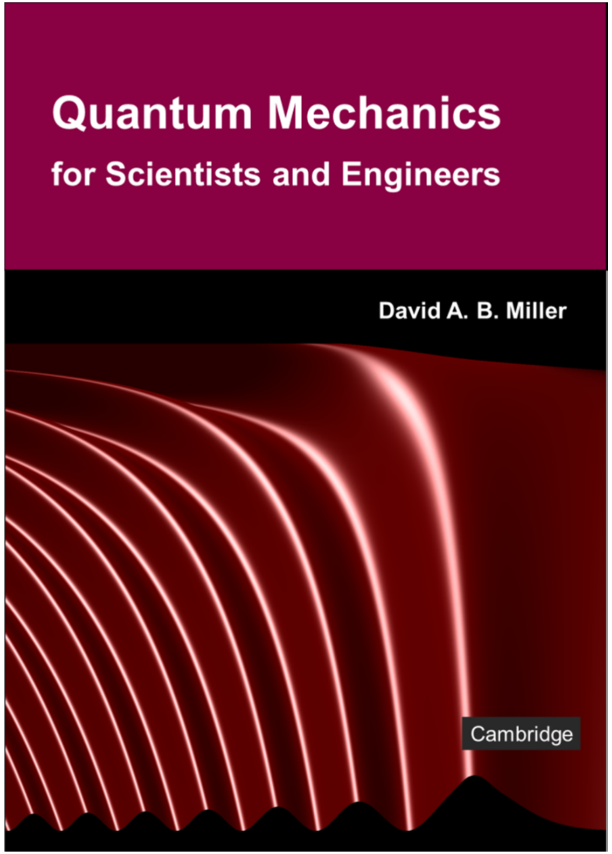
2.2 Schrödinger's wave equation

Slides: Video 2.2.4 Diffraction by two slits

Text reference: Quantum Mechanics for Scientists and Engineers

Section 2.3 (first part)



