Assignment4

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Input x_1	Input x_2	Desired Output (d)
1	1	1
1	0	1
0	1	0
-1	-1	0
-1	0	0
-1	-1	0

learning rate = 0.1
initial weights = (1,1,1)
these initial weights randomly

cost function = $J_\theta(y, d)$
- calculates the difference between good the perceptron output (y) and desired output (d)
- mean squared error

θ = neuron parameters
$(w_0, w_1, \dots, w_n)$

$$J_\theta(y,d) = \frac{1}{2N}\sum_{n=1}^{N}(d_n - y_n)^2 = \frac{1}{2N}\sum_{n=1}^{N}(d_n - \sum_{i=0}^{n} w_i \cdot x_i)^2$$

Epoch 1

x_0	x_1	x_2	d	x_0	w_1	w_2	$y = v$	$\Delta w = \eta(d-y)x$	$w = w + \Delta w$
1	1	1	1	1	1	1	$1(1)+1(1)+1(1) = 3$	$1:\ 1\cdot 1\cdot -2$	$1:\ 1-5=-3$
1	1	0	1	1	1	1	$1(1)+1(1)+0(1) = 2$	$2:\ 1\cdot 1\cdot -2$	$2:\ 1-5=-3$
1	0	1	0	1	1	1	$1(1)+0(1)+1(1) = 2$	$0:\ 1\cdot 1\cdot 0 = 0$	$0:\ 1-0=1$
1	-1	-1	0	1	1	1	$1(1)+(-1)(1)+(-1)(1) = -1$	$0:\ 1\cdot 1\cdot 0 = 0$	$0:\ 1-0=1$
1	-1	0	0	1	1	1	$1(1)+(-1)(1)+0(1) = 0$	$0:\ 1\cdot 1\cdot 0 = 0$	$0:\ 1-0=1$
1	-1	-1	0	1	1	1	$1(1)+(-1)(1)+(-1)(1) = -1$	$0:\ 1\cdot 1\cdot 0 = 0$	$0:\ 1-0=1$

$\Delta w = \frac{\eta}{N} = \frac{0.1}{6}(-5, -3, -6) = -0.083, -0.05, -0.1$

$w = w + \Delta w = (1,1,1) + (-0.083, -0.05, -0.1)$

$(0.917, .95, .9)$

$J_\theta = \frac{1}{2\cdot 6}\left((-1-3)^2 + (1+3)^2 + (0-2)^2 + (0+1)^2 + (0-0)^2 + (0-1)^2\right) = 4+1+4+1+0+1 = 11\left(\frac{1}{12}\right) = 0.917$

Epoch 2

$(0.917, .95, .9)$

x_0	x_1	x_2	d	x_0	w_1	w_2	$y = v$	$\Delta w = \eta(d-y)x$	$w = w + \Delta w$
1	1	1	1	1			$0.917 + .95 + .9 = 2.767$	$1:\ 1\cdot 1\cdot -1.767 = -1.767$	$1:\ 0.917 - 1.767 = -.850$
1	1	0	1	1			$0.917 + .95 + 0 = 1.867$	$2:\ 1\cdot 1\cdot -1.867 = -1.867$	$2:\ .95 - 1.867 = -.917$
1	0	1	0	1			$0.917 + 0 + .9 = 1.817$	$0:\ 1\cdot 1\cdot -1.817 = -1.817$	$0:\ .9 - 1.817 = -.917$
1	-1	-1	0	1			$0.917 + -.95 - .9 = -.933$	$0:\ 1\cdot 1\cdot .933 = .933$	$0:\ -.917 + .933 = .017$
1	-1	0	0	1			$0.917 + -.95 + 0 = -.033$	$0:\ 1\cdot 1\cdot -.033 = .033$	$0:\ .017 + .033 = .050$
1	-1	-1	0	1			$0.917 + -.95 + -.9 = -0.933$	$0:\ 1\cdot 1\cdot .933 = .933$	$0:\ .050 + .933 = .983$

$\Delta w = \frac{0.1}{6}(-4.352, -4.467, -5.384) = -0.07253, -0.07445, -0.0897$

$w = w + \Delta w = (0.917, .95, .9) + (-0.07253, -0.07445, -0.0897)$

$= (0.8445, 0.8756, 0.810)$

$J_\theta = \frac{1}{2\cdot 6}\left((-1.767)^2 + (-0.867)^2 + (-1.817)^2 + (.933)^2 + (0.033)^2 + (-0.967)^2\right)$

