@morganb4

The following questions were answered based on Fourier analysis of a waveform having amplitude 1 over the interval [π - π/2, π + π/2] and amplitude zero everywhere else.

1) Why does the power spectrum of the sampled waveform have so many frequencies?

There is a large number of frequencies on the power spectrum because the box wave is an example of a wave that can be represented using an infinite Fourier series. In order to produce a "boxy" waveform with sharp corners, an infinite number of wave functions with varying frequencies must be added together. The superposition of lots of curved cosine wave functions with varying frequencies can approach a wave with sharp corners.

2) Why does the magnitude of the DC component equal 0.5?

The magnitude of the DC component is 1/2 because the DC component (frequency = 0) represents the average amplitude of the sampled wave, which is 0.5 in this case because the amplitude is 0 for half the samples and 1 for the other half.

3) Why are there no sine components in the waveform's DFT?

There are no sine components in the DFT because the function sin(x) is out of phase with the box graph. On the other hand, the box graph is in phase with cosine. One way to see this is the zeroes of the function cos(x) correspond to the x-values (time values) where the sampled box wave switches from 0 to 1. This means that when the sampled wave has an amplitude of 1, the values of cosine remain on the same side of the x-axis--that is, they do not change sign. So, the summation within the Fourier formula used to calculate cosine amplitudes always has some value associated with it, even as the integer frequency of the cosine filter wave increases. However, the graph of sin(x) crosses the x-axis at t = pi seconds (no matter what frequency is used inside the sine function), which is right in the middle of the box wave. Therefore, the summation in the formula used to calculate amplitudes of the sine components will always equal zero, since half the sine function values will be positive while half are negative. This results in the absence of sine components in the Fourier transform.

Resources:
https://www.youtube.com/watch?v=WTD8jg8fXoE&t=105s
https://www.youtube.com/watch?v=aalANrz7bi8