An Introductory Text on Using R As a Statistical Computing Software

For Use in the Biomedical and Life Sciences

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Contents

Chapter 0

Preface

This text is a companion to *Introductory Statistics for the Life and Biomedical Sciences*; while the main text focuses on statistical concepts and ideas, this supplement provides details about how to use the statistical computing language R.

This is a preview text so this package is not available just yet, but when it is available, all of the necessary data will be accessible using the following commands.

```
install.packages("OIBioStat")  ## make sure to include the quotations
```

```
require(OIBioStat) ## note the lack of quotations here
```

Chapter 1

Introduction to Basic Techniques

Contents

This chapter introduces basic commands for manipulating datasets, including how to calculate numerical summaries, create tables from data, and make graphical plots.

1.1   Case study: preventing peanut allergies

The `LEAP` dataset contains the results of the "Learning Early About Peanut Allergy" (LEAP) study, an experiment conducted to assess whether early exposure to peanut products reduces the probability of peanut allergies developing. To access the documentation associated with the dataset, run the following command:

```
help(LEAP)
```

A file will appear in the Help pane that provides some basic information about the dataset:

- **Description:** A general overview of the dataset.

- **Usage:** Instructions for how to load the dataset.

- **Format:** The names and descriptions of each variable in the dataset, including information about variable type and measurement units.

- **Details:** Additional details about the conditions under which the data were collected.

- **Source**: Information about where the data originates from.

To view the dataset itself, run:

```
View(LEAP)
```

In the main console pane, a window will appear that shows the entire dataset. The variable names are displayed across the top, as the names of the columns. Data for each case in the study are contained within the rows. Note that the farthest left column shows the **indices**, which are computer-generated values that allow specific rows in the dataset to be accessed. These can be used to view a specific portion of the data.

For example, to print out data contained in the first 5 rows and first 3 columns of the data, use the following syntax:

```
LEAP[1:5,1:3]

##   participant.ID    treatment.group age.months
## 1   LEAP_100522 Peanut Consumption     6.0780
## 2   LEAP_103358 Peanut Consumption     7.5893
## 3   LEAP_105069    Peanut Avoidance     5.9795
## 4   LEAP_105328 Peanut Consumption     7.0308
## 5   LEAP_106377    Peanut Avoidance     6.4066
```

The bracket notation after the dataset name implies location; the syntax `dataset[rows,columns]` specifies rows 1 through 5 and columns 1 through 6. To access only the first 5 rows, but all the columns, leave an empty space where the columns would usually be specified:

```
## note the space (or lack of text after the comma)
LEAP[1:5, ]

##   participant.ID    treatment.group age.months    sex primary.ethnicity
## 1   LEAP_100522 Peanut Consumption     6.0780 Female             Black
## 2   LEAP_103358 Peanut Consumption     7.5893 Female             White
## 3   LEAP_105069    Peanut Avoidance     5.9795   Male             White
## 4   LEAP_105328 Peanut Consumption     7.0308 Female             White
## 5   LEAP_106377    Peanut Avoidance     6.4066   Male             White
##   overall.V60.outcome
## 1            PASS OFC
## 2            PASS OFC
## 3            PASS OFC
## 4            PASS OFC
## 5            PASS OFC
```

Alternatively, since there are 6 columns in the dataset, the command `LEAP[1:5,1:6]` would also achieve the same result. This is a common theme in R – there can be several ways to accomplish a desired result.

OI Biostat Table 1.1 shows the participant ID, treatment group, and overall outcome for five patients. It is not specified in the main text, but the data are specifically from rows 1, 2, 3, 529, and 530. The command `c()` can be used to bind the row numbers into a list, as well as to create a list of the desired columns. Columns can be referred to by name, instead of by number:

```
## OI Biostat Table 1.1
LEAP[c(1, 2, 3, 529, 530),c("participant.ID", "treatment.group",
                           "overall.V60.outcome")]

##      participant.ID     treatment.group overall.V60.outcome
## 1       LEAP_100522 Peanut Consumption            PASS OFC
## 2       LEAP_103358 Peanut Consumption            PASS OFC
## 3       LEAP_105069   Peanut Avoidance            PASS OFC
## 639     LEAP_994047   Peanut Avoidance            PASS OFC
## 640     LEAP_997608 Peanut Consumption            PASS OFC
```

Two-way summary tables organize the data according to two variables and display the number of counts matching each combination of variable categories. The following code corresponds to *OI Biostat* Table 1.2, which groups participants into categories based on treatment group and overall outcome. In the `table()` command, the first variable specifies the rows and the second variable specifies the columns. The addition of the `addmargins()` command prints the sums of the rows and columns on the sides of the table.

```
## Table 1.2
table(LEAP$treatment.group, LEAP$overall.V60.outcome)

##
##                     FAIL OFC PASS OFC
##    Peanut Avoidance       36      227
##    Peanut Consumption      5      262

addmargins(table(LEAP$treatment.group, LEAP$overall.V60.outcome))

##
##                     FAIL OFC PASS OFC Sum
##    Peanut Avoidance       36      227 263
##    Peanut Consumption      5      262 267
##    Sum                    41      489 530
```

1.2   Data Basics

Entire datasets can be partitioned into data frames using bracket notation. For example, *OI Biostat* Table 1.3 shows a data frame consisting of rows 1-3 and 150 (and all columns) from the `frog.altitude` dataset. The data frame can then be given a specific name, such as `frog.df`, and directly called on for later operations.

```
## Table 1.3
frog.df = frog.altitude.data[c(1:3, 150),]
frog.df

##      altitude latitude clutch.size body.size clutch.volume egg.size
## 1    3,462.00    34.82    181.9701  3.630781      177.8279 1.949845
## 2    3,462.00    34.82    269.1535  3.630781      257.0396 1.949845
## 3    3,462.00    34.82    158.4893  3.715352      151.3561 1.949845
## 150  2,597.00    34.05    537.0318        NA      776.2471 2.238721
```

Similarly, matrix notation can be used to create *OI Biostat* Table 1.5.

```
## Table 1.5
famuss[c(1,2,3,595),c( "sex", "age", "race", "height", "weight", "actn3.r577x",
                       "ndrm.ch")]

##           sex age      race height weight actn3.r577x ndrm.ch
## 1     Female  27 Caucasian   65.0    199          CC    40.0
## 2       Male  36 Caucasian   71.7    189          CT    25.0
## 3     Female  24 Caucasian   65.0    134          CT    40.0
## 1348 Female  30 Caucasian   64.0    134          CC    43.8
```

1.3 Numerical Data

Numerical summaries can be quickly and easily calculated using R. The summary() command is one way to access several numerical summaries at once, including the minimum and maximum values of a variable:

```
summary(frog.altitude.data$clutch.volume)

##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   151.4   609.6   831.8   882.5  1096.0  2630.0
```

1.3.1 Measures of center: mean and median

The **mean** of a numerical value is the sum of all observations divided by the number of observations, where $x_1, x_2, \ldots, x_n$ represent the n observed values:

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

This calculation can be completed in R the same way as a hand calculation is done:

```
# identify n, the number of observations in the data
n = length(frog.altitude.data$clutch.volume)
n

## [1] 431

# calculate the sum of all observations and divide by n
sum(frog.altitude.data$clutch.volume)/n

## [1] 882.474
```

Alternatively, the R function mean() can be directly applied to the variable of interest:

```
x.bar = mean(frog.altitude.data$clutch.volume)
x.bar

## [1] 882.474

## round to the first decimal
round(x.bar, 1)

## [1] 882.5
```

To identify the **median** value, use the following command:

```
median(frog.altitude.data$clutch.volume)
```

```
## [1] 831.7638
```

1.3.2 Measures of spread: standard deviation and variance

The distance of a single observation from the mean is its **deviation**. The following command produces the deviations for the 1st, 2nd, 3rd, and 431th observations in the clutch.volume variable.

```
frog.altitude.data$clutch.volume[c(1,2,3,431)]-x.bar
```

```
## [1] -704.64604 -625.43440 -731.11786    50.78032
```

The sample **standard deviation** is computed as the square root of the **variance**, which is the sum of squared deviations divided by the number of observations minus 1.

$$s = \sqrt{\frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \cdots + (x_n - \overline{x})^2}{n-1}}$$

The following steps illustrate how to calculate the variance and standard deviation using the formula:

```
# calculate all deviations
dev = frog.altitude.data$clutch.volume - x.bar

# sum the squares of the deviations and divide by n - 1
var = (sum(dev^2))/(n-1)
var
```

```
## [1] 143680.9
```

```
# take the square root of the variance
sd = sqrt(var)
sd
```

```
## [1] 379.0527
```

Alternatively, use the R functions var() and sd():

```
var(frog.altitude.data$clutch.volume)
```

```
## [1] 143680.9
```

```
sd(frog.altitude.data$clutch.volume)
```

```
## [1] 379.0527
```

Variability can also be measured using the **interquartile range (IQR)**, which equals the third quartile (the 75th percentile) minus the first quartile (the 25th percentile).

```
IQR(frog.altitude.data$clutch.volume)

## [1] 486.9009
```

1.3.3 Robust statistics

In the `frog.altitude` dataset, there are four extreme values for clutch volume that are larger than 2,000 mm^3. To illustrate how the summary statistics are influenced by extreme values, *OI Biostat* Table 1.15 shows summary statistics for the data without the four largest observations.

The subset of the data with values less than 2,000 mm^3 can be pulled out by first specifying a logical condition, which assigns TRUE or FALSE to each entry.

```
# logical condition
less.than.2000 = frog.altitude.data$clutch.volume<= 2000

# view the first 5 values
less.than.2000[1:5]

## [1] TRUE TRUE TRUE TRUE TRUE
```

Bracket notation can then be used to calculate summary statistics specifically for the values in `clutch.volume` that satisfy the logical condition.

```
## robust estimates
median(frog.altitude.data$clutch.volume[less.than.2000])

## [1] 831.7638

IQR(frog.altitude.data$clutch.volume[less.than.2000])

## [1] 493.9186

## non-robust estimates
mean(frog.altitude.data$clutch.volume[less.than.2000])

## [1] 867.9425

sd(frog.altitude.data$clutch.volume[less.than.2000])

## [1] 349.1596
```

1.3.4 Visualizing distributions of data

While most numerical summaries can be calculated by hand, R is essential for creating graphical summaries.

Histograms

A **histogram** provides a view of data density. Observations are sorted into different bins based on their value, and the histogram shows the number of observations in each bin. The following command produces a histogram of the `clutch.volume` variable.

```
hist(frog.altitude.data$clutch.volume)
```

Histogram of frog.altitude.data$clutch.volume
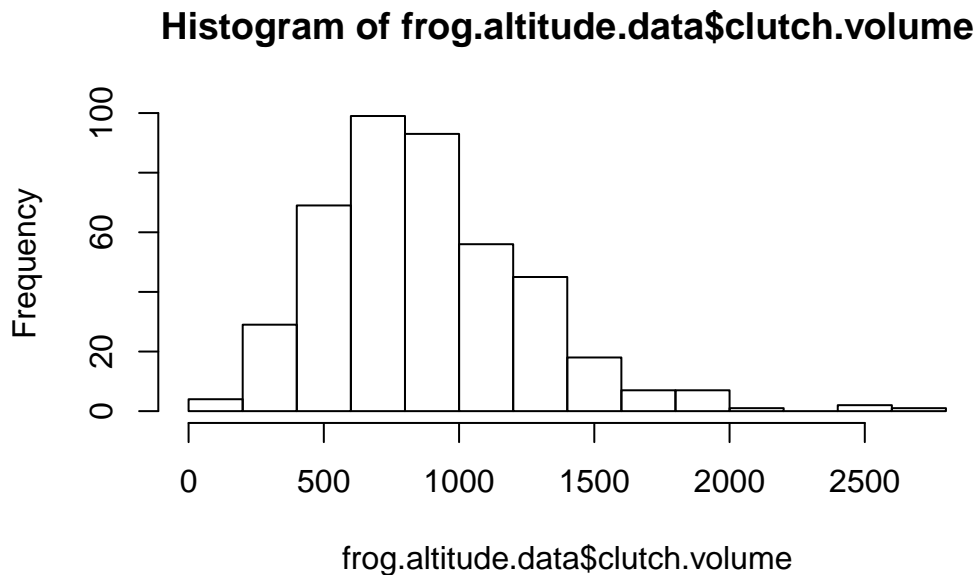
The hist() function takes several arguments:

- x: variable of interest

- breaks: number of bins

- col: color of the bars, enclosed in ""

- xlab: *x*-axis label, enclosed in ""

- ylab: *y*-axis label, enclosed in ""

- xlim: range of values for the *x*-axis, in the form c(lower bound, upper bound)

- ylim: range of values for the *y*-axis, in the form c(lower bound, upper bound)

- main: main title of the plot, enclosed in ""

- plot: if TRUE (default), a histogram is plotted; otherwise, data about the histogram is returned

As seen above, not all arguments must be specified; only the x argument is necessary. When options are not specified, R uses the default options, such as the default color of white for the histogram bars.

The following command reproduces *OI Biostat* Table 1.16 and Figure 1.17.

```
## Table 1.16
hist(frog.altitude.data$clutch.volume, breaks = 14, plot = FALSE)$counts

##  [1]  4 29 69 99 93 56 45 18  7  7  1  0  2  1

## Figure 1.17
hist(x = frog.altitude.data$clutch.volume, breaks = 14, col = "dodgerblue",
     xlab = "Clutch Volume", ylab = "Frequency", ylim = c(0, 100),
     main = "Histogram of Clutch Volume Frequencies")
```
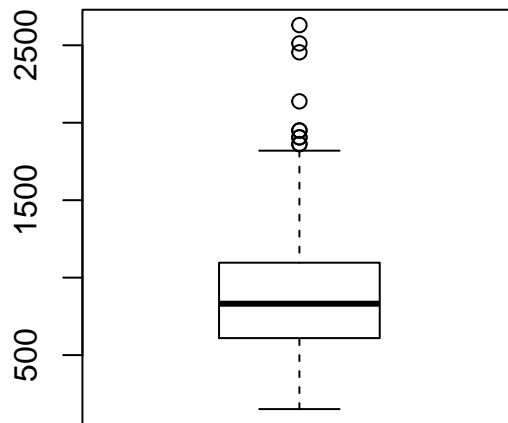
Histogram of Clutch Volume Frequencies
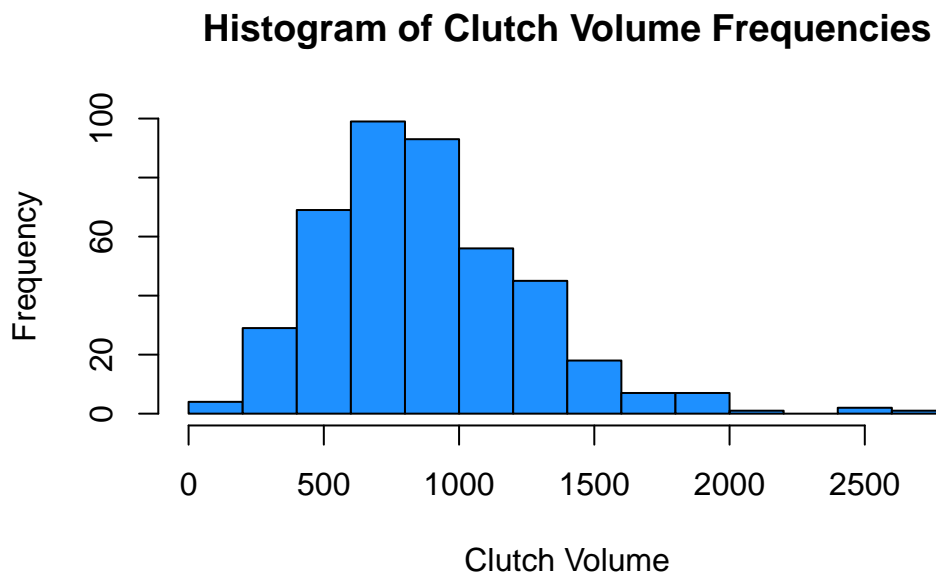
Boxplots

A **boxplot** uses five statistics to summarize a dataset, in addition to showing unusual observations. The following command produces a boxplot of the clutch.volume variable.

```
boxplot(frog.altitude.data$clutch.volume)
```

Like the `hist()` function, the `boxplot()` function also takes several arguments that allow for certain parameters to be specified:

- x: variable of interest

- axes: if TRUE, numbers are shown on the axes

- col: color of the boxplot, enclosed in ""

- xlab: *x*-axis label, enclosed in ""

- ylab: *y*-axis label, enclosed in ""

- xlim: range of values for the *x*-axis, in the form c(lower bound, upper bound)

- ylim: range of values for the *y*-axis, in the form c(lower bound, upper bound)

- main: main title of the plot, enclosed in ""

The following command reproduces a simplified version of *OI Biostat* Figure 1.19.

```
## Figure 1.19
boxplot(x = frog.altitude.data$clutch.volume, ylab = 'Clutch Volume', axes = TRUE,
        ylim = range(frog.altitude.data$clutch.volume))
```
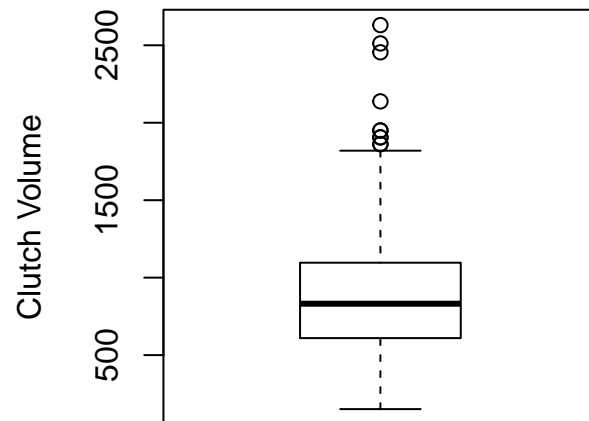
1.3.5   Scatterplots and correlation

