# Basic symbols and algebra notations

Background mathematics review

David Miller

# Basic symbols and algebra notations

## Basic mathematical symbols

Background mathematics review

David Miller

# Elementary arithmetic symbols

| | | | |
|---|---|---|---|
| $=$ | Equals | | |
| $+$ | Addition or ″plus″ | | $2+3=5$ |
| $-$ | Subtraction, “minus” or “less” | | $3-2=1$ |
| $\times$ or $\cdot$ | Multiplication | $2\times 3=6$ | $2\cdot 3=6$ |
| $\div$ or $/$ | Division | $6\div 3=2$ | $6/3=2$ |

or $\dfrac{(numerator)}{(demonimator)} \equiv \dfrac{(dividend)}{(divisor)} = (quotient)$ $\dfrac{6}{3}=2$ $\;{}^{6}\!/_{3}=2$ $\;{}^{6}\!/_{3}=2$

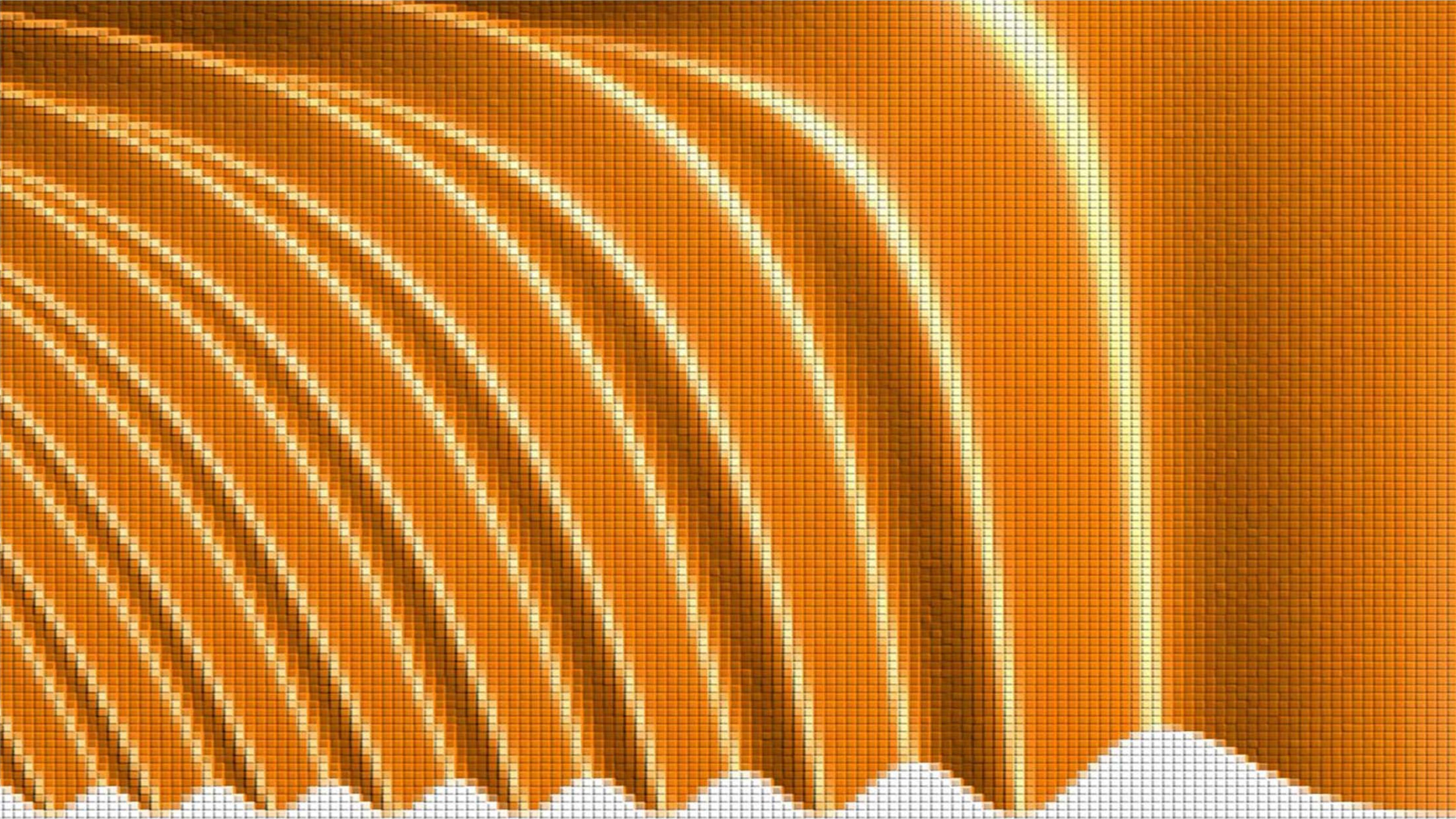
# Relational symbols

| | | |
|---|---|---|
| $\equiv$ | "Is equivalent to" | $x/y \equiv \frac{x}{y}$ |
| $\simeq$ or $\cong$ | "Is approximately equal to" | $\frac{1}{3} \simeq 0.33$ |
| $\propto$ | "Is proportional to" | $a \times x \propto x$ |
| $>$ | "Is greater than" | $3 > 2$ |
| $\geq$ | "Is greater than or equal to" | $1 + x^2 \geq 1$ |
| $<$ | "Is less than" | $2 < 3$ |
| $\leq$ | "Is less than or equal to" | $1 \leq 1 + x^2$ |
| $\gg$ | "Is much greater than" | $100 \gg 1$ |
| $\ll$ | "Is much less than" | $1 \ll 100$ |

# Greek characters used as symbols

| Upper case | Lower case | Name | Roman equiv. | Key-board |
|---|---|---|---|---|
| Α | α | alpha | a | a |
| Β | β | beta | b | b |
| Γ | γ | gamma | g | g |
