CAP 6635 Artificial Intelligence
Homework 5 [10+2 pts, Due April 26 2023]

[Homework solutions must be submitted through Canvas. Only pdf, word, and txt files are allowed. If you have multiple pictures, please include all pictures in one Word/pdf file. You can always update your submissions before due date, but only the latest version will be graded.]

Question 1 [2 pts]: Table 1 shows probability values of different events. Using the table to calculate following values and show proof:

- The probability that a persona has no cavity [0.25 pt]
- The probability of no toothache [0.25 pt]
- The joint probability of cavity and no toothache [0.25 pt]
- Calculate conditional probability of no cavity, given the patient has toothache [0.25 pt]
- Calculate conditional probability of no cavity, given the patient does not have toothache [0.25 pt]
- Determine whether cavity and toothache are independent or not, why [0.25 pt]
- Given a patient has cavity, determine whether the tooth probe catch is conditionally independent of toothache or not, why [0.25 pt]
- Given a patient does not have cavity, determine whether the tooth probe catch is conditionally independent of toothache or not, why [0.25 pt]

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	**.108**	**.012**	**.072**	**.008**
¬ cavity	**.016**	**.064**	**.144**	**.576**

Table 1

HW5.1

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	toothache		¬ toothache	
	catch	¬ catch	catch	¬ catch
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

1. Probability that a person has no cavity:

$.016 + .064 + .144 + .576 = .80$

2. Probability no toothache

$.072 + .008 + .144 + .576 = .80$

3. Joint probability of cavity and no toothache.

$.072 + .008 = 0.08$

4. No cavity, has toothache

$\dfrac{.016 + .064}{.108 + .012 + .016 + .064} = \dfrac{0.08}{0.2} = 0.4$

5. No cavity, given no toothache.

$\dfrac{.144 + .576}{.072 + .144 + .008 + .576} = \dfrac{.72}{.8} = 0.9$

6. Are cavity and toothache independent?

ind of A & B

is P of Both A & B = P(A and B)

ind = P(A and B) = P(A)P(B)

cavity = $.108 + .012 + .072 + .008 = 0.2$

toothache = $.108 + .012 + .016 + .064 = 0.2$

$0.2 + 0.2 = 0.4 \neq 0.2 \cdot 0.2 = 0.04$

$0.4 \neq 0.04$

No, they are not independent by proof above.

7. Given a patient has cavity, determine whether probe catch is conditionally ind. of toothache or not, why?

Def of

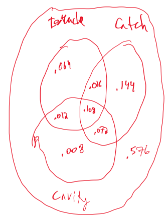
Ind. in original unconditional model

$P(A \cap B) = P(A)P(B)$

Conditional model

$P(A \cap B | C) = P(A|C)P(B|C)$

$= P(A \cap C)P(C)P(B \cap C)P(C)$



A = Catch

B = toothache

C = cavity

$P(A|C) = .108 + .072 = .18$

$P(B|C) = .012 + .108 = .12$

$P(A \cap B | C) = .108$

$.108 = .18 * .12 = 0.0216$

$P(A \cap B | C) = P(A|C)P(B|C)$

$.108 \neq 0.0216$

Are not conditionally independent

8. Given a patient does not have cavity, determine whether probe is cond. ind. of toothache or not, why?

A = catch

B = toothache

C = ¬ cavity

$P(A|C) = .144 + .016 = .16$

$P(B|C) = .064 + .016 = .08$

$P(A \cap B | C) = .016$

$P(A \cap B | C) = P(A|C)P(B|C)$

$.016 = (.16)(.08)$

$.016 \neq 0.0128$

Are not conditionally independent

8.

