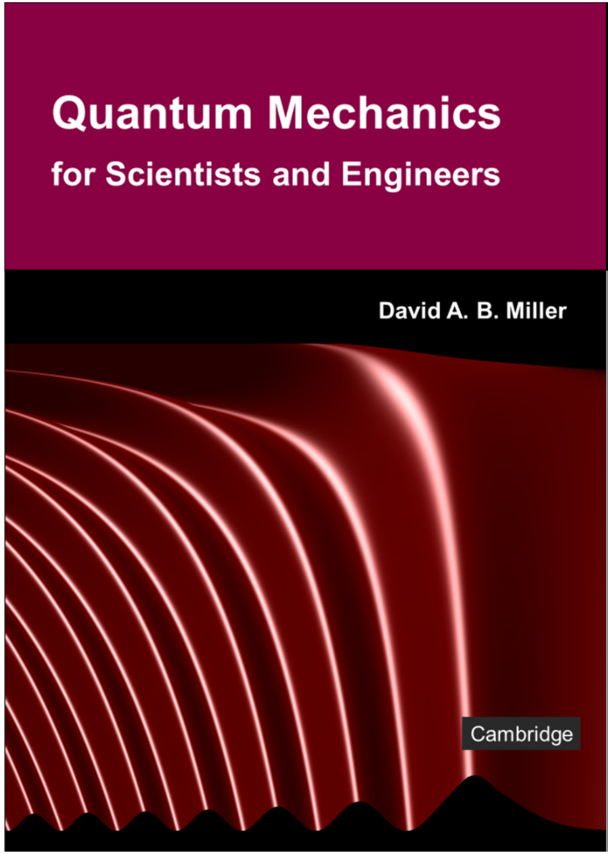
4.1 Time evolution of superpositions

Slides: Video 4.1.6 The coherent state

Text reference: Quantum Mechanics for Scientists and Engineers

Section 3.6 (“Coherent state”)



Time evolution of superpositions

The coherent state

Quantum mechanics for scientists and engineers

David Miller

The coherent state

The coherent state for a harmonic oscillator of frequency ω is

$$\Psi_N(\xi,t) = \sum_{n=0}^{\infty} c_{Nn} \exp\left[-i\left(n+\frac{1}{2}\right)\omega t\right]\psi_n(\xi)$$

where

$$c_{Nn} = \sqrt{\frac{N^n \exp(-N)}{n!}}$$

and the $\psi_n(\xi)$ are the harmonic oscillator eigenstates

The coherent state

Incidentally, note that for the expansion coefficients c_{Nn}

$$\left|c_{Nn}\right|^2 = \frac{N^n \exp(-N)}{n!}$$

This is the Poisson distribution from statistics

with mean N and standard deviation $\sqrt{N}$

We will make no direct use of this here

but in the end it explains, e.g., the Poissonian distribution of photons in a laser beam

Coherent state

Coherent state oscillations

with

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$$c_{Nn} = \sqrt{\frac{N^n \exp(-N)}{n!}}$$

