TOC_Assignment4

Tuesday December 8th, 2020 7:44 PM

Q1. For $\Sigma = \{a, b\}$

Find a regular expression for the complement of the following language.

$L = L(a^*b^*)^*$

$\varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon = \varepsilon$

$\overline{L} = (\overline{a^*b^*})^*$

ε
ab
aab
aa b
aabaaaaaab
aaaaaa
aaaaaab
ε ε ε ε ε
ε ε ε ε ba
ε ε ε ba ε
ε ε ba ε ε
ε ba ε ε ε
ba ε ε ε ε

$\overline{L_1} = \Sigma^* / L$

$\overline{L} = ba$

Q2. Let $L_1 = L(a^*ba)$,
$L_2 = L(a^*b^*a^*)$

Find a regular expression for

$(L_1 \cup L_2)^* \cap L_3$

$((a^*ba)^* + (a^*b^*a^*))^* (a^*b^*a^*)$

Q3. Show that if a language family is closed under union and complementation, it must also be closed under intersection.

By Demorgan's law,

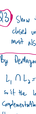
$L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}$

So if the language family is closed under complementation and union, $L_1 \cap L_2$ must be in the family.

Q4. $L_1(a,b^*,a^*) \cap L_2(a^*,b^*,a)$

L_1 - DFA

L_2 - DFA



$q_0 p_0, a \to q_1 p_1$
$q_0 p_0, b \to q_2 p^2$
$q_1 p_1, a \to q_3 p^1$
$q_1 p_1, b \to q_1 p^2$
$q_3 p^1, b \to q_3 p^3$
$q_3 p^2, a \to q_3 p^4$
$q_3 p^1, a \to q_3 p^1$
$q_3 p_1, b \to q_3 p^2$
$q_1 p^2, a \to q_3 p^3$
$q_1 p^2, b \to q_1 p^2$
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$q_2 p^4, b \to q_2 p^4$

$q_0 p_0, a \to p^1$
$q_0 p_0, b \to q^3$
$p^1, a \to q^3$
$p^1, b \to p^2$
$q_1 p_1 \to q_1$
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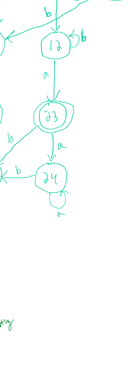
9 states



* Union → one state must be accepted
# Intersection → both states must be accepted

	q^1	q^2	q^3	q^4
p^0				
p^1		X	X	
p^2				
p^3		X	X	
p^4				

Q5. The nor of two languages is
$nor(L_1, L_2) = \{w : w \notin L_1 \text{ and } w \notin L_2\}$

Show that the family of regular languages is closed under the nor operation.

$nor(L_1, L_2) = \overline{L_1} \cap \overline{L_2}$

The result then follows from closure under intersection and complementation.

Q6.

$cor(L_1, L_2) = \{w : w \in \overline{L_1} \text{ or } w \in \overline{L_2}\}$

Show that the family of reg. lang. is closed under the cor operation.

Assume the DFAs represented by:

$M_1 = (Q_1, \Sigma, \delta_1, q_0, F_1)$

$M_2 = (Q_2, \Sigma, \delta_2, p_0, F_2)$

accept L_1 and L_2,

then,

$\overline{M_1} = (Q_1, \Sigma, \delta_1, q_0, Q_1 - F_1)$

$\overline{M_2} = (Q_2, \Sigma, \delta_2, p_0, Q_2 - F_2)$

will accept $\overline{L_1}$ and $\overline{L_2}$.

Therefore $cor(L_1, L_2)$ is regular.

Q7. $string = (a_1 a_2 \dots a_n)$

$shift(string) = (a_2 \dots a_n a_1)$

$shift(L) = \{v : v = shift(w) \text{ for some } w \in L\}$

→ It doesn't matter where the λ is, it will still be accepted.

String = (abbaa)

Shift(abbaa) = (bbaaa)

abbaa
bbaaa
baaab
aaabb
aabba

↖ Will accept all of these (every shift).

Q8. Show that the language

$L = \{a^n b^n a^n : n \geq 0, k \geq 0\}$

is not regular

Suppose that L is regular and we are given an that we split $w = a^n b^n$ in L. Now because $|xy| \leq n$ and we know that $y = a^n$. So the only possible choice for some $k \geq 1$, then the pumped string will:

$w_i = a^n (b^n)^k b^n \in L_1$

for all $i \geq 0$, i.e. however,

$w_2 = a^{n+k} b^{n+k} \notin L$

This contradiction implies that L is not a regular language.

Q9. Prove or disprove:

If L_1 & L_2 are nonregular languages, then $L_1 \cup L_2$ is also non regular.

Let $L_1 = \{a^m b^n \mid m \geq n\}$ ← Non regular

Let $L_2 = \{a^n b^n \mid m < n\}$ ← Non regular

$L_1 \cup L_2 = a^* b^*$ → which is regular