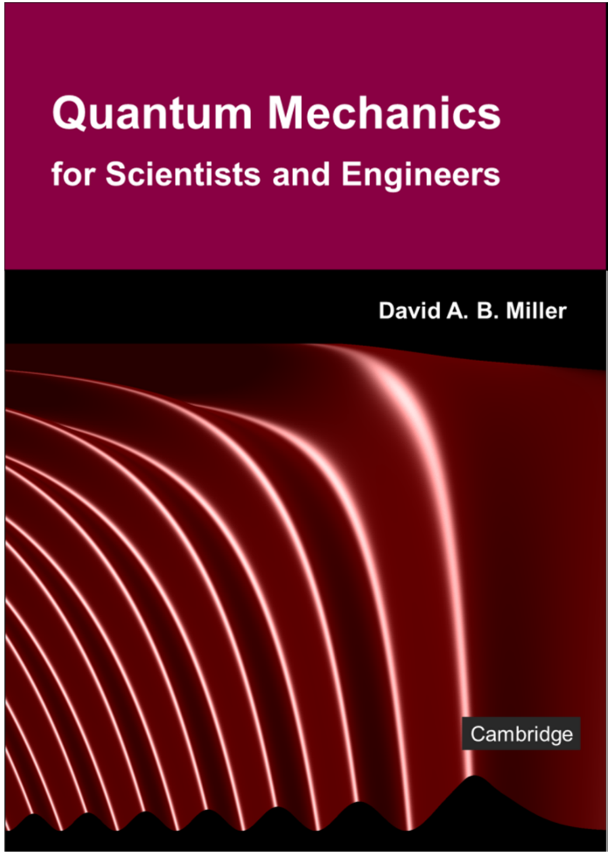
3.1 Particles and barriers

Slides: Video 3.1.1 Sets of functions

Text reference: Quantum Mechanics for Scientists and Engineers

Section 2.7 (“Completeness of sets – Fourier series”)

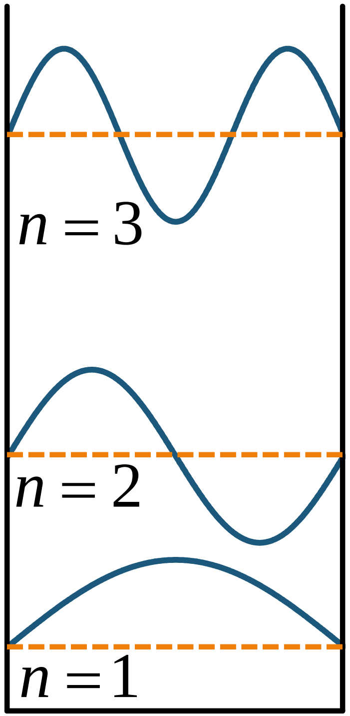


Particles and barriers

Sets of functions

Quantum mechanics for scientists and engineers

David Miller



$n = 3$
$n = 2$
$n = 1$

Fourier series

Suppose we are interested in the behavior of some function

such as a loudspeaker cone displacement

from time 0 to time t_o

presuming it starts and ends at 0 displacement

An appropriate Fourier series would be

$$f(t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi t}{t_o}\right)$$

where a_n are the amplitudes of these frequency components

Fourier series

Similarly, if we had any function $f(z)$

over the distance L_z from $z = 0$ to $z = L_z$

and that started and ended at 0 height

we could similarly write it as

$$f(z) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi z}{L_z}\right)$$

for some set of numbers or "amplitudes" a_n

which we would have to work out

Fourier series and eigenfunctions

We remember our set of normalized eigenfunctions

$$\psi_n(z) = \sqrt{\frac{2}{L_z}} \sin\left(\frac{n\pi z}{L_z}\right)$$

With a minor change

we could use these instead of the sines

$$f(z) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi z}{L_z}\right) = \sum_{n=1}^{\infty} b_n \psi_n(z)$$

where $b_n = \sqrt{L_z / 2}\, a_n$

Expansion in eigenfunctions

Now we can restate the same mathematics in new words

$$f(z)=\sum_{n=1}^{\infty} b_n \psi_n(z)$$

is now the

expansion of $f(z)$ in the complete set of (normalized) eigenfunctions $\psi_n(z)$

Note that, though we have illustrated this by connecting to Fourier series

this will work for other sets of eigenfunctions also

Basis sets of functions

A set of functions such as the $\psi_n(z)$

that can be used in this way to represent a function such as $f(z)$

is referred to as

a "basis set of functions"

or, more simply,

a "basis"

The set of "expansion coefficients" (amplitudes) b_n is then

the "representation" of $f(z)$ in the basis $\psi_n(z)$