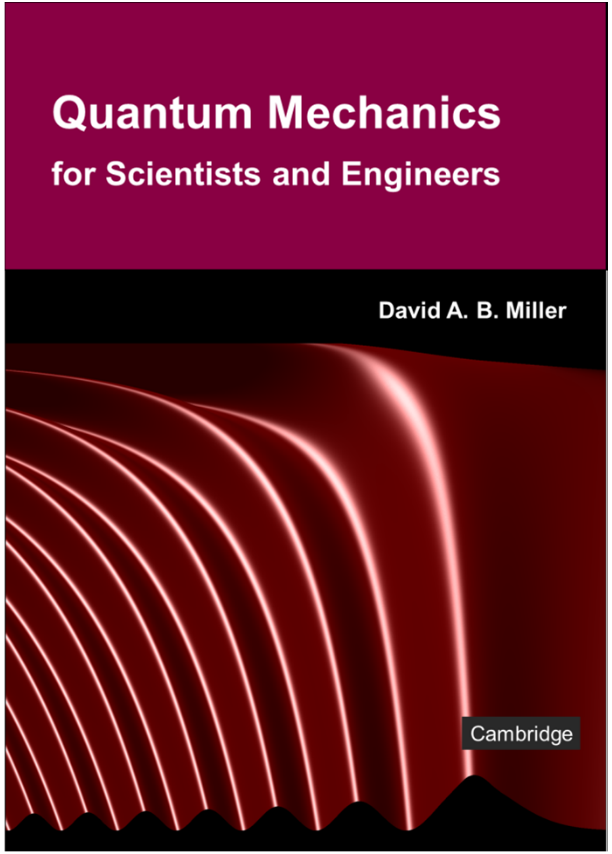
9.2 Time-dependent perturbation theory

Slides: Video 9.2.5 Transition probabilities

Text reference: Quantum Mechanics for Scientists and Engineers

Section 7.2 (second part)



Time-dependent perturbation theory

Transition probabilities

Quantum mechanics for scientists and engineers

David Miller

Transition probability calculation

In this model the probability $P(j)$

of finding the system in state $|\psi_j\rangle$ is $P(j) = \left|a_j^{(1)}\right|^2$

i.e.,

$$P(j) \simeq$$

$$\frac{t_o^2}{\hbar^2}\left|\langle\psi_j|\hat{H}_{po}|\psi_m\rangle\right|^2\left\{\begin{array}{l}\left[\dfrac{\sin\left[(\omega_{jm}-\omega)t_o/2\right]}{(\omega_{jm}-\omega)t_o/2}\right]^2+\left[\dfrac{\sin\left[(\omega_{jm}+\omega)t_o/2\right]}{(\omega_{jm}+\omega)t_o/2}\right]^2\\ +2\cos(\omega t_o)\dfrac{\sin\left[(\omega_{jm}-\omega)t_o/2\right]}{(\omega_{jm}-\omega)t_o/2}\dfrac{\sin\left[(\omega_{jm}+\omega)t_o/2\right]}{(\omega_{jm}+\omega)t_o/2}\end{array}\right\}$$

Transition probability calculation

$\sin(x)/x$ falls off rapidly for arguments $\gg 1$

Hence, for sufficiently long t_o

either one or the other of the two $\sin(x)/x$ functions in the last term will be small

$$P(j) \simeq \frac{t_o^2}{\hbar^2} \left| \left\langle \psi_j \middle| \hat{H}_{po} \middle| \psi_m \right\rangle \right|^2 \left\{ \left[\frac{\sin\left[\left(\omega_{jm} - \omega\right) t_o / 2\right]}{\left(\omega_{jm} - \omega\right) t_o / 2} \right]^2 + \left[\frac{\sin\left[\left(\omega_{jm} + \omega\right) t_o / 2\right]}{\left(\omega_{jm} + \omega\right) t_o / 2} \right]^2 + 2\cos\left(\omega t_o\right) \frac{\sin\left[\left(\omega_{jm} - \omega\right) t_o / 2\right]}{\left(\omega_{jm} - \omega\right) t_o / 2} \frac{\sin\left[\left(\omega_{jm} + \omega\right) t_o / 2\right]}{\left(\omega_{jm} + \omega\right) t_o / 2} \right\}$$

