Homework 2

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Problems from 8^{th} edition (Chapter 2): 3a, 3b, 5a, 5b, 5c, 5f, 5g, 5i, 7a

9 solutions for 9 problems.

1 Problem Set 1

1. 3a. If our experiment consists of tossing a coin and if we assume that a head is as likely to appear as a tail, what would we have?

$$P(H) = P(T) = \frac{1}{2} \tag{1}$$

If the coin were biased and we felt that a head were twice as likely to appear as a tail, what would it be?

$$P(H) = \frac{2}{3}, P(T) = \frac{1}{3} \tag{2}$$

2. 3b. If a die is rolled and we suppose that all six sides are equally likely to appear, then we would have P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = $\frac{1}{6}$. From Axiom 3, what would the probability of rolling an even number be?

$$P(2,4,6) = P(2) + P(4) + P(6) \tag{3}$$

$$P(2,4,6) = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \tag{4}$$

$$P(2,4,6) = \frac{3}{6} = \frac{1}{2} \tag{5}$$

2 Problem Set 2

1. 5a. If two dice are rolled, what is the probability that the sum of the upturned faces will equal 7?

$$1, 2, 3, 4, 5, 6 \text{ and } 1, 2, 3, 4, 5, 6 \tag{6}$$

$$\text{Total Possible Outcomes } = 6 * 6 = 36 \tag{7}$$

$$\text{Outcomes equal to 7 } = (1, 6), (2, 5), (3, 4), (6, 1), (5, 2), (4, 3) \tag{8}$$

$$\frac{\text{Total Outcomes equal to 7}}{\text{Total Possible Outcomes}} = \frac{6}{36} = \frac{1}{6} \tag{9}$$

2. 5b. If 3 balls are "randomly drawn" from a bowl containing 6 white and 5 black balls, what is the probability that one of the balls is white and the other two black?

$$\text{Total Outcomes } = 11 * 10 * 9 = 990 \tag{10}$$

Possible Orders

$$(White, Black, Black) = (6 * 5 * 4) = 120 \tag{11}$$

$$(Black, White, Black) = (5 * 6 * 4) = 120 \tag{12}$$

$$(Black, Black, White) = (5 * 4 * 6) = 120 \tag{13}$$

$$\frac{Sum of Possible Orders}{Total Outcomes} = \frac{120 + 120 + 120}{990} = \frac{4}{11} \tag{14}$$

3. 5c. A committee of 5 is to be selected from a group of 6 men and 9 women. If the selection is made randomly, what is the probability that the committee consists of 3 men and 2 women?

$$\text{committee total } = \binom{TotalPeople}{PeopleSelected} = \binom{15}{5} \tag{15}$$

$$\text{men } = \binom{TotalMen}{MenSelected} = \binom{6}{3} \tag{16}$$

$$\text{women } = \binom{TotalWomen}{WomenSelected} = \binom{9}{2} \tag{17}$$

$$\frac{\binom{6}{3}\binom{9}{2}}{\binom{15}{5}} \tag{18}$$

$$\frac{6!}{3! * 3!} = \frac{6 * 5 * 4}{3 * 2 * 1} = \frac{120}{6} = 20 \tag{19}$$

$$\frac{9!}{7! * 2!} = \frac{9 * 8}{2 * 1} = \frac{72}{2} = 36 \tag{20}$$

$$\frac{15!}{10! * 5!} = \frac{15 * 14 * 13 * 12 * 11}{5 * 4 * 3 * 2 * 1} = \frac{360,360}{120} = 3003 \tag{21}$$

$$\frac{36 * 20}{3003} \tag{22}$$

$$\frac{720}{3003} \tag{23}$$

$$\frac{240}{1001} \tag{24}$$

4. 5f. A poker hand consists of 5 cards. If the cards have distinct consecutive values and are not all of the same suit, we say that the hand is a straight. For instance, a hand consisting of the five of spades, six of spades, seven of spades, eight of spades, and nine of hearts is a straight. What is the probability that one is dealt a straight?

$$\text{Assume all possible poker hands are equally likely: } \binom{52}{5} = v \tag{25}$$

$$\text{There are 4 of each number/ face card.} = x \tag{26}$$

$$\text{Five cards will be drawn.} = y \tag{27}$$

$$\text{4 of these combinations will results in a straight flush.} = z \tag{28}$$

$$\text{There are 10 possible ways to get a straight} = w \tag{29}$$

$$\frac{w(x^y - z)}{v} = \frac{10(4^5 - 4)}{\binom{52}{5}} \tag{30}$$

$$\frac{10,200}{\binom{311,875,200}{120}} = \frac{10,200}{2,598,960} \approx .0039 \tag{31}$$

5. 5g. A 5-card poker hand is said to be a full house if it consists of 3 cards of the same denomination and 2 other cards of the same denomination (of course, different from the first denomination). Thus, one kind of full house is three of a kind plus a pair. What is the probability that one is dealt a full house?

$$\text{Assume all possible poker hands are equally likely: } \binom{52}{5} = v \tag{32}$$

$$\text{There are 13 different suits} = x \tag{33}$$

$$\text{12 different suits after 1st selection} = u \tag{34}$$

$$\text{Denominations are in sets of 4} = y \tag{35}$$

$$\text{3 of a kind} = z \tag{36}$$

$$\text{Pair } = w \tag{37}$$

$$\frac{x * u * \binom{y}{z} * \binom{y}{w}}{v} = \frac{13 * 12 * \binom{4}{3} * \binom{4}{2}}{\binom{52}{5}} \tag{38}$$

$$\frac{13 * 12 * 24 * 1}{2,598,960} = \frac{3,744}{2,598,960} \approx 0.0014 \tag{39}$$

6. 5i. If n people are present in a room, what is the probability that no two of them celebrate their birthday on the same day of the year? How large need n be so that this probability is less than $\frac{1}{2}$?

$$\text{3 people possible birthday combos} = 365 * 365 * 365 = 365^3 \tag{40}$$

$$\text{first person chooses * subtract 1 for each additional person} \tag{41}$$

$$\text{i.e. 3 people} = 365 * 364 * 363 \tag{42}$$

$$\frac{\text{3 people}}{\text{3 b-day combos}} = \frac{365 * 364 * 363}{365^3} \tag{43}$$

$$P(N) = [365 * 364 * ... * (365 - N + 1)]/365^N \tag{44}$$

$$P(N) < 0.5 = 23 \tag{45}$$

At 23, the probability that 2 people have the same birthday becomes greater than 50 percent. Which means that the chance they all have separate birthdays becomes less than 50 percent.

3 Problem Set 3

1. 7a. Suppose that in a 7-horse race, you feel that each of the first 2 horses has a 20 percent chance of winning, horses 3 and 4 each have a 15 percent chance, and the remaining 3 horses have a 10 percent chance each. Would it be better for you to wager at even money that the winner will be one of the first 3 horses or to wager, again at even money, that the winner will be one of the horses 1,5,6 and 7?

$$\text{First bet } = .2 + .2 + .15 = .55 \tag{46}$$

$$\text{Second bet } = .2 + .1 + .1 + .1 = .50 \tag{47}$$

This means the first wager is better.

Stochastic Processes and Random Signals

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