1. In how many ways can 3 coins, 3 mathematics books and 1 chemistry book be arranged in a bookcase?

(a) the books are arranged in any order?

$3 + 3 + 1 = 6$

$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

(b) The coin and math books must be together.

$3 \cdot 2 \cdot 1 \quad 3! \cdot 1$

3 coins, 2 math books, 1 chemistry book → 3 coins, 3 math books, 1 chemistry book

Number the ways the coins can be arranged in order:

6 - chemistry $(3)(2)(1)(2)(3)$;
5 - math
4 - chemistry $(3)(2)(1)(1)(2)$

6 5 4
6 1 3
1 5 6
3 5 1
3 1 6

→ 6 different ways the 3 coins can be arranged, therefore:

$(3!)(3!)(6) = 72$

c. The coins must be together, but the other books can be arranged in any order?

$\underbrace{\frac{3 \cdot 2 \cdot 1}{N}}_{\text{coins}} \cdot \underbrace{\frac{3 \cdot 2 \cdot 1}{\text{others}}}_{\text{math}} = 6 \times 6 = 72$

Total: 72

2. A committee of 5 persons is selected randomly from a group of 5 men and 6 women.

(a) Find the probability that the committee consists of 3 men and 2 women.

$_nC_r = \frac{n!}{r!(n-r)!}$ = Number of combinations

n: total objects in set
r: number of chosen objects in set

men: $_5C_3 = \frac{5!}{3!(5-3)!} = \frac{5!}{3!2!} = \frac{120}{6} = 10$

women: $_6C_2 = \frac{6!}{2!(6-2)!} = \frac{6 \cdot 5 \cdot 4 \cdot 3}{4} = \frac{720}{6} = 120$

Total combinations: $_{11}C_5 = \frac{11!}{5!(11-5)!} = \frac{\text{numerator}_{11}}{\text{denominator}_5} = \frac{39916800}{720} = 55440$

$\frac{10 \times 120}{55440} = 0.3116 \approx 0.40$

(b)

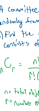
women: $_6C_5 = \frac{6!}{5!(6-5)!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5!} = \frac{30720}{120} = 252$

$\frac{252}{55440} = 0.0833 \approx 0.01$

3. Draw Venn diagrams and shade the areas corresponding to the following sets.

(A) $(A \cup B \cup C) \cap (\bar{A} \cup \bar{B} \cup \bar{C})$

(b) $[(A \cup B) \cap C] \cup (\overline{A \cup C})$



4. A customer visiting a certain department store will purchase a suit with a probability of 0.23, a shirt with probability 0.30, and a tie with probability 0.32. The customer will purchase both a suit and a shirt with prob. 0.11, both a suit and tie with prob. 0.14, and both a shirt and tie w/ prob. 0.10. A customer will purchase all 3 items with prob. 0.06

(a) What is the prob. that the customer purchases at least one of these items?

$P(\text{Suit}) = 0.23$

$P(\text{Shirt}) = 0.30$

$P(\text{Tie}) = 0.32$

Add all these together =

$0.23 + 0.30 + 0.32 = 0.85$

(b) What is the probability that the customer purchases exactly two of these items?

$P(A)$ = Purchases a suit and shirt = 0.11
$P(B)$ = Purchases a suit and tie = 0.14
$P(C)$ = Purchases a shirt and tie = 0.10
$P(D)$ = Purchases all three = 0.06

$P(A) - P(D) = 0.11 - 0.06 = 0.05$
$P(B) - P(D) = 0.14 - 0.06 = 0.08$
$P(C) - P(D) = 0.10 - 0.06 = 0.04$

Sum these up:

$0.05 + 0.08 + 0.04 = 0.17$

5. In an experiment, A, B, C, and D are events w/ probabilities $P(A \cup B) = \frac{5}{8}$

$P(A) = \frac{3}{8}$
$P(C \cap D) = \frac{1}{3}$
$P(C) = \frac{1}{2}$

(a) The intersection two disjoint sets is 0, so

$A \cap B = 0$

(b) A and B are disjoint

$P(A \cup B) = \frac{5}{8}$
$P(A) = \frac{3}{8}$
and
$P(A \cap B) = 0$ ∴ $P(B) = P(\bar{A}) = \frac{5}{8}$

(c) $P(A \cap \bar{B})$

The two are disjoint so the intersection of $P(A \cap B) = 0$ ∴ $P(A \cap \bar{B}) =$

$P(A) = \frac{3}{8}$

(d) $P(C \cup D)$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ $\quad P(B) \neq 0$

↓ (if you that it has occurred) Multiplicate

$P(A \cap B) = P(A|B)P(B)$

B has occured

★ Independent, so Multiplicate

$P(C|D) = \frac{P(C \cap D)}{P(D)}$

$P(D|C) = \frac{P(D \cap C)}{P(C)} =$

$P(D|C) = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{1}{3} \cdot \frac{2}{1} = \frac{2}{3}$

$P(D|C) = \frac{2}{3}$

$P(C \cap D) = P(C) \quad P(D) = \frac{1}{3}$

$P(C) = \frac{1}{2}$

$\frac{1}{2} \times P(D) = \frac{1}{3}$
$\frac{1}{2} \quad \frac{1}{2}$

$P(D) = \frac{1}{3} \cdot \frac{2}{1} = \frac{2}{3}$

$P(C \cap D) = P(C)P(D)$

$\frac{1}{3} = \frac{1}{2} \cdot P(D)$
$\frac{1}{2}$

$P(C \cup D) = (P(C) + P(D))$

$P(C \cup D) = (\frac{1}{3} + \frac{2}{3}) - (\frac{1}{2} \times \frac{2}{3}) =$

$\cup \qquad \cap$

$\frac{1}{2} + \frac{2}{3} - \frac{1}{3} =$

$\frac{1}{2} + \frac{1}{3} =$

$\frac{3}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{2}{3}$

$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

$P(C \cup D) = \frac{5}{6}$

(e) By definition,

disjoint means

$P(A \cap B) = P(A)P(B) = 0$

independent means

$P(A \cap B) = P(A)P(B) \neq 0$

We are given that A and B are disjoint, therefore they cannot be independent.