Coherent state

Coherent state oscillations

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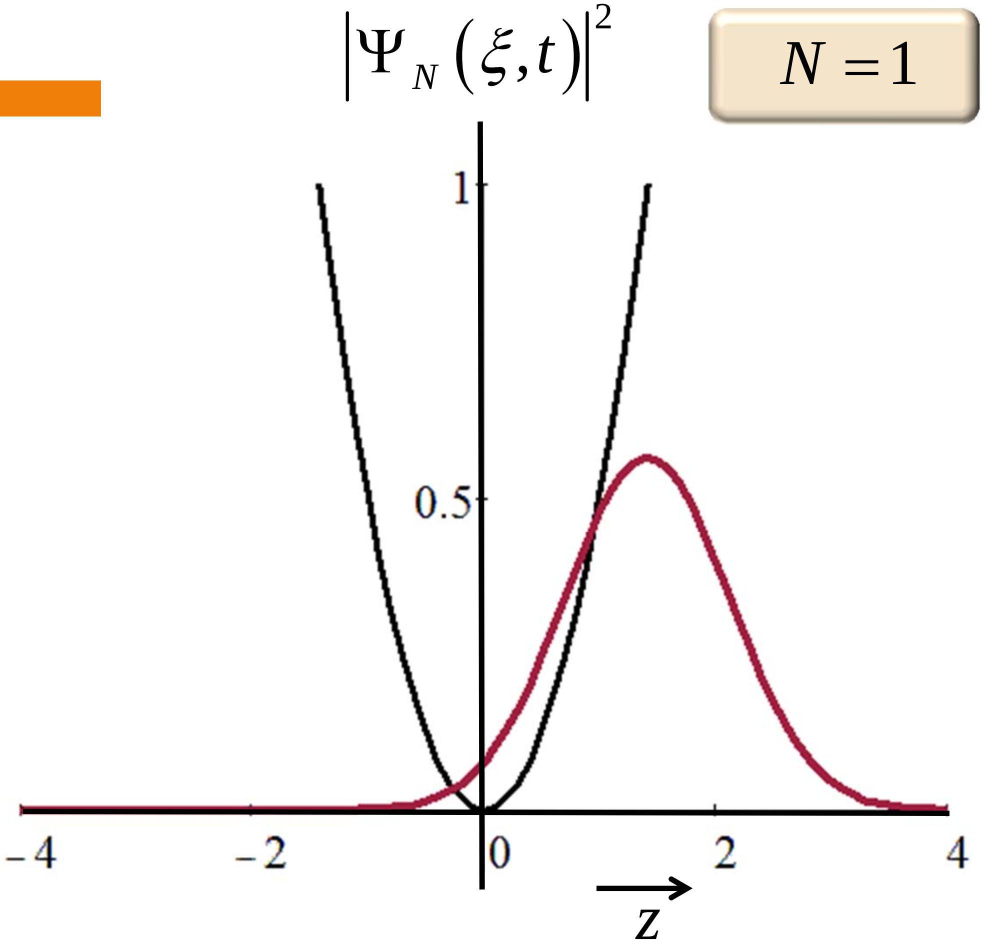
$N = 1$



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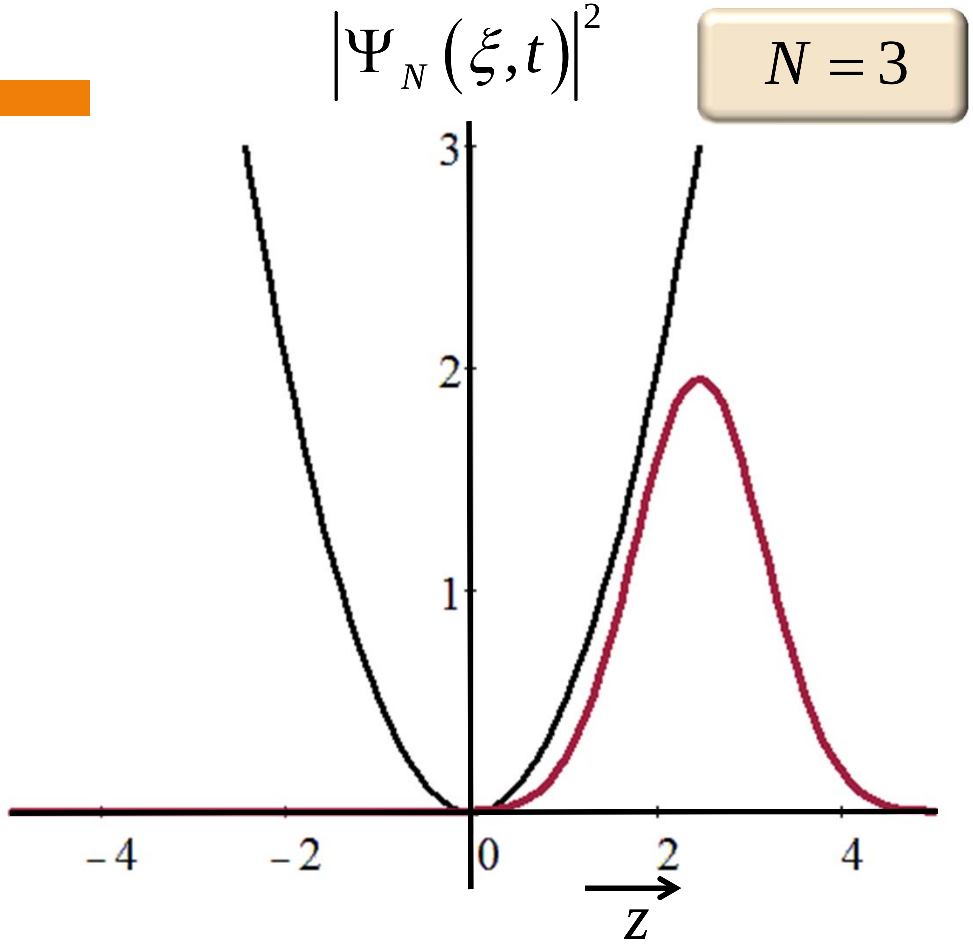