| Δ | δ | delta | d | d |
| Ε | ε | epsilon | e | e |
| Ζ | ζ | zeta | z | z |
| Η | η | eta | (e) | h |
| Θ | θ | theta | th | q |
| Ι | ι | iota | i | i |
| Κ | κ | kappa | k | k |
| Λ | λ | lambda | l | l |
| Μ | μ | mu | m | m |

| Upper case | Lower case | Name | Roman equiv. | Key-board |
|---|---|---|---|---|
| Ν | ν | nu | n | n |
| Ξ | ξ | xi | x | x |
| Ο | ο | omicron | (o) | o |
| Π | π | pi | p | p |
| Ρ | ρ | rho | r | r |
| Σ | σ | sigma | s | s |
| Τ | τ | tau | t | t |
| Υ | υ | upsilon | u | u |
| Φ | ϕ | phi | ph | f |
| Χ | χ | chi | ch | c |
| Ψ | ψ | psi | psy | y |
| Ω | ω | omega | o | w |



# Basic symbols and algebra notations

## Basic mathematical operations

Background mathematics review

David Miller

# Conventions for multiplication

For multiplying numbers

We explicitly use the multiplication sign "×"

$$2 \times 3 = 6$$

For multiplying variables

We can use the multiplication sign

But where there is no confusion

We drop it

$$a \times b = c$$

might be simply replaced by

$$ab = c$$

# Use of parentheses and brackets

When we want to group numbers or variables

- We can use parentheses (or brackets)

$$2 \times (3+4) = 2 \times 7 = 14$$

For such grouping, we can alternatively use

- square brackets

$$2 \times [3+4] = 2 \times 7 = 14$$

- or curly brackets

$$2 \times \{3+4\} = 2 \times 7 = 14$$

When used this way, there is no difference in the mathematical meaning of these brackets

