TOC_Assignment5

Tuesday, March 28, 2023 2:53 PM

Q1. Consider the languages below. For each, make a conjecture whether or not it is regular. Then prove your conjecture.

(a) $L = \{a^n b^k : n + k \geq 5\}$

The language is regular. This can be seen by splitting the problem into cases such as $k=0, k=0, n \geq 5$. This can easily be constructed into a regular expression.

(b) $L = \{a^n b^k a^n ; n > 5, l > 3, k \leq l\}$

~~Not~~ Not regular.

(1) $xy^i z \in A$ for every $i \geq 0$

(2) $|y| > 0$

(3) $|xy| \leq p$

$w = a^{4m} b^{4m} a^{4m}$ and $w \in L$

$y = a^k$ for $1 \leq k \leq 6$, $x = a^{6-k}$, $z = b^{4m} a^{4m}$

$w_0 = a^{6-k} b^{4m} a^{4m} \notin L$, because $6-k \leq 5$

$y = b^k$ for $k \geq 1$ $= a^6 b^{4m-k}$, $z = a^{4m}$

$w_0 = a^6 b^{4m-k} a^{4m} \notin L$, because $4m-k \leq 4m$

(e) $L = \{a^n b^l : n \leq l \leq 2n\}$

This language is not regular. Suppose m is given, we pick $w = a^m b^{m+1}$. So our opponent can only choose $y = a^k$ for some $1 \leq k \leq m$. The pumped strings are: $w_i = a^{m+(i-1)k} b^{m+1}$. So for example, $w_2 = a^{m+2k} b^{m+1}$ violates the required condition since $m+1 < m+2k$.

(f) $L = \{a^n b^l : n \geq 100, l \leq 100\}$

Regular

Q2. Find context-free grammars for the following languages.

(a) $L = a^n b^n$, n is even

$S \to aaSbb \mid \lambda$

(b) $L = a^n b^n$, n is odd

$S \to aAb$

$A \to aaAbb \mid \lambda$

(c) $L = a^n b^n$, n is a multiple of three

$S \to aaaSbbb \mid \lambda$

Q3. Show a derivation tree for $w = aaabbaaa$ using the grammar $V = \{S\}$, $T = \{a, b\}$ and productions.

