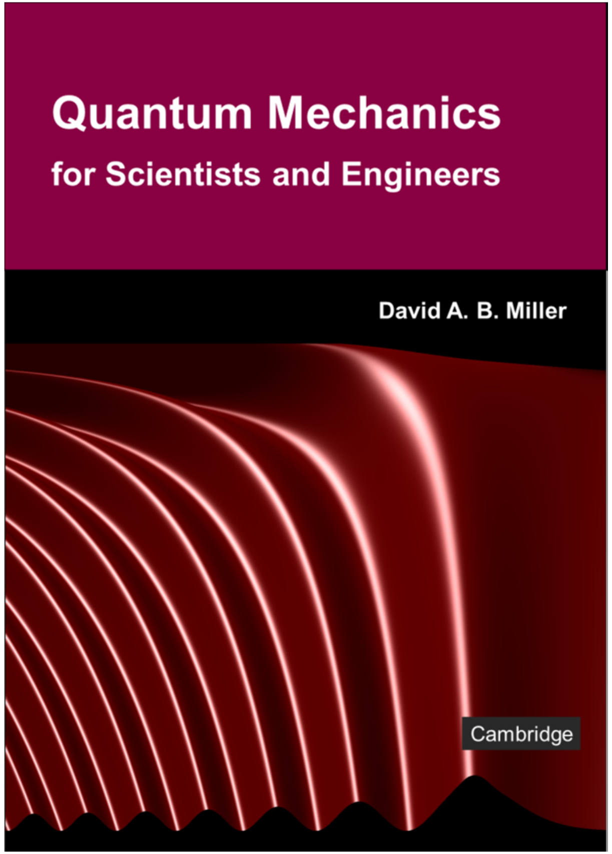
4.1 Time evolution of superpositions

Slides: Video 4.1.4 Superposition for the harmonic oscillator

Text reference: Quantum Mechanics for Scientists and Engineers

Section 3.6 (“Harmonic oscillator example”)



Time evolution of superpositions

Superposition for the harmonic oscillator

Quantum mechanics for scientists and engineers

David Miller

Superpositions and oscillation

Quite generally

if we make a linear combination of two energy eigenstates

with energies E_a and E_b

the resulting probability distribution will oscillate at the (angular) frequency

$$\omega_{ab} = \left|E_a - E_b\right| / \hbar$$

Superpositions and oscillation

So, if we have a superposition wavefunction

$$\Psi_{ab}(\mathbf{r},t) = c_a \exp\left(-i\frac{E_a}{\hbar}t\right)\psi_a(\mathbf{r}) + c_b \exp\left(-i\frac{E_b}{\hbar}t\right)\psi_b(\mathbf{r})$$

then the probability distribution will be

$$\left|\Psi_{ab}(\mathbf{r},t)\right|^2 = \left|c_a\right|^2 \left|\psi_a(\mathbf{r})\right|^2 + \left|c_b\right|^2 \left|\psi_b(\mathbf{r})\right|^2$$