Schrödinger's wave equation

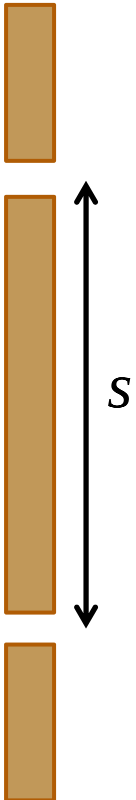
Diffraction by two slits

Quantum mechanics for scientists and engineers

David Miller

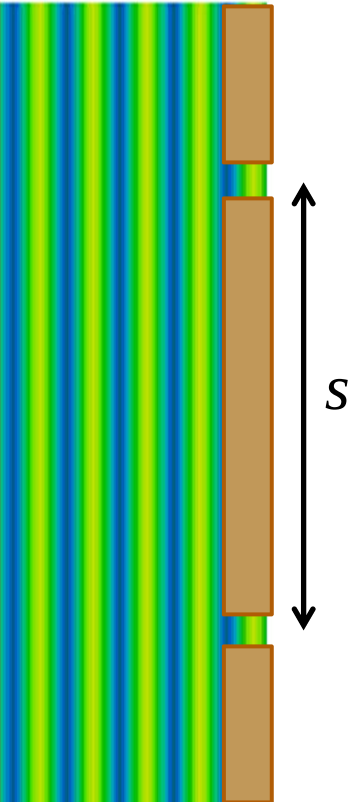
Young's slits

An opaque mask has two slits cut in it, a distance s apart