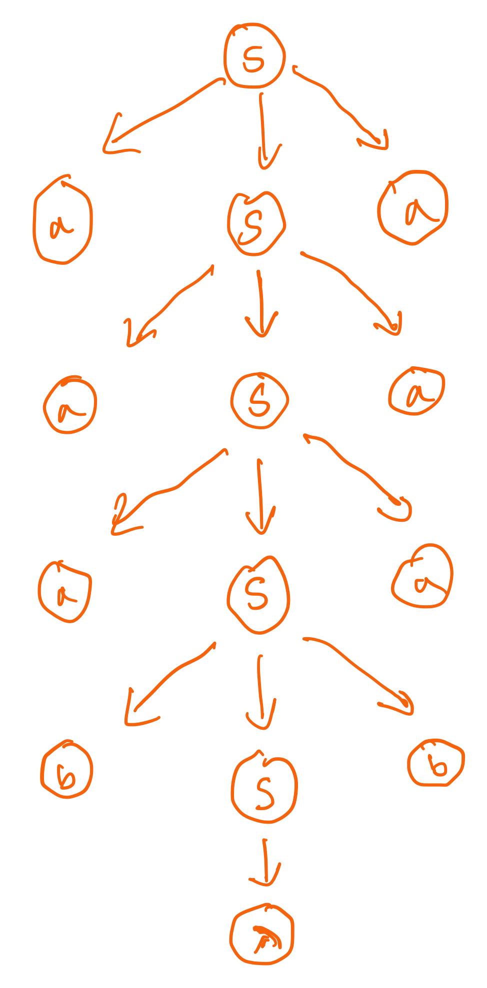
$S \to aSa$

$S \to bSb$

$S \to \lambda$

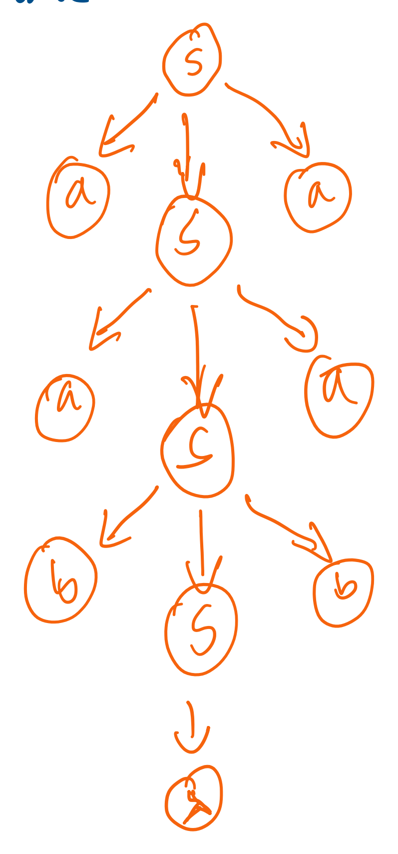
The language generated by the grammar is $\{ww^R : w \in \{a,b\}^*\}$

Palindromes of even length in $\{a,b\}^*$



Q4. Draw the derivation tree corresponding to

$S \Rightarrow aSa \Rightarrow aaSaa \Rightarrow aabSbaa \Rightarrow aabbaa$



Q5. Find context-free grammars for the following languages (with $n \geq 0$, $m \geq 0$)

(a) $S \to aSb \mid A \mid B$

$A \to \lambda \mid a \mid aa \mid aaa$

$B \to bB \mid b$

(c) $L = \{a^n b^m : n \neq 2m\}$

$S \to aaSb \mid aA \mid Bb$

$A \to aA \mid \lambda$

$B \to Bb \mid \lambda$

(h) $L = \{w \in \{a,b\}^* : n_a(w) = n_b(w) + 2\}$

$S \to AaAaA$

$A \to AabA \mid AbaA \mid \lambda$

Q6. (a) $L = \{a^n b^m c^k : n = m \text{ or } m \leq k\}$

$S \to S_1 C \mid A S_2$

$S_1 \to a S_1 b \mid \lambda$

$S_2 \to b S_2 c \mid C$

$A \to aA \mid \lambda$

$C \to cC \mid \lambda$

(b) $L = \{a^n b^m c^k : n = m \text{ or } m \neq k\}$

$S \to S_1 C \mid A S_2$

$S_1 \to a S_1 b \mid \lambda$

$S_2 \to b S_2 c \mid bB \mid cC$

$B \to bB \mid \lambda$

$C \to cC \mid \lambda$

$A \to aA \mid \lambda$

Q7. $S \to aS \mid \cancel{AB} \mid \lambda$

~~$A \to bA$~~

~~$B \to AA$~~

Remove anything that doesn't lead to a terminal string.

$S \to aS \mid \lambda$

generates language $L(a^*)$

$L = \{a^n : n \geq 0\}$

Q8. $S \to aSSS$

$S \to bb \mid \lambda$

||

$S \to aSSS \mid aSS \mid aS \mid a$

$S \to bb$

Q9. $S \to aSS \mid a \mid b$ into Chomsky normal form.

$S \to A_a SS$	$S \to A_a D_1$
$S \to a$	$D_1 \to SS$
$S \to b$	$S \to a$
$A_a \to a$	$S \to b$
	$A_a \to a$

Q10. Convert

$S \to aSb \mid Sab \mid ab$ into Chomsky normal form.

$S \to B_1 S B_2$	$S \to B_1 D$
$S \to S B_1 B_2$	$D \to S B_2$
$S \to B_1 B_2$	$S \to S S$
$B_1 \to a$	$S \to B_1 B_2$
$B_2 \to b$	$B_1 \to a$
	$B_2 \to b$