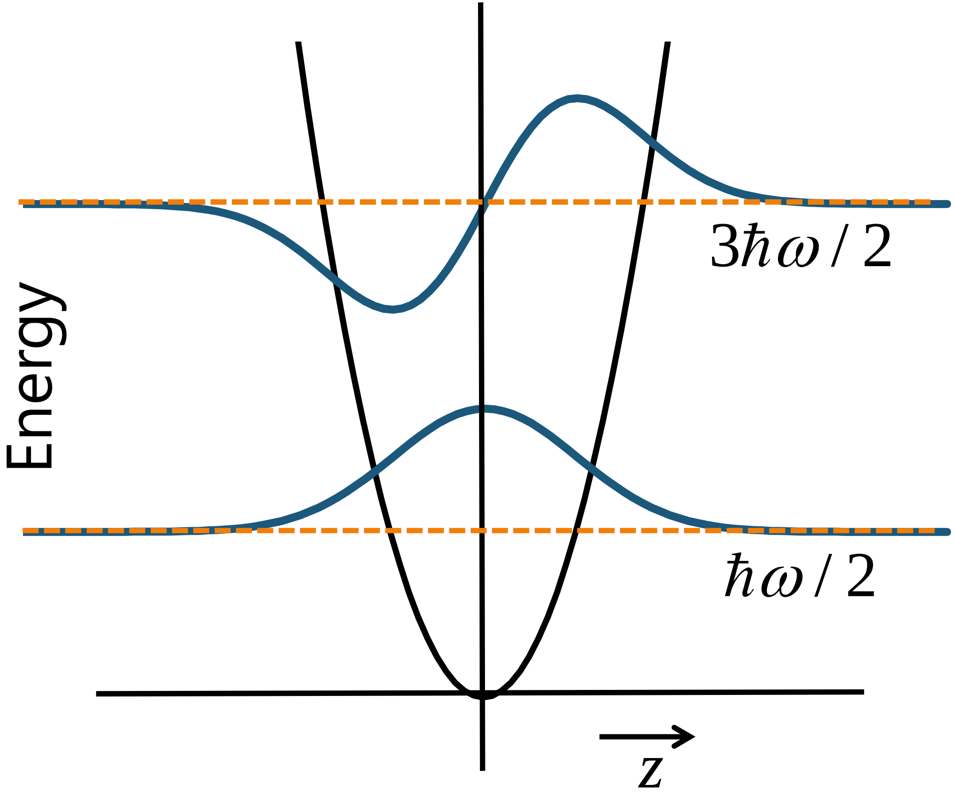
$$+2\left|c_a^*\psi_a^*(\mathbf{r})c_b\psi_b(\mathbf{r})\right|\cos\left[\frac{(E_a - E_b)t}{\hbar} - \theta_{ab}\right]$$

where $\theta_{ab} = \arg\left(c_a\psi_a(\mathbf{r})c_b^*\psi_b^*(\mathbf{r})\right)$

Harmonic oscillator

As a reminder

- here are the first two harmonic energy levels
- and their associated wavefunctions
- plotted with the orange dashed lines as horizontal axes



