Stochastic Models for Computer Science

Morgan Benavidez

August 2022

1 Combinatorial Analysis

Combinatorial Analysis - The mathematical theory of counting.

2 The Basic Principle of Counting

The basic principle of counting - Suppose that two experiments are to be performed. Then if experiment 1 can result in any one of m possible outcomes and if, for each outcome of experiment 1, there are n possible outcomes of experiment 2, then together there are mn possible outcomes of the two experiments.
The generalized basic principle of counting - If r experiments that are to be performed are such that the first one may result in any of *n1* possible outcomes; and if, for each of these *n1* possible outcomes, there are *n2* possible outcomes of the second experiment; and if, for each of the possible outcomes of the first two experiments, there are *n3* possible outcomes of the third experiment; and if ..., there there is a total of *n1 * n2 ... nr* possible outcomes of the r experiments.

1. 2a. A small community consists of 10 women, each of whom has 3 children. If one woman and one of her children are to be chosen as mother and child of the year, how many different choices are possible?

$$10 * 3 = 30 \tag{1}$$

2. 2b. A college planning committee consists of 3 freshmen, 4 sophomores, 5 juniors, and 2 seniors. A subcommittee of 4, consisting of 1 person from each class, is to be chosen. How many different subcommittees are possible?

$$3 * 4 * 5 * 2 = 120 \tag{2}$$

3. 2c. How many different 7-place license plates are possible if the first 3 places are to be occupied by letters and the final 4 by numbers?

$$26 * 26 * 26 * 10 * 10 * 10 * 10 = 175,760,000 \tag{3}$$

4. 2d. How many functions defined on n points are possible if each functional value is either 0 or 1? Let the points be i = 1,2,...,n. Since *f(i)* must be either 0 or 1 for each i, it follows that there are 2^n possible functions.

$$2^n \tag{4}$$

5. 2e. In example 2c, how many license plates would be possible if repetition among letters or numbers were prohibited?

$$26 * 25 * 24 * 10 * 9 * 8 * 7 = 78.624,000 \tag{5}$$

Thermodynamics - The study of the flow of energy.

Kinetic Energy - Energy associated with motion.

Potential Energy - Energy associated with position or composition.

Law of Conservation of Energy - Energy can be neither created or destroyed; the total energy of the universe is constant.

Endothermic - Absorbs heat.

Exothermic - Releases heat.

Specific Heat Capacity - The amount of heat required to raise the temperature of 1 g of a substance by 1 K.

When energy flows into a system from the surroundings, the energy of the system *increases*, and the energy of the surroundings *decreases*.

When energy flows out of a system into the surroundings, the energy of the system *decreases*, and the energy of the surroundings *increases*.

Examples of types of energy:

Kinetic Energy	*Potential Energy*
Thermal	Gravitational
Mechanical	Chemical Potential
Electrical	Electrostatic
Sound	-

Formula to calculate heat:

$$q = mc\Delta T \tag{6}$$

q = heat added to or released by a sample (J)
m = mass of the sample (g)
c = specific heat of the substance $\frac{J}{g*K}$

ΔT = temperature change (K)

Heat is the transfer of energy from hotter to colder and it is an extensive property.

Temperature is the measure of the intensity of heat and it is an intensive property. Concentration of energy.

3 Specific Heat Practice

1. If the temperature of 34.4 g of ethanol increases from 25°C to 78.8°C, how much heat has been absorbed by the ethanol? The specific heat of ethanol is 2.44 J/g* °C.

$$q = mc\Delta T \tag{7}$$

$$\Delta T = 78.8°C - 25°C = 53.8°C \tag{8}$$

$$q = (34.4g) * (2.44\frac{J}{g *° C}) * (53.8°C) \tag{9}$$

$$q = 4,515.76J \tag{10}$$

2. A 40 g sample of water absorbs 500 Joules of energy. How much did the water temperature change?

$$q = mc\Delta T \tag{11}$$

$$c_{H_2O} = 4.184\frac{J}{g * K} \tag{12}$$

$$\Delta T = \frac{q}{mc} = \frac{500J}{(40g) * (4.184\frac{J}{g*K})} \tag{13}$$

$$\Delta T = 2.987K \approx 3K \tag{14}$$

3. Equal amounts of heat are absorbed by 10 g solid samples of four different metals: aluminum, lead, tin, and iron. Of these four, which will exhibit the smallest change in temperature?

$$q = mc\Delta T \tag{15}$$

Aluminum Specific Heat

$$c_{Al} = 0.9\frac{J}{g * K} \tag{16}$$

Lead Specific Heat

$$c_{Pb} = 0.13\frac{J}{g*K} \tag{17}$$

Tin Specific Heat

$$c_{Sn} = 0.23\frac{J}{g*K} \tag{18}$$

Iron Specific Heat

$$c_{Fe} = 0.45\frac{J}{g*K} \tag{19}$$

$$\Delta T_{Al} = \frac{q}{mc} = \frac{100J}{(10g)*(0.9\frac{J}{g*K})} \tag{20}$$

$$\Delta T_{Al} = 11.11K \approx 11K \tag{21}$$

$$\Delta T_{Pb} = \frac{q}{mc} = \frac{100J}{(10g)*(0.13\frac{J}{g*K})} \tag{22}$$

$$\Delta T_{Pb} = 76.92K \approx 77K \tag{23}$$

$$\Delta T_{Sn} = \frac{q}{mc} = \frac{100J}{(10g)*(0.23\frac{J}{g*K})} \tag{24}$$

$$\Delta T_{Sn} = 43.48K \approx 43K \tag{25}$$

$$\Delta T_{Fe} = \frac{q}{mc} = \frac{100J}{(10g)*(0.45\frac{J}{g*K})} \tag{26}$$

$$\Delta T_{Fe} = 22.22K \approx 22K \tag{27}$$

Since the specific heat for Aluminum is highest and all other numbers remain constant, this means the denominator will always be higher which will make the Delta T always lower out of these choices. Aluminum is the answer.

4. An unknown substance with a mass of 100 g absorbs 1000 J while undergoing a temperature increase of 15 K. What is the specific heat of the substance?

$$q = mc\Delta T \tag{28}$$

$$c = \frac{q}{m * \Delta T} \tag{29}$$

$$c = \frac{1,000J}{1,500g * K} \tag{30}$$

$$c = 0.667\frac{J}{g * K} \approx 0.7\frac{J}{g * K} \tag{31}$$

5. What mass of water will change its temperature by 3 K when 525 J of heat is added to it?

$$q = mc\Delta T \tag{32}$$

$$m = \frac{q}{c * \Delta T} \tag{33}$$

$$m = \frac{525J}{4.184\frac{J}{g*K} * 3K} \tag{34}$$

$$m = 41.826g \approx 42g \tag{35}$$