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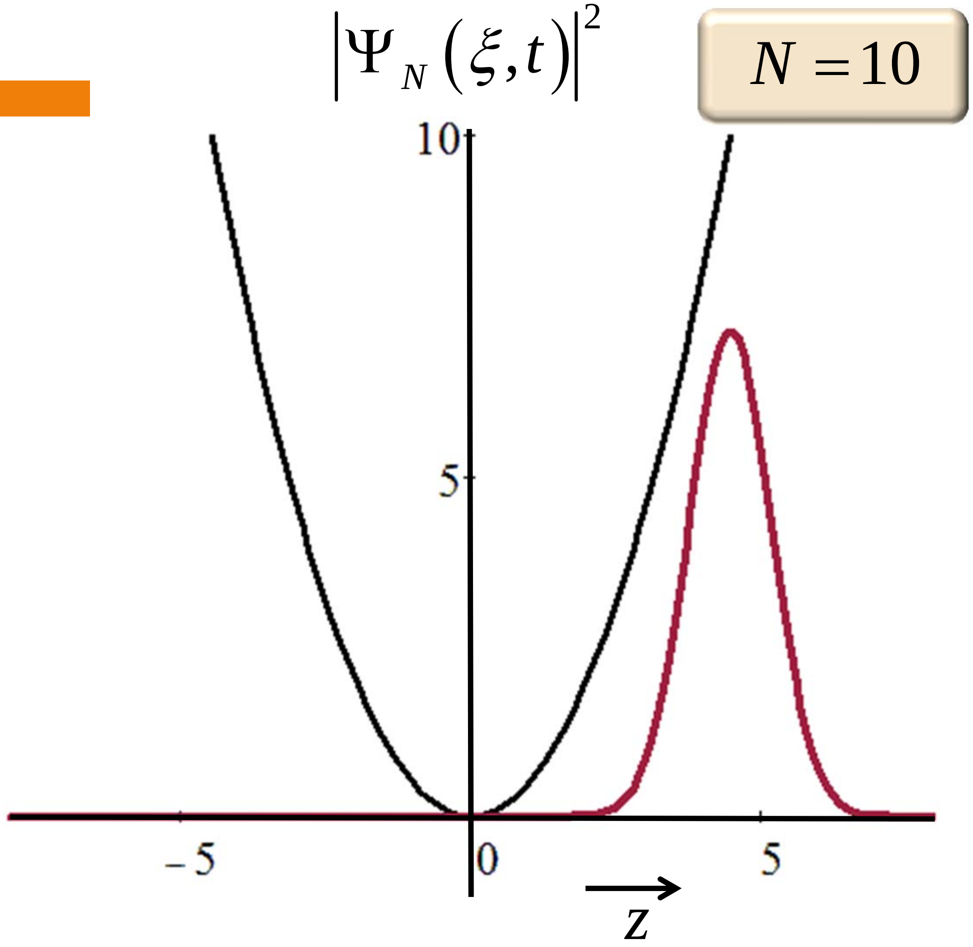

