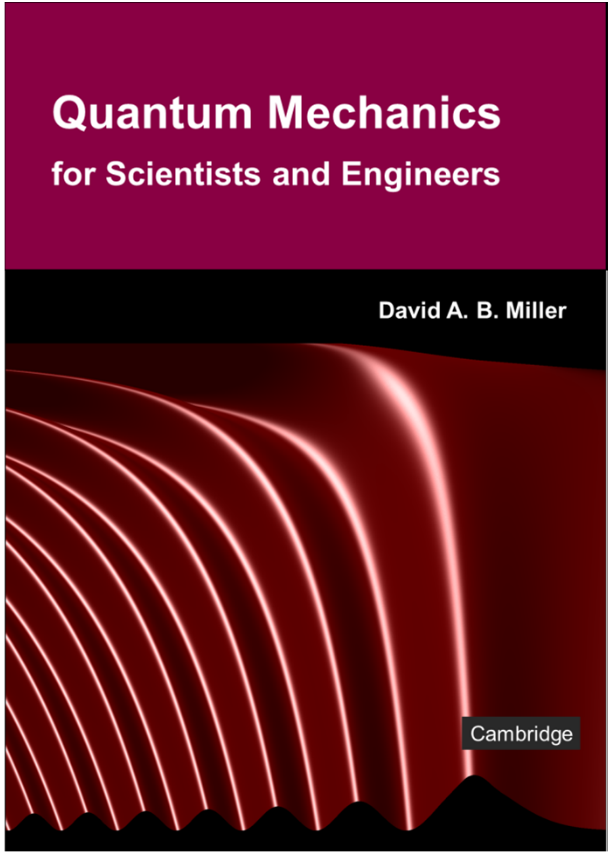
5.2 Functions and Dirac notation

Slides: Video 5.2.2 Functions as vectors

Text reference: Quantum Mechanics for Scientists and Engineers

Section 4.1 (up to "Dirac bra-ket notation")



Functions and Dirac notation

Functions as vectors

Quantum mechanics for scientists and engineers

David Miller

Functions as vectors

One kind of list of arguments would be the list of all real numbers

which we could list in order as

$x_1, x_2, x_3 \ldots$

and so on

This is an infinitely long list

and the adjacent values in the list

are infinitesimally close together

but we will regard these infinities as details!

Functions as vectors

If we presume that we know this list of possible arguments of the function

we can write out the function as the corresponding list of values, and

we choose to write this list as a column vector

$$\begin{bmatrix} f(x_1) \\ f(x_2) \\ f(x_3) \\ \vdots \end{bmatrix}$$

Functions as vectors

For example

we could specify the function at points spaced by some small amount δx

with $x_2 = x_1 + \delta x$, $x_3 = x_2 + \delta x$ and so on

We would do this

for sufficiently many values of x and

over a sufficient range of x

to get a sufficiently useful representation for some calculation

such as an integral

Functions as vectors

The integral of $\left|f(x)\right|^2$

could then be written as

$$\int \left|f(x)\right|^2 dx \cong \begin{bmatrix} f^*(x_1) & f^*(x_2) & f^*(x_3) & \cdots \end{bmatrix} \begin{bmatrix} f(x_1) \\ f(x_2) \\ f(x_3) \\ \vdots \end{bmatrix} \delta x$$

