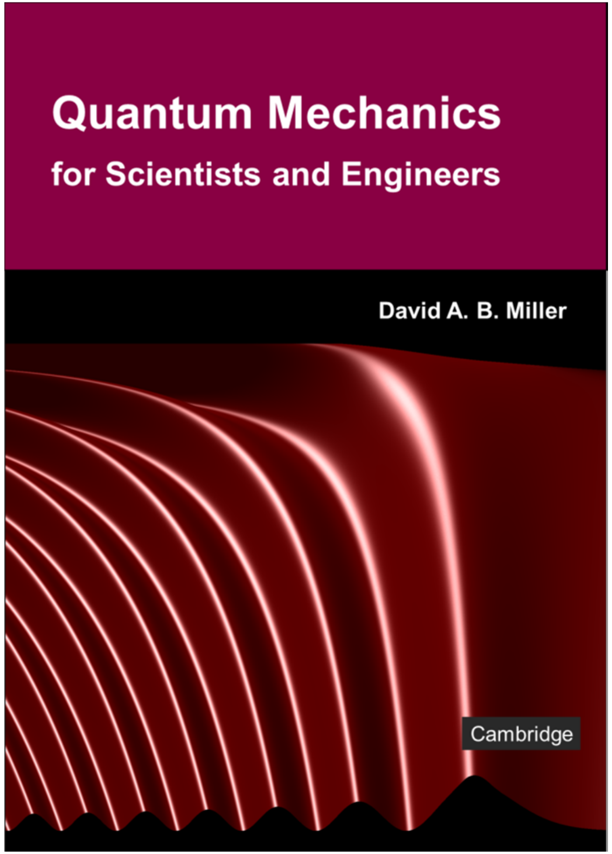
8.2 Approximation methods

Slides: Video 8.2.2 Potential well with field

Text reference: Quantum Mechanics for Scientists and Engineers

Section 6.1



Approximation methods

Potential well with field

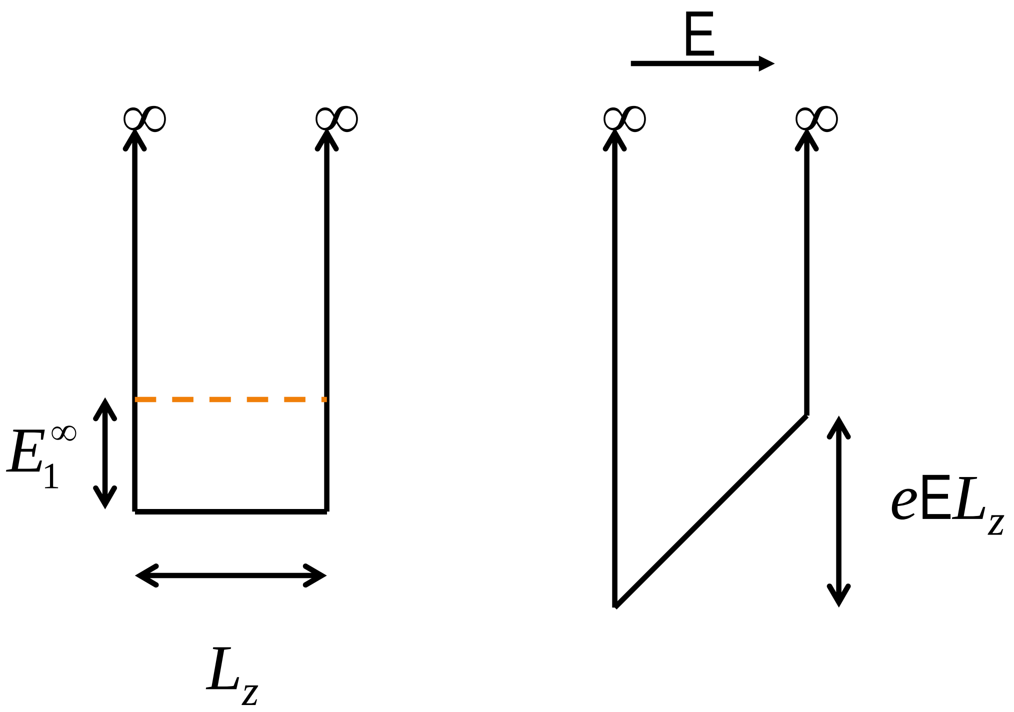
Quantum mechanics for scientists and engineers

David Miller

Potential well with field

We are considering
an electron in
a potential well
with infinitely
high walls
and with an
applied
electric field E

without field
with field



Construction of Hamiltonian

The energy of an electron in an electric field E

- simply increases linearly with distance
  - A positive electric field in the positive z direction
    - pushes the electron in the negative z direction
      - with a force of magnitude eE

So the potential energy of the electron

- increases in the positive z direction
  - with the form eEz

Construction of the Hamiltonian

We choose the potential to be zero in the middle of the well

Hence, within the well

the potential energy is

$$V(z) = e\mathrm{E}(z - L_z/2)$$

and the Hamiltonian becomes

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dz^2} + e\mathrm{E}(z - L_z/2)$$

Construction of the Hamiltonian

We can usefully define dimensionless units

A convenient unit of energy is

the confinement energy of the first state of the original infinitely deep well

$$E_1^{\infty} = \frac{\hbar^2}{2m}\left(\frac{\pi}{L_z}\right)^2$$

and in those units the eigenenergy of the nth state will be

$$\eta_n = \frac{E_n}{E_1^{\infty}}$$

Construction of the Hamiltonian

A convenient unit of field E_o

gives one energy unit of potential change from one side of the well to the other

$$E_o = \frac{E_1^{\infty}}{eL_z}$$

So, the (dimensionless) field will be

$$f = E / E_o$$

A convenient distance unit is the thickness L_z

so the dimensionless distance will be

$$\xi = z / L_z$$

Construction of the Hamiltonian

From the original Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dz^2} + e\mathsf{E}\left(z - L_z/2\right)$$

dividing by E_1^∞ and using dimensionless units gives

$$\hat{H} = -\frac{1}{\pi^2}\frac{d^2}{d\xi^2} + \mathsf{f}\left(\xi - 1/2\right)$$

and a time-independent Schrödinger equation

$$\hat{H}\phi(\xi) = \eta\phi(\xi)$$

Construction of the Hamiltonian

For the "unperturbed" problem without field

we write the "unperturbed" Hamiltonian within the well as

$$\hat{H}_o = -\frac{1}{\pi^2}\frac{d^2}{d\xi^2}$$

The normalized solutions of the corresponding Schrödinger equation

$$\hat{H}_o \psi_n = \varepsilon_n \psi_n$$

are then

$$\psi_n(\xi) = \sqrt{2}\sin(n\pi\xi)$$