$= \frac{1}{12}(3.12 + 0.7519 + 3.301 + .8707 + .0017 + .750)$

$= \frac{1}{12}(8.7961) \approx 0.733$

$(1,1)\ \ 0.917 + 1(.95) + 1(.9) = 2.767$

$(1,0)\ \ 0.917 + 1(.95) + 0 = 1.867$

$(0,1)\ \ 0.917 + 0 + .9 = 1.817$

$(-1,-1)\ \ 0.917 + -.95 - .9 = -.933$

$(-1,0)\ \ 0.917 + -.95 + 0 = -.033$

$(-1,1)\ \ 0.917 + -.95 + .9 = 0.867$

Epoch 3

$(0.845, 0.876, 0.810)$

x_0	x_1	x_2	d	x_0	w_1	w_2	$y = v$	$\Delta w = \eta(d-y)x$	$w = w + \Delta w$
1	1	1	1	1			$.845 + .876 + .810 = 2.531$	$1:\ 1\cdot 1\cdot -1.531 = -1.531$	$1:\ .845 - 1.531 = -.686$
1	1	0	1	1			$.845 + .876 + 0 = 1.721$	$2:\ 1\cdot 1\cdot -1.721 = -1.721$	$2:\ .876 - 1.721 = -.845$
1	0	1	0	1			$.845 + 0 + .810 = 1.655$	$0:\ 1\cdot 1\cdot -1.655 = -1.655$	$0:\ .810 - 1.655 = -.845$
1	-1	-1	0	1			$.845 + -.876 - .810 = -.841$	$0:\ 1\cdot 1\cdot .841 = .841$	$0:\ -.845 + .841 = -.004$
1	-1	0	0	1			$.845 + -.876 + 0 = -.031$	$0:\ 1\cdot 1\cdot .031 = .031$	$0:\ -.004 + .031 = .027$
1	-1	-1	0	1			$.845 + -.876 - .810 = -.841$	$0:\ 1\cdot 1\cdot .841 = .841$	$0:\ .027 + .841 = .868$

$\Delta w = \frac{0.1}{6}(-3.194, -3.395, -4.886) = -0.0532, -0.0565, -0.0814$

$w = w + \Delta w = (0.845, 0.876, 0.810) + (-0.0532, -0.0565, -0.0814)$

$= (.7814, .8197, 0.7279)$

$J_\theta = \frac{1}{2\cdot 6}\left((-1.531)^2 + (-0.721)^2 + (-1.655)^2 + (0.041)^2 + (0.031)^2 + (-0.729)^2\right)$

$= \frac{1}{12}(2.34 + 0.5198 + 2.74 + .007 + .0016 + .6618)$

$= \frac{1}{12}(6.915) \approx 0.576$

$(1,1)\ \ 0.845 + 1(.876) + 1(.810) = 2.531$

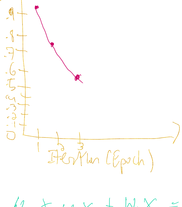
$(1,0)\ \ 0.845 + 1(.876) + 0 = 1.721$

$(0,1)\ \ 0.845 + 0 + .810 = 1.655$

$(-1,-1)\ \ 0.845 + -.876 - .810 = -0.841$

$(-1,0)\ \ 0.845 + -.876 + 0 = -0.031$

$(-1,1)\ \ 0.845 + -.876 + .810 = .079$



The Neuron is learning because the cost function is decreasing. As more epochs ensue, the cost function should approach 0.

$w_0 + w_1x_1 + w_2x_2 = 0$

$0.845 + \cancel{0} + .810x_2 = 0$

$-.845 \qquad -.845$

$\frac{.810x_2}{.810} = \frac{-.845}{.810}$

$x_2 = -1.043$

$(0, -1.043)$

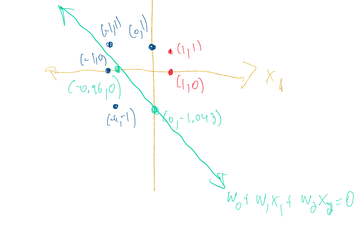
$w_0 + w_1x_1 + w_2x_2 = 0$

$.845 + .876x_1 + \cancel{0} = 0$

$\frac{.876x_1}{.876} = \frac{-.845}{.876}$

$x_1 = -0.96$

$(-0.96, 0)$



	1	0	(f+1-1)
1	2	0	2
0	2	2	4

	1	0
1	TP	FN
0	FP	TN

$\text{Accuracy} = \frac{TP + TN}{All} = \frac{4}{6} = 0.667$

$\text{Sensitivity} = \frac{TP}{TP + FN} = \frac{2}{2} = 1$

$\text{Specificity} = \frac{TN}{TN + FP} = \frac{2}{4} = \frac{1}{2} = .5$