

Q1. Consider the languages below. For each, make a conjecture whether or not it is regular. Then prove your conjecture.

(a) $L = \{a^n b^l a^K : n+l+K \geq 5\}$

The language is regular. This can be seen by splitting the problem into cases such as $L=0, K=0, n \geq 5$. This can easily be constructed into a regular expression.

(b) $L = \{a^n b^l a^K : n \geq 5, l \geq 3, K \leq L\}$

Not regular.

$w = a^6 b^{4+m} a^{4+m}$ and $w \in L$

$y = a^K$ for $1 \leq K \leq 6$,

$x = a^{6-K}, z = b^{4+m} a^{4+m}$

$w_0 = a^{6-K} b^{4+m} a^{4+m} \notin L$, because

$6-K \leq 5$

$y = b^K$ for $K \geq 1 = a^6 b^{4+m-K}, z = a^{4+m}$

$w_0 = a^6 b^{4+m-K} a^{4+m} \notin L$, because

$4+m-K \leq 4+m$

(c) $L = \{a^n b^l : n \leq l \leq 2n\}$

This language is not regular. Suppose

n is given, we pick $w = a^n b^{n+1}$

So our opponent can only choose $y = a^K$ for

some $1 \leq K \leq n$. The pumped strings are:

$w_i = a^{n(i-1)K} b^{n+1}$. So for example,

$w_3 = a^{n(3-1)K} b^{n+1}$ violates the required

condition since $n+1 < n+2K$.

(f) $L = \{a^n b^l : n \geq 100, l \leq 100\}$

Regular

Q2. Find context-free grammars for the following languages.

(a) $L = \{a^n b^n, n \text{ is even}\}$

$S \rightarrow aaSbb \mid \lambda$

(b) $L = \{a^n b^n, n \text{ is odd}\}$

$S \rightarrow aAb$

$A \rightarrow aaAbbb \mid \lambda$

(c) $L = \{a^n b^n, n \text{ is a multiple of three}\}$

$S \rightarrow aaaSbbb \mid \lambda$

Q3. Show a derivation tree for $w = aabbbbaa$ using the grammar $V = \{S\}, T = \{a, b\}$ and productions.

