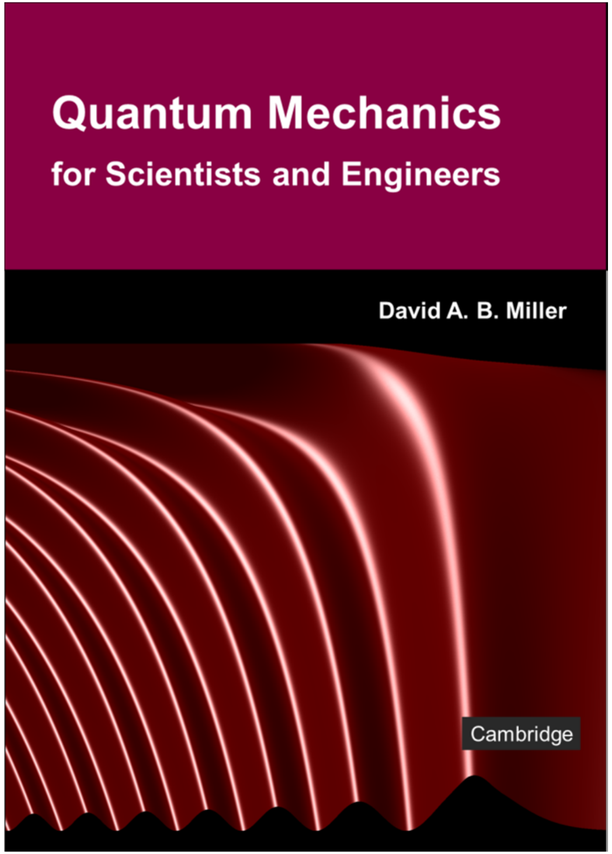
4.2 Wavepackets

Slides: Video 4.2.4 Group velocity for a free electron

Text reference: Quantum Mechanics for Scientists and Engineers

Section 3.7 (“Group velocity” second part)



Wavepackets

Group velocity for a free electron

Quantum mechanics for scientists and engineers

David Miller

Group velocity in optics

The small dispersion in glass gives significant effects in long fibers

Large dispersions are found near absorption lines

In waveguides, different spatial forms (modes) propagate at different velocities

- dispersion from geometry or structure

Any structure whose physical properties

- such as refractive index
  - change on a scale comparable with a wavelength
    - will also show strong "structural" dispersion

Group velocity for a free electron

For a free electron

the frequency ω is not proportional to the wavevector magnitude k

For a free electron,

i.e., one for which the potential $V(z)=0$

in one direction z the Schrödinger equation is

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dz^2}=E\psi$$

with solutions $\psi(z)\propto\exp(\pm ikz)$ and $E=\dfrac{\hbar^2k^2}{2m}$

Group velocity for a free electron

But this means that

$$\omega = \frac{E}{\hbar} = \frac{\hbar k^2}{2m}$$

so

$$\omega \propto k^2$$

not $\omega \propto k$

So the free electron group velocity is

$$v_g = \frac{d\omega}{dk} = \frac{1}{\hbar}\frac{dE}{dk} = \frac{\hbar k}{m} = \sqrt{\frac{\hbar^2 k^2}{m^2}} = \sqrt{\frac{2}{m}\frac{\hbar^2 k^2}{2m}} = \sqrt{\frac{2E}{m}}$$

Group velocity of a free electron

This group velocity $v_g = \sqrt{\frac{2E}{m}}$

does give us $E = \frac{1}{2} m v_g^2$

which corresponds with our classical ideas of velocity and kinetic energy

This suggests it is meaningful to think of the electron as moving at the group velocity

Phase velocity and energy

Note that the phase velocity $v_p = \omega / k$ does not give us this kind of relation

With $E \equiv \hbar\omega = \hbar^2 k^2 / 2m$, we have $\dfrac{\omega}{k} \equiv v_p = \dfrac{\hbar k}{2m}$

i.e.,

$$v_p^2 = \left(\frac{\hbar k}{2m}\right)^2 = \frac{1}{2m}\frac{\hbar^2 k^2}{2m} = \frac{1}{2m}E$$

i.e.,

$$E = 2mv_p^2$$

which does nor correspond to the classical relation between energy and velocity

The electron does not move at the phase velocity