Transition probability calculation

As the time t_o is increased

these two $\sin(x)/x$ line functions get sharper

and they will eventually not overlap for ω

$$P(j) \simeq$$

$$\frac{t_o^2}{\hbar^2}\left|\left\langle \psi_j \middle| \hat{H}_{po} \middle| \psi_m \right\rangle\right|^2 \left\{ \begin{array}{l} \left[\dfrac{\sin\left[\left(\omega_{jm}-\omega\right)t_o/2\right]}{\left(\omega_{jm}-\omega\right)t_o/2}\right]^2 + \left[\dfrac{\sin\left[\left(\omega_{jm}+\omega\right)t_o/2\right]}{\left(\omega_{jm}+\omega\right)t_o/2}\right]^2 \\ +2\cos\left(\omega t_o\right)\dfrac{\sin\left[\left(\omega_{jm}-\omega\right)t_o/2\right]}{\left(\omega_{jm}-\omega\right)t_o/2}\dfrac{\sin\left[\left(\omega_{jm}+\omega\right)t_o/2\right]}{\left(\omega_{jm}+\omega\right)t_o/2} \end{array} \right\}$$

Transition probability calculation

Presuming we take t_o sufficiently large, we are left with

$$P(j) \simeq \frac{t_o^2}{\hbar^2}\left|\left\langle \psi_j \middle| \hat{H}_{po} \middle| \psi_m \right\rangle\right|^2 \left\{ \left[\frac{\sin\left[\left(\omega_{jm}-\omega\right)t_o/2\right]}{\left(\omega_{jm}-\omega\right)t_o/2} \right]^2 + \left[\frac{\sin\left[\left(\omega_{jm}+\omega\right)t_o/2\right]}{\left(\omega_{jm}+\omega\right)t_o/2} \right]^2 \right\}$$

We now have some finite probability that the system

has changed state from its initial state $\left|\psi_m\right\rangle$

to another "final" state $\left|\psi_j\right\rangle$

Transition probability calculation

$$P(j) \simeq \frac{t_o^2}{\hbar^2} \left| \left\langle \psi_j \middle| \hat{H}_{po} \middle| \psi_m \right\rangle \right|^2 \left\{ \left[\frac{\sin\left[\left(\omega_{jm} - \omega \right) t_o / 2 \right]}{\left(\omega_{jm} - \omega \right) t_o / 2} \right]^2 + \left[\frac{\sin\left[\left(\omega_{jm} + \omega \right) t_o / 2 \right]}{\left(\omega_{jm} + \omega \right) t_o / 2} \right]^2 \right\}$$

This probability depends on

the strength of the perturbation squared, and the

modulus squared of the perturbation matrix element

between the initial and final states

Transition probability calculation

$$P(j) \simeq$$

$$\frac{t_o^2}{\hbar^2} \left| \left\langle \psi_j \middle| \hat{H}_{po} \middle| \psi_m \right\rangle \right|^2 \left\{ \left[\frac{\sin\left[\left(\omega_{jm} - \omega \right) t_o / 2 \right]}{\left(\omega_{jm} - \omega \right) t_o / 2} \right]^2 + \left[\frac{\sin\left[\left(\omega_{jm} + \omega \right) t_o / 2 \right]}{\left(\omega_{jm} + \omega \right) t_o / 2} \right]^2 \right\}$$

With an oscillating electric field acting on an electron

this probability is $\propto$ the square of the field amplitude E_o^2

which is proportional to the intensity I (Power/Area)

so the probability of making a transition is proportional to the intensity I

Absorption and emission terms

In

$$P(j) \simeq \frac{t_o^2}{\hbar^2} \left| \langle \psi_j | \hat{H}_{po} | \psi_m \rangle \right|^2 \left\{ \left[\frac{\sin\left[\left(\omega_{jm} - \omega \right) t_o / 2 \right]}{\left(\omega_{jm} - \omega \right) t_o / 2} \right]^2 + \left[\frac{\sin\left[\left(\omega_{jm} + \omega \right) t_o / 2 \right]}{\left(\omega_{jm} + \omega \right) t_o / 2} \right]^2 \right\}$$

what is the meaning of the two different terms?