$S \rightarrow aSa$

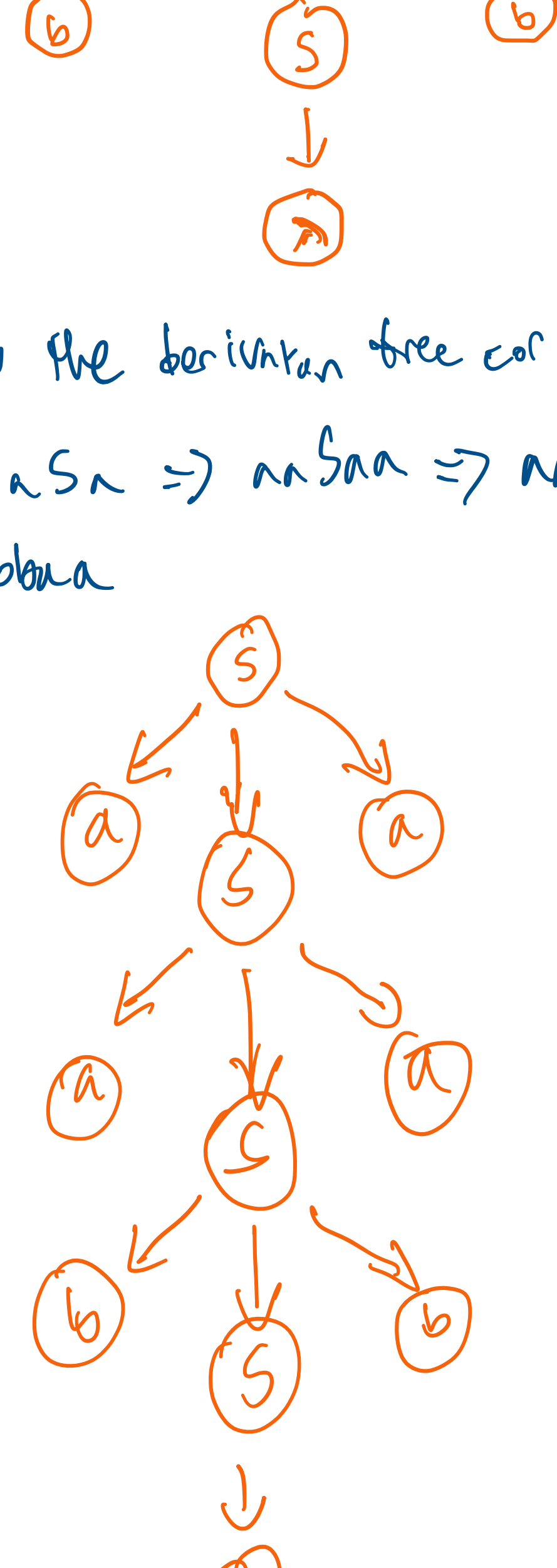
$S \rightarrow bSb$

$S \rightarrow \lambda$

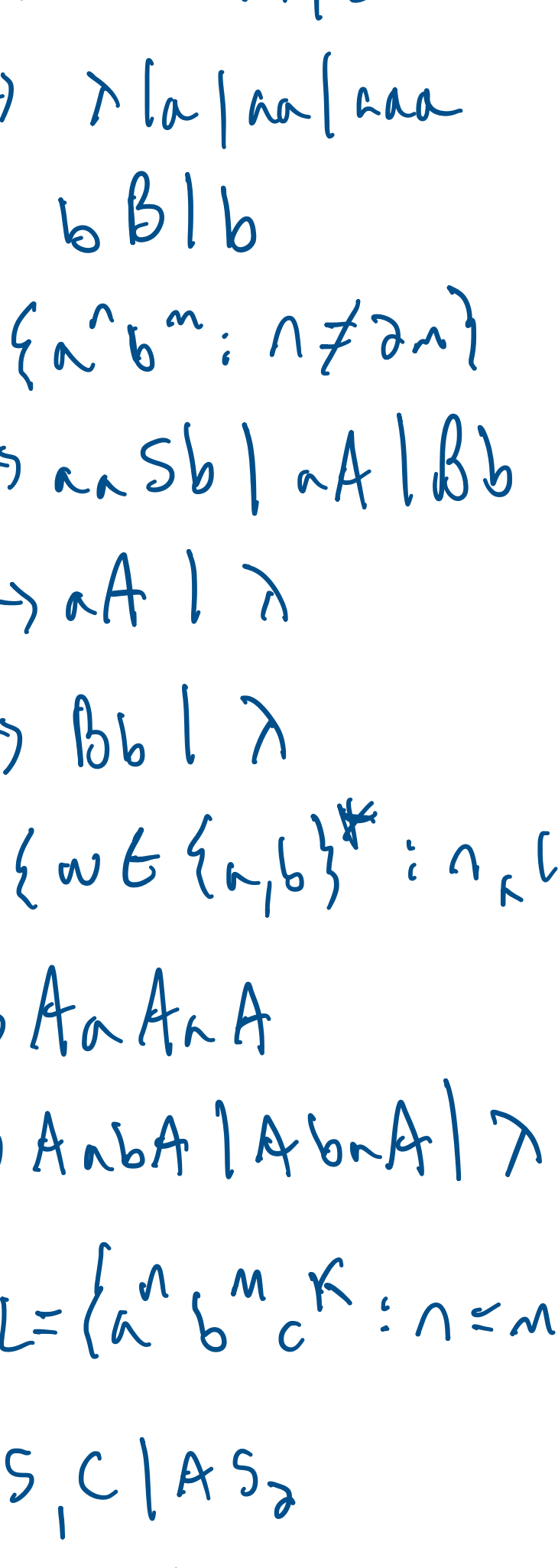
The language generated by the grammar is

$\{w w^R : w \in \{a, b\}^*\}$

Palindromes of even length in $\{a, b\}^*$



Q4. Draw the derivation tree corresponding to $S \Rightarrow aSa \Rightarrow aaSaa \Rightarrow aabSbaa \Rightarrow aabbaa$



Q5. Find context-free grammars for the following languages (with $n \geq 0, m \geq 0$)

(a) $S \rightarrow aSb \mid A \mid B$

$A \rightarrow \lambda \mid a \mid aa$

$B \rightarrow bB \mid b$

(c) $L = \{a^n b^m : n \neq 2m\}$

$S \rightarrow aSb \mid aA \mid Bb$

$A \rightarrow aA \mid \lambda$

$B \rightarrow bB \mid \lambda$

(h) $L = \{w \in \{a, b\}^* : n_a(w) = n_b(w) + 2\}$

$S \rightarrow AaAaA$

$A \rightarrow AabA \mid AbaA \mid \lambda$

Q6. (a) $L = \{a^n b^m c^K : n \leq m \text{ or } m \leq K\}$

$S \rightarrow S_1 C \mid A S_2$

$S_1 \rightarrow a S_1 b \mid \lambda$

$S_2 \rightarrow b S_2 c \mid C$

$A \rightarrow aA \mid \lambda$

$C \rightarrow cC \mid \lambda$

(b) $L = \{a^n b^m c^K : n = m \text{ or } m \neq K\}$

$S \rightarrow S_1 C \mid A S_2$

$S_1 \rightarrow a S_1 b \mid \lambda$

$S_2 \rightarrow b S_2 c \mid bB \mid cC$

$B \rightarrow bB \mid \lambda$

$C \rightarrow cC \mid \lambda$

$A \rightarrow aA \mid \lambda$

Q7. $S \rightarrow aS \mid AB \mid \lambda$

~~$A \rightarrow bA$~~

~~$B \rightarrow AA$~~

Remove anything that doesn't lead to a terminal string.

$S \rightarrow aS \mid \lambda$

generates language

$L(a^*)$

Q8. $L = \{a^n : n \geq 0\}$

Q8. $S \rightarrow aSSS$

$S \rightarrow bbb \mid \lambda$

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$S \rightarrow aSSS \mid aSS \mid aS \mid a$

$S \rightarrow bbb$

Q9. $S \rightarrow aSS \mid a \mid b$ into Chomsky normal form.

$S \rightarrow A_1 SS \quad S \rightarrow A_1 D_1$

$S \rightarrow a \quad D_1 \rightarrow SS$

$S \rightarrow b \quad S \rightarrow a$

$A_1 \rightarrow a \quad S \rightarrow b$

$A_1 \rightarrow a \quad A_1 \rightarrow a$

Q10. Convert

$S \rightarrow aSb \mid Sab \mid ab$ into Chomsky normal form.

$S \rightarrow B_1 S B_2 \quad S \rightarrow B_1 D$

$S \rightarrow S B_1 B_2 \quad D \rightarrow S B_2$

$S \rightarrow B_1 B_2 \quad S \rightarrow SS$

$B_1 \rightarrow a \quad S \rightarrow B_1 B_2$

$B_2 \rightarrow b \quad B_1 \rightarrow a$

$B_2 \rightarrow b \quad B_2 \rightarrow b$