Homework 1

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1 Problem Set 1

1. 2a. A small community consists of 10 women, each of whom has 3 children. If one woman and one of her children are to be chosen as mother and child of the year, how many different choices are possible?

$$10 * 3 = 30 \tag{1}$$

2. 2b. A college planning committee consists of 3 freshmen, 4 sophomores, 5 juniors, and 2 seniors. A subcommittee of 4, consisting of 1 person from each class, is to be chosen. How many different subcommittees are possible?

$$3 * 4 * 5 * 2 = 120 \tag{2}$$

3. 2c. How many different 7-place license plates are possible if the first 3 places are to be occupied by letters and the final 4 by numbers?

$$26 * 26 * 26 * 10 * 10 * 10 * 10 = 175,760,000 \tag{3}$$

4. 2d. How many functions defined on n points are possible if each functional value is either 0 or 1? Let the points be i = 1,2,...,n. Since *f(i)* must be either 0 or 1 for each i, it follows that there are 2^n possible functions.

$$2^n \tag{4}$$

5. 2e. In example 2c, how many license plates would be possible if repetition among letters or numbers were prohibited?

$$26 * 25 * 24 * 10 * 9 * 8 * 7 = 78.624,000 \tag{5}$$

2 Problem Set 2

1. 3a. How many different batting orders are possible for a baseball team consisting of 9 players?

$$n(n-1)(n-2)...3*2*1 = n! \tag{6}$$

$$9! = 9*8*7*6*5*4*3*2*1 = 362,880 \tag{7}$$

There are 362,880 possible batting orders.

2. 3b. A class in probability theory consists of 6 men and 4 women. An examination is given, and the students are ranked according to their performance. Assume that no two students obtain the same score.
(a) How many different rankings are possible?

$$6+4 = 10 \tag{8}$$

$$10! = 3,628,800 \tag{9}$$

(b) If the men are ranked just among themselves and the women just among themselves, how many different rankings are possible?

$$\text{men} = 6! \tag{10}$$

$$\text{women} = 4! \tag{11}$$

$$6!*4! = (720)(24) = 17,280 \tag{12}$$

3. 3c. Ms. Jones has 10 books that she is going to put on her bookshelf. Of these, 4 are mathematics books, 3 are chemistry books, 2 are history books, and 1 is a language book. Ms. Jones wants to arrange her books so that all the books dealing with the same subject are together on the shelf. How many different arrangements are possible?

$$\text{mathematics books} = 4! \tag{13}$$

$$\text{chemistry books} = 3! \tag{14}$$

$$\text{history books} = 2! \tag{15}$$

$$\text{language books} = 1! \tag{16}$$

$$\text{total possible subject arrangements} = 4! \tag{17}$$

$$4!*4!*3!*2!*1! = 24*24*6*2*1 = 6,912 \tag{18}$$

4. 3d. How many different letter arrangements can be formed from the letters PEPPER?

$$\text{total permutations (if they were distinct)} = 6! \tag{19}$$

$$\text{letter P permutations (alike)} = 3! \tag{20}$$

$$\text{letter E permutations (alike)} = 2! \tag{21}$$

$$\text{letter R permutations (alike)} = 1! \tag{22}$$

$$\frac{6!}{3! * 2! * 1!} = \frac{720}{12} = 60 \tag{23}$$

5. 3e. A chess tournament has 10 competitors, of which 4 are Russian, 3 are from the United States, 2 are from Great Britain, and 1 is from Brazil. If the tournament result lists just the nationalities of the players in the order in which they placed, how many outcomes are possible?

$$\text{Total competitors (if they were distinct)} = 10! \tag{24}$$

$$\text{Russian permutations (alike)} = 4! \tag{25}$$

$$\text{US permutations (alike)} = 3! \tag{26}$$

$$\text{Great Britain permutations (alike)} = 2! \tag{27}$$

$$\text{Brazil permutations (alike)} = 1! \tag{28}$$

$$\frac{10!}{4! * 3! * 2! * 1!} = \frac{3,628,800}{288} = 12,600 \tag{29}$$

6. 3f. How many different signals, each consisting of 9 flags hung in a line, can be made from a set of 4 white flags, 3 red flags, and 2 blue flags if all flags of the same color are identical?

$$\text{Total flags (if they were distinct)} = 9! \tag{30}$$

$$\text{white flags (alike)} = 4! \tag{31}$$

$$\text{red flags (alike)} = 3! \tag{32}$$

$$\text{blue flags (alike)} = 2! \tag{33}$$

$$\frac{9!}{4! * 3! * 2!} = \frac{362,880}{288} = 1,260 \tag{34}$$

3 Problem Set 3

1. 4a. A committee of 3 is to be formed from a group of 20 people. How many different committees are possible?

$$\binom{n}{r} = \frac{n!}{(n-r)! * r!} \tag{35}$$

$$\binom{20}{3} = \frac{20!}{17! * 3!} \tag{36}$$

$$\binom{20}{3} = \frac{20 * 19 * 18}{3 * 2 * 1} = 1,140 \tag{37}$$

2. 4b. From a group of 5 women and 7 men, how many different committees consisting of 2 women and 3 men can be formed? What if 2 of the men are feuding and refuse to serve on the committee together?

