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$$1. \bar{S}_1 = \{1, \cancel{4}, \cancel{6}, \cancel{8}, \cancel{9}, 10\}$$

$$S_2 = \{2, 4, 5, 8, 9\}$$

$$\bar{S}_1 \cup S_2 = \{1, 2, 4, 5, 8, 9, 10\}$$

$$2. S_1 = \{2, 3, 5, 7\}$$

$$S_2 = \{2, 4, 5, 8, 9\}$$

$$S_1 \times S_2 =$$

$$\{(2,2)(2,4)(2,5)(2,8)(2,9), \\ (3,2)(3,4)(3,5)(3,8)(3,9), \\ (5,2)(5,4)(5,5)(5,8)(5,9), \\ (7,2)(7,4)(7,5)(7,8)(7,9)\}$$

$$3. S_1 = \{1, 2\} \quad S_2 = \{2, 1\}$$

$$U = \{1, 4\}$$

$$S_1 \subseteq S_2$$

$$\bar{S}_1 = \{3, 4\} \quad \bar{S}_2 = \{3, 4\}$$

$$\bar{S}_2 \subseteq \bar{S}_1$$

$$4. ww^R w, \quad w = aabbaab$$

$$w^R = baabbaa$$

$$ww^R w = \underline{aabbbaabbaabbaa} \underline{aabbaab}$$

There are two substrings of 'aab' in the above, underlined with

Note: $|\Sigma|$ = Number of symbols in an alphabet

n = length of string

$|\Sigma|^n$ = number of strings @ length 'n' possible w/ alphabet Σ

$$L^* = \varepsilon L^+$$

$$\Sigma^* = \{g\}^*$$

$$\{\varepsilon, g, gg, ggg, gggg, \dots\}$$

$$g^+$$

$$\{g, gg, ggg, gggg, \dots\}$$

Σ^* = power set, Every possible string.

$$L \subseteq \Sigma^*$$

* includes ε (empty set)

$$5. \overline{L^*} \rightarrow L^* \text{ includes } \varepsilon$$

so its complement would not include ε .

$$(\bar{L}) \rightarrow L \text{ may or may not include } \varepsilon$$

so its complement may or may not include ε , But when you add the * on the outside \rightarrow

$$(\bar{L})^*, \text{ this will now definitely include } \varepsilon.$$

Therefore:

$$\overline{L^*} \neq (\bar{L})^* \text{ in any scenario.}$$

Notes: Grammars = G

$$G = (V, T, S, P)$$

V = Finite set of objects called variables (uppercase letters)

T = Finite set of objects called terminal symbols (lowercase)

S = Start symbol ($S \in V$)

P = Production rules; specify how the grammar transforms one string to another.

Disjoint sets = no elements in common. Intersection of 2 disjoint sets is ε

Sets V & T are non-empty and disjoint.

$$\lambda = \varepsilon = \text{empty string}$$

$$S \Rightarrow aSb \quad S \rightarrow aSb \rightarrow ab$$

$$S \rightarrow \lambda \quad S \rightarrow aSb \rightarrow aaSb \rightarrow \dots$$

All strings that start with a and end with b .

$$\Sigma = \{a, b\}$$

$$S \rightarrow aAb \quad S \rightarrow aaaSb \rightarrow aaaSb \rightarrow \dots$$

$$A \rightarrow aA \mid bA \mid \lambda$$

All strings with at least 3 a 's.

$$\Sigma = \{a, b\}$$

$$S \rightarrow AaAaAaA$$

$$A \rightarrow aA \mid Ab \mid \lambda$$

6. $\Sigma = \{a, b\}$

$$G = (\{S, A\}, \{a, b\}, S, P)$$

$$(a) P =$$

$$S \rightarrow AaAaA$$

$$A \rightarrow bA \mid \lambda$$

$$(b) P =$$

$$S = AaAaA$$

$$A = aA \mid bA \mid \lambda$$

$$(c) P =$$

$$S \rightarrow bS \mid aA \mid \lambda$$

$$A \rightarrow bA \mid aB \mid \lambda$$

$$B \rightarrow bB \mid aC \mid \lambda$$

$$C \rightarrow bC \mid \lambda$$

$$7. S \rightarrow aA \mid \lambda$$

$$A \rightarrow bS$$

$$aA \rightarrow aabS \rightarrow aabaaA \rightarrow aabaaabS \rightarrow \dots$$

Every string in the language will be 2 a 's followed by a b of various different lengths, or it could go on forever.

$$L = \{ \lambda, \\ aab, \\ aabab, \\ aababab, \\ aabababab, \dots \}$$