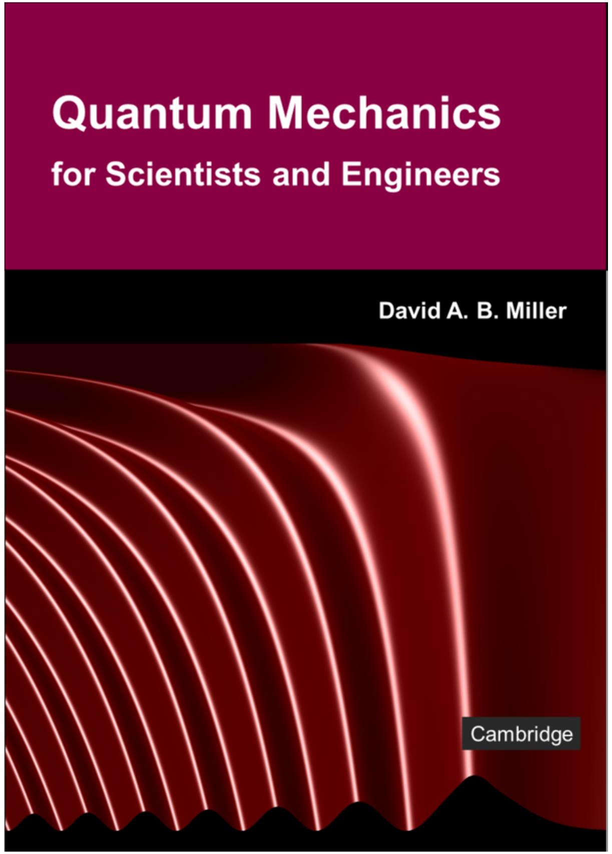
# 4.3 Measurement and expectation values

Slides: Video 4.3.1 Quantum-mechanical measurement

Text reference: Quantum Mechanics for Scientists and Engineers

Section 3.8



# Measurement and expectation values

## Quantum-mechanical measurement

Quantum mechanics for scientists and engineers

David Miller

# Probabilities and expansion coefficients

Suppose we take some (normalized) quantum mechanical wave function $\Psi(\mathbf{r},t)$

and expand it in some complete orthonormal set of spatial functions $\psi_n(\mathbf{r})$

At least if we allow the expansion coefficients $c_n$ to vary in time

we know we can always do this

$$\Psi(\mathbf{r},t) = \sum_n c_n(t)\psi_n(\mathbf{r})$$

# Probabilities and expansion coefficients

Then the fact that $\Psi(\mathbf{r},t)$ is normalized means

we know the answer for the normalization integral

$$\int_{-\infty}^{\infty} \left|\Psi(\mathbf{r},t)\right|^2 d^3\mathbf{r} = \int_{-\infty}^{\infty} \left[\sum_n c_n^*(t)\psi_n^*(\mathbf{r})\right] \times \left[\sum_m c_m(t)\psi_m(\mathbf{r})\right] d^3\mathbf{r} = 1$$

Because of the orthogonality of the basis functions

only terms with $n = m$ survive the integration

Because of the orthonormality of the basis functions

the result from any such term will simply be $\left|c_n(t)\right|^2$

Hence we have $\sum_n \left|c_n\right|^2 = 1$

# Measurement postulate

On measurement of a state

the system collapses into the $n$th eigenstate of the quantity being measured

with probability $P_n = |c_n|^2$

In the expansion of the state

in the eigenfunctions

of the quantity being measured

$c_n$ is the expansion coefficient

of the $n$th eigenfunction

# Expectation value of the energy

Suppose do an experiment to measure the energy $E$ of some quantum mechanical system

We could repeat the experiment many times

and get a statistical distribution of results

Given the probabilities $P_n$ of getting a specific energy eigenstate, with energy $E_n$

we would get an average answer

$$\langle E \rangle = \sum_n E_n P_n = \sum_n E_n |c_n|^2$$

where we call this average $\langle E \rangle$ the "**expectation value**"

# Energy expectation value example

For example, for the coherent state discussed above with parameter $N$, we have

$$\langle E \rangle = \sum_{n=0}^{\infty} E_n \frac{N^n \exp(-N)}{n!} = \hbar\omega \left[ \sum_{n=0}^{\infty} n \frac{N^n \exp(-N)}{n!} \right] + \frac{1}{2}\hbar\omega = \left( N + \frac{1}{2} \right)\hbar\omega$$