Scatterplots can be used to visualize the relationship between two numerical variables. In the `plot()` command, either a comma or a tilde can be used between the variable names; i.e., `plot(x,y)` versus `plot(y ~ x)`.

```r
plot(frog.altitude.data$body.size, frog.altitude.data$clutch.volume)
```
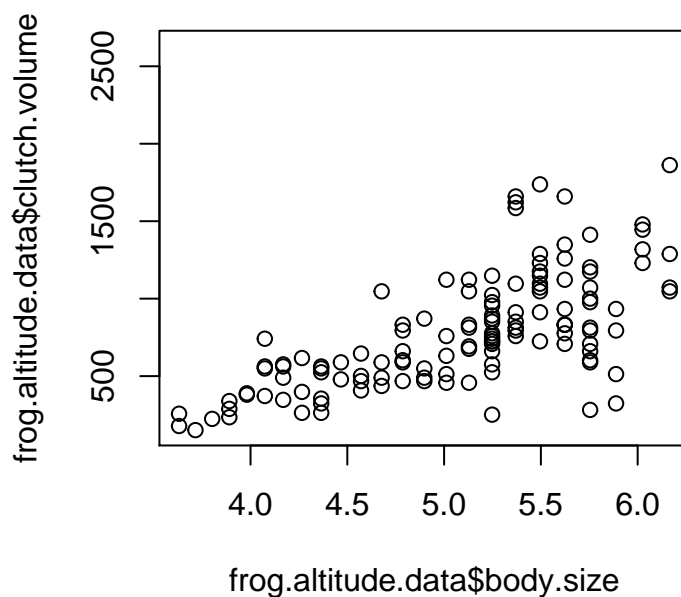
The `plot()` command takes the following arguments:

- x: variable defining the x-coordinates

- y: variable defining the y-coordinates

- col: color of the dots, enclosed in ""

- type: by default, data points are marked with dots; other options include "l" which draws a line chart, or "b" which includes both dots and lines

- xlab: x-axis label, enclosed in ""

- ylab: y-axis label, enclosed in ""

- xlim: range of values for the x-axis, in the form c(lower bound, upper bound)

- ylim: range of values for the y-axis, in the form c(lower bound, upper bound)

- main: main title of the plot, enclosed in ""

The following command reproduces a simplified version of *OI Biostat* Figure 1.20. The additional argument pch changes the appearance of the dots to filled dots, which is specified by 19.

```
## Figure 1.20
plot(frog.altitude.data$clutch.volume~frog.altitude.data$body.size, col = "dodgerblue",
    pch = 19, xlab = "Female Body Size (cm)", ylab = expression("Clutch Volume" ~ (mm^3)))
```
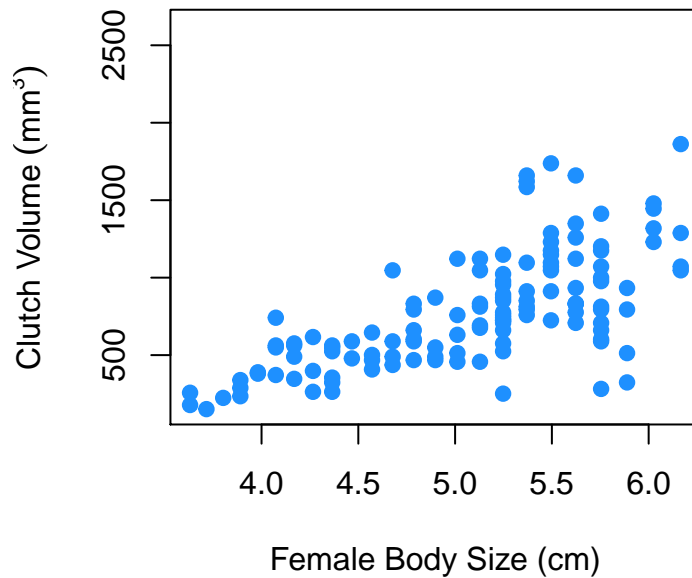
Simplified versions of *OI Biostat* Figures 1.21, 1.22, and 1.23 can also be reproduced using `plot()`.

1.3.6 Correlation

Correlation is a numerical measure of the strength of a linear relationship. The formula for correlation uses the pairing of the two variables, just as the scatterplot does in a graph, so the data used in calculating correlation is a set of n ordered pairs $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$.

The correlation between two variables x and y is given by:

$$r = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i - \overline{x}}{s_x} \right) \left(\frac{y_i - \overline{y}}{s_y} \right)$$

where $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ are the n paired values of x and y, and s_x and s_y are the sample standard deviations of the x and y variables, respectively.

The following illustrates the calculations for finding the correlation coefficient of (1,5), (2, 4), and (3,0), as shown in *OI Biostat* Example 1.13.

```
# define variables
x = c(1, 2, 3)
y = c(5, 4, 0)

# calculate sample means
x.bar = mean(x)
y.bar = mean(y)

# calculate sample sd's
sd.x = sd(x)
```

```
sd.y = sd(y)

# calculate products and sum of products
x.component = (x - x.bar)/(sd.x)
y.component = (y - y.bar)/(sd.y)
products = x.component * y.component
products.sum = sum(products)

# divide by n - 1
n = length(x)    ## n also equals length(y)
r = products.sum / (n - 1)
r

## [1] -0.9449112
```

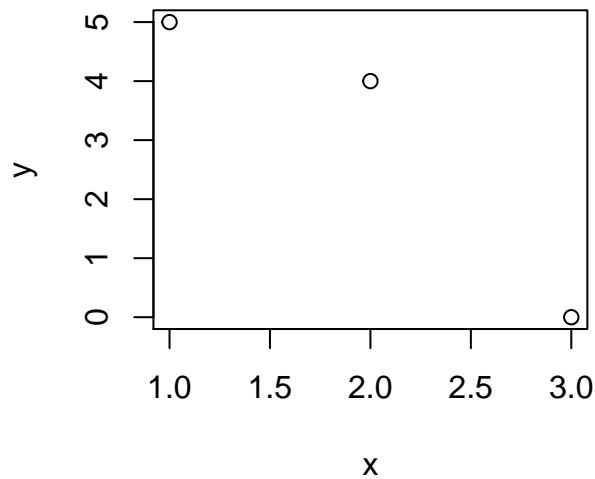It is much easier to use the cor() function to find correlation.

```
## Figure 1.22
cor(x,y)

## [1] -0.9449112

plot(x,y)
```


The correlation can also be calculated for the income versus life expectancy data plotted in *OI Biostat* Figure 1.23. In this case, the command includes the argument use = "complete.obs" to allow for the computation to disregard any missing values in the dataset.

```
cor(life.expectancy.income$income, life.expectancy.income$life.expectancy,
    use = "complete.obs")

## [1] 0.6308783
```

1.3.7 Transforming Data

A **transformation** is a rescaling of data using a function. The figure below shows the original plot of income data as well as the plot of the log-transformed data. Note that the `log` command in R computes natural logarithms.

The function `par()` creates partitions in the graphing output. In this case, specifying `mfrow = c(1,2)` produces 1 row and 2 columns.

```
## Figure 1.26
par(mfrow = c(1, 2))  ## this line allows two plots to print at the same time
hist(life.expectancy.income$income, breaks = 12, xlab = "Income (USD)",
    ylab = "Frequency", ylim = c(0, 120), main = "Untransformed")

hist(log(life.expectancy.income$income), breaks = 12, xlab = "Income (log USD)",
    ylab = "Frequency", ylim = c(0, 30), main = "Log Transformed")
```
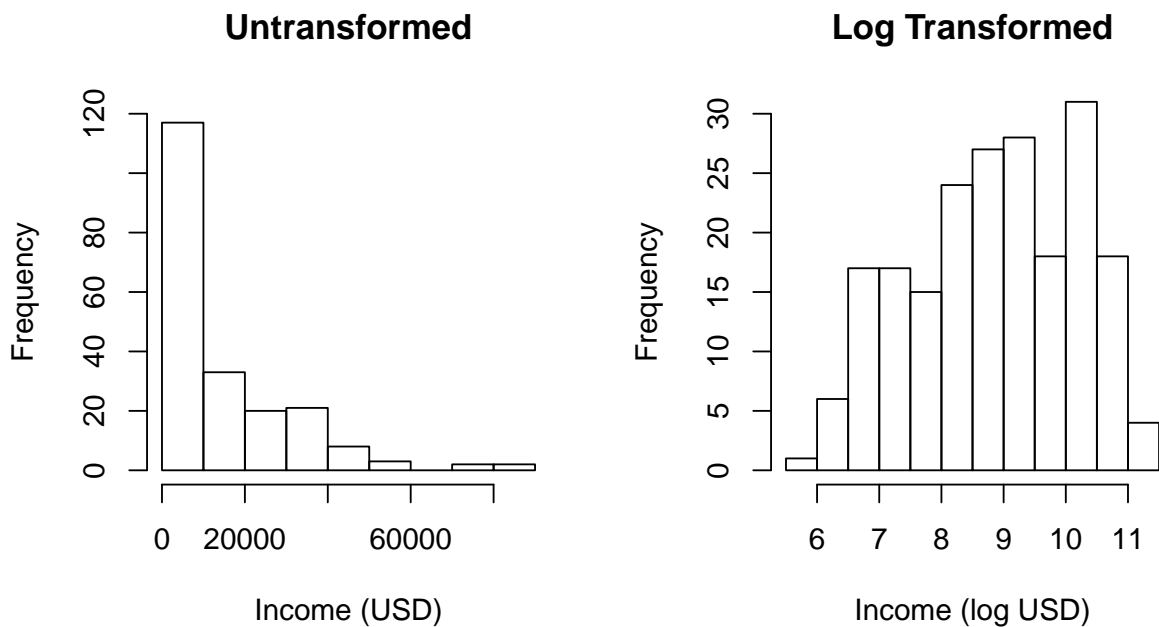

Transformations can also be applied to both variables when exploring a relationship. The following figure shows simplified versions of *OI Biostat* Figures 1.23 and 1.27.

```
## Figure 1.23 and Figure 1.27
par(mfrow = c(1, 2))
plot(life.expectancy.income$income, life.expectancy.income$life.expectancy,
    ylab = "Life Expectancy (years)", xlab = "Per Capita Income (USD)",
    main = "Untransformed")
plot(log(life.expectancy.income$income), log(life.expectancy.income$life.expectancy),
    ylab = "Life Expectancy (log years)", xlab = "Per Capita Income (log USD)",
    main = "Transformed")
```

Untransformed — Life Expectancy (years) vs Per Capita Income (USD)

Transformed — Life Expectancy (log years) vs Per Capita Income (log USD)
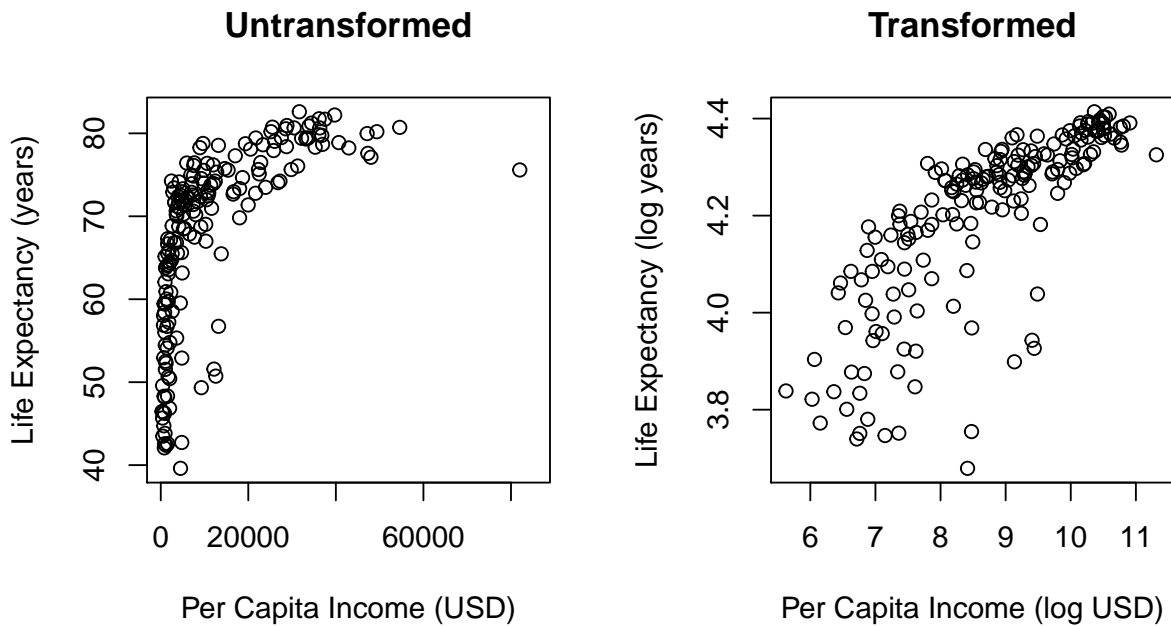
1.4 Categorical Data

1.4.1 Contingency Tables

A **frequency table** shows the counts for each category within a variable. The following table() command produces a frequency table for the actn3.r577x variable.

```
## Table 1.28
addmargins(table(famuss$actn3.r577x))

##
##  CC  CT  TT Sum
## 173 261 161 595
```

A **contingency table** summarizes data for two categorical variables, such as race and genotype in *OI Biostat* Table 1.29.

```
## Table 1.29
addmargins(table(famuss$race, famuss$actn3.r577x))

##
##               CC  CT  TT Sum
##   African Am  16   6   5  27
##   Asian       21  18  16  55
##   Caucasian  125 216 126 467
##   Hispanic     4  10   9  23
##   Other        7  11   5  23
##   Sum        173 261 161 595
```

OI Biostat Table 1.30 shows a contingency table with row proportions, computed as the counts divided by their row totals. The prop.table() command produces a table with row proportions, as specified by the 1 in the argument.

```
## Table 1.30
counts.table = table(famuss$race, famuss$actn3.r577x)
row.prop.table = prop.table(counts.table, 1)
row.prop.table

##
##                     CC        CT        TT
##   African Am 0.5925926 0.2222222 0.1851852
##   Asian      0.3818182 0.3272727 0.2909091
##   Caucasian  0.2676660 0.4625268 0.2698073
##   Hispanic   0.1739130 0.4347826 0.3913043
##   Other      0.3043478 0.4782609 0.2173913
```

Alternatively, a contingency table can be created with column proportions by changing the 1 in prop.table() to a 2.

```
## Table 1.31
counts.table = table(famuss$race, famuss$actn3.r577x)
col.prop.table = prop.table(counts.table, 2)
col.prop.table

##
##                      CC         CT         TT
##   African Am 0.09248555 0.02298851 0.03105590
##   Asian      0.12138728 0.06896552 0.09937888
##   Caucasian  0.72254335 0.82758621 0.78260870
##   Hispanic   0.02312139 0.03831418 0.05590062
##   Other      0.04046243 0.04214559 0.03105590
```

1.4.2   Bar Plots

A **bar plot** is a common way to display a single categorical variable, either from count data or from proportions. The barplot() command requires values to be input in a table format. In the below example, the table() command is nested within the barplot() command.

```
## Figure 1.32
par(mfrow = c(1, 2))
barplot(table(famuss$actn3.r577x))  ## count barplot
barplot(table(famuss$actn3.r577x)/sum(table(famuss$actn3.r577x)))  ## frequency barplot
```
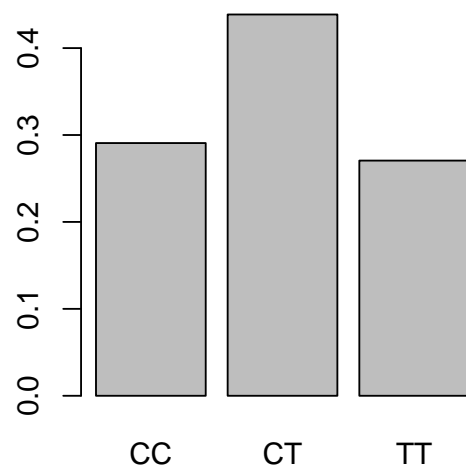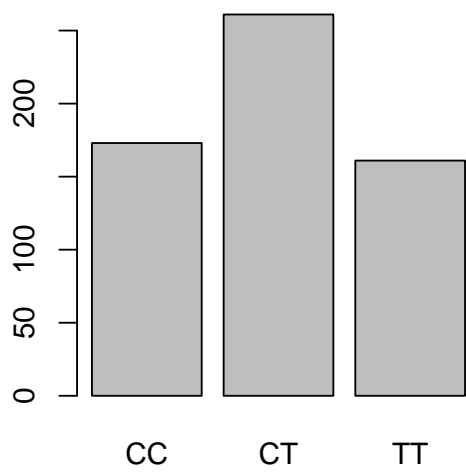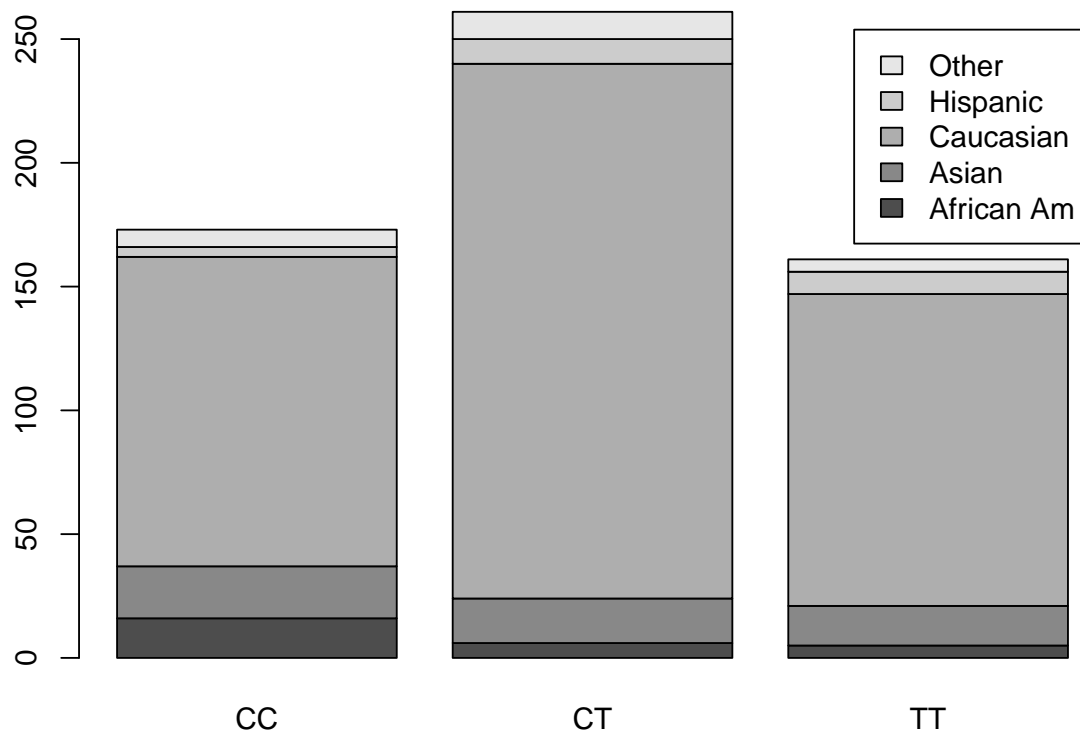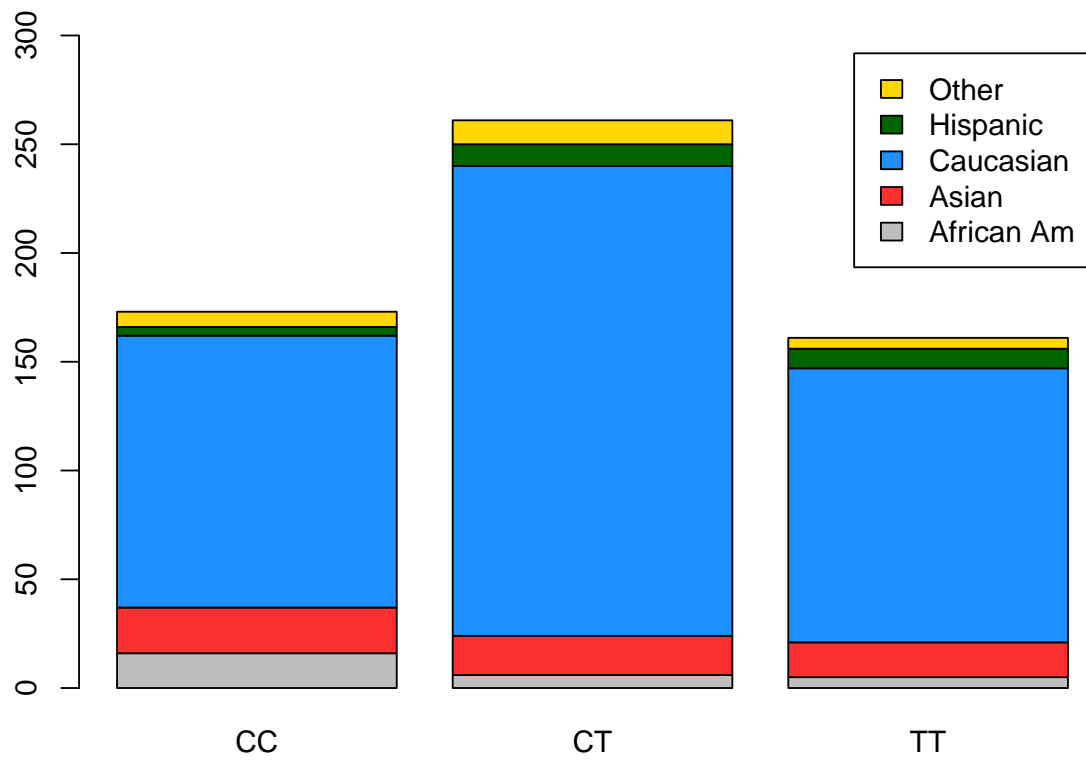
1.4.3 Segmented Bar Plot