$$\text{women} = \binom{\text{total women}}{\text{women selected}} = \binom{5}{2} \tag{38}$$

$$\binom{5}{2} = \frac{5!}{3! * 2!} \tag{39}$$

$$\binom{5}{2} = \frac{5 * 4}{2 * 1} = 10 \tag{40}$$

$$\text{women} = 10 \tag{41}$$

$$\text{men} = \binom{\text{total men}}{\text{men selected}} = \binom{7}{3} \tag{42}$$

$$\binom{7}{3} = \frac{7!}{4! * 3!} \tag{43}$$

$$\binom{7}{3} = \frac{7 * 6 * 5}{3 * 2 * 1} = \frac{210}{6} = 35 \tag{44}$$

$$\text{men} = 35 \tag{45}$$

$$\text{women} * \text{men} = 10 * 35 = 350 \tag{46}$$

There are 350 possible committees of 2 women and 3 men

Second Part

$$\binom{\text{men that are feuding}}{\text{feuding men selected}} = \binom{2}{2} = \frac{2!}{0! * 2!} = \frac{2!}{2!} = 1 \tag{47}$$

$$\binom{\text{total remaining men}}{\text{men selected}} = \binom{5}{1} = \frac{5!}{4! * 1!} = \frac{5}{1!} = \frac{5}{1} = 5 \tag{48}$$

$$\binom{2}{2} * \binom{5}{1} = 1 * 5 = 5 \tag{49}$$

Total groups of men (including those feuding):

$$\binom{7}{3} = \frac{7 * 6 * 5}{3 * 2 * 1} = \frac{210}{6} = 35 \tag{50}$$

$$\text{Total groups of men - groups that contain the feuding men} = \tag{51}$$

$$35 - 5 = 30 = \text{groups without feuding men} \tag{52}$$

$$\text{women} * \text{groups without feuding men} = 10 * 30 = 300 \tag{53}$$

There are 300 possible committees of 2 women and 3 men that aren't feuding.

4 Problem Set 4

1. 5a. A police department in a small city consists of 10 officers. If the department policy is to have 5 of the officers patrolling the streets, 2 of the officers working full time at the station, and 3 of the officers on reserve at the station, how many different divisions of the 10 officers into the 3 groups are possible?

$$\text{Total officers} = 10! \tag{54}$$

$$\text{Patrolling} = 5! \tag{55}$$

$$\text{Station} = 2! \tag{56}$$

$$\text{Reserve} = 3! \tag{57}$$

$$\frac{10!}{5! * 2! * 3!} = \frac{3,628,800}{1,440} = 2,520 \tag{58}$$

2. 5b. Ten children are to be divided into an A team and a B team of 5 each. The A team will play in one league and the B team in another. How many different divisions are possible?

$$\text{Total kids} = 10! \tag{59}$$

$$\text{Team A} = 5! \tag{60}$$

$$\text{Team B} = 5! \tag{61}$$

$$\frac{10!}{5! * 5!} = \frac{3,628,800}{14,400} = 252 \tag{62}$$

3. 5c. In order to play a game of basketball, 10 children at a playground divide themselves into two teams of 5 each. How many different divisions are possible?

Different than the previous one because the order is irrelevant, no Team A or Team B. So we will divide by (number of teams)!

$$\text{Total kids} = 10! \tag{63}$$

$$\text{a group} = 5! \tag{64}$$

$$\text{a group} = 5! \tag{65}$$

$$\text{number of teams} = 2! \tag{66}$$

$$\frac{\frac{10!}{5!*5!}}{2!} = \frac{\frac{3,628,800}{14,400}}{2} = \frac{252}{2} = 126 \tag{67}$$

5 Problem Set 5

1. 6a. How many distinct non-negative integer-valued solutions of $x_1+x_2 = 3$ are possible?

$$x_1 + x_2 + ... + x_r = n \tag{68}$$

$$\binom{n+r-1}{r-1} \tag{69}$$

$$\binom{3+2-1}{2-1} \tag{70}$$

$$\frac{4}{1} = 4 \text{ possible solutions} \tag{71}$$

$$(0,3)(1,2)(2,1)(3,0) \tag{72}$$

2. 6b. An investor has 20 thousand dollars to invest among 4 possible investments. Each investment must be in units of a thousand dollars. If the total 20 thousand is to be invested, how many different investment strategies are possible? What if not all the money need to be invested?

$$x_1 + x_2 + ... + x_r = n \tag{73}$$

$$\binom{n+r-1}{r-1} \tag{74}$$

$$x_1 + x_2 + x_3 + x_4 = 20 \tag{75}$$

$$\binom{20+4-1}{4-1} \tag{76}$$

$$\binom{23}{3} \tag{77}$$

$$\binom{23!}{3! * 20!} \tag{78}$$

$$\binom{23 * 22 * 21}{3 * 2 * 1} = \frac{10,626}{6} = 1,771 \tag{79}$$

There are 1,771 possible investment strategies.

If not all the money needs to be invested, you include another slot that represents unused money. (x_5)

$$x_1 + x_2 + x_3 + x_4 + x_5 = 20 \tag{80}$$

$$\binom{20+5-1}{5-1} \tag{81}$$

$$\binom{24}{4} \tag{82}$$

$$\left(\frac{24!}{4! * 20!}\right) \tag{83}$$

$$\left(\frac{24 * 23 * 22 * 21}{4 * 3 * 2 * 1}\right) = \frac{255,024}{24} = 10,626 \tag{84}$$

There are 10,626 possible investment strategies when not all the money needs to be invested.

3. 6c. How many terms are there in the multinomial expansion of $(x_1 + x_2 + ... + x_r)^n$?

$$\text{I need help with this question.} \tag{85}$$