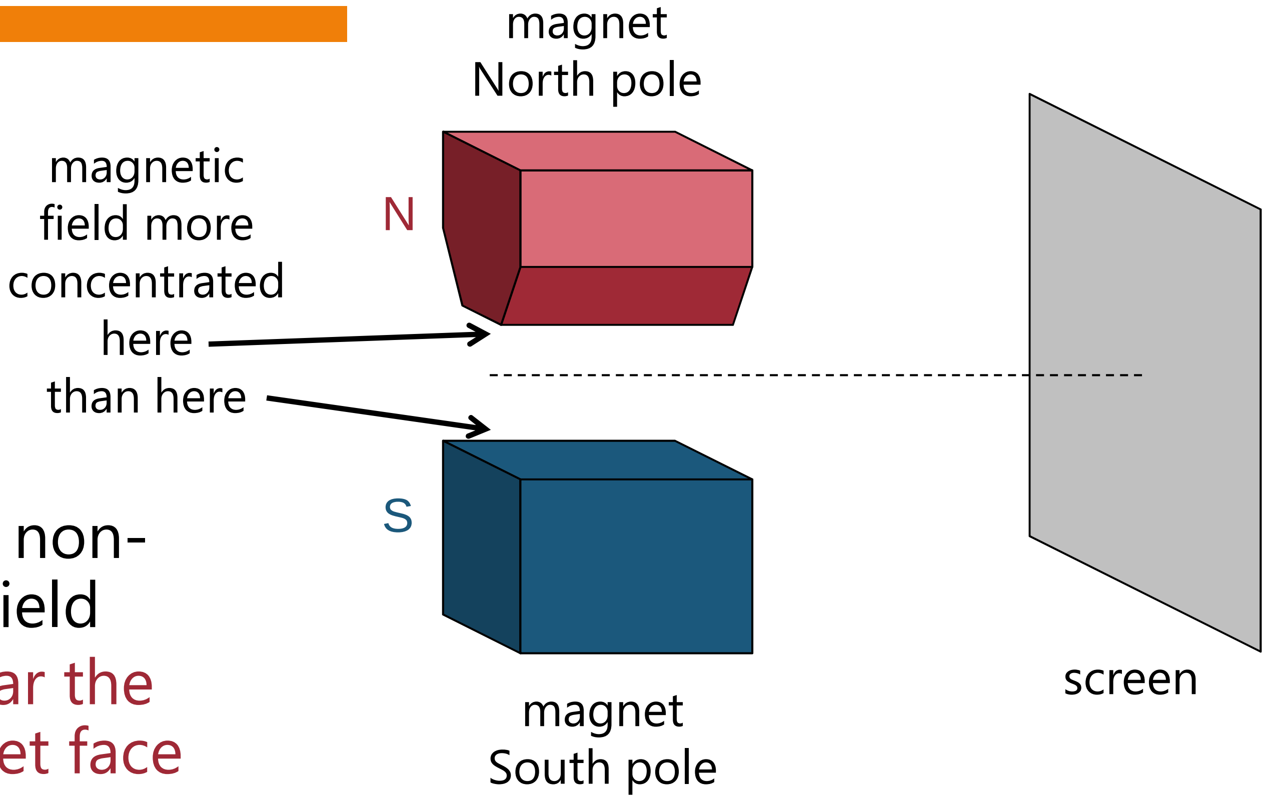
where we use the result that the average in a Poisson statistical distribution is just the parameter $N$

Note that N does not have to be an integer

This is an expectation value, not an eigenvalue

We can have states with any expectation value we want

# Stern-Gerlach experiment




This apparatus has a non-uniform magnetic field

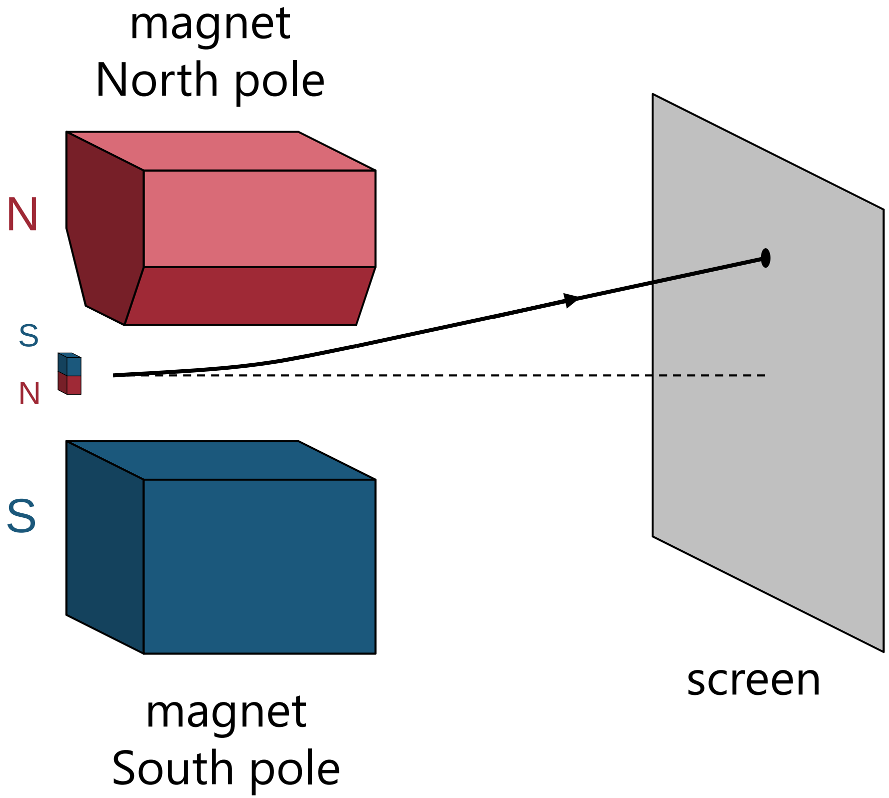
locally stronger near the North pole magnet face