A **segmented bar plot** draws from a contingency table to display information about two categorical variables. The following code reproduces *OI Biostat* Figure 1.33a, in which a bar plot was created using the actn3.r577x variable, with each group divided by the levels of race. The simplified version of the plot uses the default greyscale shading; it is also possible to specify a list of colors using c().

The argument legend can be used to specify whether the row names or the column names are used for the legend.

```
## Figure 1.31a
counts.table = (table(famuss$race, famuss$actn3.r577x))
barplot(counts.table, legend = rownames(counts.table))
```
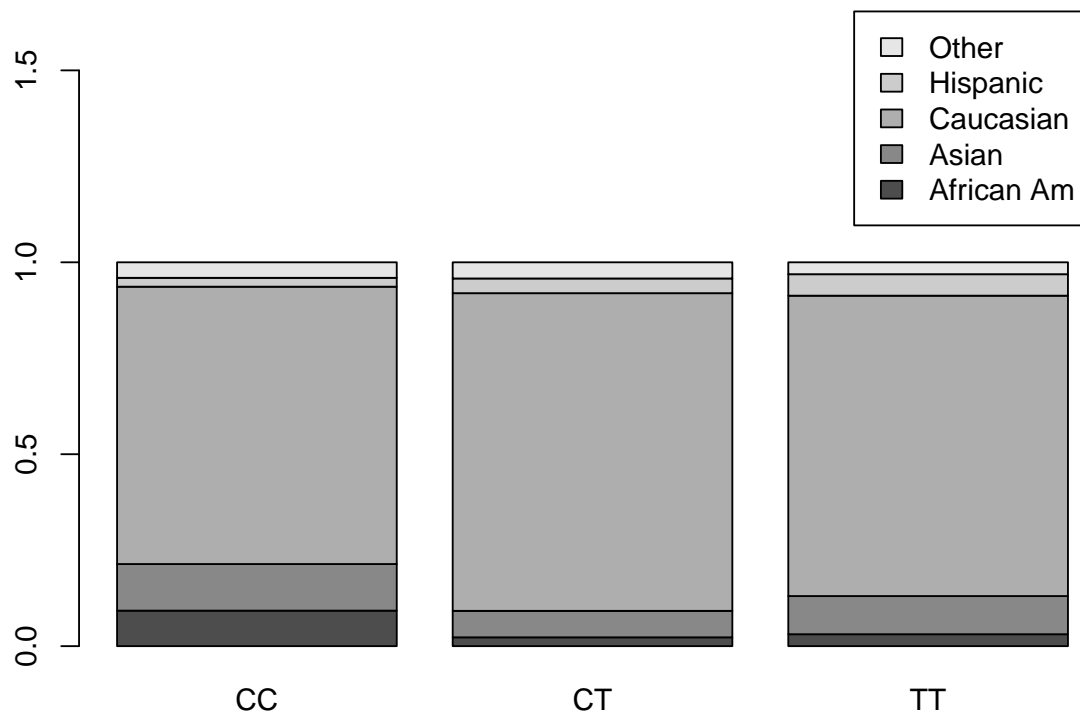


```
barplot(counts.table, col = c("gray", "firebrick1", "dodgerblue",
    "darkgreen", "gold"), ylim = c(0, 300), legend = rownames(counts.table))
```

A **standardized segmented bar plot** uses proportions to scale the data, and draws from a contingency table with proportions. Setting `ylim` from 0 to 1.7 allows for enough empty space on the plot so that the legend does not overlap the bars.

```
## Figure 1.33b
counts.table = (table(famuss$race, famuss$actn3.r577x))
row.prop.table = prop.table(counts.table, 2)
barplot(row.prop.table, ylim = c(0, 1.7), legend = rownames(counts.table))
```



```
barplot(row.prop.table, col = c("gray", "firebrick1", "dodgerblue",
    "darkgreen", "gold"), ylim = c(0, 1.7), legend = rownames(counts.table))
```
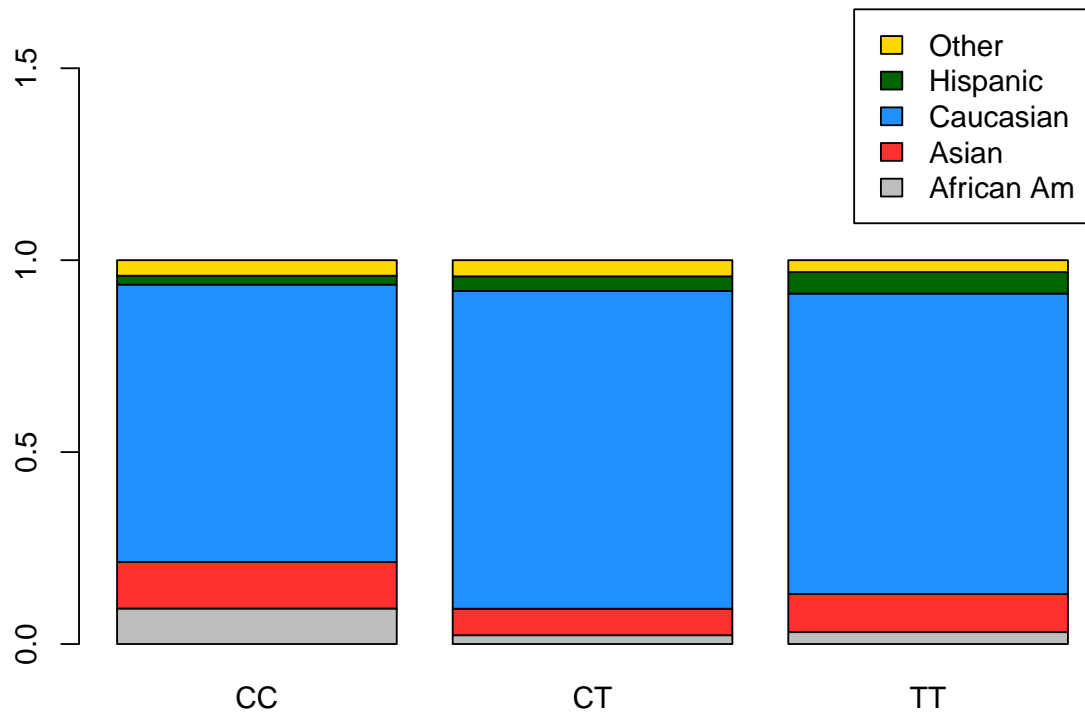
In *OI Biostat* Figure 1.34, the data from the contingency table are organized differently, with each bar representing a level of race. To make this change, reverse the order of the variables in counts.table

```
## Figure 1.34a
counts.table = (table(famuss$actn3.r577x, famuss$race))  ## change variable order

barplot(counts.table, col = c("dodgerblue", "darkgreen", "gold"),
    ylim = c(0, 500), legend = rownames(counts.table))
```

```
## Figure 1.34b
row.prop.table = prop.table(counts.table, 2)
barplot(row.prop.table, col = c("dodgerblue", "darkgreen", "gold"),
    ylim = c(0, 1.8), legend = rownames(counts.table))
```
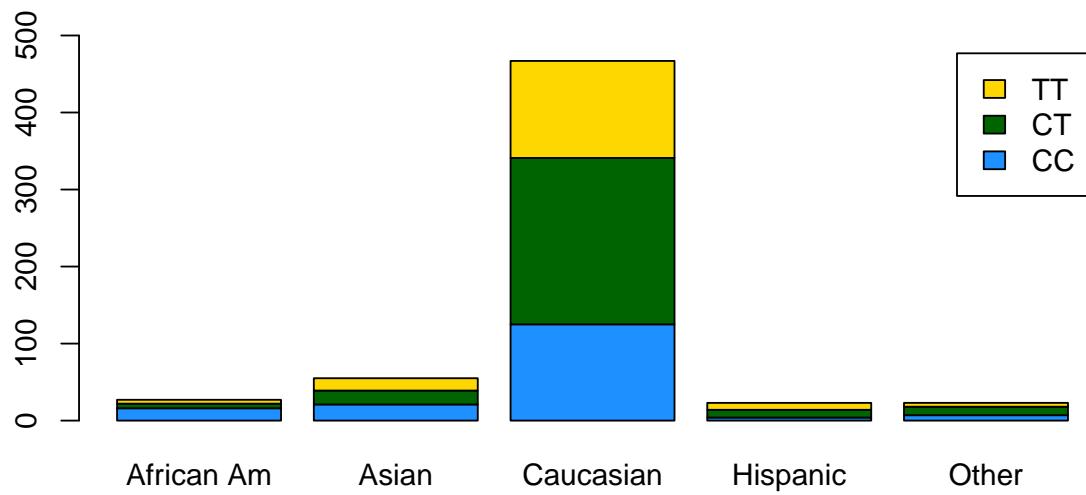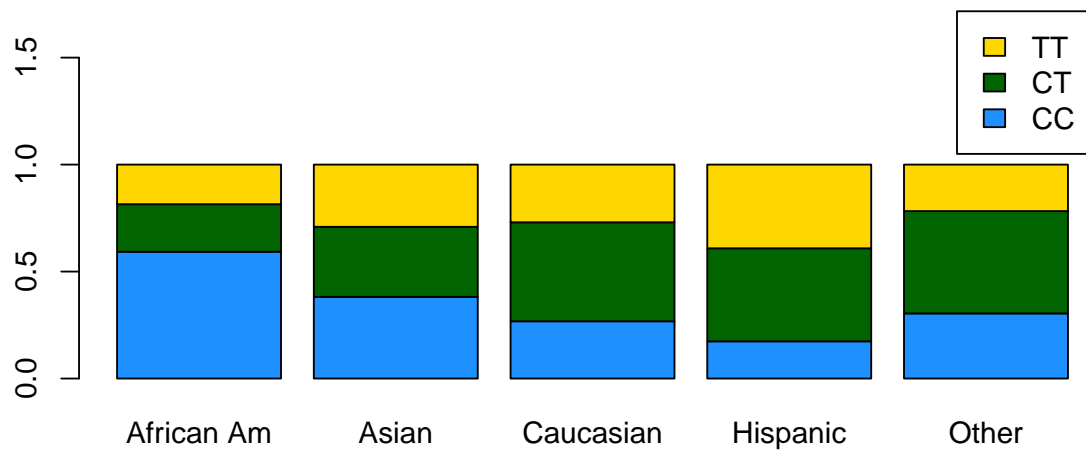
1.4.4 Comparing numerical data across groups

Side-by-side boxplots

The **side-by-side** boxplot is a useful tool for comparing the distribution of numerical data across categories. *OI Biostat* Figure 1.35a shows a side-by-side boxplot for percent change in non-dominant arm strength grouped by genotype. The response variable y comes before the tilde and the explanatory variable x; in this case, the response variable is percent change in strength.

```
## Figure 1.335(a)
boxplot(famuss$ndrm.ch ~ famuss$actn3.r577x,
    ylab = "% Change in Non-Dominant Arm Strength",
    xlab = "Genotype")
```
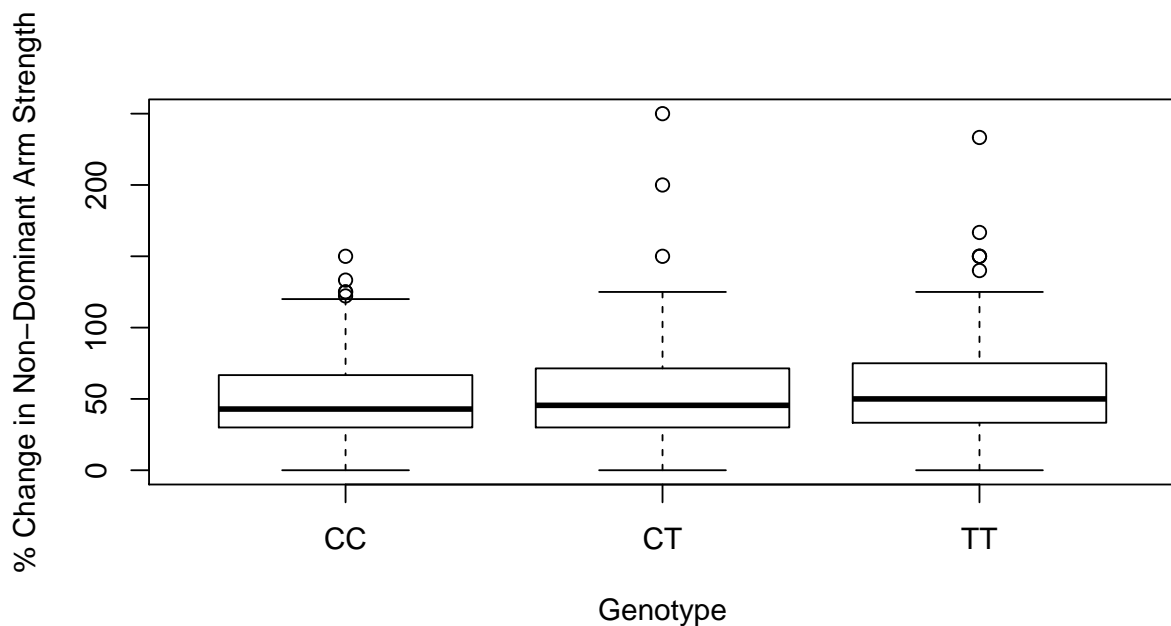

OI Biostat Figure 1.36 shows a larger side-by-side boxplot which compares the distribution of frog clutch sizes for different altitudes.

```
## Figure 1.36
boxplot(frog.altitude.data$clutch.volume ~ frog.altitude.data$altitude,
    xlab = "Clutch Volume", ylab = "Altitude")
```
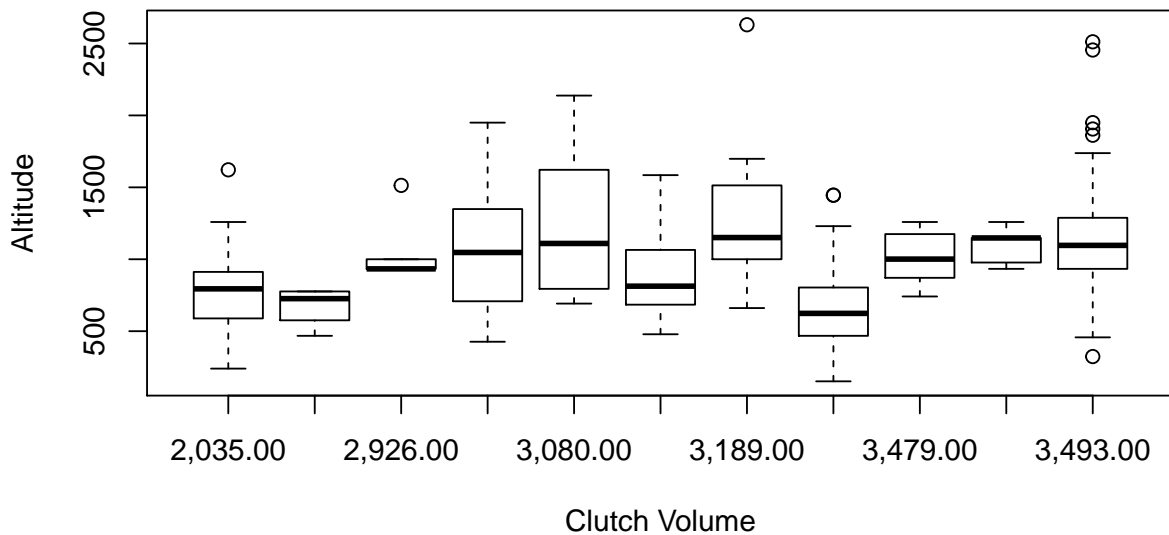
Hollow Histograms