A = catch

B = toothache

C = ¬ cavity

$P(A) = 0.34$

$P(B) = 0.2$

$P(C) = 0.8$

$P(A \cap C) = .016 + .144 = 0.16$

$P(B \cap C) = .016 + .064 = 0.08$

$P(B|C) =$

Question 2: [1 pts]: Figure 1 shows a Bayesian network, using first letter to denote each named variable, e.g, using T to denote tampering, and complete following questions. Assume x ⊥ y denotes that x are independent of y, x ⊥ y | z denotes that x and y are conditionally independent, given z. Complete Table 2, and use √ to mark correct answers. [1 pt]

	True	**False**
$T \perp F$		
$A \perp S$		
$T \perp S$		
$R \perp A$		
$R \perp A \mid L$		
$L \perp S$		
$L \perp S \mid F$		
$S \perp A \mid F$		
$R \perp S \mid L$		

Table 2

Figure 1

HW5.2

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2

	True	False
$T \perp F$	✓	
$A \perp S$		✓
$T \perp S$	✓	
$R \perp A$		✓
$R \perp A \mid L$	✓	
$L \perp S$		✓
$L \perp S \mid F$	✓	
$S \perp A \mid F$	✓	
$R \perp S \mid L$	✓	

Question 3: [2 pts]: Figure 2 shows a Bayesian network where *r* denotes "rain", *s* denotes "sprinkler", and *w* denotes "wet lawn" (each variable takes binary values 1 or 0). The prior probabilities of rain and sprinkler, and the conditional probabilities values are given as follows:

$p(r = 1) = 0.10$
$p(s = 1) = 0.20$
$p(w = 1|r = 0, s = 0) = 0.001$
$p(w = 1|r = 0, s = 1) = 0.97$
$p(w = 1|r = 1, s = 0) = 0.90$
$p(w = 1|r = 1, s = 1) = 0.99$

- Show joint probability value formula of the whole network, and calculate the joint probability value of $P(r=1, s=1, w=1)$. [0.25 pt]
- Calculate overall probability of lawn is wet, ie., $P(w=1)$ [0.25 pt]
- After observing the lawn is wet, calculate the probability that the sprinkler was left off (i.e., $s=0$). [0.5 pt]
- After observing the lawn is wet, please calculate the probability that there was rain (i.e., $r=1$). [0.5 pt]

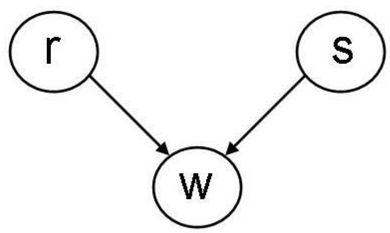

Figure 2

HW5.3

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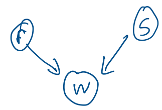


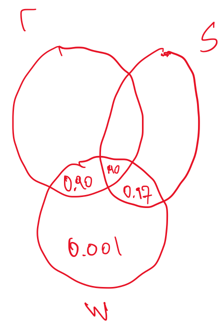
Nodes = random variables
Edges = direct dependence

Convergent path

$P(r=1) = 0.10$ $P(r=0) = 0.90$
$P(s=1) = 0.20$ $P(s=0) = 0.80$

① $P(w=1 \mid r=0, s=0) = 0.001$
② $P(w=1 \mid r=0, s=1) = 0.97$
③ $P(w=1 \mid r=1, s=0) = 0.90$
④ $P(w=1 \mid r=1, s=1) = 0.99$

A: r=1 0.10
B: s=1 0.20
C: w=1
D: r=0 0.90
E: s=0 0.80



1) $P(r,s,w) = P(w|r,s)P(r)P(s)$
$= (0.99)(.1)(.2) = 0.0198$

2)
① $P(r,s,w) = P(w|r,s)P(r)P(s)$
$= (0.001)(0.90)(0.80) = 0.00072$

② $P(r,s,w) = P(w|r,s)P(r)P(s)$
$= (0.97)(0.90)(0.20) = 0.1746$

③ $P(r,s,w) = P(w|r,s)P(r)P(s)$
$= (0.90)(0.10)(0.80) = 0.072$

④ $P(r,s,w) = P(w|r,s)P(r)P(s)$
$= (0.99)(0.1)(0.2) = 0.0198$

$P(w=1) = 0.00072 + 0.1746 + 0.072 + 0.0198$
$= 0.26712 = W$

$.26712(.80) = .213696$
$.26712(.90) = .240408$
$\frac{.26712}{.80} = .3339$
$\frac{.26712}{.9} \approx .2968$