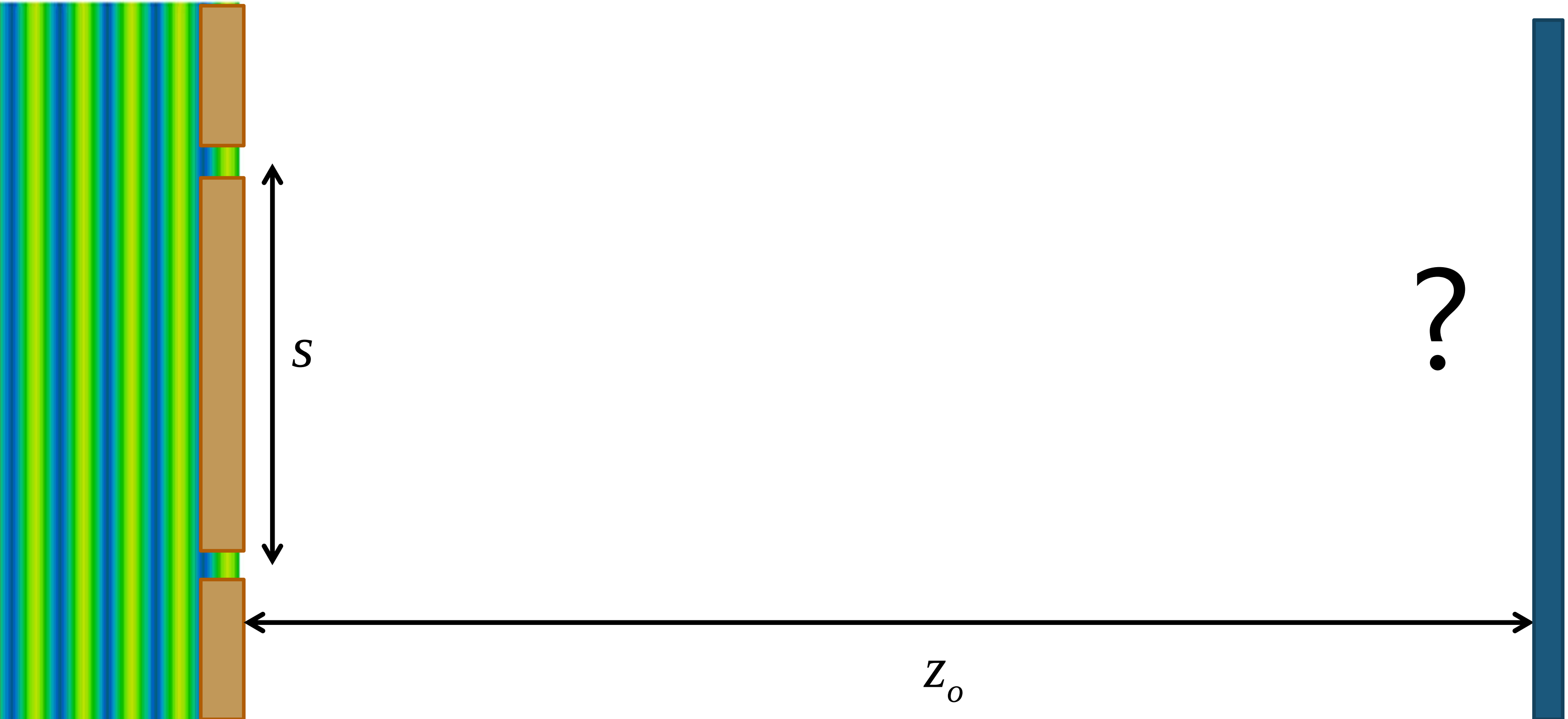
Young's slits

We shine a plane wave on the mask from the left



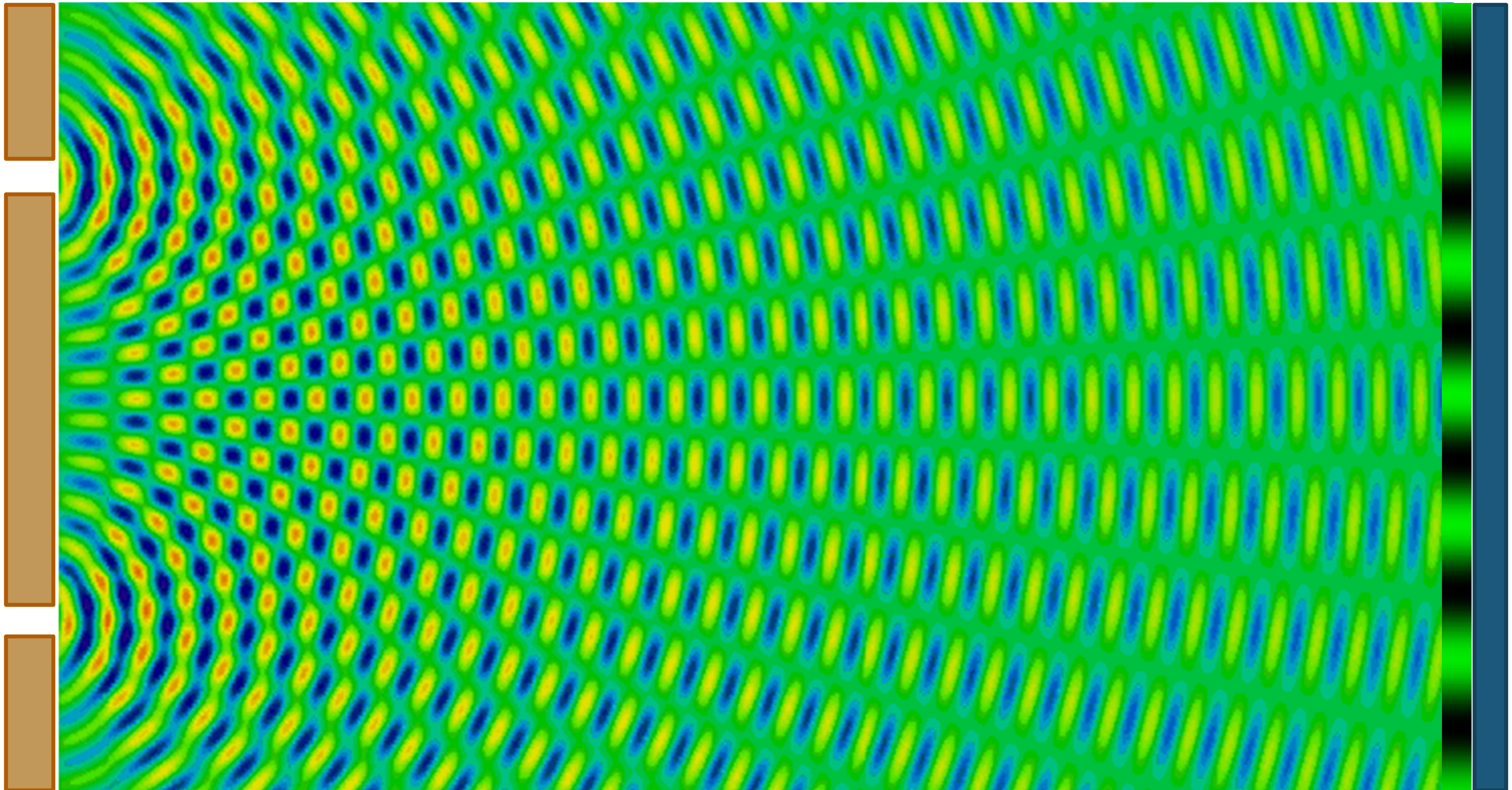
Young's slits

What will be the pattern on a screen at a large distance z_o?