A **hollow histogram** plots the outlines of histograms for each group onto the same axes. To plot a hollow histogram, specify any axis limits and labels in the command for the first histogram; the subsequent histograms require the argument add = T in order for them to be overlaid on top of the first plot.

```r
## Figure 1.35(b)
hist(famuss$ndrm.ch[famuss$actn3.r577x == "CC"], breaks = 20, border = "dodgerblue",
     xlab = "% Change in Non-Dominant Arm Strength", ylim = c(0,50), xlim = c(0,250),
     main = "Percent Change in Non-Dominant Arm Strength by Genotype")
hist(famuss$ndrm.ch[famuss$actn3.r577x == "CT"], breaks = 20, border = "darkgreen",
     add = T)
hist(famuss$ndrm.ch[famuss$actn3.r577x == "TT"], breaks = 20, border = "firebrick1",
     add = T)

legend('topright',
       col = c("dodgerblue", "darkgreen", "firebrick1"),
       lwd = c(1, 1, 1),      ## line width
       legend = c('CC', 'CT', 'TT'))    ##legend labels
```

Percent Change in Non−Dominant Arm Strength by Genotype
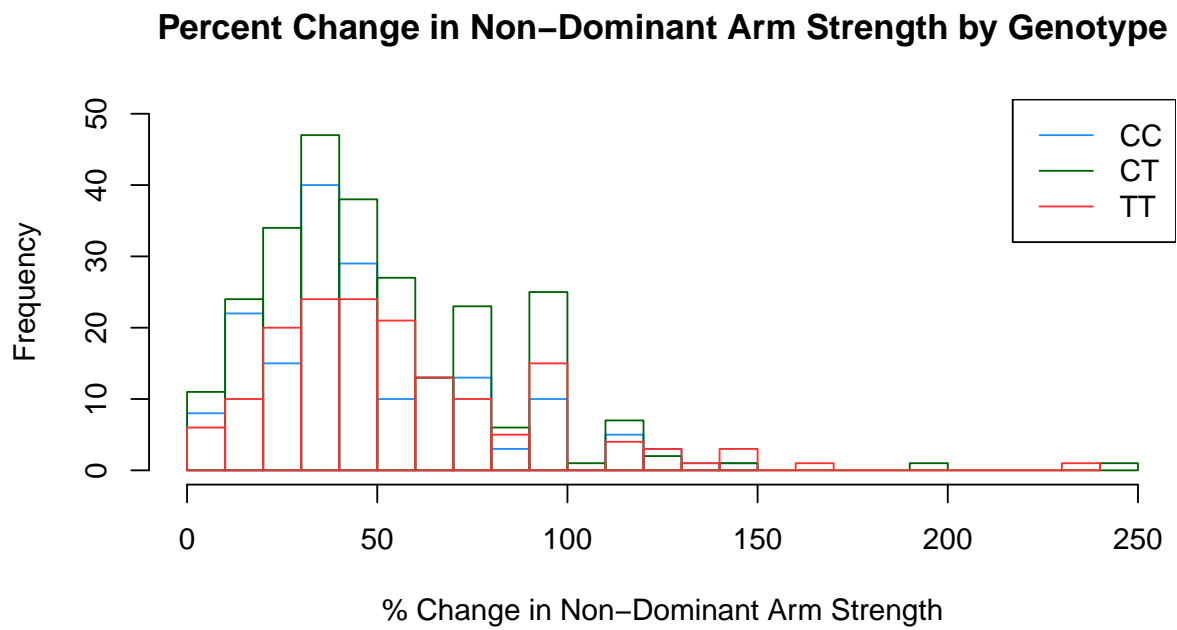
1.5 Genomic Data

```
boxplot(Golub[, 7:9])
```
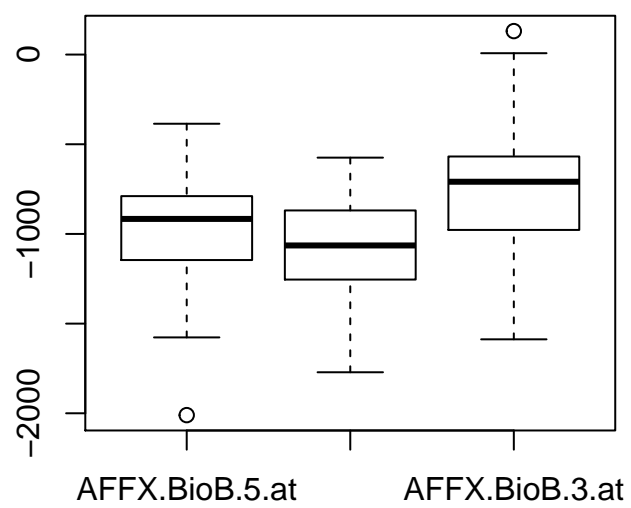
Chapter 2

Using R for Probabilities

Contents

This chapter focuses on the material covered in *OI Biostat* Chapter 2, Section 2.5. While simple conditional probability problems are easily solved by hand with a basic calculator, R can be useful for more elaborate scenarios like those that involve Bayes' Theorem.

What are the chances that a woman with a positive mamogram has breast cancer? This question can be rephrased as the conditional probability that a woman has breast cancer, given that her mammogram is abnormal, otherwise known as the **positive predictive value** of a mammogram. Two methods discussed in this chapter, using Bayes' Theorem and creating a contingency table, are also explained in *OI Biostat*. This chapter will also cover an additional approach: modeling the problem scenario by running a simulation.

> *OI Biostat* **Example 2.37.** In Canada, about 0.35% of women over 40 will develop breast cancer in any given year. A common screening test for cancer is the mammogram, but it is not perfect. In about 11% of patients with breast cancer, the test gives a **false negative**: it indicates a woman does not have breast cancer when she does have breast cancer. Similarly, the test gives a **false positive** in 7% of patients who do not have breast cancer: it indicates these patients have breast cancer when they actually do not. If a randomly selected woman over 40 is tested for breast cancer using a mammogram and the test is positive – that is, the test suggests the woman has cancer – what is the probability she has breast cancer?

2.1 Bayes' Theorem

Bayes' Theorem states that the conditional probability $P(A_1|B)$ can be identified as the following fraction:

$$\frac{P(A_1 \text{ and } B)}{P(B)} = \frac{P(B|A_1)P(A_1)}{P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \cdots + P(B|A_k)P(A_k)}$$

where A_2, A_3, ..., and A_k represent all other possible outcomes of the first variable.

The expression can also be written in terms of diagnostic testing language, where $D = \{$has disease$\}$, $D^c = \{$does not have disease$\}$, $T^+ = \{$positive test result$\}$, and $T^- = \{$negative test result$\}$.

$$P(D|T^+) = \frac{P(D \text{ and } T^+)}{P(D)}$$

$$= \frac{P(T^+|D) \times P(D)}{[P(T^+|D) \times P(D)] + [P(T^+|D^c) \times P(D^c)]}$$

$$\text{PPV} = \frac{\text{sensitivity} \times \text{prevalence}}{[\text{sensitivity} \times \text{prevalence}] + [(1 \text{ - specificity}) \times (1 \text{ - prevalence})]}$$

R can be used to store values for prevalence, sensitivity, and specificity so that calculations are less tedious. Recall that the **sensitivity** is the probability of a positive test result when disease is present, which is the complement of a false negative. The **specificity** is the probability of a negative test result when disease is absent, which is the complement of a false positive.

```
prevalence = 0.0035
sensitivity = 1 - 0.11
specificity = 1 - 0.07

ppv.num = (sensitivity*prevalence)  ## numerator
ppv.den = ppv.num + ((1-specificity)*(1-prevalence))  ## denominator
ppv = ppv.num / ppv.den

ppv

## [1] 0.04274736
```

2.2 Contingency Table

The PPV can also be calculated by constructing a two-way contingency table for a hypothetical population and calculating conditional probabilities by conditioning on rows or columns. While this method results in an estimate of PPV, using a large enough population size such as 100,000 produces an empirical estimate that is very close to the exact value found through using Bayes' Theorem.

	D+	D-	Total
T+			
T-			
Total			100,000

First, calculate the expected number of disease cases and non-disease cases in the population:

```
population.size = 100000
expected.cases = prevalence * population.size
expected.cases

## [1] 350

expected.noncases = (1 - prevalence) * population.size
expected.noncases

## [1] 99650
```

	D+	D-	Total
T+			
T-			
Total	350	99,650	100,000

Next, calculate the expected number of cases of true positives and the expected number of cases of false positives:

```
expected.true.positives = expected.cases * sensitivity
expected.true.positives

## [1] 311.5

expected.false.positives = expected.noncases * (1 - specificity)
expected.false.positives

## [1] 6975.5

total.expected.positives = expected.true.positives + expected.false.positives
total.expected.positives

## [1] 7287
```

	D+	D-	Total
T+	311.5	6,975.5	7,287
T-			
Total	350	99,650	100,000

Finally, calculate the positive predictive value:

```
ppv = expected.true.positives/total.expected.positives
ppv

## [1] 0.04274736
```

2.3   Simulation

R can be used to simulate a population of 100,000 individuals that fits the parameters specified by the problem, i.e., a population where 0.35% of women have breast cancer. Afterwards, using the known specificity and sensitivity of the diagnostic test, individuals can be assigned a test result of either positve or negative. This results in a simulated dataset of 100,000 individuals that each have a disease status and test result.

1. Define the parameters of the simulation: disease prevalence, test sensitivity, test specificity, and hypothetical population size.

2. In order for the results of the simulation to be reproducible, it is necessary to use set.seed() to associate the particular set of results with a "seed". Any integer can be used as the seed. Different seeds will produce a different set of results.

3. Using the vector() command, create two empty lists that will be used to store the results of the simulation: one will store disease status and the other will store test outcome. The results will be in the form of numbers; specifically, either 0 or 1.

4. Assign disease status by using the `sample()` command, which takes a sample of a specified size from a list x. In this context, the goal is to sample between 0 and 1 100,000 times, where 0 represents an individual without breast cancer and 1 an individual with breast cancer. The argument prob defines the probability that either number is sampled; a 1 should be sampled with the same probability as the prevalence, since the prevalence indicates how many members of the population have disease. Similarly, a 0 should be sampled with probability (1 - prevalence). The argument replace = TRUE allows for sampling with replacement; in other words, allowing the numbers 0 and 1 to be assigned multiple times.

5. Assign test result by using the `sample()` command in combination with a for loop and conditional statements. In this step, a test result is assigned with different probability depending on whether the disease status is a 0 or a 1; this relates to the sensitivity and specificity of the diagnostic test. The loop allows for this process to be repeated for each of the 100,000 individuals being simulated.

   - The first line sets up the loop, with a variable ii that starts at 1 and finishes at `population.size`, which was specified earlier as 100,000. This allows for each test result to be assigned one at a time.

   - Two if statements are in the loop which direct R to take a different action depending on the value of `disease.status`. The double equals signs == imply a conditional statement, allowing `if(disease.status[ii] == 1)` to make the statement "For this loop, if disease status is equal to 1, then do the following...".

   - Depending on disease status, R will execute one of the two `sample()` functions in the loop. One sample function has probabilities weighted based on sensitivity and the other has probabilities defined by specificity.

6. Make calculations using the two lists, `disease.status` and `disease.outcome`, which are now filled with values. Since the value 1 was assigned to a positive test result and to an individual with disease, the `sum()` command can be used to determine the total number of individuals with disease and the number of positive test results.

```
## 1. set parameters
prevalence = 0.0035
sensitivity = 1 - 0.11
specificity = 1 - 0.07
population.size = 100000

## 2. set seed
set.seed(2016)

## 3. create empty lists
disease.status = vector("numeric", population.size)
test.outcome = vector("numeric", population.size)

## 4. assign disease status
disease.status = sample(x = c(0,1), size = population.size,
                        prob = c(1 - prevalence, prevalence),  ## matches order of x
                        replace = TRUE)

## 5. assign test result
for (ii in 1:population.size) {
  if (disease.status[ii] == 1) {  ## note the ==
```

```
      test.outcome[ii] = sample(c(0, 1), size = 1, ## test results assigned 1 at a time
                                 prob = c(1 - sensitivity, sensitivity))}
  if (disease.status[ii] == 0) {  ## note the ==
      test.outcome[ii] = sample(c(0, 1), size = 1, ## test results assigned 1 at a time
                                 prob = c(specificity, 1 - specificity))}

}

## 6. calculate ppv
num.disease = sum(disease.status)  ## total number of individuals with disease
num.pos.test = sum(test.outcome)   ## total number of positive tests

# identify the number of individuals with disease who tested positive
num.true.pos = sum(test.outcome[disease.status == 1])

# calculate ppv
ppv = num.true.pos / num.pos.test
ppv

## [1] 0.04273973
```

Chapter 3

Distributions

Contents

Chapter 3 presents the idea of several known and commonly used distributions. R is a powerful source for accessing and using these distributions.

3.1 Normal Distribution

We have learned that the **normal distribution** can be written as

$$N(\mu, \sigma)$$

where μ is the mean of the distribution and σ is the standard deviation. Using this idea, a standard normal distribution can be created in one of two ways:

3.1.1 Direct Plot

There is a function in R called the *dnorm* function, which, for a given posible value of the distribution, returns the probability of the distribution holding that value.

```
## getting a list of values to evaluate distribution
X = seq(-4, 4, 0.01)
## getting normal distribution value of the given X values
Y = dnorm(X, mean = 0, sd = 1)
## plotting results
plot(X,Y, type = "l")
```
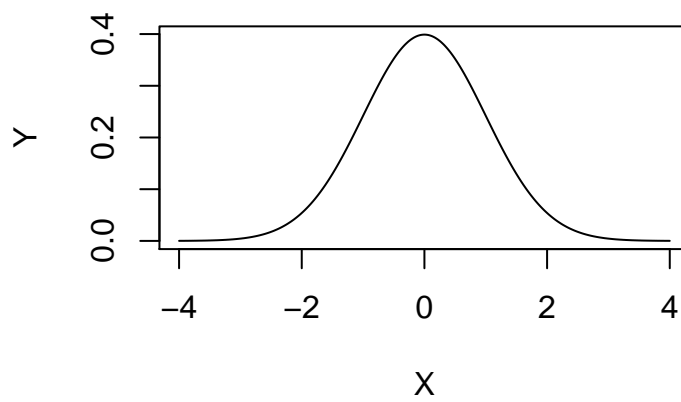

3.1.2 Through Simulation

The second method involves sampling randomly from the known normal distribution to simulate the prob-
abilities that we drew directly in the above method. In this case, the function *rnorm* takes *n* number of
random samples from the normal distribution with the given mean and standard deviation. In the *hist*
command, the specification that *freq = FALSE* should be included to make the plot reflect percentages
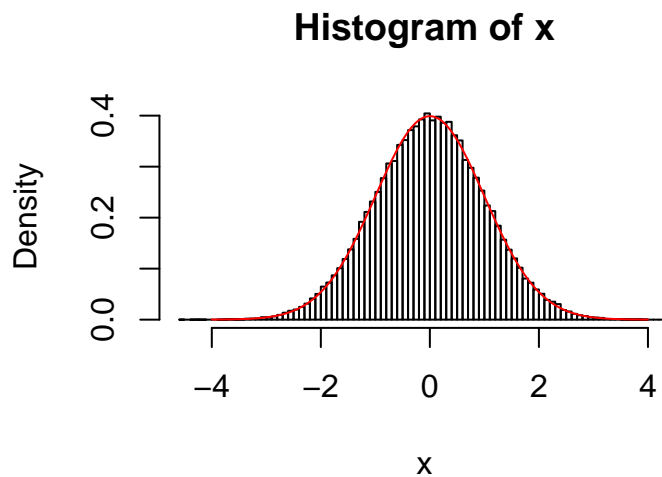rather than frequency counts.

```
set.seed(1)
x <- rnorm(n = 100000, mean = 0, sd = 1)
hist(x, breaks = 100, freq = FALSE)
```

Histogram of x
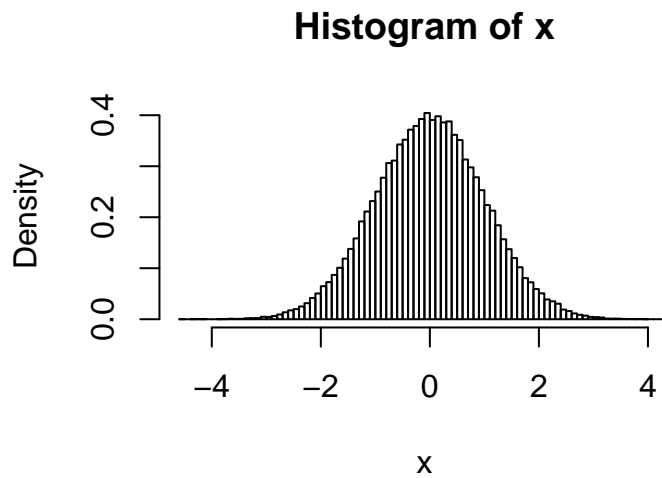
3.1.3 A Comparison

Plotting both methods on the same graphs is useful for comparison as follows.

```
hist(x, freq = FALSE, breaks = 100)
lines(X,Y, col = "red")
```


Histogram of x

Note that the two show very similar results. For practical purposes, *dnorm* is typically used as it only requires one command and is the asymptotic result of simulations, rather than an approximation.