because it is narrower

# Stern-Gerlach experiment

Imagine firing some small magnets

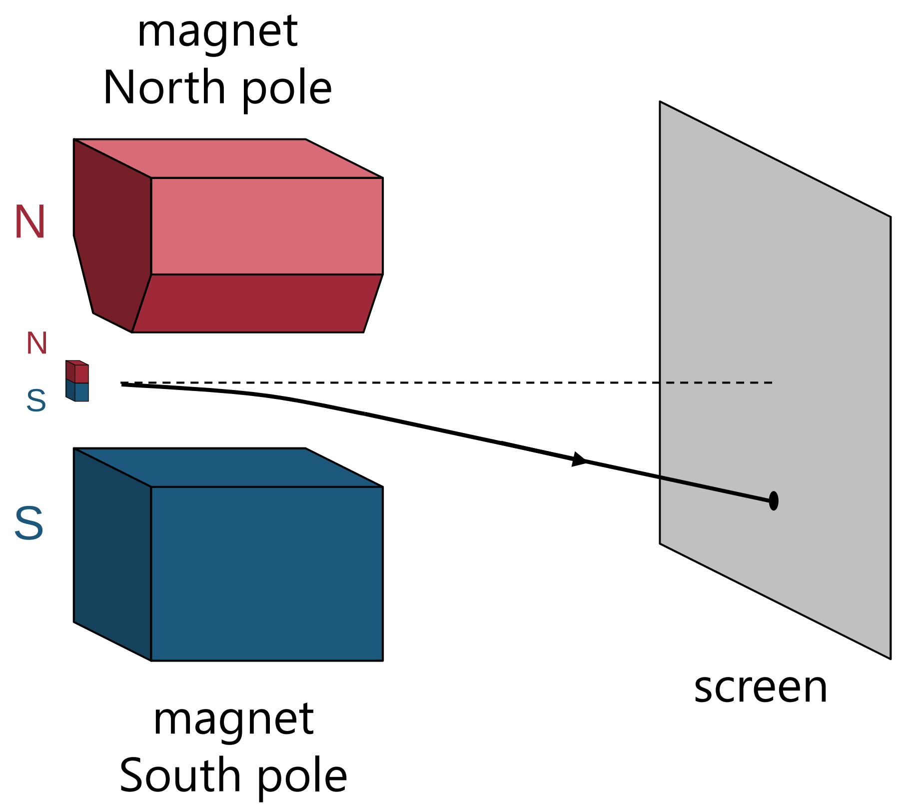
- initially along the dashed line
  - Because the magnetic field is non-uniform
    - stronger near the North pole than near the South pole
      - a vertical magnet will be deflected up