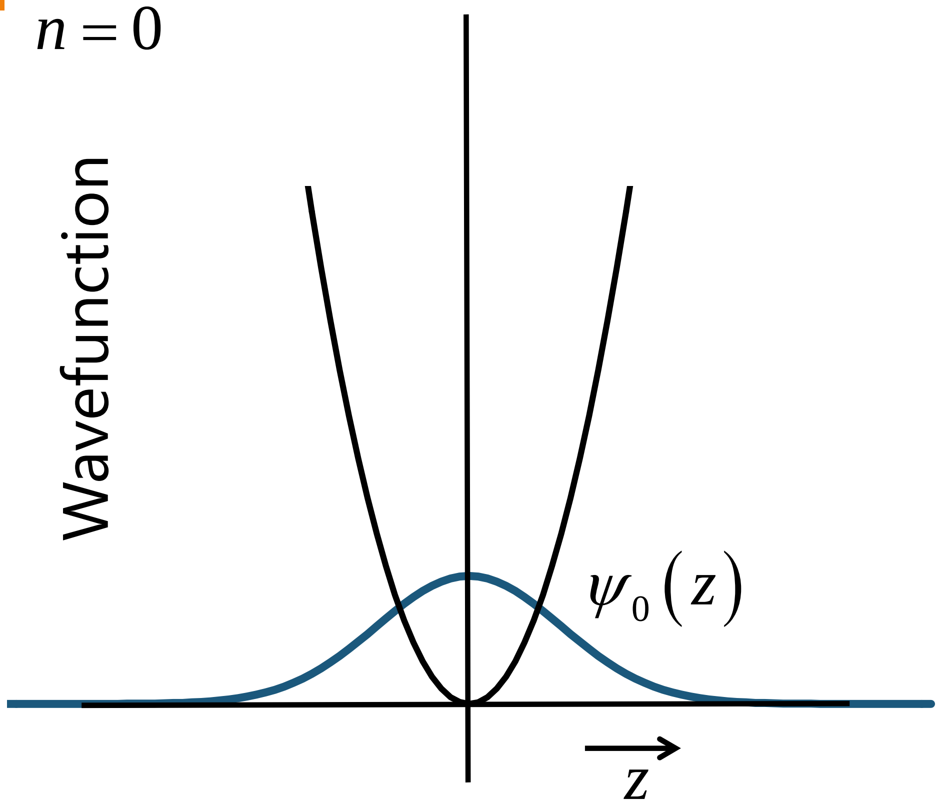
Superposition

The $n = 0$ spatial eigenfunction

$\psi_0(z)$

is plotted here

with the bottom of the parabolic well as its horizontal axis



Superposition

For the probability density

$$|\psi_0(z)|^2$$

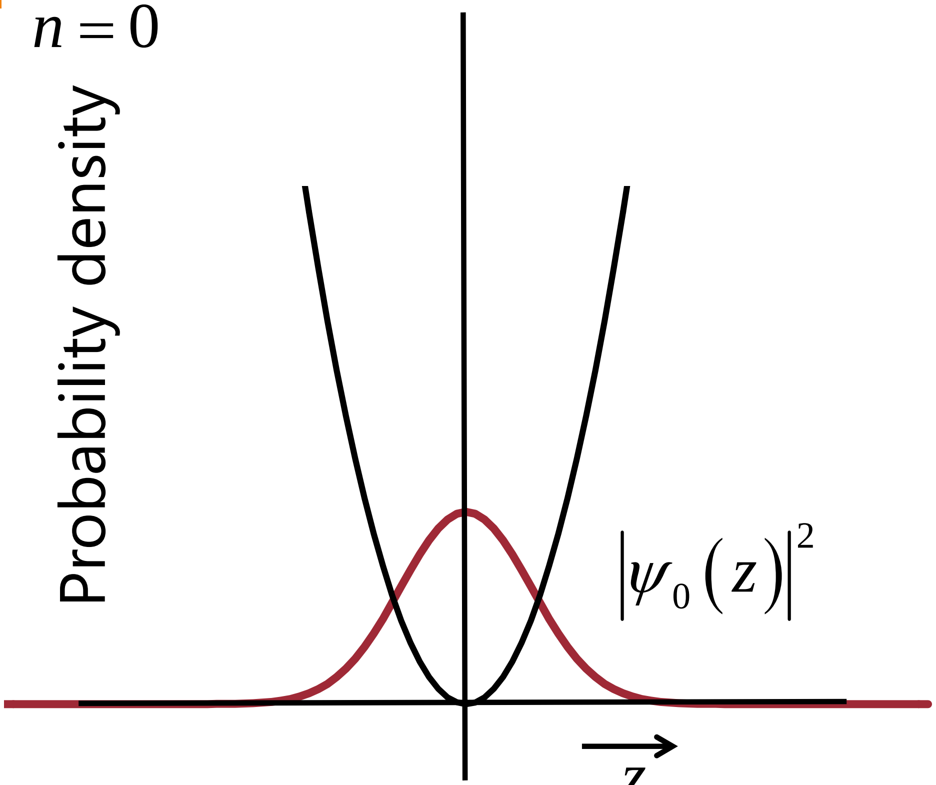
note the narrower shape

Multiplying $\psi_0(z)$ by the time dependent factor gives

$$\Psi_0(z,t) = \exp\left(-i\frac{E_0}{\hbar}t\right)\psi_0(z)$$

The probability densities are the same

$$|\Psi_0(z,t)|^2 = |\psi_0(z)|^2$$