Provided we choose δx sufficiently small

and the corresponding vectors therefore sufficiently long

we can get an arbitrarily good approximation to the integral

Visualizing a function as a vector

Suppose the function $f(x)$ is approximated by its values at three points

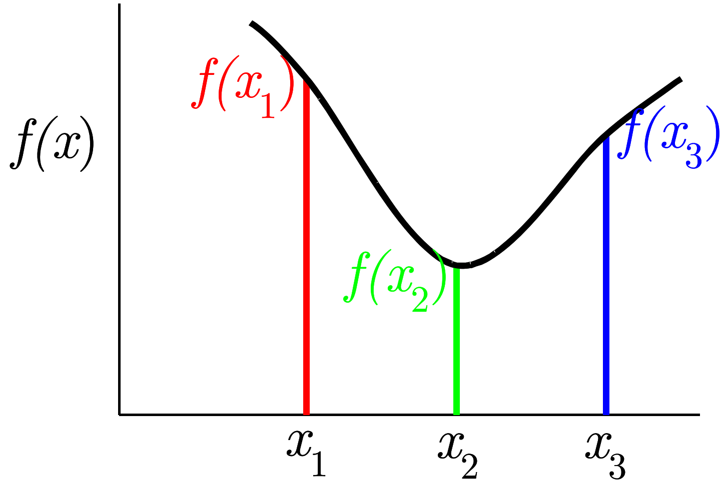
x_1, x_2, and x_3

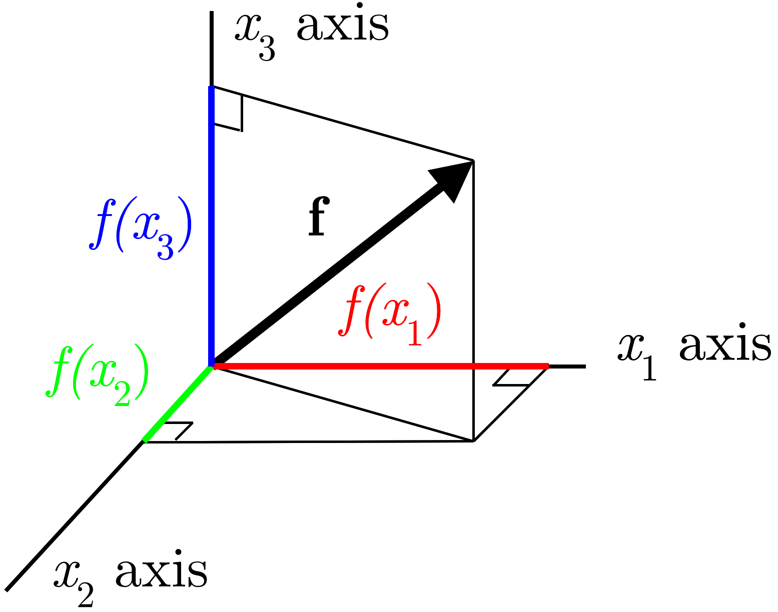
and is represented as a vector

$$\mathbf{f} \equiv \begin{bmatrix} f(x_1) \\ f(x_2) \\ f(x_3) \end{bmatrix}$$

then we can visualize the function as a vector in ordinary geometrical space

Visualizing a function as a vector