# Associative property

Operations are associative if it does not matter how we group them

e.g., addition of numbers is associative

$$(a+b)+c=a+(b+c)$$

e.g., multiplication of numbers is associative

$$(a\times b)\times c=a\times(b\times c)$$

But

division of numbers is not associative

$$(8/4)/2=2/2=1 \text{ but } 8/(4/2)=8/2=4$$

# Distributive property

- Property where terms within parentheses can be “distributed” to remove the parentheses

$$a \times (b + c) = a \times b + a \times c$$

  - Here, multiplication is said to be distributive over addition
    - Many other conceivable operations are not distributive, however
      - E.g., addition is not distributive over multiplication

$$3 + (2 \times 5) = 13 \neq (3 + 2) \times (3 + 5) = 40$$

# Commutative property

Property where the order can be switched round

e.g., addition of numbers is commutative

$$a + b = b + a$$

e.g., multiplication of numbers is commutative

$$a \times b = b \times a$$

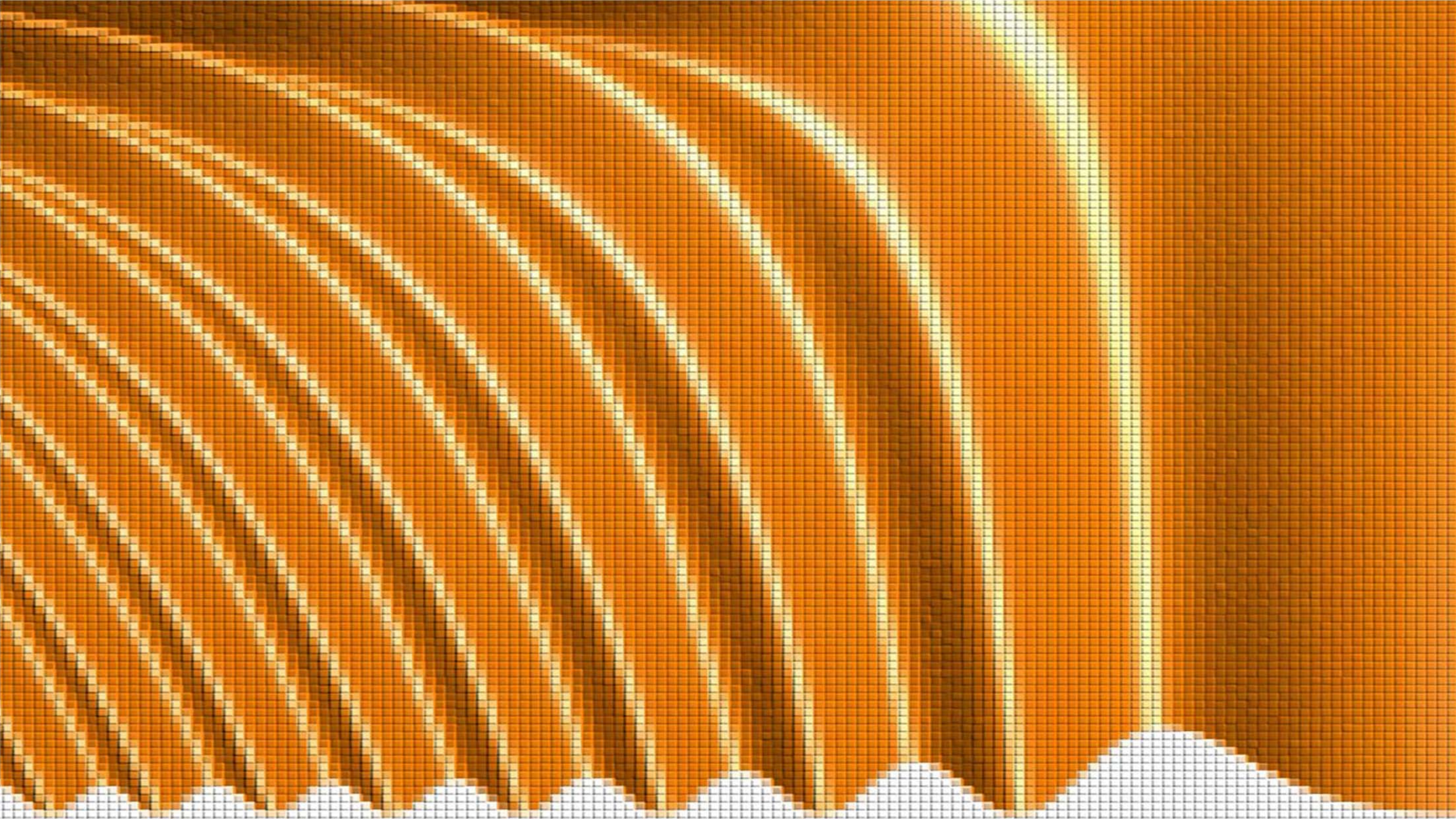
But

e.g., subtraction is not commutative

$$5 - 3 = 2 \neq 3 - 5 = -2$$

e.g., division is not commutative

$$6 / 3 = 2 \neq 3 / 6 = \tfrac{1}{2}$$



# Basic symbols and algebra notations

## Algebra notation and functions

Background mathematics review

David Miller

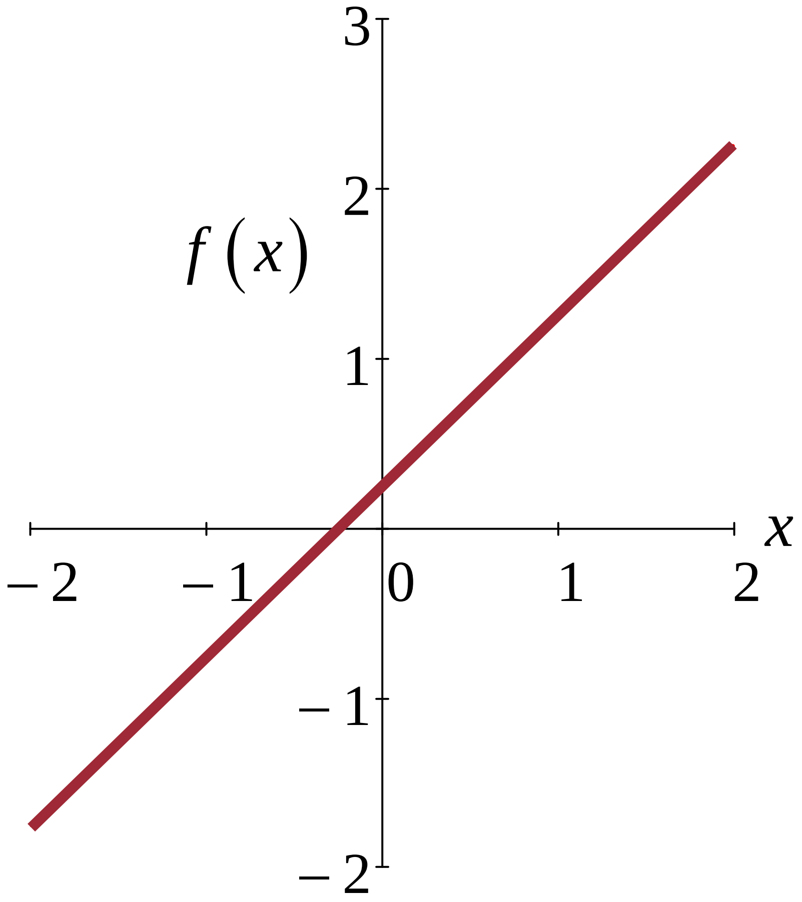
# Parentheses and functions

A function is something that relates or "maps"

- One set of values
  - Such as an "input" variable or "argument" $x$
- To another set of values
  - which we could think of as an "output"