3.1.4 Probability Functions

For the normal distribution, there are two more functions that are very useful, *pnorm* and *qnorm*.

pnorm

The function *pnorm* tells the percentage of the normal distribution that is less than or equal to the value that is put in. This corresponds to the gray area in the below plot and can be written as $pnorm(x, mean = \mu, sd = \sigma)$ which is equivalent to $P(X < x)$ where $X \sim N(\mu, \sigma)$ and x is the point of interest.
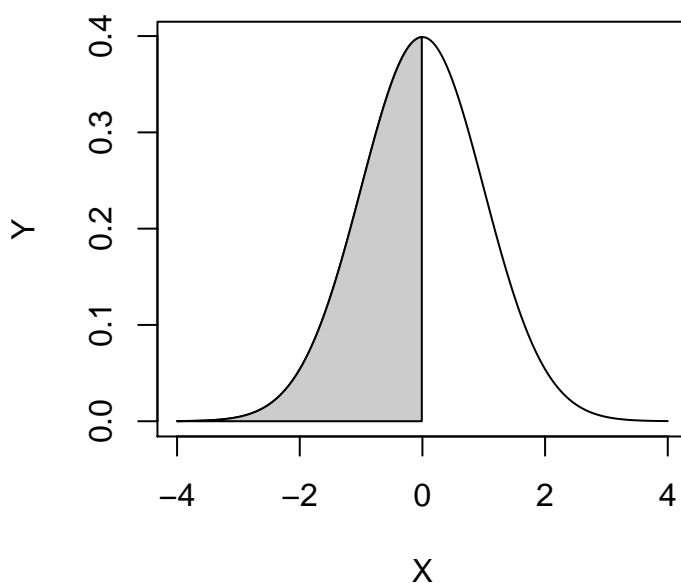
For some example scenarios, where the goal is to find a proportion of the normal distribution given x or z, the following configurations of *pnorm()* can be used:

- $pnorm(z) = P(Z \leq z)$

- $pnorm(z, lower.tail = FALSE) = P(Z > z)$

- $pnorm(x, \mu, \sigma) = P(X \leq x)$ where $X = \sigma Z + \mu$

- $pnorm(x, \mu, \sigma, lower.tail = FALSE) = P(X > x)$ where $X = \sigma Z + \mu$

Several examples that are shown in the text can be evaluated as follows. Keep note that for every question, there are multiple methods to arrive at the same answer. The two main methods involve approximating to a standard normal distribution and allowing R to do all the work for you.

```
## Example 3.5
pnorm(q = 1)

## [1] 0.8413447
```

```
## Example 3.7
# using the approximation to the standard normal
pnorm(q = 0.43, lower.tail = FALSE)

## [1] 0.3335978

1 - pnorm(q = 0.43)

## [1] 0.3335978

# using the actual normal distribution
pnorm(q = 1630, mean = 1500, sd = 300, lower.tail = FALSE)

## [1] 0.3323863

1 - pnorm(q = 1630, mean = 1500, sd = 300)

## [1] 0.3323863
```

```
## Example 3.10 using standard normal approximation
pnorm(1.21, lower.tail = TRUE) - pnorm(-0.3, lower.tail = TRUE)

## [1] 0.504772

# using actual distribution
pnorm(q = 74, mean = 70, sd = 3.3, lower.tail = TRUE) - pnorm(q = 69, mean = 70,
    sd = 3.3, lower.tail = TRUE)

## [1] 0.5063336
```

qnorm

On the other hand, the *qnorm* function is the inverse function, instead giving the X value such that the given percentage of the distribution is less than or equal to that value. This function takes a known probability and returns a value on the normal distribution that is corresponding.

Some sample configurations of using *qnorm()* can be as follows,

- $qnorm(p) = P(Z \leq z)$

- $qnorm(p, lower.tail = FALSE) = P(Z > z)$

- $qnorm(p, \mu, \sigma) = P(X \leq x)$ where $X = \sigma Z + \mu$

- $qnorm(p, \mu, \sigma, lower.tail = FALSE) = P(X > x)$ where $X = \sigma Z + \mu$

Working through an example in the main text,

```
## Example 3.12
# using standard normal
70 + 3.3*qnorm(0.4)

## [1] 69.16395
```

```
# one step function
qnorm(p = .4, mean = 70, sd = 3.3)

## [1] 69.16395
```

3.1.5 Evaluating the Normal Approximation

It is of interest to test the normality of a distriubtion in order to determine if approximation with a normal distribution is appropriate. There are two methods in *R* that can be used to test this. The first involves plotting the data with a histogram and then superimposing a line on top with the corresponding mean and standard deviation of the data. The more closely that the histogram matches the line, the more appropriate a normal approximation would be.

The second method utilizes the function *qqnorm()*, which plots the residual differences between the observed data and the theoretical normal distribution that would match. A line can be plotted on top of this plot, using *qqline()*, which demonstrates how a perfect match would appear. Deviation of the dots on the plot from the line shows lack of fit for the normal approimation.

Both of these methods can be seen below in this recreation of *Figure 3.10* from the main text. The rows show the first and second methods respectively, while the columns show a progressively better fit to the normal from left to right.

```
obs1 <- c(-0.01, 1.39, 1.598, 0.383, -0.084, -0.475, -0.105, -1.062, 0.607,
    0.539, 0.871, 0.45, -0.039, 1.256, 0.55, -0.333, -0.252, 1.187, -0.916,
    0.677, -1.324, -1.773, 1.813, 0.023, -2.291, -1.361, 0.642, 0.249, -0.132,
    1.345, 0.629, -1.274, 0.435, 0.043, 0.55, -0.465, 0.756, -0.396, -0.767,
    1.348)

obs2 <- c(-0.78, -0.552, -0.027, 0.33, -0.964, 0.865, -0.112, 0.075, -0.392,
    0.365, -1.738, 0.491, -0.245, 0.436, 0.016, -0.202, 1.322, 0.515, 0.333,
    -0.28, -0.843, 0.181, -0.284, -0.409, 0.542, 0.117, -0.194, -0.415, 1.362,
    0.826, 1.099, -1.243, -0.265, -0.387, 0.901, 0.706, -0.353, -1.05, 0.081,
    -1.102, -2.705, -0.142, -0.608, 0.661, -0.616, 1.025, -1.384, 0.337, 1.14,
    0.523, -0.662, 0.19, 1.468, 2.38, -0.351, -2.125, 1.141, 1.149, -0.448,
    1.166, -0.269, -0.145, -1.319, -0.445, 0.34, -1.789, -0.626, -1.302, 2.441,
    -2.016, 0.333, -0.019, 0.457, -0.706, 0.236, 0.496, -0.02, -0.228, -1.756,
    -1.309, 0.564, 1.597, -0.172, -0.369, 0.478, -0.854, -0.52, -0.045, 1.654,
    1.122, -0.155, 0.281, 0.205, 0.096, -2.303, -1.399, -0.877, -1.205, 0.02,
    0.563)

obs3 <- c(0.673, -0.538, -0.703, -0.004, -1.395, -0.354, 0.415, 1.097, 0.74,
    0.657, -0.759, 0.415, 2.094, -0.662, -0.161, 0.293, 0.057, -1.487, -1.822,
    1.192, 2.186, -0.26, 0.453, -0.267, -0.049, -1.075, -0.959, -2.338, 0.512,
    2.365, 0.56, -0.812, 0.363, -0.731, -0.644, -0.448, -0.024, -0.024, -1.133,
    -0.692, 1.233, 0.566, 1.109, -2.215, 0.494, 0.011, -2.785, -0.59, -0.895,
    -1.084, -0.848, -0.129, 1.132, -0.851, -0.419, -0.232, -0.789, -2.018, 1.302,
    -1.276, -0.592, 1.578, 0.474, -0.437, 0.3, 0.145, 0.263, -2.189, -0.265,
    -0.02, 0.85, 1.523, 0.938, 0.651, -1.866, -2.202, 0.083, -0.816, 1.082,
    1.448, -1.563, -0.145, -1.168, 1.633, -0.472, 0.173, -1.592, 0.623, -0.674,
    -1.336, -0.059, 0.209, 0.152, -0.345, -0.686, 2.494, -0.616, 0.615, -0.718,
    1.748, 0.011, -0.936, -0.196, 0.839, -0.099, 0.216, -0.036, -0.687, 1.126,
    -0.024, -0.239, 1.475, 1.548, -1.254, -1.513, 0.635, 0.482, 1.068, -0.814,
```

```
    1.171, -0.509, -0.733, -0.32, 0.05, -0.359, -0.22, 0.269, 1.581, -0.967,
    1.725, 0.322, -0.176, -0.655, 2.362, -0.004, -1.209, 0.622, -1.125, 1.767,
    -0.053, 0.066, 0.049, 0.45, 0.302, -0.172, -1.237, 0.072, -1.007, 0.312,
    -0.647, -0.759, 0.753, -1.179, 0.984, 1.947, -0.653, -0.34, -0.669, -0.066,
    -1.774, -1.409, -0.875, -0.225, -0.775, -0.657, 0.812, 2.278, 0.25, 1.206,
    -0.646, -0.693, -0.545, -1.44, -1.548, -0.924, 0.408, 1.135, 1.173, 1.472,
    -0.578, -0.04, 0.422, -0.214, 0.983, -1.605, -0.6, -0.641, -0.501, 0.795,
    0.542, -1.471, 0.185, 0.504, 3.559, 0.487, 0.406, 0.318, 1.485, 0.217, 0.409,
    -0.005, -0.351, -0.932, 1.504, 0.528, -2.061, -1.405, -0.256, 2.293, 0.385,
    0.363, 0.928, 1.455, -0.317, 0.804, -1.358, 1.137, -1.072, 0.015, -0.905,
    1.768, -0.562, -1.268, 0.284, -0.952, -1.163, -0.352, 0.507, 0.194, 0.579,
    0.345, 1.171, -1.009, 0.622, -1.311, 0.407, 0.277, -0.191, -1.417, 0.089,
    -1.607, 0.012, -1.355, 0.267, 2.723, -1.16, -2.613, 0.161, -0.371, -0.331,
    0.726, 1.389, 1.111, -0.911, -0.74, -0.818, 1.667, 0.714, -0.262, 0.045,
    0.009, -0.022, -0.508, -1.423, 0.229, 0.765, -0.549, 0.587, 0.183, -0.091,
    0.501, 0.452, 1.927, -0.237, 0.697, 0.181, -1.044, 0.605, 1.178, 1.047,
    1.405, 1.686, -0.382, 1.217, 0.499, 0.271, 0.662, 0.562, 0.528, 0.684, -0.751,
    1.843, 1.063, -1.828, 1.345, 0.077, 0.943, 1.048, 1.637, -0.535, 0.664,
    0.433, -0.559, -0.141, 0.663, 0.777, 1.442, -0.685, -0.451, 0.265, 0.727,
    -1.206, 0.339, -0.32, 0.079, -0.052, 0.097, 0.827, -2.121, 0.57, -2.563,
    0.663, -1.115, 0.176, 0.786, 0.783, 1.068, -1.734, 1.255, 0.79, -0.721,
    -0.028, 0.514, -0.963, -2.052, -1.195, 0.091, 0.187, 0.61, -1.61, -0.066,
    2.733, -0.853, -1.175, 1.079, -0.58, 0.033, 1.213, 0.367, -0.567, -0.107,
    0.188, -0.091, -0.932, -0.11, -1.312, 0.968, 0.698, 1.089, 0.695, 1.309,
    1.017, 0.677, 0.471, -1.524, -1.82, 0.256, 0.397, -0.489, 1.734, -0.297,
    0.075, 0.533, 0.165, 0.814, -1.915, -0.332, 1.035, -0.858, 1.07, 0.532,
    -0.016, 1.932, -0.564, -0.018, -0.542, 1.048, 0.759, 1.575, -1.263, -0.667,
    -1.195, -0.179, 0.337, 0.257, 0.356, 0.375, -1.326, 0.509, 0.975)

par(mfrow = c(2, 3))

hist(obs1, breaks = 12, xlim = c(-3, 3), freq = FALSE, col = "light blue", main = " ")
x1 <- seq(min(obs1) - 2, max(obs1) + 2, 0.01)
y1 <- dnorm(x1, mean(obs1), sd(obs1))
lines(x1, y1, lwd = 1.5)

hist(obs2, breaks = 12, xlim = c(-3, 3), freq = FALSE, col = "light green",
    main = " ")
x2 <- seq(min(obs2) - 2, max(obs2) + 2, 0.01)
y2 <- dnorm(x2, mean(obs2), sd(obs2))
lines(x2, y2, lwd = 1.5)

hist(obs3, breaks = 12, xlim = c(-3, 3), freq = FALSE, col = "yellow", main = " ")
x3 <- seq(min(obs3) - 2, max(obs3) + 2, 0.01)
y3 <- dnorm(x3, mean(obs3), sd(obs3))
lines(x3, y3, lwd = 1.5)

qqnorm(obs1, col = "light blue", main = " ")
qqline(obs1)

qqnorm(obs2, col = "light green", main = " ")
```
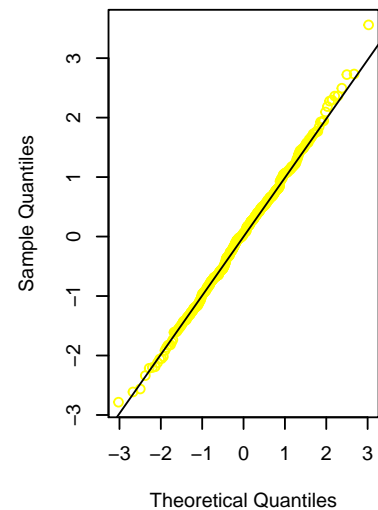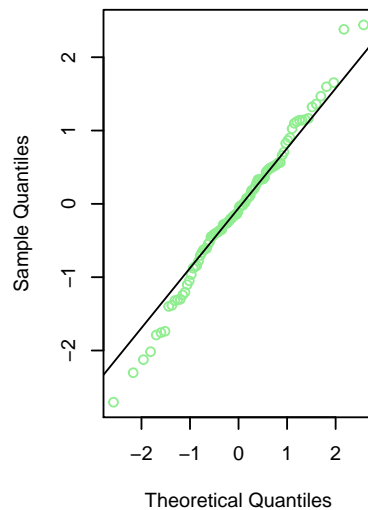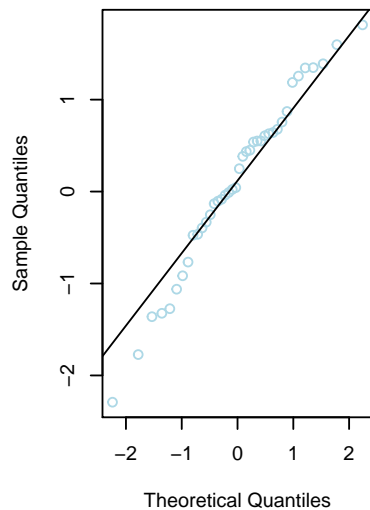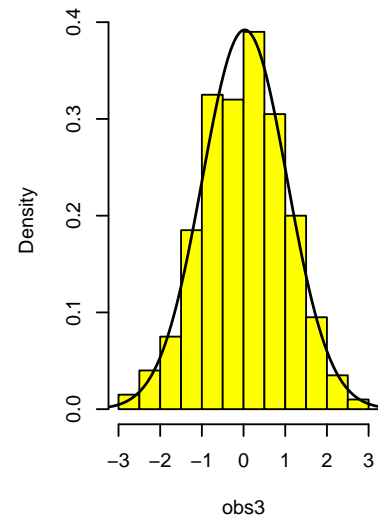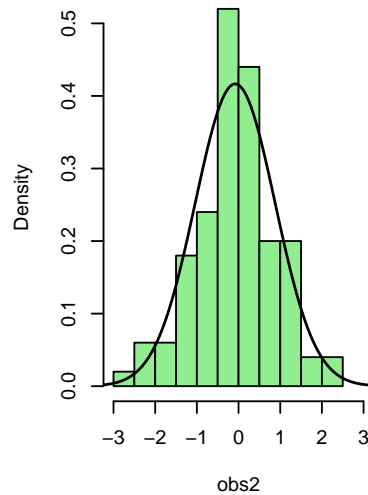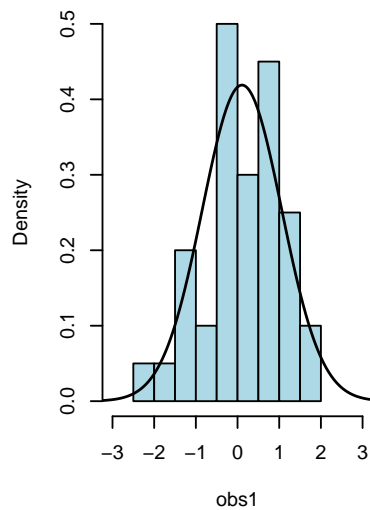
```
qqline(obs2)

qqnorm(obs3, col = "yellow", main = " ")
qqline(obs3)
```
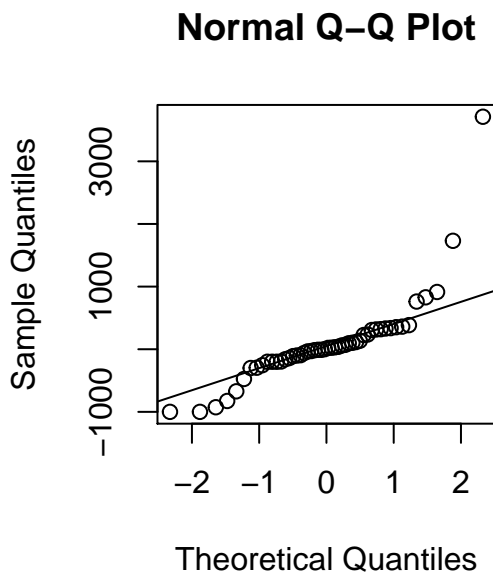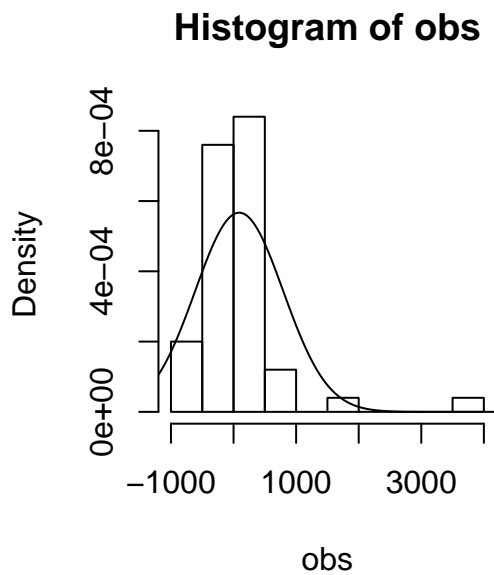


```
par(mfrow = c(1, 2))
obs <- c(-110, -9, -60, 316, -200, -196, 320, -160, 31, 331, 1731, 21, -926,
    -475, 914, -300, -15, 1, -29, 829, 761, 227, -141, -672, 352, 385, 24, 103,
    -826, 95, 115, 39, -9, -1000, -35, -200, -200, 235, 70, 307, 135, 60, -100,
    -295, -1000, 361, -95, 337, 3712, -255)
```

```
x <- seq(min(obs) - 3000, max(obs) + 3000, 1)
y <- dnorm(x, mean(obs), sd(obs))

hist(obs, freq = FALSE)
lines(x, y)

qqnorm(obs)
qqline(obs)
```


3.2 Binomial Distribution

A random variable which follows a **binomial distribution** can be written as

$$X \sim Binom(n, p)$$

where n represents the sample size and p represents the probability of success.