Absorption and emission terms

$$P(j) \simeq$$

$$\frac{t_o^2}{\hbar^2}\left|\left\langle \psi_j \left| \hat{H}_{po} \right| \psi_m \right\rangle\right|^2 \left\{ \left[\frac{\sin\left[\left(\omega_{jm} - \omega\right) t_o / 2\right]}{\left(\omega_{jm} - \omega\right) t_o / 2} \right]^2 + \left[\frac{\sin\left[\left(\omega_{jm} + \omega\right) t_o / 2\right]}{\left(\omega_{jm} + \omega\right) t_o / 2} \right]^2 \right\}$$

The first term above is significant if $\omega_{jm} \approx \omega$

i.e., if $\hbar\omega \approx E_j - E_m$

Since we chose ω to be a positive quantity

this term is significant if we are absorbing energy

raising from a lower energy state $\left|\psi_m\right\rangle$

to a higher energy state $\left|\psi_j\right\rangle$

Absorption and emission terms

$$P(j) \simeq$$

$$\frac{t_o^2}{\hbar^2}\left|\left\langle \psi_j \left| \hat{H}_{po} \right| \psi_m \right\rangle\right|^2 \left\{ \left[\frac{\sin\left[\left(\omega_{jm} - \omega\right)t_o / 2\right]}{\left(\omega_{jm} - \omega\right)t_o / 2} \right]^2 + \left[\frac{\sin\left[\left(\omega_{jm} + \omega\right)t_o / 2\right]}{\left(\omega_{jm} + \omega\right)t_o / 2} \right]^2 \right\}$$

We note that

the amount of energy we are absorbing is $\hbar\omega$

This first term behaves as we would require

for absorption of a photon

Absorption and emission terms

$$P(j) \simeq$$

$$\frac{t_o^2}{\hbar^2}\left|\langle \psi_j | \hat{H}_{po} | \psi_m \rangle\right|^2 \left\{ \left[\frac{\sin\left[\left(\omega_{jm} - \omega\right) t_o / 2\right]}{\left(\omega_{jm} - \omega\right) t_o / 2} \right]^2 + \left[\frac{\sin\left[\left(\omega_{jm} + \omega\right) t_o / 2\right]}{\left(\omega_{jm} + \omega\right) t_o / 2} \right]^2 \right\}$$

The second term above is significant if $-\omega_{jm} \approx \omega$

i.e., if $\hbar\omega \approx E_m - E_j$

Since we chose ω to be a positive quantity

this term is significant if we are emitting energy

falling from a higher energy state $|\psi_m\rangle$

to a lower energy state $|\psi_j\rangle$

Absorption and emission terms

$$P(j) \simeq$$

$$\frac{t_o^2}{\hbar^2}\left|\langle \psi_j | \hat{H}_{po} | \psi_m \rangle\right|^2 \left\{ \left[\frac{\sin\left[(\omega_{jm} - \omega)t_o / 2\right]}{(\omega_{jm} - \omega)t_o / 2} \right]^2 + \left[\frac{\sin\left[(\omega_{jm} + \omega)t_o / 2\right]}{(\omega_{jm} + \omega)t_o / 2} \right]^2 \right\}$$

We note that the amount of energy we are emitting is $\hbar\omega$

This second term corresponds to

stimulated emission of a photon

the process used in lasers

The spontaneous emission of normal light requires

quantizing the electromagnetic field as well