Coherent state

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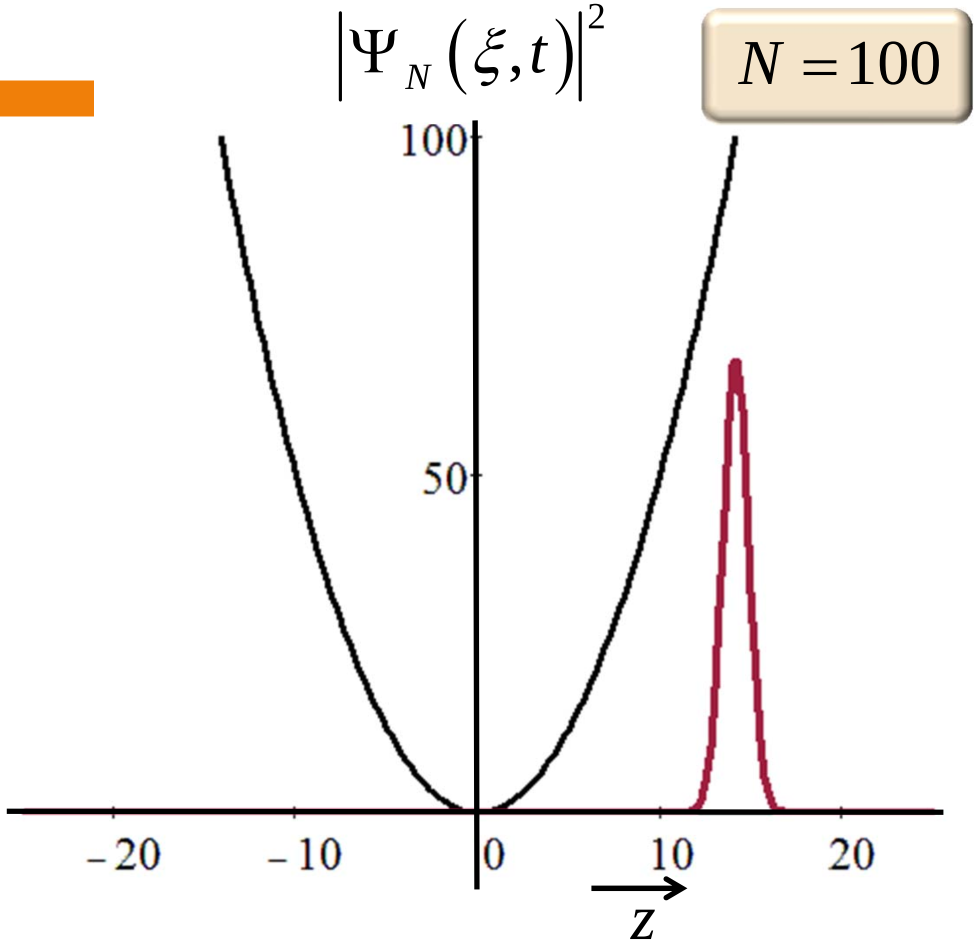


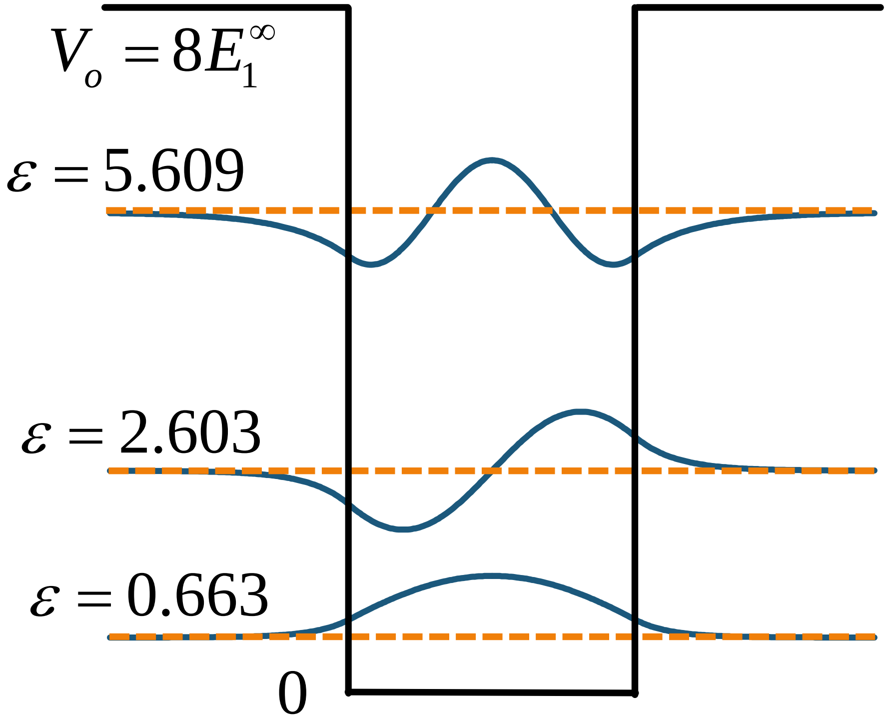
Finite well superposition

Make an equal superposition of the first three states of a finite potential well

as in our previous example

Because the energies are not rationally related

the superposition never repeats



Finite well superposition

Make an equal superposition of the first three states of a finite potential well

as in our previous example

Because the energies are not rationally related

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e.g., in the probability density in time