# Stern-Gerlach experiment

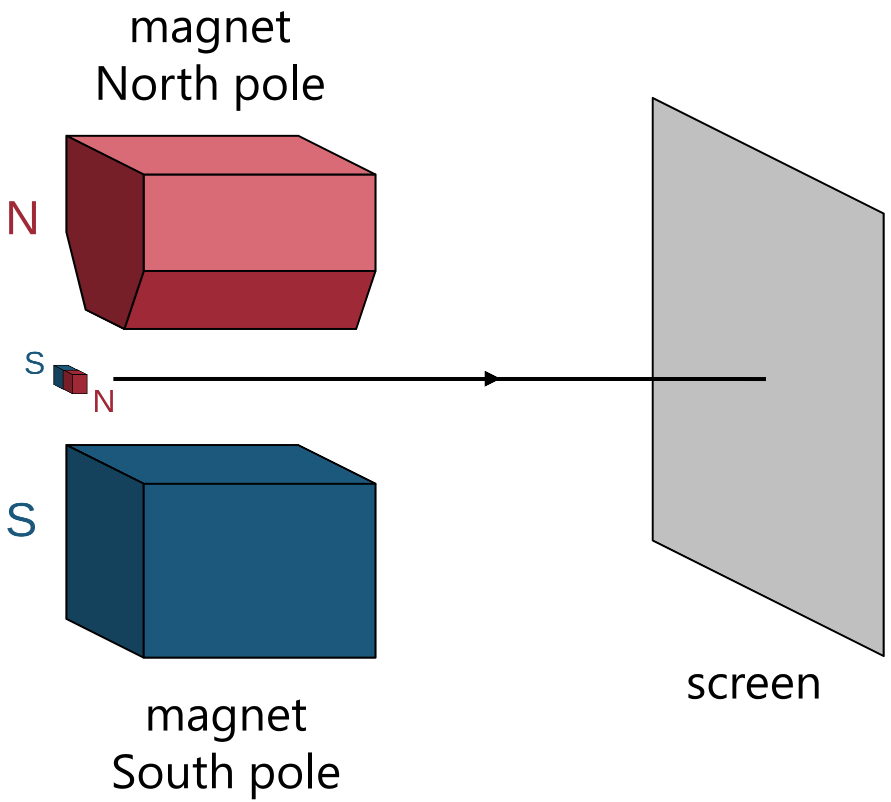
Imagine firing some small magnets

- initially along the dashed line
  - Because the magnetic field is non-uniform
    - stronger near the North pole than near the South pole
      - a vertical magnet will be deflected up or down



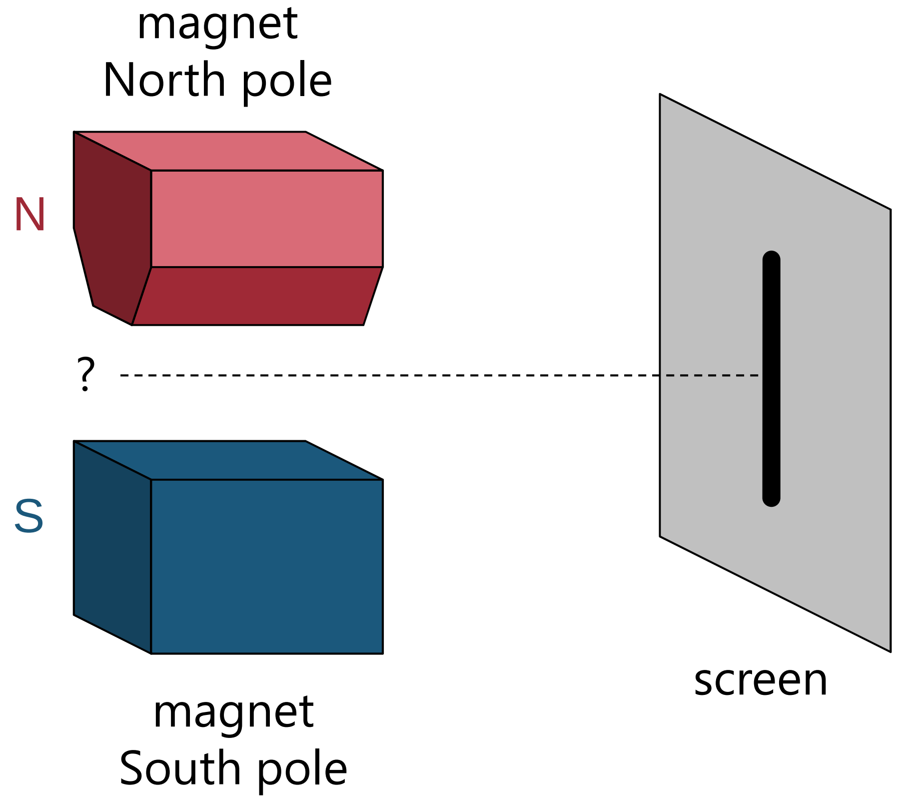
# Stern-Gerlach experiment

A horizontally-oriented magnet will not be deflected



# Stern-Gerlach experiment

- A horizontally-oriented magnet will not be deflected
  - and magnets of other orientations
    - should be deflected by intermediate amounts
- After "firing" many randomly oriented magnets
  - we should end up with a line on the screen



# Electrons and the Stern-Gerlach experiment

- Electrons have a quantum mechanical property called spin
  - It gives them a “magnetic moment”
    - just like a small magnet
- What will happen if we fire electrons
  - with no particular “orientation” of their spin
    - into the Stern-Gerlach apparatus?
      - We might expect the “line” on the screen
- (Note: the actual experiment used silver atoms, which behave the same as electrons in this case)

# Stern-Gerlach experiment

With electrons

- we get two dots!

"Explanation"

- We are measuring the vertical component of the spin
  - There are two eigenstates of this component
    - up and down
      - so we have collapse to the eigenstates