For example, the function

$$f(x) = x + \frac{1}{4}$$



# Parentheses and functions

Conventionally, we say

"f of x" when we read $f(x)$

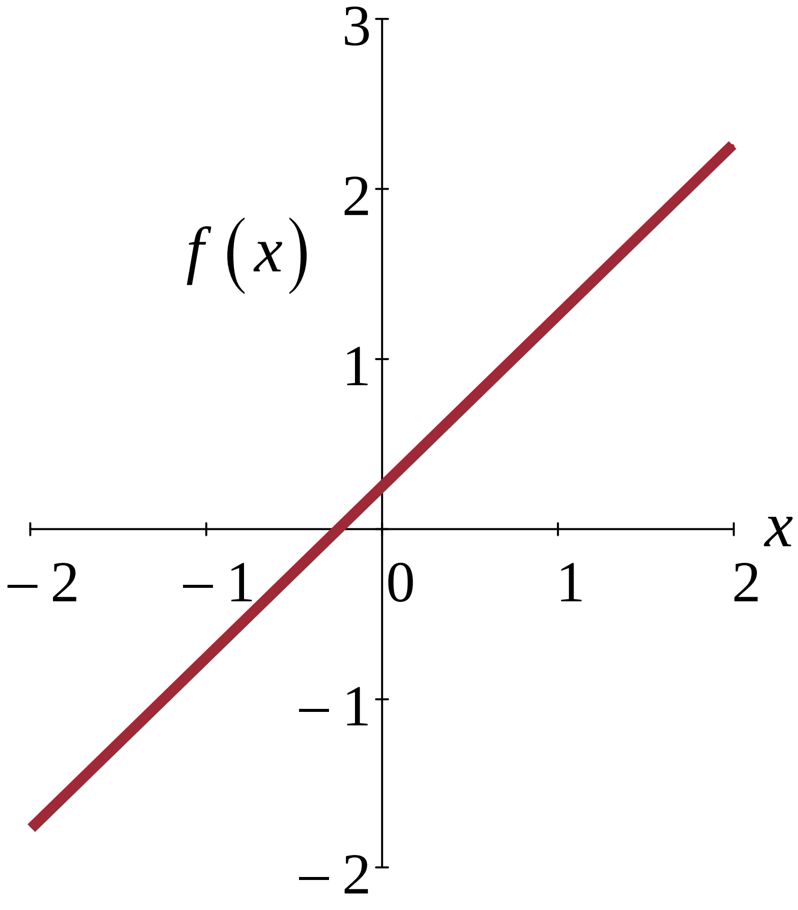
Here obviously

$f(x)$ is not "f times x"

Most commonly

Only parentheses are used around the argument $x$

not square [ ] or curly { } brackets



# Parentheses and functions

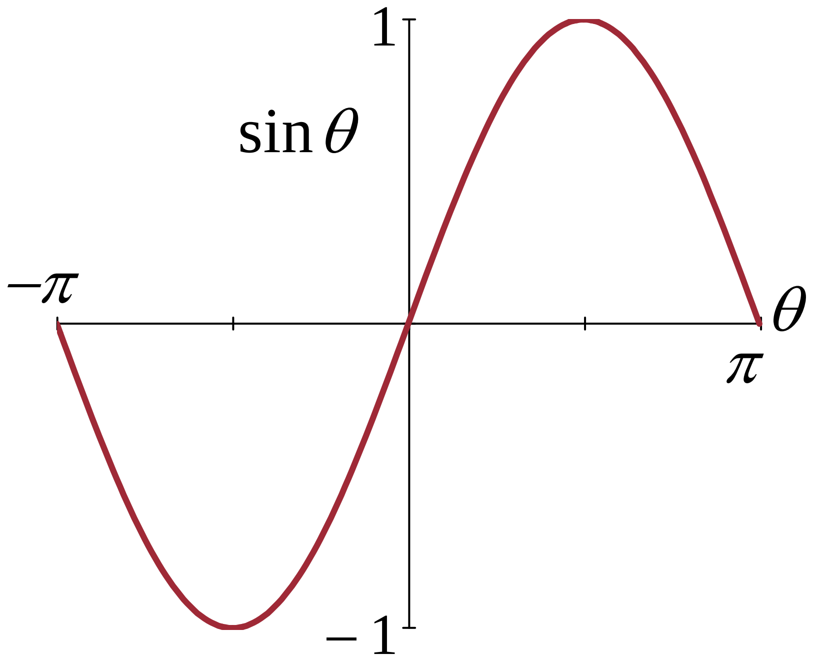
For a few very commonly used functions

- Such as the trigonometric functions
  - The parentheses are optionally omitted when the argument is simple