The functions seen above for the normal distribution can be similarly used for the binomial as follows where the necessary arguments are *size* and *prob*, corresponding respectively to n and p in our above representation of the binomial.

- *dbinom*

- *rbinom*

- *pbinom*

- *qbinom*

Similar to the above example with the normal distribution, these functions can be used in multiple ways to work with the binomial distribution.

3.2.1 Accessing the Binomial Distribution

As above, the binomial distribution can be accessed directly with *dbinom* as follows:

```r
## Splitting the graphics window into two panes
par(mfrow=c(1,2))
## Directly plotting the binomial distribution
 # getting a list of values to evaluate distribution
X = seq(0, 10, 1)
 # getting normal distribution value of the given X values
Y = dbinom(X, size = 10, prob = .25)
 # plotting results
plot(X,Y, type = "s", main = "Direct Plot", xlim = c(0,10))

## Instead Using Simulation
set.seed(1)
x <- rbinom(n = 100000, size = 10, prob = .25)
hist(x, breaks = 10, freq = FALSE,right = FALSE, main = "Simulation", xlim = c(0,10))
box()
```
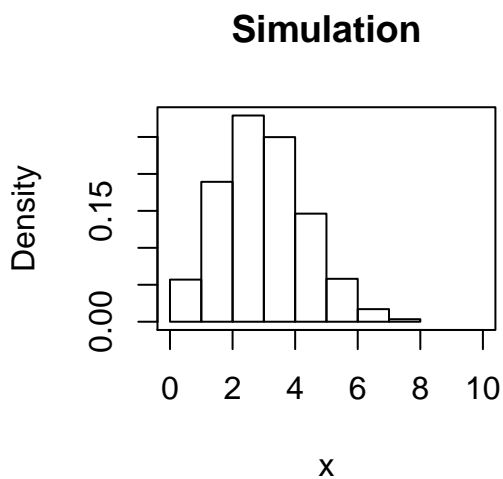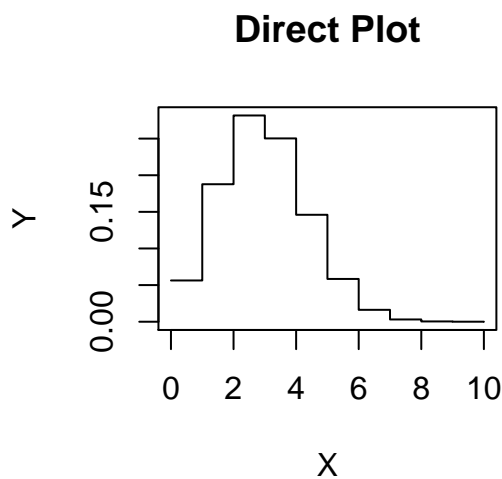
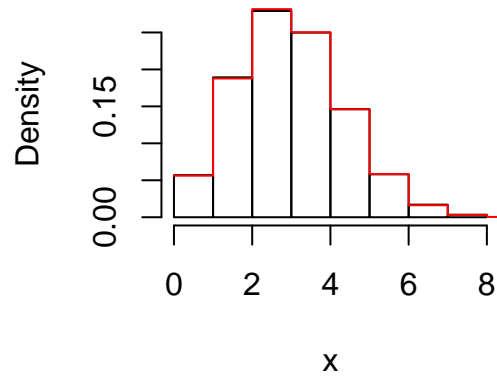

3.2.2 Comparison of the Methods

Superimposing the graphs is another useful mechanism for direct comparison of the two methods.

```r
hist(x, breaks = 10, freq = FALSE,right = FALSE)
lines(X,Y, type = "s", col = "red")
```

Histogram of x

3.2.3 Continuous and Discrete Distributions

Using the binomial distribution highlights an important feature of distributions that must be considered: whether the random variable is **discrete** or **continuous**. A discrete distribution can only take on specified values, typically the integers. The binomial is a great example of this - it can be any integer greater than or equal to zero. For example, the possible values of a binomial are {0, 1, 2, 3, ...}.

On the other hand, a continuous distribution can take on any real number within a specified range. For example, looking at the standard normal distribution, a list of some possible values could be {0, 0.1, 0.01, 0.001, 0.0001, ...}. This is a limited list of the infinite list of possible values that a continuous distribution could take on.

The reason we care about the classification of a random variable as discrete or continous is that it affects the implementation of probability functions in R. Let's walk through an example to see the difference. Consider a normally distributed random variable, X, and a binomially distributed random variable, Y.

$$X \sim N(0,1) \qquad Y \sim Binom(15, 0.5)$$

If the goal is to determine the probability of being less than 2 for each of these two distributions, this would be equivalent to $P(X < 2)$ and $P(Y < 2)$. However, for the binomial distribution $P(Y < 2) = P(Y \leq 1) = P(Y = 0) + P(Y = 1)$ because there is a discrete set of possible values that Y can take on. Y can only take on the integer values, and less than 2, that just leaves 0 and 1, whereas the normal distribution can take on any non-integer value in that interval, which is an infinite number of values. For the normal distribution, solving for $P(Y < 2)$ is mathematically equivalent to $P(Y \leq 2)$ because it is continuous.

To solve for these probabilities, use *pnorm* and *pbinom*, noting that for the discrete binomial case, these functions automatically give the less than or equal to probability, not just the less than probability. The correct commands can be seen through the following examples from the text.

3.2.4 Examples of the Binomial

To summarize, the binomial probaility functions can be used as follows,

- $dbinom(x, n, p) = P(X = x)$

- $pbinom(x, n, p) = P(X \leq x)$

- $pbinom(x, n, p, lower.tail = FALSE) = P(X > x)$

```
## Example 3.19
dbinom(1, size = 4, prob = 0.35)

## [1] 0.384475
```

```
## Example 3.23
dbinom(3, size = 8, prob = 0.35)

## [1] 0.2785858
```

```
## Example 3.24
pbinom(3, size = 8, prob = 0.35, lower.tail = TRUE)

## [1] 0.7063994
```

```
## Example 3.27
pnorm(59, mean = (400*0.2), sd = sqrt(400*0.2*(1-0.2)), lower.tail = TRUE)

## [1] 0.004332448

pbinom(59, prob = .2, size = 400)

## [1] 0.004111644
```

3.3   Other Distributions

3.3.1   Poisson Distribution

Similarly, the functions for the **poisson distribution** are the following where the necessary argument is *lambda*

- $dpoisson(x, lambda) = P(X = x)$

- $ppois(x, lambda) = P(X \le x)$

- $ppois(x, lambda, lower.tail = FALSE) = P(X > x)$

Example of the Poisson

```
## Example 3.28
dpois(x = 2, lambda = 4.4*7)

## [1] 1.99435e-11
```

3.3.2 Geometric Distribution

The same pattern can be applied for the **geometric distribution** to get the following where the necessary argument is *prob*

- $dgeom(x, prob) = P(X = x)$

- $pgeom(x, prob) = P(X \leq x)$

Examples of the Geometric

Note here that using the dgeom function requires an input of $k - 1$, because this function returns the probability of k failures before the first success, ending with a total of k turns.

```
## Example 3.31
dgeom(x = 1-1, prob = 0.35)

## [1] 0.35

dgeom(x = 2-1, prob = 0.35)

## [1] 0.2275

dgeom(x = 3-1, prob = 0.35)

## [1] 0.147875
```

```
## Example 3.35
pgeom(q = (4-1), prob = 0.35)

## [1] 0.8214938
```

3.3.3 Negative Binomial Distribution

Finally, for the **negative binomial distribution**, the necessary arguments are *prob* and *size* and can be implemented as follows,

- $dnbinom(x, n, p) = P(X = x)$

- $pnbinom(x, n, p) = P(X \leq x)$

Example of the Negative Binomial

```
## Example 3.38
dnbinom(x = 4, size = 6, prob = 0.8)

## [1] 0.05284823
```

Chapter 4

Foundations for Inference

Contents

4.1   Taking a Sample of a Dataset

The main text talks through the importance of using a sample of data as a reflection of the population as a whole. The two steps for doing this are as follows

1. Of the total number of datapoints you have (equal to the length of the dataset), select a random sampling of them, of size *n*. This is done with the *sample()* command below. *x* is a list of the row indeces, and *sample* returns a random list of values in that list of length *n*.

2. Taking the output of the *sample* command, pair these indeces with the dataset to get the corresponding rows (or columns).

```
set.seed(5011)
# Step 1: Take a sample of the row indeces
indeces = sample(x = (1:nrow(yrbss)), size = 100, replace = FALSE)
# Step 2: Pull those corresponding individuals from the dataset
yrbss.sample = yrbss[indeces,]
```

Standard histograms plots of some of the variables on this dataset can be plotted.

```
## Figure 4.4
par(mfrow = c(2, 2))
hist(yrbss.sample$height, xlab = "Height (meters)", main = "", breaks = 15)
hist(yrbss.sample$weight, xlab = "Weight (kilograms)", main = "", breaks = 5)
hist(yrbss.sample$physically.active.7d, xlab = "Physical Activity in Past Week",
    main = "", breaks = -1:7 + 0.5)
```

```r
hist(yrbss.sample$strength.training.7d, xlab = "Lifting Weights in Past Week",
    main = "", breaks = -1:7 + 0.5)
```
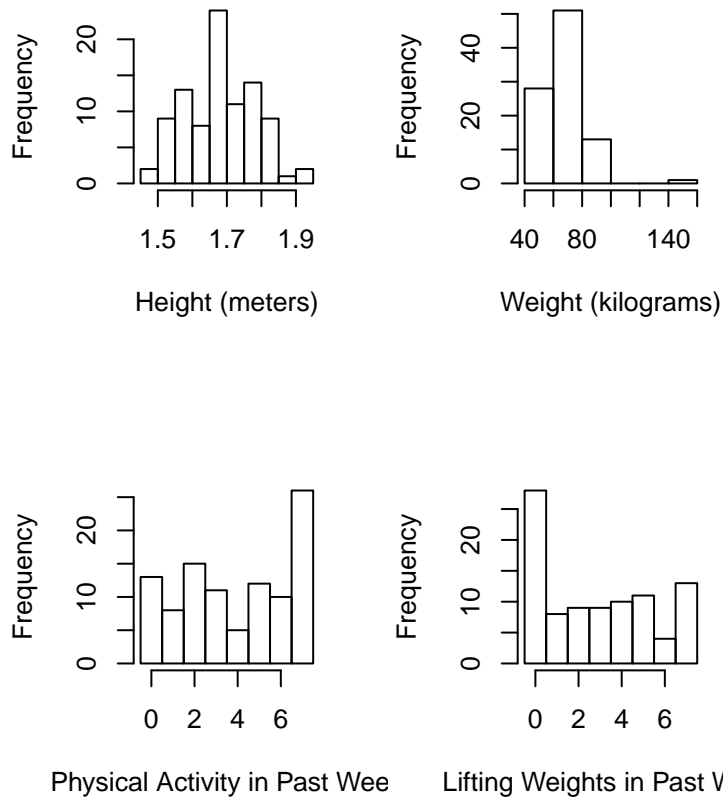

4.2 Variability in Estimates

The sample parameters can be calculated as well as the population parameters. These correspond to the data seen in Table 4.5 in the main text.

```r
mean(yrbss.sample$physically.active.7d, na.rm = TRUE)
```

```
## [1] 3.93
```

```r
mean(yrbss$physically.active.7d, na.rm = TRUE)
```

```
## [1] 3.903005
```

```r
median(yrbss.sample$physically.active.7d, na.rm = TRUE)
```

```
## [1] 4
```

```r
median(yrbss$physically.active.7d, na.rm = TRUE)
```

```
## [1] 4

sd(yrbss.sample$physically.active.7d, na.rm = TRUE)

## [1] 2.535625

sd(yrbss$physically.active.7d, na.rm = TRUE)

## [1] 2.564105
```

4.2.1   Sampling Distribution for the Mean

The concept of a sampling distribution highlights the fact that sampling is a random process, and every sample is likely to be quite different than any other. For this reason, sampling distributions are created, which represent the accumulated information of a large number of random samples.

The following steps can be used to create a **sampling distribution of the sample mean**.

```
## Figure 4.7
# Step 1: Create an empty list to put values in
means = rep(NA, 1000)

# Step 2: Use a "for" loop to collect 1000 samples
for(ii in 1:1000){
  # Step 2a: Get the random sample
  indeces = sample(x = (1:nrow(yrbss)), size = 100, replace = FALSE)
  sample = yrbss[indeces,]
  # Step 2b: Take the mean of the sample and store it
  means[ii] = mean(sample$physically.active.7d, na.rm = TRUE)
}

# Step 3: Plot your results
par(mfrow = c(1,2))
hist(means, breaks = 30)
qqnorm(means)
qqline(means)
```

Histogram of means **Normal Q–Q Plot**
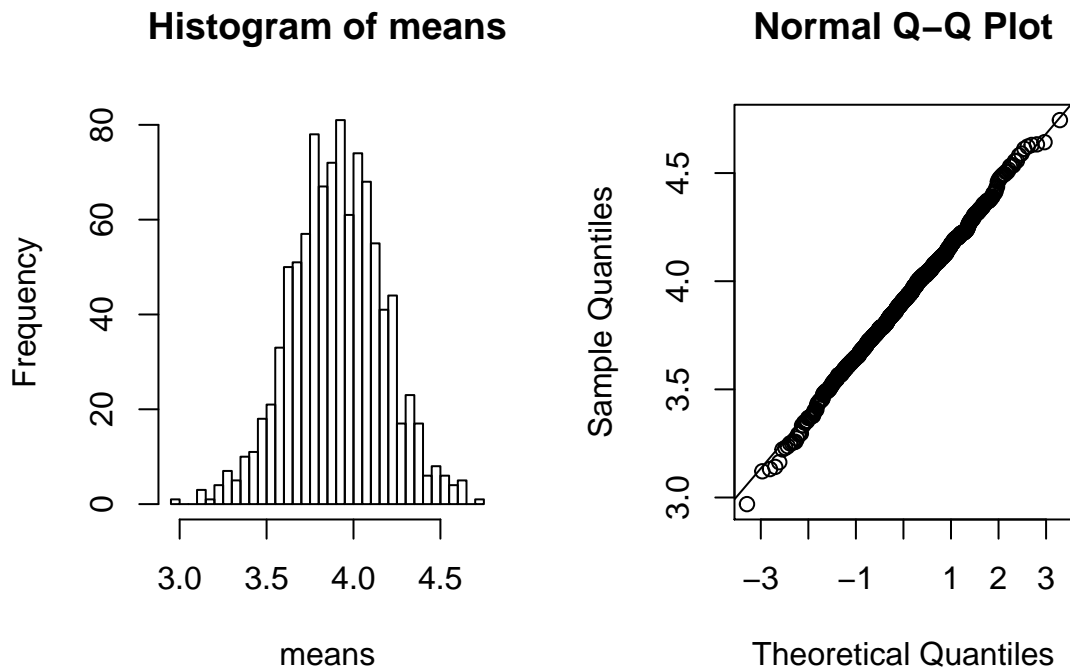
As discussed in chapter 3, the above plots are used to determine the normality of the sample. The left plot is just a histogram, which can be inspected visually to see that it approximates a normal distribution. The plot on the right is a more powerful tool for determining the normality - the dots represent deviation of data points from a theoretical normal distribution, as represented by the line on the plot. The above plot shows a fairly normal distribution because of how closely the dots match the line.