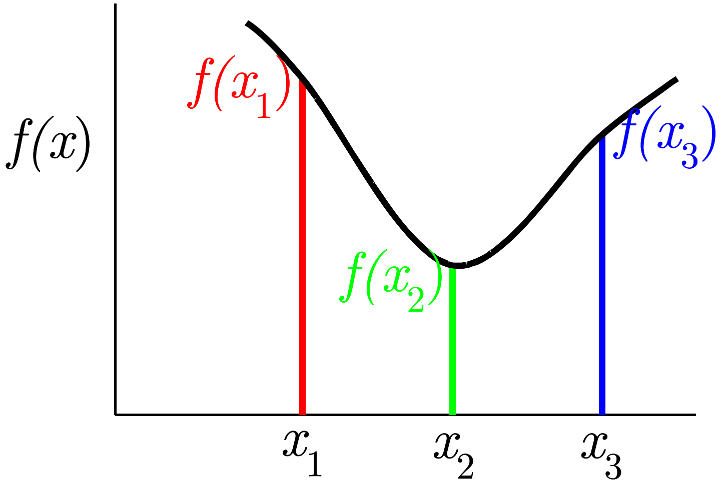


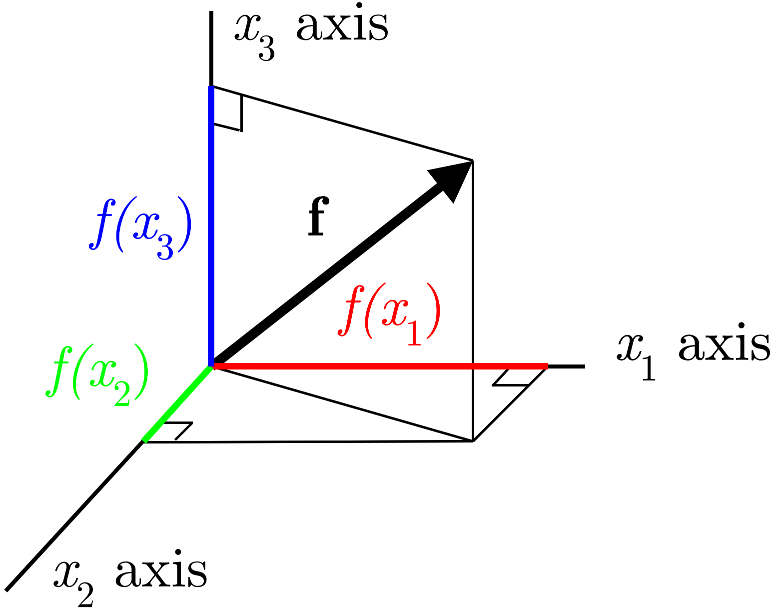
We could draw a vector

whose components along three axes

were the values of the function at these three points

Visualizing a function as a vector





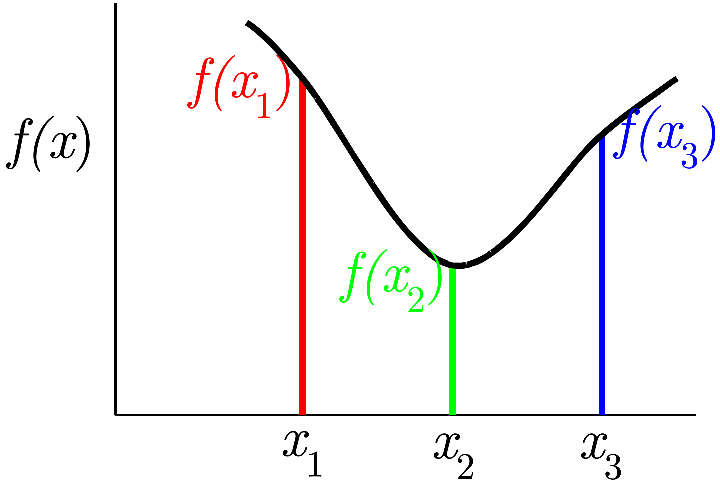
In quantum mechanics

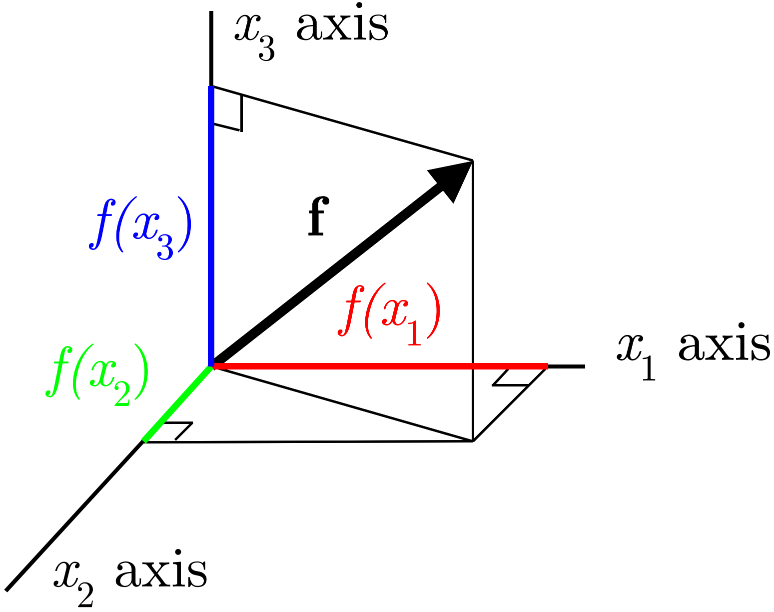
the functions are complex, not merely real

and there may be many elements in the vector

possibly an infinite number

Visualizing a function as a vector





But we will still visualize the function

and, more generally, the quantum mechanical state

as a vector in a space