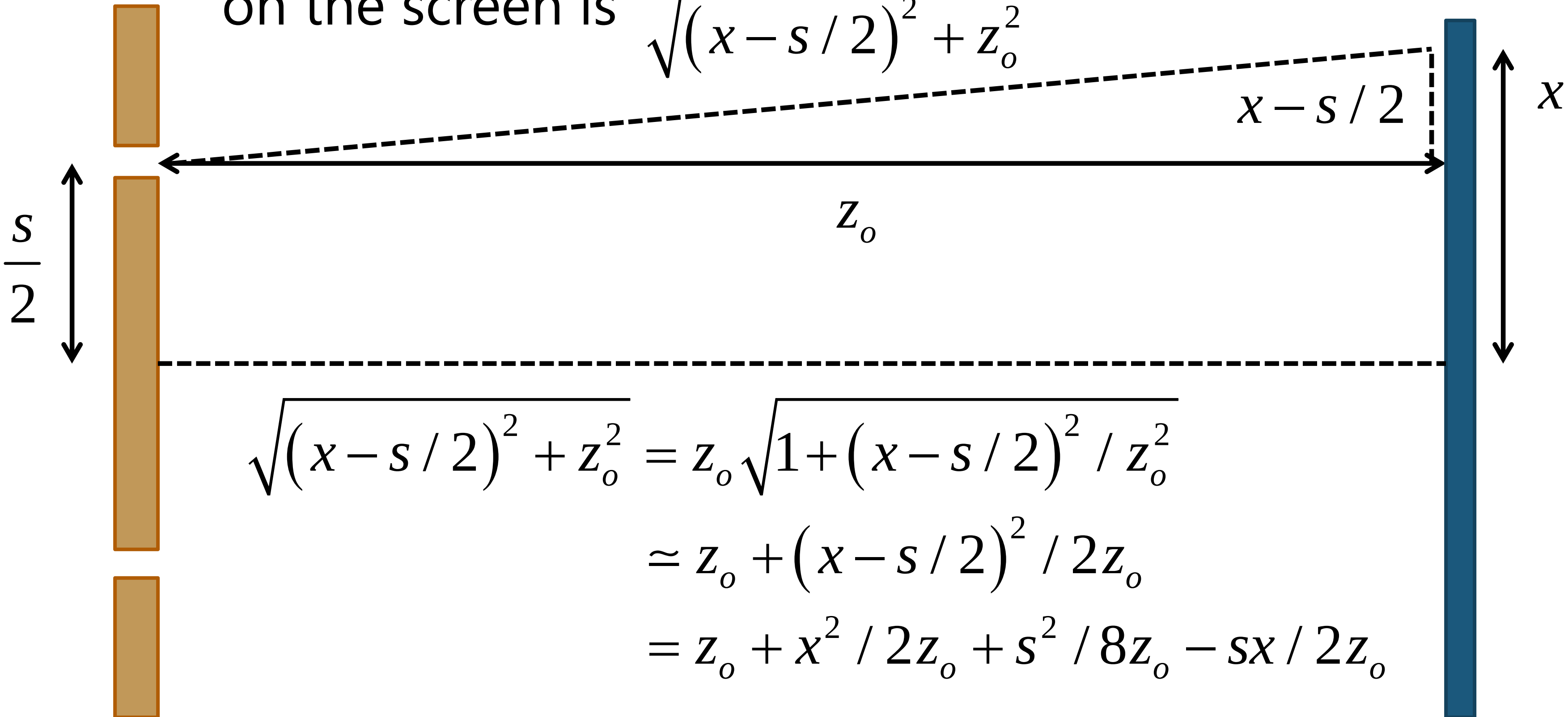
Young's slits

The slits as point sources give an interference pattern



Young's slits

The distance from the upper source to point x on the screen is $\sqrt{(x-s/2)^2+z_o^2}$