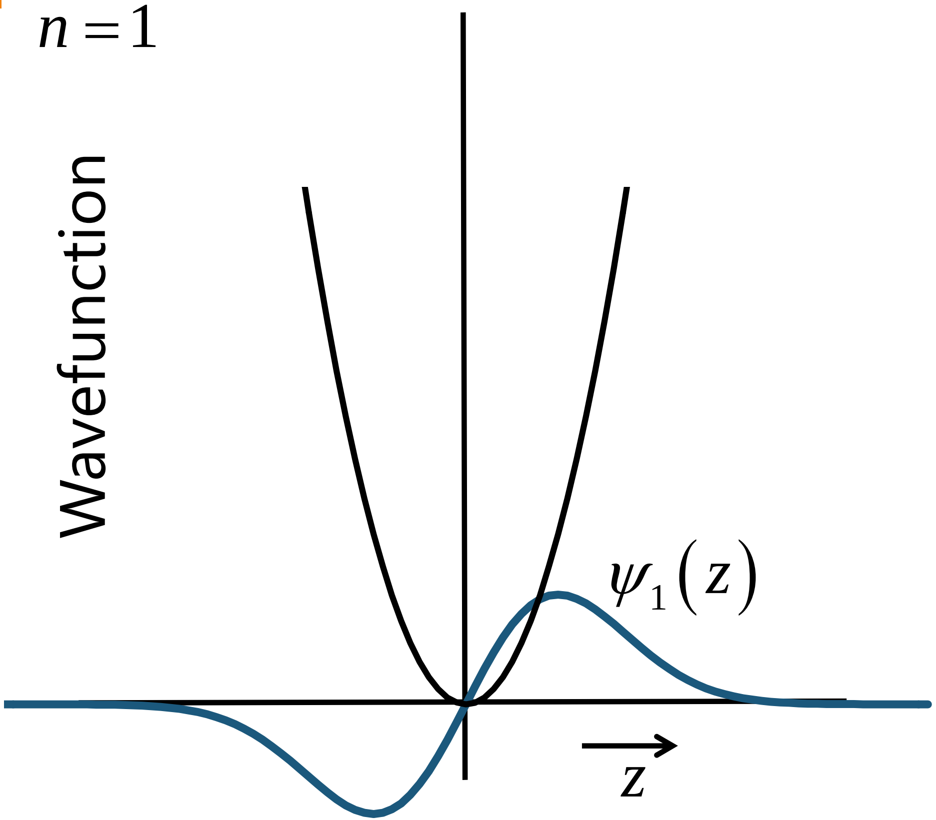
Superposition

The $n = 1$ spatial eigenfunction

$\psi_1(z)$

is plotted here

with the bottom of the parabolic well as its horizontal axis



Superposition

For the probability density

$$\left|\psi_1(z)\right|^2$$

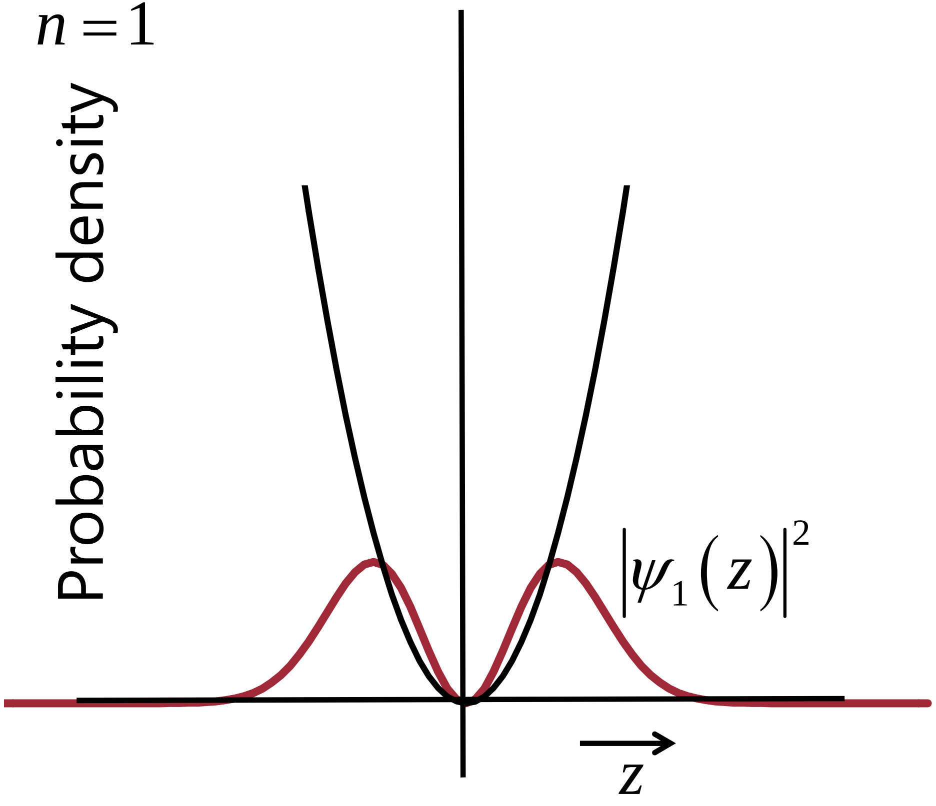
note it is positive

Multiplying by the time dependent factor gives

$$\Psi_1(z,t) = \exp\left(-i\frac{E_1}{\hbar}t\right)\psi_1(z)$$

The probability densities are the same

$$\left|\Psi_1(z,t)\right|^2 = \left|\psi_1(z)\right|^2$$