    $\sin\theta$ instead of $\sin(\theta)$

Note, incidentally,

$$\sin(-\theta) = -\sin(\theta)$$



# Parentheses and functions

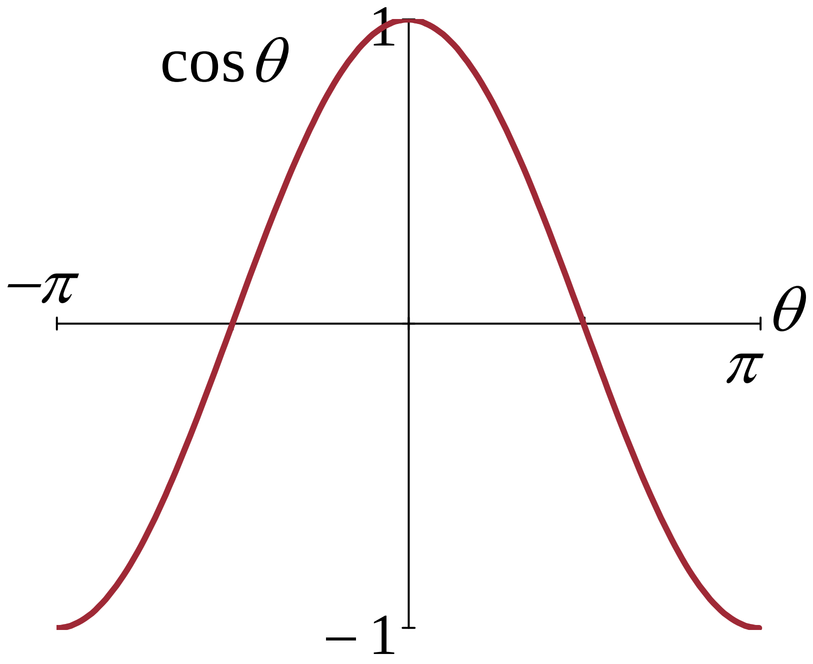
For a few very commonly used functions

- Such as the trigonometric functions
  - The parentheses are optionally omitted when the argument is simple

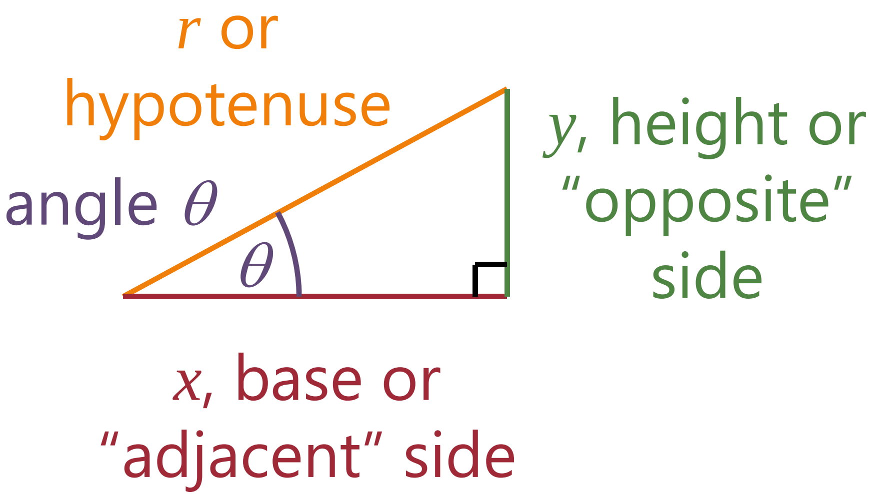
    $\cos\theta$ instead of $\cos(\theta)$

Note, incidentally

$$\cos(-\theta) = \cos(\theta)$$



# Sine, cosine, and tangent




Defined using a right-angled triangle

$$\sin\theta = \frac{y}{r} \quad \cos\theta = \frac{x}{r} \quad \tan\theta = \frac{y}{x}$$

