(1)

Rotation - Transformation Matrix

(1) Applying series of equation become much complex, especially if you are performing multiple operation such as "translation after Rotation".

(a) It is also time consuming to determine the mathematical expression every time you need a set of transformation & then implement in Vertex Shader.

Solution: THE TRANSFORMATION MATRIX

ROTATION

STEP-1 UNDERSTANDING MATRIX MULTIPLICATION

Eq 1 $\longrightarrow$ $$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The multiplication of Matrix & Vector is shown in Eq. 1.

A Vector is an object represented by an n-tuple of numbers such a Vertex Cordinates [0.0, 0.5, 1.0]

② We define x', y', z' using the element of the matrix $\&$ vector.

Eq. 2 $\longrightarrow$

$$x' = ax + by + cz$$
$$y' = dx + ey + fz$$
$$z' = gx + hy + iz$$

No. of columns of a matrix must matches the number of rows in a vector.

Lets have a look at our Rotation equation which we saw earlier.

Eq. 3 $\longrightarrow$

$$x' = x\cos\beta - y\sin\beta$$
$$y' = x\sin\beta + y\cos\beta$$
$$z' = z$$

Let us compare Eq. 2 $\&$ Eq. 3

(3)

$$X' = ax + bY + cZ$$

$$X' = X\cos\beta - Y\sin\beta$$

if we sent $a = \cos\beta$, $b = -\sin\beta$ and $c = 0$ the equation become same.

Similarly

$$Y' = dx + eY + fz$$

$$Y' = x\sin\beta + Y\cos\beta$$

Here, $d = \sin\beta$, $e = \cos\beta$ and $f = 0$, we get the same equation.

Similarly, we can compute for Z'.
$Z' = Z$ $(g=0, h=0, i=1)$.

Assigning result to equation (1)

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

See Program "RotatedTriangle_Matrix.JS")

(4) TRANSFORMATION MATRIX : TRANSLATION

Let us compare our translation ~~re~~equation with Eq. 2

$$x' = ax + by + cz$$

$$x' = x + T_x$$

This equation contains a constant term T_x which doesn't exist in the first equation above. We cannot represent the second equation using 3×3 matrix. Therefore, we will use 4×4 matrix to represent the constant term.

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$x' = ax + by + cz + d$$
$$y' = ex + fy + gz + h$$
$$z' = ix + jy + kz + l$$
$$1 = mx + ny + oz + p$$

(5) We already know that

$$X' = X + T_x$$
$$Y' = Y + T_y$$
$$Z' = Z + T_z$$

Now Putting $a=1, b=0, c=0$ & $d=T_x$, Similarly for Y', we have to Put $e=0, f=1, g=0$ & $h=T_y$

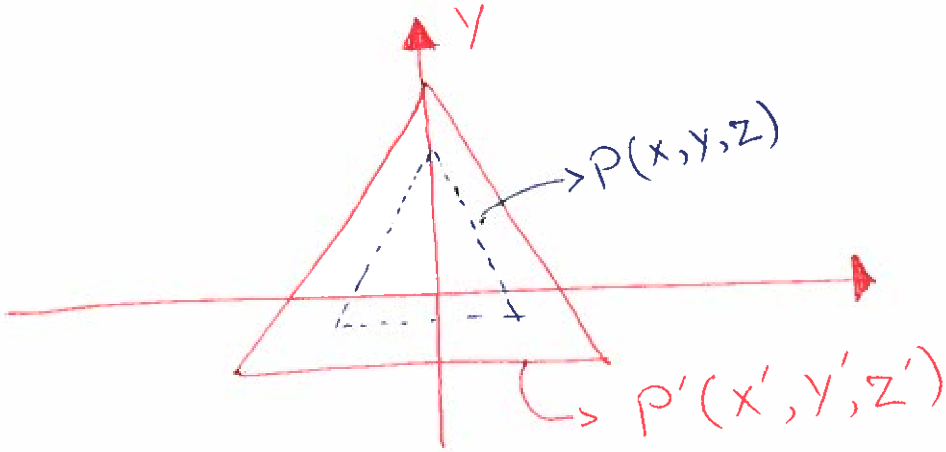
Thus, we can write the matrix as

$$\begin{bmatrix} X' \\ Y' \\ Z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

The above matrix can also be used for both rotation & translation at same time.

$a = \cos\beta$; $b = -\sin\beta$, $c = 0$, $d = T_x$
$e = \sin\beta$, $f = \cos\beta$, $g = 0$, $h = T_y$
& so on.

⑥ Transformation Matrix (Scaling).



$$x' = S_x \times x$$

$$y' = S_y \times y$$

$$z' = S_z \times z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Triangles can be scaled in horizontal as well as vertical directions.