Superposition

An equal superposition of the two oscillates

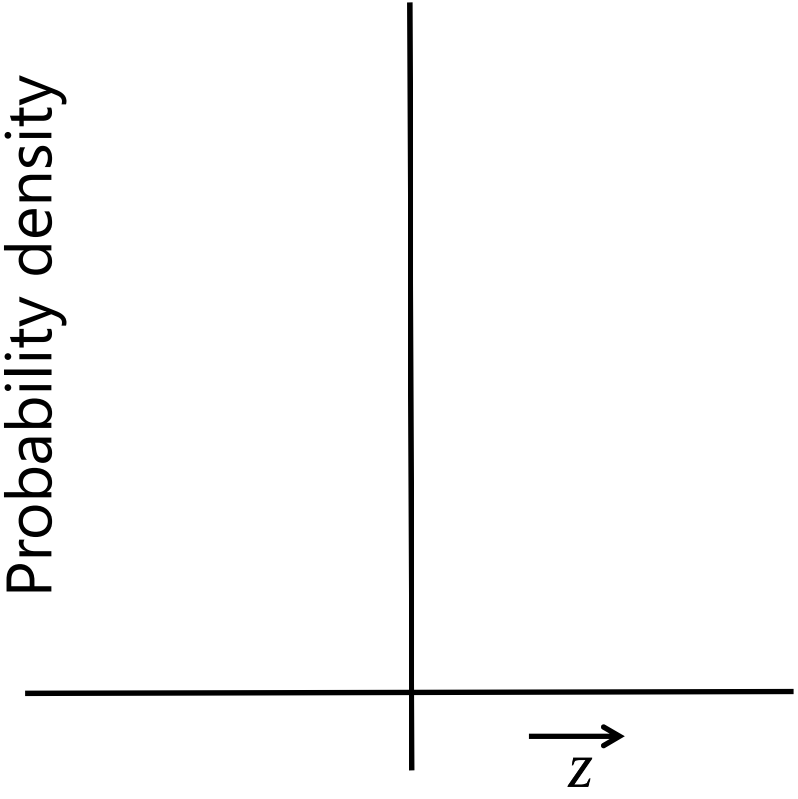
at the angular frequency

$$\omega = \left(E_1 - E_0\right)/\hbar$$

$$\left|\Psi\left(z,t\right)\right|^2 = \left|\Psi_0\left(z,t\right) + \Psi_1\left(z,t\right)\right|^2$$

$$= \left|\psi_0\left(z\right)\right|^2 + \left|\psi_1\left(z\right)\right|^2$$

$$+2\cos\left(\omega t\right)\psi_0\left(z\right)\psi_1\left(z\right)$$



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