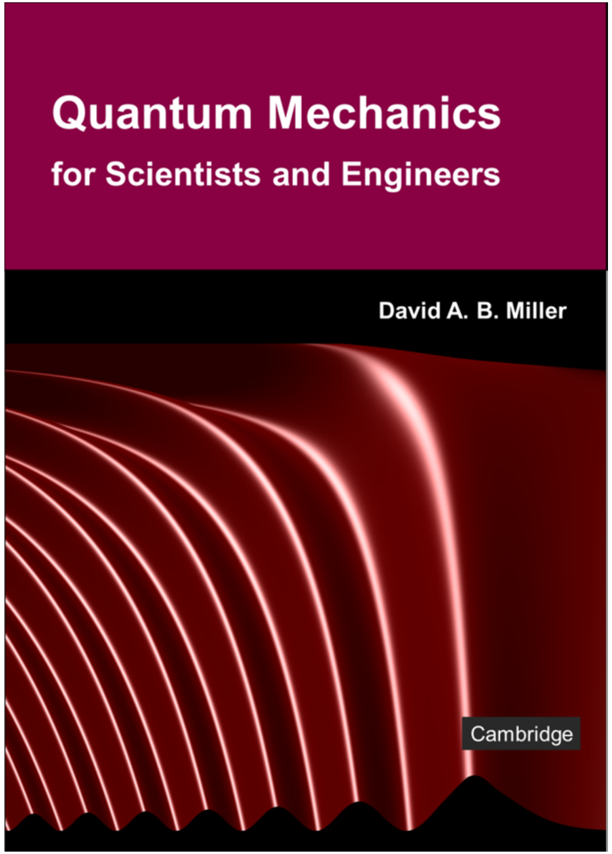
3.3 The time-dependent Schrödinger equation

Slides: Video 3.3.4 Solutions of the time-dependent Schrödinger equation

Text reference: Quantum Mechanics for Scientists and Engineers

Section 3.3



The time-dependent Schrödinger equation

Solutions of the time-dependent Schrödinger equation

Quantum mechanics for scientists and engineers

David Miller

Contrast to classical wave equation

The common classical wave equation has a different form

$$\nabla^2 f = \frac{k^2}{\omega^2}\frac{\partial^2 f}{\partial t^2}$$

for which

$$f \propto \exp\left[i\left(kz - \omega t\right)\right]$$

would also be a solution

Note the classical equation has a second time derivative

as opposed to the first time derivative in Schrödinger's time-dependent equation

Schrödinger's complex waves

Note that Schrödinger's use of a complex wave equation

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t)+V(\mathbf{r},t)\Psi(\mathbf{r},t)=i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t}$$

with the "i" on the right hand side

means that generally the wave Ψ is required to be a complex entity

For example, for $V=0$

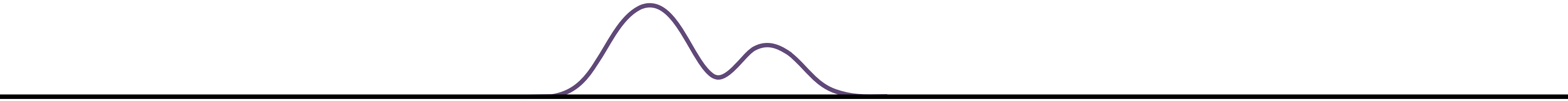
though $\exp\left[i\left(kz-Et/\hbar\right)\right]$ is a solution

$\sin\left(kz-Et/\hbar\right)$ is ***not*** a solution

Wave equation solutions

With the classical wave equation

- if at some time we see a particular shape of wave
  - e.g., on a string



Wave equation solutions

With the classical wave equation

if at some time we see a particular shape of wave

e.g., on a string

we do not know if it is going

to the right $f\left(z-ct\right)$

Wave equation solutions

With the classical wave equation

if at some time we see a particular shape of wave

e.g., on a string

we do not know if it is going

to the right $f(z-ct)$

or to the left $g(z+ct)$

or even some combination of the two

Time evolution from Schrödinger's equation

In Schrödinger's equation, for a known potential V

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t)+V(\mathbf{r},t)\Psi(\mathbf{r},t)=i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t}$$

if we knew the wavefunction $\Psi(\mathbf{r},t_o)$ at every point in space at some time t_o

we could evaluate the left hand side of the equation at that time for all $\mathbf{r}$

so we would know $\partial\Psi(\mathbf{r},t)/\partial t$ for all $\mathbf{r}$

so we could integrate the equation to deduce $\Psi(\mathbf{r},t)$ at all future times

Time evolution from Schrödinger's equation

Explicitly

knowing $\partial\Psi(\mathbf{r},t)/\partial t$ we can calculate

$$\Psi(\mathbf{r},t_o+\delta t)\simeq\Psi(\mathbf{r},t_o)+\left.\frac{\partial\Psi}{\partial t}\right|_{\mathbf{r},t_o}\delta t$$

that is, we can know the new wavefunction in space at the next instant in time

and we can continue on to the next instant

and so on

predicting all future evolution of the wavefunction