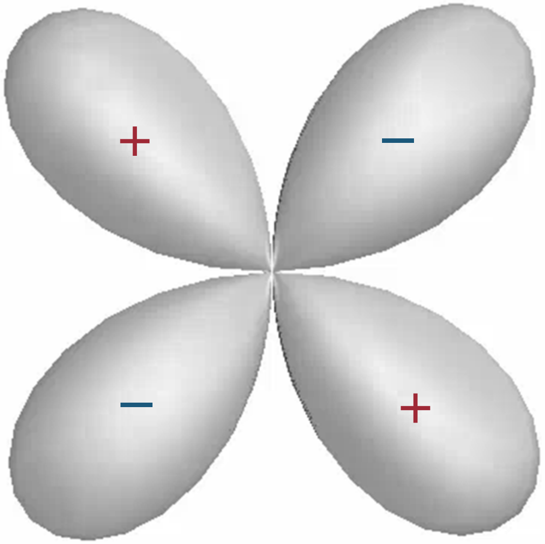


$l = 2$
$m = 1$