3) $P(w|r,s)P(r)P(s)$
$(0.26712)(0.1)(0.80) = 0.0213696$

Sprinkler off:

$P\left(\frac{0.26712}{(.80*2)}\right) = 0.16695$

Was Rain:

$P(0.26712*(0.1*2)) = 0.053$

Question 4 [2 pts]: Given the following toy dataset with 15 Instances

- Please manually construct a Naïve Bayes Classifier (list the major steps, including the values of the priori probability [0.5 pt] and the conditional probabilities [1.0 pt]. Use *m*-estimate to calculate the conditional probabilities (*m*=1, and *p* equals to 1 divided by the number of attribute values for each attribute).
- Please use your Naïve Bayes classifier to determine whether a person should play tennis or not, under conditions that "Outlook=Overcast & Temperature=Hot & Humidity =Normal& Wind=Weak". [0.5 pt]

ID	Outlook	Temperature	Humidity	Wind	Class
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
3	Overcast	Hot	High	Weak	Yes
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
7	Overcast	Cool	Normal	Strong	Yes
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
10	Rain	Mild	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes
12	Overcast	Mild	High	Strong	Yes
13	Overcast	Mild	Normal	Weak	No
14	Rain	Hot	High	Strong	Yes
15	Rain	Mild	High	Strong	No

HW5.4

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Outlook

	N	Y
Sunny	3/6	2/9
overcast	1/6	3/9
rain	2/6	4/9

	N	Y
S	1/2	2/9
O	1/6	1/3
r	1/3	4/9

No's = 6 = $\frac{6}{15} = \frac{2}{5} = 0.4$

Yes's = 9 = $\frac{9}{15} = \frac{3}{5} = 0.6$

Data set = 15

sunny = $\frac{5}{15} = \frac{1}{3}$

overcast = $\frac{4}{15} = \frac{4}{15}$

rain = $\frac{6}{15} = \frac{2}{5}$

Temperature

		N	Y
Hot	= 4/15	1/3	2/9
cool	= 4/15	1/6	3/9 = 1/3
Mild	= 7/15	1/2	4/9

Humidity

		N	Y
High	= 8/15	4/6 = 2/3	4/9
Normal	= 7/15	1/3	5/9

Wind

		N	Y
weak	= 8/15	3/6 = 1/2	5/9
strong	= 7/15	3/6 = 1/2	4/9

Overcast Hot Normal Weak

NO

$\frac{4}{15}\ \frac{1}{6}\quad \frac{4}{15}\ \frac{1}{3}\quad \frac{7}{15}\ \frac{1}{3}\quad \frac{8}{15}\ \frac{1}{2} = \frac{896}{5467500} = 0.00016$

Yes

$\frac{4}{15}\ \frac{1}{3}\quad \frac{4}{15}\ \frac{2}{9}\quad \frac{7}{15}\ \frac{5}{9}\quad \frac{8}{15}\ \frac{5}{9} = \frac{44800}{110,716,875} = 0.0004$

Yes, they should play tennis

it has a higher value

Question 5 [1 pt]: An agent (or a player) is playing a game which involves a fair *n*-sided dice (*i.e.*, a dice with *n* faces, where n=2, 3, 4, …. E.g., A dice with two faces would be a coin). The agent will start the game until it stops, per rules below:
For each round r = 1, 2, 3, …..

- The agent chooses to stay or quit.
- If the agent quits, he/she receives \$10 and the game stops.
- If the agent stays, he/she receives \$4 and then rolls the *n*-sided dice.
  - If the dice results in one specific face, the game stops (the specific face is pre-determined before the game starts).
  - Otherwise, continue to the next round.