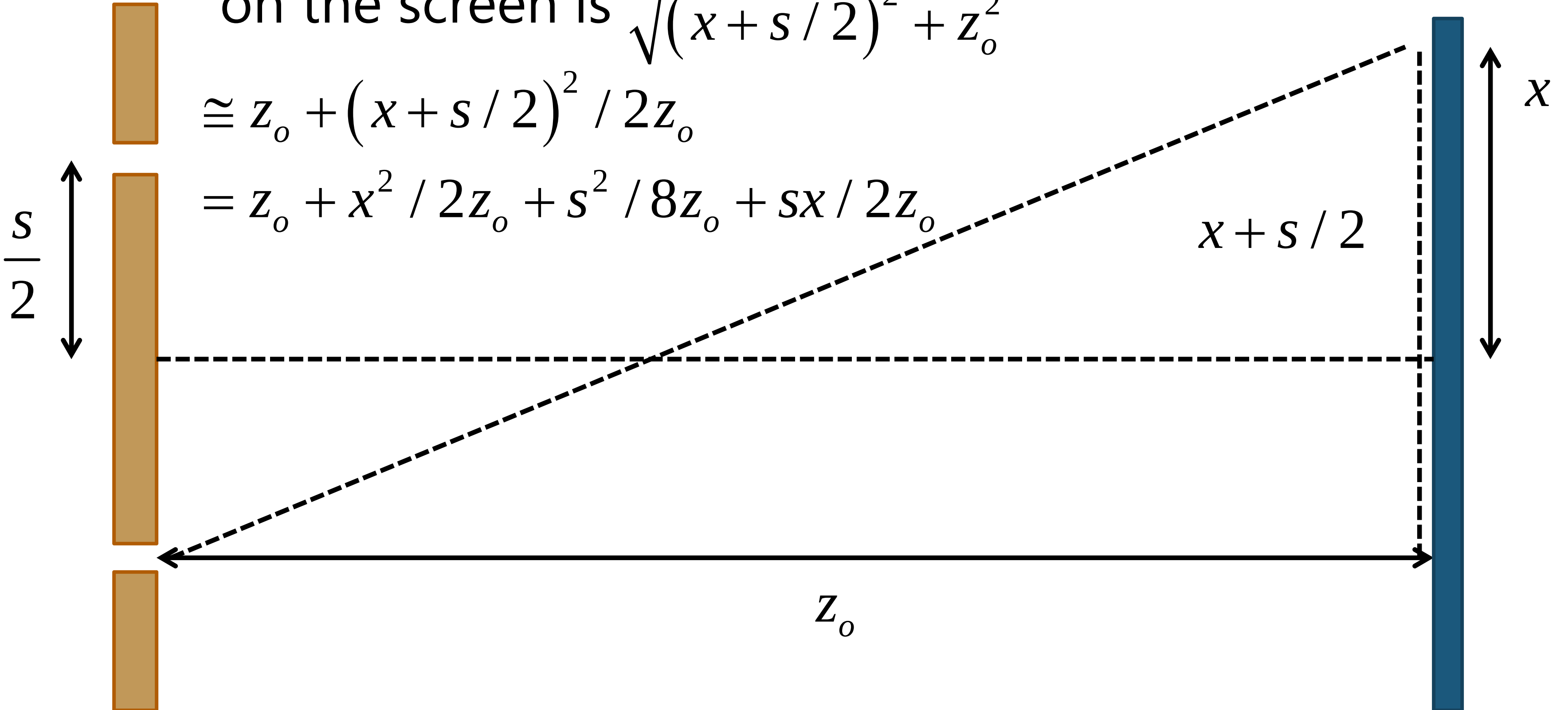
$$\sqrt{(x-s/2)^2+z_o^2}=z_o\sqrt{1+(x-s/2)^2/z_o^2}$$
$$\simeq z_o+(x-s/2)^2/2z_o$$
$$=z_o+x^2/2z_o+s^2/8z_o-sx/2z_o$$

Young’s slits

The distance from the lower source to point x on the screen is $\sqrt{\left(x+s/2\right)^2+z_o^2}$

$$\cong z_o+\left(x+s/2\right)^2/2z_o$$

$$=z_o+x^2/2z_o+s^2/8z_o+sx/2z_o$$



Young's slits

For large z_o the waves are approximately uniformly "bright"

i.e., using exponential waves for convenience

$$\psi_s(x) \propto \exp\left[ik\sqrt{(x-s/2)^2+z_o^2}\right]+\exp\left[ik\sqrt{(x+s/2)^2+z_o^2}\right]$$

Using our approximate formulas for the distances gives

$$\psi_s(x) \propto \exp(i\alpha)\left\{\exp\left[ik(sx/2z_o)\right]+\exp\left[-ik(sx/2z_o)\right]\right\}$$

where $\alpha = k\left(z_o + x^2/2z_o + s^2/8z_o\right)$

Young's slits

Now $\exp(i\theta)+\exp(-i\theta)=2\cos(\theta)$

so $$\psi_s(x)\propto\exp(i\alpha)\left[\exp\left(ik\frac{sx}{2z_o}\right)+\exp\left(-ik\frac{sx}{2z_o}\right)\right]$$