The more samples are taken, the more accurately will the distribution be depicted. Figure 4.8 demonstrates this below

```
## Figure 4.8
means = rep(NA, 100000)
for(ii in 1:100000){
  indeces = sample(x = (1:nrow(yrbss)), size = 100, replace = FALSE)
  sample = yrbss[indeces,]
  means[ii] = mean(sample$physically.active.7d, na.rm = TRUE)
}

par(mfrow = c(1,2))
hist(means, breaks = 30)
qqnorm(means)
qqline(means)
```

4.3   Confidence Intervals

The first formula introduced in the text for calculating confidence intervals is as follows,

$$\text{point estimate} \pm 1.96 \cdot \text{SE}$$

```
## Example 4.3
xbar = mean(yrbss.sample$physically.active.7d, na.rm = TRUE)
std.error = sd(yrbss.sample$physically.active.7d, na.rm = TRUE)/sqrt(length(yrbss.sample$physically.active.7d)
ci = c(xbar - 1.96*std.error, xbar + 1.96*std.error)
ci

## [1] 3.433018 4.426982
```

To generalize this formula, a standard normal distribution can be used to obtain z^*, giving the following

$$\bar{x} \pm z^* \cdot \text{SE}$$

R can be used to calculate z^* and then using that value, to solve for the confidence interval. A key point here is that we want the middle 95% of the distribution, which divides the remaining 5% between the two tails. This is equivalent to

```
## Example 4.3 (version 2)
perc = .95
z = qnorm(p = perc + (1-perc)/2, lower.tail = TRUE)
z

## [1] 1.959964

ci = c(xbar - 1.96*std.error, xbar + 1.96*std.error)
ci

## [1] 3.433018 4.426982
```

```
## Example 4.6
perc = .99
z = qnorm(p = perc + (1-perc)/2, lower.tail = TRUE)
z

## [1] 2.575829
```

```
## Example 4.8
ci = c(xbar - z*std.error, xbar + z*std.error)
ci

## [1] 3.276866 4.583134
```

Example 4.10 is an excellent example of how to clean up data. The main text restricts the sample to adults over 21 with reported BMI values. The steps below show how to find the individuals who are children or who have missing data and how to remove them from the sample. Often times, when using a sample of a large population, a process similar to this one must be used. Missing data can be problematic for analyses of the data, so understanding how to clean up the data appropriately is quite important.

```
## Example 4.10
# Collect the sample of size 200
```
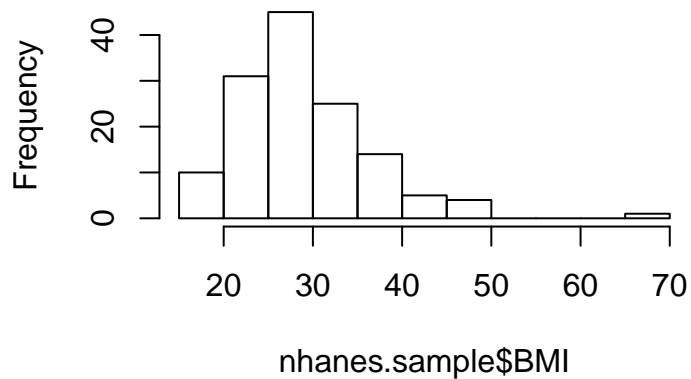
```
set.seed(5011)
indeces = sample(1:length(NHANES$ID), size = 200)
nhanes.sample = NHANES[indeces,]

# First remove the children from the sample
children = which(nhanes.sample$Age < 21)   #Find children
nhanes.sample = nhanes.sample[-children, ]        #Remove them from the sample

hist(nhanes.sample$BMI, breaks = "FD") ## This gives Figure 4.10
```
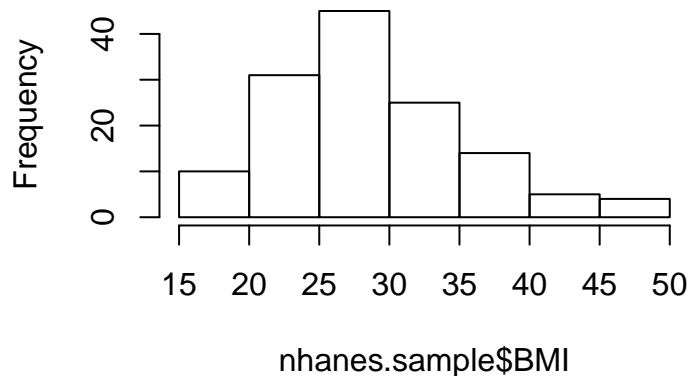
Histogram of nhanes.sample$BMI


```
# Locate where the outlier is occuring
x = which(nhanes.sample$BMI == max(nhanes.sample$BMI, na.rm = TRUE))
# Remove the outlier
nhanes.sample = nhanes.sample[-x,]
# Plot again to confirm that it was removed
hist(nhanes.sample$BMI)
```

Histogram of nhanes.sample$BMI


```r
# Calculate sample statistics and confidence interval
xbar = mean(nhanes.sample$BMI, na.rm = TRUE)
n = length(nhanes.sample$BMI)
s = sd(nhanes.sample$BMI, na.rm=TRUE)
se = s/sqrt(n)
perc = .95
z = qnorm(p = perc + (1-perc)/2, lower.tail = TRUE)
ci = c(xbar - z*se, xbar + z*se)
ci
```

```
## [1] 27.66076 29.94282
```

4.4 Hypothesis Testing

If you have calculated your test statistic by hand, *R* can easily be used to get the p-value using the function *pnorm()* as in Chapter 3. The example that is worked through in the main text can be completed as follows. Note that *lower.tail = FALSE* because the alternative hypothesis here is

$$H_A : \mu_{bmi} > 21.7$$

which implies that the upper tail must be considered.

```r
mu = 21.7
t = (xbar - mu)/(s/sqrt(n))
t
```

```
## [1] 12.19889
```

```r
pnorm(t, lower.tail = FALSE)
```

```
## [1] 1.575536e-34
```

In *R*, all of the steps of hypothesis testing can be done with one single function, *t.test()*. This gives the test statistic, the p-value, and the confidence interval. The function takes the data as an argument, *x*, the null hypothesis value, as *mu*, and the alternative hypothesis type, as *alternative*, which can be *"less"*, *"greater"*, or *"two.sided"*.

The example shown below gives a comparison of doing the method by hand, as well as using the function.

```r
## Example 4.13
set.seed(5011)
indeces = sample(1:length(NHANES$ID), size = 200)
nhanes.sample = NHANES[indeces,]

# First remove the children from the sample
children = which(nhanes.sample$Age < 21)  #Find children
nhanes.sample = nhanes.sample[-children, ]

# Method 1
xbar = mean(nhanes.sample$SleepHrsNight, na.rm = TRUE)
s = sd(nhanes.sample$SleepHrsNight, na.rm = TRUE)
mu = 7
n = length(nhanes.sample$SleepHrsNight)
t = (xbar - mu)/(s/sqrt(n))
t

## [1] -0.864111

p = pnorm(t, lower.tail = TRUE)
p

## [1] 0.1937634

# Method 2
t.test(x = nhanes.sample$SleepHrsNight, mu = mu, alternative = "less")

##
##  One Sample t-test
##
## data:  nhanes.sample$SleepHrsNight
## t = -0.8641, df = 134, p-value = 0.1945
## alternative hypothesis: true mean is less than 7
## 95 percent confidence interval:
##      -Inf 7.095073
## sample estimates:
## mean of x
##  6.896296
```

Chapter 5

Inference For Numerical Data

Contents

5.1 The t-distribution

The main text introduces the **t-distribution** and explains under what circumstances it should be used. Using it in in R is very similar to many concepts seen through Chapter 4 thus far.

The following plot shows how the t-distribution approximates the normal distribution as the degrees of freedom grow. The use of the function *dt()* parallels the use of *dnorm()* as has been used in previous chapters.

```
## Figure 5.2 getting the x-axis values
x = seq(-5, 9, 0.1)

# finding the corresponding distribution values for several distributions
t1 = dt(x, df = 1)
t2 = dt(x, df = 2)
t4 = dt(x, df = 4)
t8 = dt(x, df = 8)
norm = dnorm(x)

# plotting the results, with custom colors
plot(x, t1, type = "l", ylim = c(0, 0.4), xlab = "", ylab = "", col = rgb(0,
    0, 1, alpha = 0.1))
lines(x, t2, col = rgb(0, 0, 1, alpha = 0.2))
lines(x, t4, col = rgb(0, 0, 1, alpha = 0.3))
```
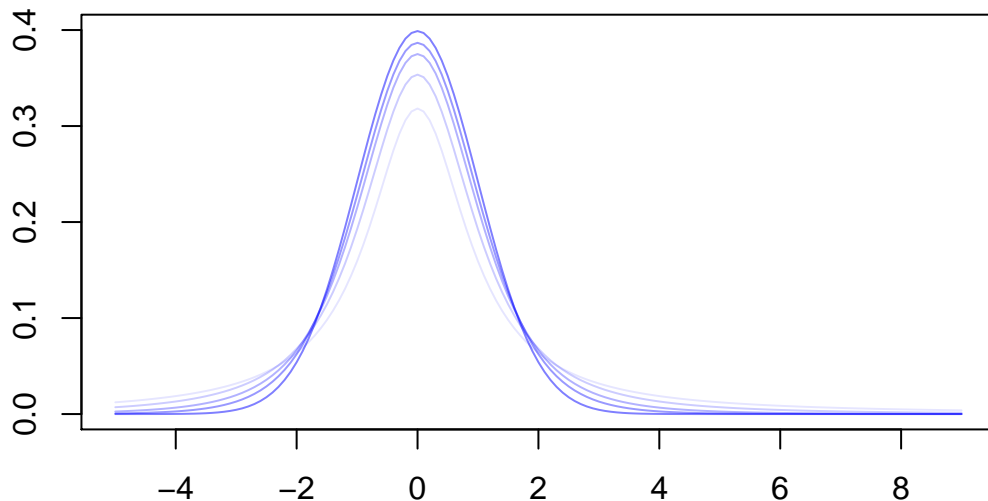
```
lines(x, t8, col = rgb(0, 0, 1, alpha = 0.4))
lines(x, norm, col = rgb(0, 0, 1, alpha = 0.5))
```


5.1.1 The Alternative to t-Tables

The main text talks through the use of a **t-table**, but this text will go through examples of using software to do the same calculations. In *R*, the command of interests here are *pt()* to get the percentage of the distribution above or below a certain value and *qt()* to get distribution values for which a certain percentage of the distribution falls above. The command *pt()* takes values inside the t-table to return the values on the x-axis of the table, whereas *qt()* returns values within the table when given values along the upper x-axis.

An example of a one-sided calculation using the t-distribution can be viewed in Example 5.1 below. Note the use of the stipulation *lower.tail = TRUE* which ensures that the calculation is of values in the lower tail, i.e. those values which are less than -2.1.

```
## Example 5.1
pt(-2.1, df = 18, lower.tail = TRUE)

## [1] 0.0250452
```

Example 5.2 shows a two sided calculation using the t-distribution. In order to do this in *R*, each tail can be calculated separately and then the two values summed. Alternatively, the symmetry of the t-distribution can be utilized to calculate one tail and multiply it by two.

```
## Example 5.2
# Method 1: calculate each tail and sum
pt(1.65, df = 20, lower.tail = FALSE) + pt(-1.65, df = 20, lower.tail = TRUE)
```

```
## [1] 0.1145608

# Method 2: calculate one tail and multiply by 2
2*pt(1.65, df = 20, lower.tail = FALSE)

## [1] 0.1145608
```

5.1.2 Confidence Intervals

As in chapter 4, confidence intervals using the t-distribution can be set up in the same way with a t^* value instead of a z^* value.

```
## Example 5.3
n = 19
xbar = 4.4
s = 2.3
std.error = s/sqrt(n)
perc = .95
t.star = qt(p = perc + (1-perc)/2, df = n-1, lower.tail = TRUE)
ci = c(xbar - t.star*std.error, xbar + t.star*std.error)
ci

## [1] 3.291435 5.508565
```

```
## Example 5.5
mu = 0.5
n = 15
xbar = 0.287
s = 0.069
std.error = s/sqrt(n)
t = (xbar - mu)/std.error
pt(t, df = n-1)

## [1] 4.903809e-09
```

5.2 Paired Data

When working with paired data, several methods can be used to measure the effect. The first method is to calculate by hand differences between the pairs and then to perform a standard t-test as follows.

```
## Example 5.6
diff = swim$wet.suit.velocity - swim$swim.suit.velocity
t.test(diff, mu = 0, alternative = "two.sided")

##
##  One Sample t-test
##
## data:  diff
## t = 3.7019, df = 11, p-value = 0.00349
```

```
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.02534062 0.09965938
## sample estimates:
## mean of x
##    0.0625
```

The second method involves using a **paired t-test**, which performs the same operation without the direct calculation of the differences being required first. It can be seen that the output of this test is the exact same as the test above. This is a nice example of *R* taking out a lot of the computational work.

```
t.test(swim$wet.suit.velocity, swim$swim.suit.velocity, paired = TRUE)

##
##  Paired t-test
##
## data:  swim$wet.suit.velocity and swim$swim.suit.velocity
## t = 3.7019, df = 11, p-value = 0.00349
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.02534062 0.09965938
## sample estimates:
## mean of the differences
##                  0.0625
```

5.3   Difference of Two Means

5.3.1   Hypothesis Tests for a Different in Means

In order to perform a **two-sample t-test**, there are again two methods. The first involves dividing the data into two samples (unless the data has already come separated) and performing a t-test. The second has *R* do the entire process. Note that for this method, the tilde signifies the grouping variable. So in this case, the *weight* data is grouped by the variable *smoke*.

```
## Example 5.11
# Method 1: divide the data by hand
smoker = baby.smoke$weight[baby.smoke$smoke == "smoker"]
non.smoker = baby.smoke$weight[baby.smoke$smoke == "nonsmoker"]
t.test(non.smoker, smoker)

##
##  Welch Two Sample t-test
##
## data:  non.smoker and smoker
## t = 1.4967, df = 89.277, p-value = 0.138
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.1311663  0.9321663
## sample estimates:
## mean of x mean of y
##    7.1795    6.7790
```

```
# Method 2: R does it all
t.test(baby.smoke$weight~baby.smoke$smoke)

##
##  Welch Two Sample t-test
##
## data:  baby.smoke$weight by baby.smoke$smoke
## t = 1.4967, df = 89.277, p-value = 0.138
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.1311663  0.9321663
## sample estimates:
## mean in group nonsmoker     mean in group smoker
##                   7.1795                   6.7790
```

5.3.2 Pooled t-test

Performing what is known as a **pooled t-test** is very simple in *R*. The procedure is the same as a two-sample t-test, with the addition of the clause *var.equal = TRUE* in the *t.test()* command. This is because a pooled t-test relies on the assumption of nearly equal variance between the two samples. For example, performing the same test as above but using a pooled t-test would look like the following,

```
t.test(baby.smoke$weight~baby.smoke$smoke, var.equal = TRUE)

##
##  Two Sample t-test
##
## data:  baby.smoke$weight by baby.smoke$smoke
## t = 1.5517, df = 148, p-value = 0.1229
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.1095531  0.9105531
## sample estimates:
## mean in group nonsmoker     mean in group smoker
##                   7.1795                   6.7790
```

5.4 ANOVA

The main text walks through the arithmetic procedures of the ANOVA test and calculating the F-statistic. *R* makes this process much simpler, such that the entire calculation can be performed with just one line of code. The command *anova()* takes, as its input, a linear regression. Chapter 6 will go into much more detail about linear regressions, but for now, note that the first one variable before the tilde is the one for which the means are being compared, while the second variable after the tilde specifies the groups. The command below works out Example 5.22 from the main text, getting the same *F* statistic as the hand calculated value.

```
## Example 5.22
anova(lm(famuss$ndrm.ch ~ famuss$actn3.r577x))

## Analysis of Variance Table
```

```
## 
## Response: famuss$ndrm.ch
##                    Df Sum Sq Mean Sq F value  Pr(>F)
## famuss$actn3.r577x   2   7043  3521.6  3.2308 0.04022 *
## Residuals          592 645293  1090.0
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Chapter 6

Inference For Numerical Data

Contents

6.1 Checking Assumptions

Before going into a discussion of linear regression itself, an investigation into the appropriateness of its use must first be conducted. The main text walks through the four assumptions necessary to use an approximation to linearity, which are as follows

1. Linearity

2. Constant Variability

3. Independent Observations

4. Residuals that are approximately normally distributed

There are various methods to check each of these assumptions, which should be investigated before the use of linear regression. This next section goes through these methods and several examples.

6.1.1 Linearity

To check for linearity, a simple examination of a scatterplot of the data will suffice. The goal is to look for data such that a straight line can be drawn which appropriately represents the data. The plot of Figure 6.1 below shows that *age* and *RFFT* appear to have a linear relationship. The inclusion of the command *cex = 0.4* simply changes the size of the dots on the scatterplot, making them 40% of their original size.

```
## Figure 6.1 First collecting the data sample
set.seed(5011)
row.num = sample(1:nrow(statins), 500, replace = FALSE)
statins.samp = statins[row.num, ]

# Then plotting it
plot(statins.samp$RFFT ~ statins.samp$Age, cex = 0.4, ylab = "RFFT Score", xlab = "Age (yrs)",
    main = "")
```
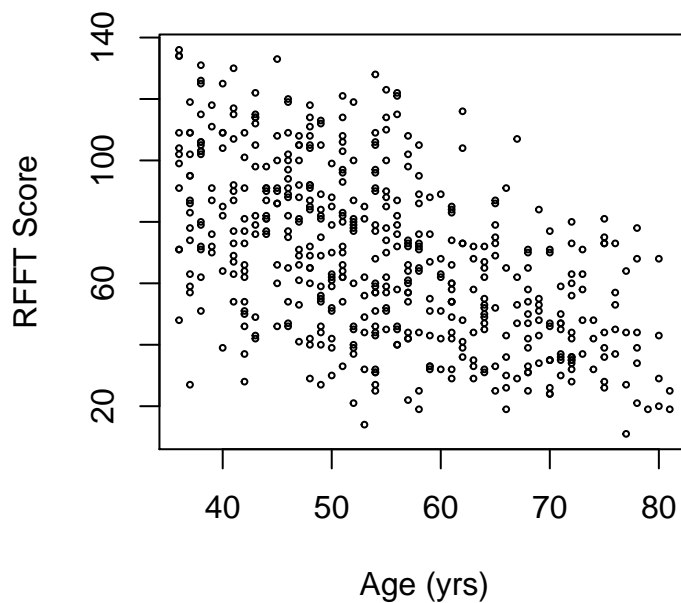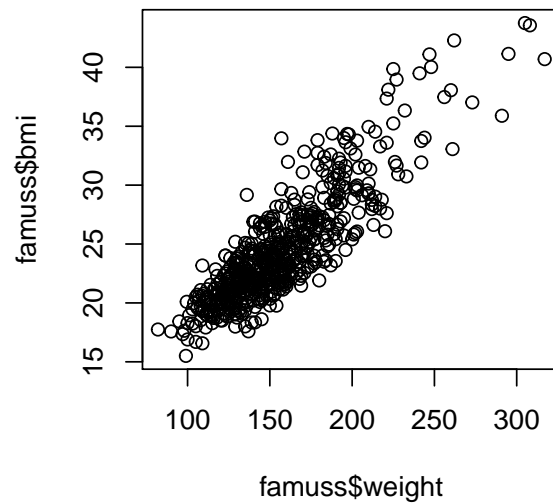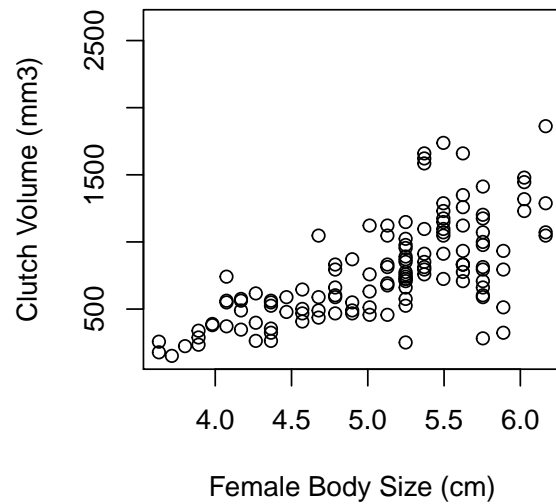

As further examples, the figures below show other linear relatonships.