- Write total reward function (i.e., the total \$ the agent receives) for games with exact one, two, and three rounds, respectively [0.5 pt]
- Define and draw the game as a Markov decision process (MDP). Your solutions must have states (S), actions (A), transition probabilities (P), and rewards (R) [0.5 pt]

Question 6 [2 pts]: Figure 3.a shows a 4x4 robot navigation field. The shade squares are obstacles, and the two cells [4,2] and [4,3] are terminal states, and the values showing are the reward of the terminal states (each cell is also a state). The reward for each of the rest states (except the obstacles) is -0.05. In order to train a robot to navigate in the field, a stochastic transition model showing in Figure 3.b is used. At any particular location, say [1,1], if the robot cannot move in a certain direction (e.g., there is wall or obstacle), it will remain in the same position. For example, when the robot is at [1,1], it cannot move to the left because of the wall. The discount γ=1, and the initial utility values of each state are 0.

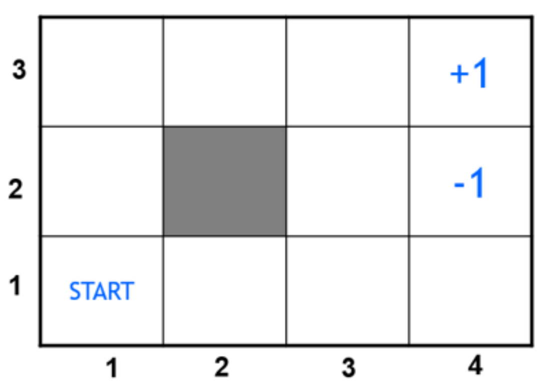

Figure 3.a

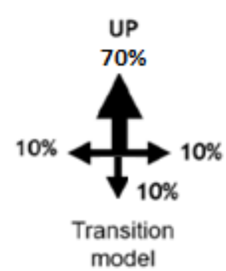

Figure 3.b

(1) What is the meaning of the transition model showing in Figure 3.b **[0.25 pt]**

(2) What are the objective of using such as transition model (with stochastic process) for learning **[0.25 pt]**

(3) Use value iteration algorithm to find all cells' utility values after the FIRST iteration (exclude terminal states and obstacles). Solutions must show calculations for cells [3,1], [3,2], [3,3] and [4,1], respectively. For remaining cells, you can just report their results **[1.5 pt]**

Extra Credit [2 pts]: Figure 4 shows a Bayesian network which is similar to Figure 2, but w_1 denotes your lawn, and w_2 denotes neighbor's lawn (each variable takes binary values 1 or 0). In this case, rain will cause both yours and your neighbor's lawn being wet, whereas your sprinkler will only cause your lawn to be wet. The prior probabilities and conditional probability values are given as follows:

$p(r = 1) = 0.10$

$p(s = 1) = 0.20$

$p(w_1 = 1|r = 0, s = 0) = 0.001$

$p(w_1 = 1|r = 0, s = 1) = 0.97$

$p(w_1 = 1|r = 1, s = 0) = 0.90$

$p(w_1 = 1|r = 1, s = 1) = 0.99$

$p(w_2 = 1|r = 1) = 0.90$

$p(w_2 =1|r=0)=0.1$

- Show joint probability value formula of the whole network, and calculate the joint probability value of $P(r=1, s=1, w_1=1, w_2=1)$. [0.25 pt]
- Calculate overall probability of yours and your neighbor's lawn are wet, i.e., $P(w_1=1, w_2=1)$ [0.25 pt]
- After observing that yours and your neighbors' lawn are both wet, calculate the probability that the sprinkler was left on (i.e., s=1). [0.5 pt]
- After observing that yours and your neighbors' lawn are both wet, calculate the probability that there was rain (i.e., r=1). [0.5 pt]

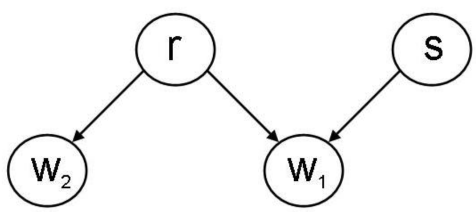

Figure 4