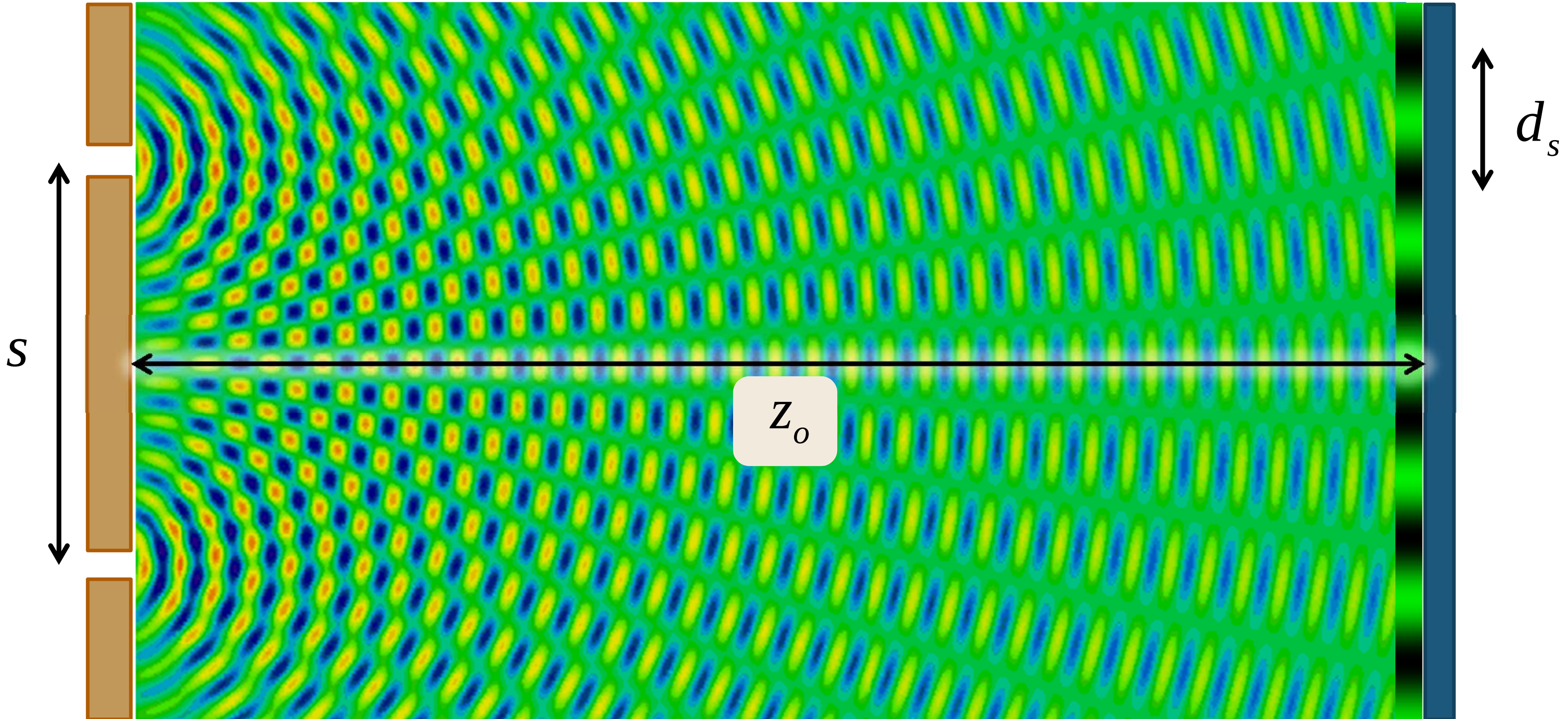
$$\propto\exp(i\alpha)\cos\left(k\frac{sx}{2z_o}\right)=\exp(i\alpha)\cos\left(\frac{\pi sx}{\lambda z_o}\right)$$

so the "intensity" of the beam

$$|\psi_s(x)|^2\propto\cos^2(\pi sx/\lambda z_o)=\frac{1}{2}\left[1+\cos(2\pi sx/\lambda z_o)\right]$$

Young's slits

The interference fringes are spaced by $d_s = \lambda z_o / s$



Young's slits

This allows us to measure small wavelengths $\lambda = d_s s / z_o$