```
par(mfrow = c(1, 2))
## Figure 6.2
plot(frog.altitude.data$clutch.volume ~ frog.altitude.data$body.size, ylab = "Clutch Volume (mm3)",
    xlab = "Female Body Size (cm)")

plot(famuss$weight, famuss$bmi)
```

For comparison, the following plot shows data that a linear relationship is not appropriate for. The first example shows data that appears to have no relationship, as indicated by the lack of trend in the scatterplot. The second example shows a pattern that would be difficult to capture with a single straight line.

```
par(mfrow = c(1, 2))
plot(famuss$height, famuss$age)
plot(frog.altitude.data$body.size, frog.altitude.data$egg.size)
```
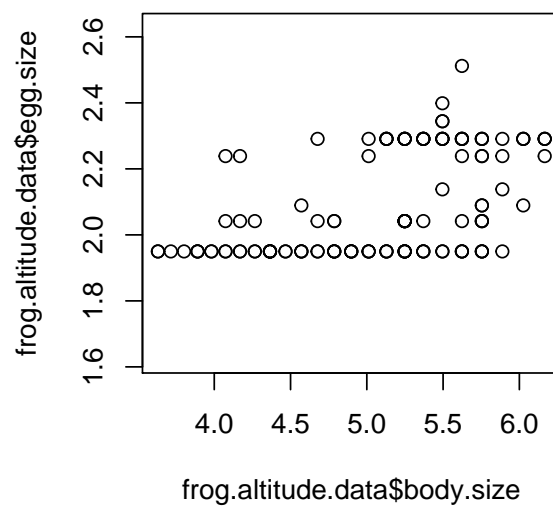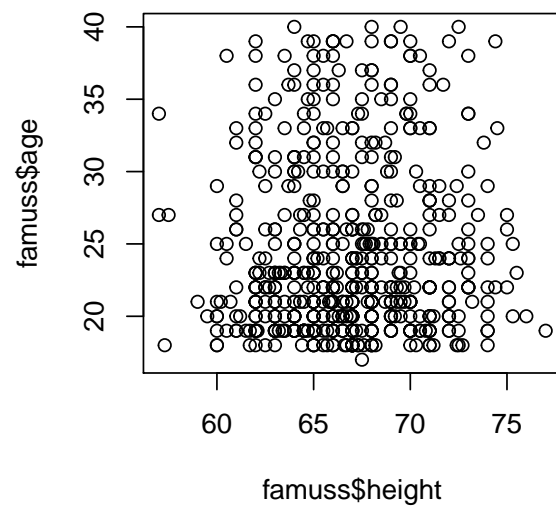
6.1.2   Constant Variability

To check for **constant variability**, a scatterplot is again a good resource. Both Figure 6.1 and Figure 6.2 above show strong examples of constant variability. For comparison, a few examples of non-constant variability are included here. Notice how in the plot on the left, there is less variability in the clutch volumes with small egg size in comparison to the values with higher egg size. Similarly, there is more variability in change in arm strength for lower weight individuals than for higher weight individuals.

```
par(mfrow = c(1, 2))
plot(frog.altitude.data$clutch.volume ~ frog.altitude.data$egg.size, main = "Variance Increasing with Egg Size
    xlab = "Egg Size", ylab = "Clutch Volume")
plot(famuss$ndrm.ch ~ famuss$weight, main = "Variance Decreasing with Weight",
    xlab = "Weight", ylab = "Change in Arm Strength")
```
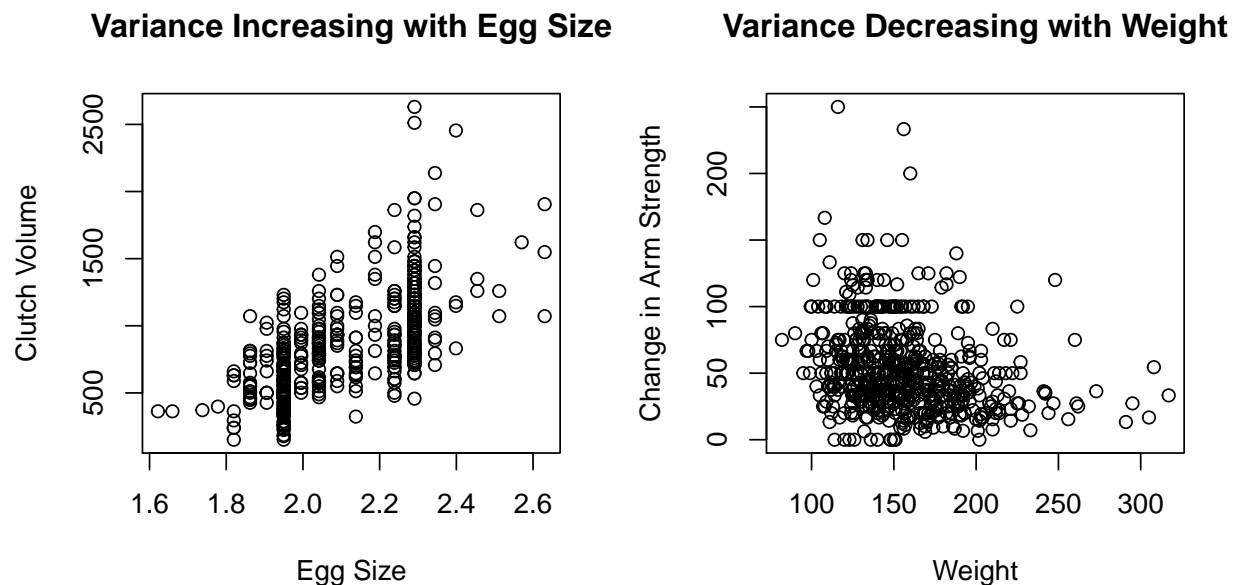

6.1.3   Independence of Observations

The assumption of **independence of observations** is less easily understood through physical study of the data. Instead, the nature of the data and its parameters must be understood. For example, some questions to consider include

- Do the data points depend on each other?

- Is there a known relationship between some or all data points?

- Are the data points from the same individual, region, or time?

6.1.4   Normally Distributed Residuals

In *R*, a plot of the residuals can be visualized to determine if this assumption is met. First, to obtain the residuals, the linear regression must be obtained, using the command *lm()*. Then, the residuals of this model can be calculated using the command *resid()* on the linear regression.

These residuals can be visualized in a few ways: as a histogram, as a plot, or using *qnorm()* and *qqline()*.

Two examples of this procedure are included below. In the first example, the residuals appear very close to normal. This is a strong sign that the assumption is met.

```r
par(mfrow = c(1, 3))
residuals = resid(lm(statins.samp$RFFT ~ statins.samp$Age))

hist(residuals, main = "", xlab = "Residuals")

plot(residuals)
abline(h = 0, col = "red", lty = 2)

qqnorm(residuals)
qqline(residuals)
```
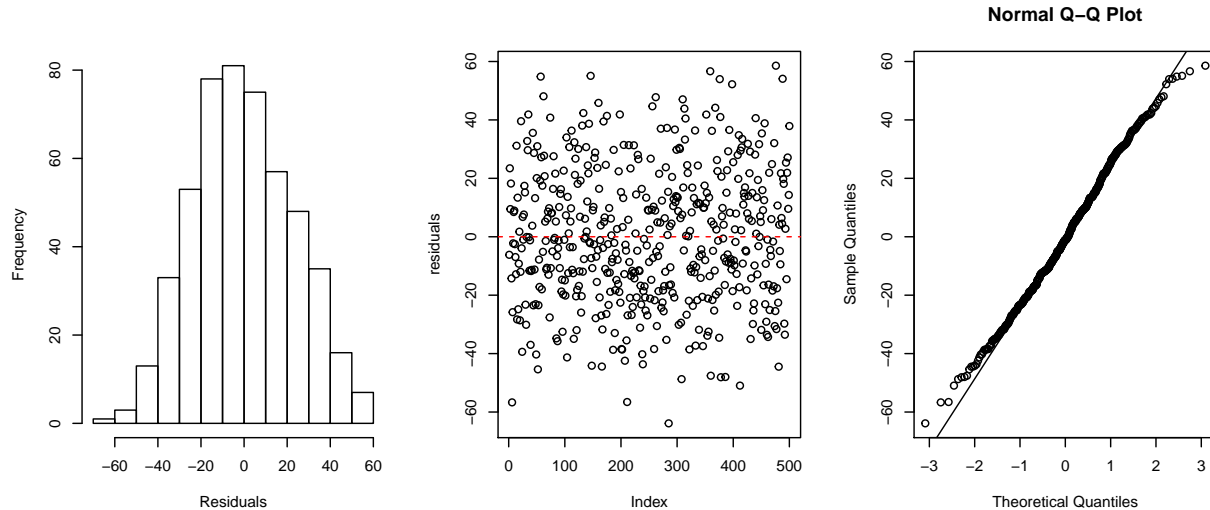

In the second example, the data appears somewhat normal, but with some outliers. This does not appear to be an egregious violation, but is something that should be kept in mind as the results of the linear regression are interpreted.

```r
par(mfrow = c(1, 3))
## Guided Practice 6.1
residuals = resid(lm(frog.altitude.data$clutch.volume ~ frog.altitude.data$body.size))

hist(residuals, main = "", xlab = "Residuals")

plot(residuals)
abline(h = 0, col = "red", lty = 2)

qqnorm(residuals)
qqline(residuals)
```
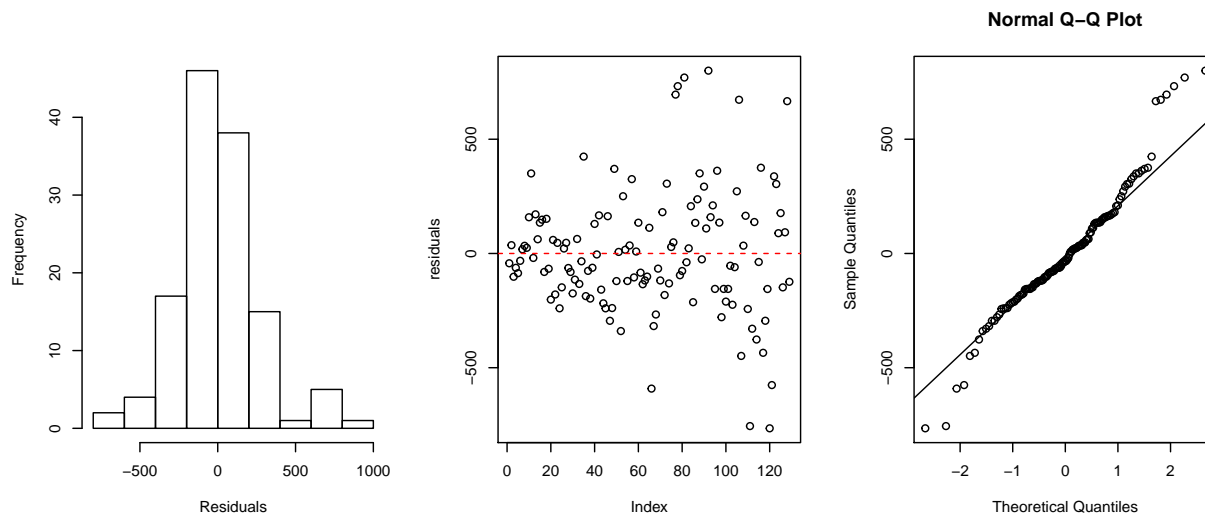
6.2 Creating a Linear Regression

Using *R* to create a linear regression is quite simple, and several examples will be shown here to illustrate that. As background, there is one command needed for linear regressions in *R*, *lm()*. This function takes two arguments, the **explanatory variable** and the **response variable** in the form *lm(response ∼ explanatory)* where the symbol between those two variables is a tilde. For example, a regression for the statins data can be calculated as follows

```
lm(statins.samp$RFFT~statins.samp$Age)

##
## Call:
## lm(formula = statins.samp$RFFT ~ statins.samp$Age)
##
## Coefficients:
##      (Intercept)   statins.samp$Age
##          137.550            -1.261
```

This output implies the following regression formula

$$RFFT = 137.55 - 1.261 \cdot Age$$

To get more information about the regression, the *summary* command can be used as follows,

```
summary(lm(statins.samp$RFFT~statins.samp$Age))

##
## Call:
## lm(formula = statins.samp$RFFT ~ statins.samp$Age)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -63.879 -16.845  -1.095  15.524  58.564
```

```
## 
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)      137.54972    5.01614   27.42   <2e-16 ***
## statins.samp$Age  -1.26136    0.08953  -14.09   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 23.19 on 498 degrees of freedom
## Multiple R-squared:  0.285,Adjusted R-squared:  0.2836
## F-statistic: 198.5 on 1 and 498 DF,  p-value: < 2.2e-16
```

Note that the summary of the linear regression is much more useful, providing not only the regression model, but standard errors, p-values, the R^2 value, and a few other useful statistics which will be discussed in greater deal in future chapters.

With this linear regression in mind, it would be helpful to be able to visualize this on the scatterplot. This can be done using the command *abline()*, which takes as its arguments the model itself.

```
plot(statins.samp$RFFT ~ statins.samp$Age, cex = 0.4)
abline(lm(statins.samp$RFFT ~ statins.samp$Age), col = "red")
```
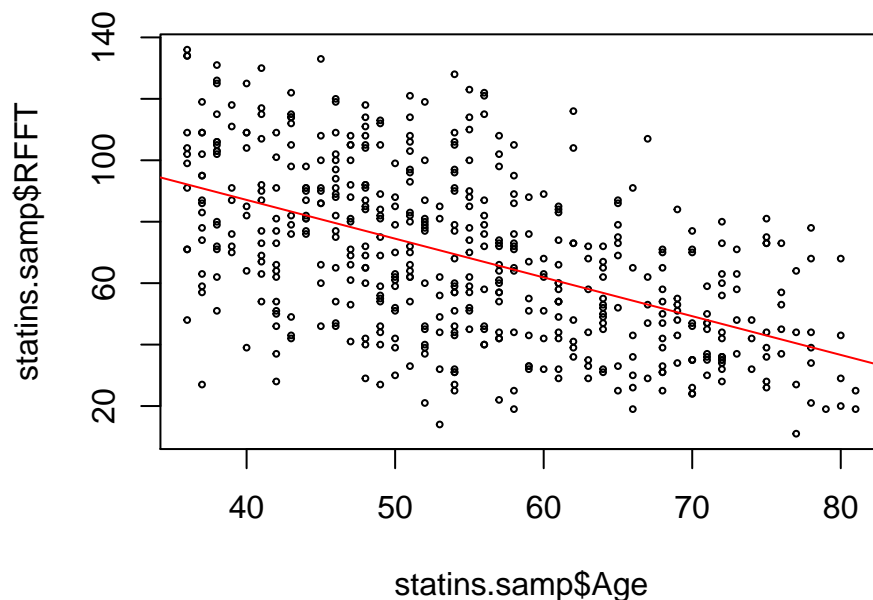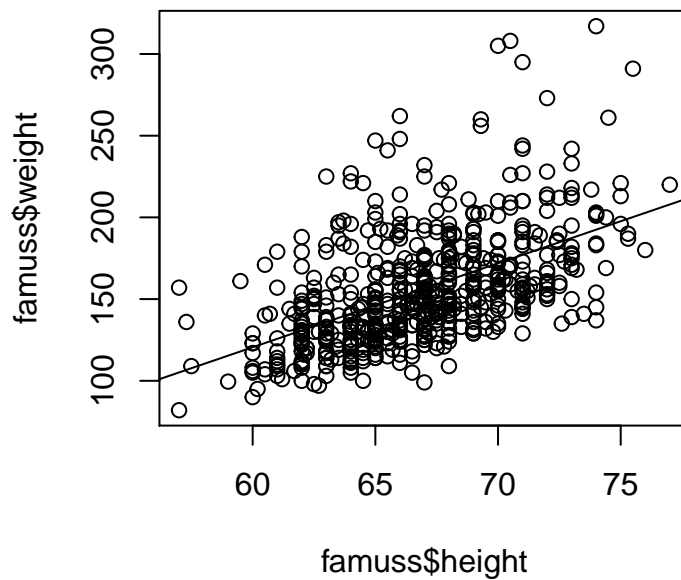

6.2.1 FAMuSS Regression

Performing a similar regression for the FAMuSS dataset between height and weight gives us the following model

$$weight = -187.79 + 5.14 \cdot height$$

```
plot(famuss$weight ~ famuss$height)
abline(lm(famuss$weight ~ famuss$height))
```



```
summary(lm(famuss$weight ~ famuss$height))

##
## Call:
## lm(formula = famuss$weight ~ famuss$height)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -57.514 -19.452  -6.959  13.882 133.500
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -187.7908    22.5454  -8.329 5.64e-16 ***
## famuss$height    5.1389     0.3369  15.255  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 29.34 on 593 degrees of freedom
## Multiple R-squared:  0.2818,Adjusted R-squared:  0.2806
## F-statistic: 232.7 on 1 and 593 DF,  p-value: < 2.2e-16
```

6.2.2 WDI Regression with Two Levels

Although the interpretation of such a model can be different, the method of performing a regression on a variable with two levels is the same.

```
summary(lm(log(wdi.2011$inf.mort)~wdi.2011$sanit.access))

##
## Call:
## lm(formula = log(wdi.2011$inf.mort) ~ wdi.2011$sanit.access)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.42680 -0.48562  0.02123  0.40440  1.70517
##
## Coefficients:
##                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)            4.930188   0.119070   41.41   <2e-16 ***
## wdi.2011$sanit.access -0.029873   0.001568  -19.05   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6107 on 161 degrees of freedom
##   (2 observations deleted due to missingness)
## Multiple R-squared:  0.6926,Adjusted R-squared:  0.6907
## F-statistic: 362.8 on 1 and 161 DF,  p-value: < 2.2e-16
```