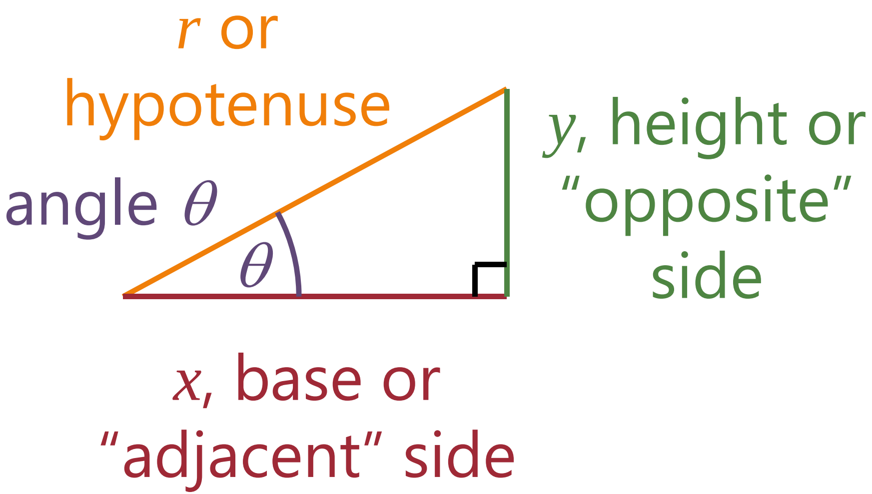
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Natural units for angles in mathematics are radians

$2\pi$ radians in a circle

1 radian ~ 57.3 degrees

# Cosecant, secant, and cotangent




## Cosecant

$$\operatorname{cosec}\theta \equiv \csc\theta = \frac{r}{y} = \frac{1}{\sin\theta}$$

## Secant

$$\sec\theta = \frac{r}{x} = \frac{1}{\cos\theta}$$

## Cotangent

$$\operatorname{cotan}\theta \equiv \cot\theta = \frac{x}{y} = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

# Inverse sine function

The inverse sine function $\sin^{-1}(a)$

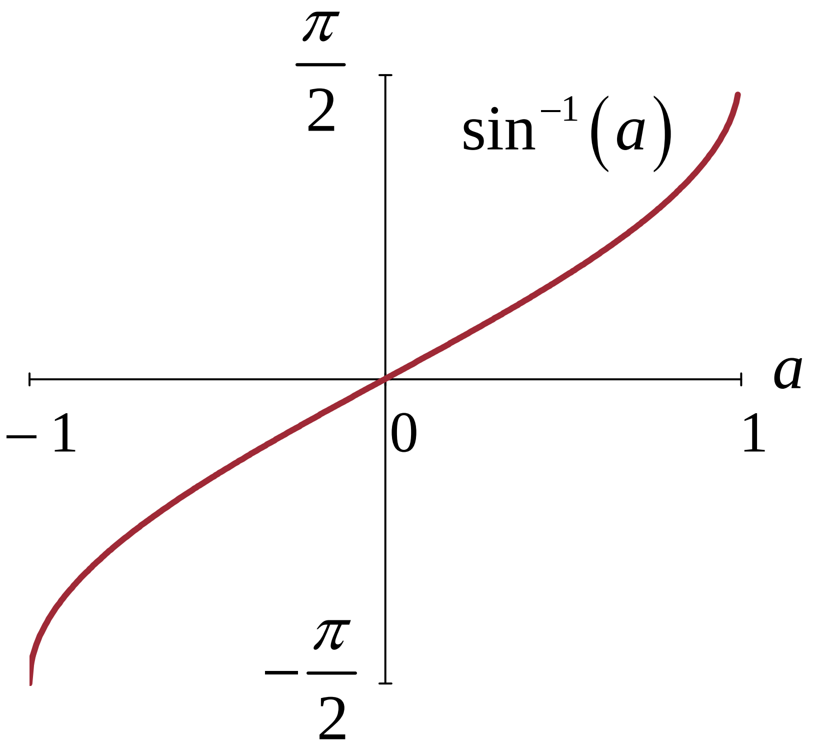
or arcsine function

Pronounced "arc-sine"

works backwards to give the angle from the sine value

If $a = \sin\theta$ then

$$\arcsin(a) = \operatorname{asin}(a) = \sin^{-1}(a) = \theta$$




Note $\sin^{-1}(a)$ does not mean $1/\sin(a)$

The "$^{-1}$" here means "inverse function" not "reciprocal"

# $\sin^2$ and $\cos^2$ functions

However

$$\sin^2\theta = \sin\theta \times \sin\theta = (\sin\theta)^2$$

Not

$$\sin(\sin\theta)$$

Similarly $\cos^2\theta = (\cos\theta)^2$

Only trigonometric functions and their close relatives commonly use this notation