Introductory Statistics for the Life and Biomedical Sciences

Derivative of
OpenIntro Statistics
Third Edition

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Preface

This book provides an introduction to statistics and its applications in the life sciences, and biomedical research. It is based on the freely available *OpenIntro Statistics, Third Edition*, and, like *OpenIntro* it may be downloaded as a free PDF at **Need location**. The text adds substantial new material, revises or eliminates sections from *OpenIntro*, and re-uses some material directly. Readers need not have read *OpenIntro*, since this book is intended to be used independently. We have retained some of the exercises from *OpenIntro* that may not come directly from medicine or the life sciences but illustrate important ideas or methods that are commonly used in fields such as biology.

Introduction to Statistics for the Life and Biomedical Sciences is intended for graduate and undergraduate students interested in careers in biology or medicine, and may also be profitably read by students of public health. It covers many of the traditional introductory topics in statistics used in those fields, but also adds some newer methods being used in molecular biology. Statistics has become an integral part of research in medicine and biology, and the tools for displaying, summarizing and drawing inferences from data are essential both for understanding the outcomes of studies and for incorporating measures of uncertainty into that understanding. An introductory text in statistics for students considering careers in medicine, public health or the life sciences should be more than the usual introduction with more examples from biology or medical science. Along with the value of careful, robust analyses of experimental and observational data, it should convey some of the excitement of discovery that emerges from the interplay of science with data collection and analysis. We hope we have conveyed some of that excitement here.

We have tried to balance the sometimes competing demands of mastering the impor-

tant technical aspects of methods of analysis with gaining an understanding of important concepts. The examples and exercises include opportunities for students to build skills in conducting data analyses and to state conclusions with clear, direct language that is specific to the context of a problem. We also believe that computing is an essential part of statistics, just as mathematics was when computing was more difficult or expensive. The text includes many examples where software is used to aid in the understanding of the features of a data as well as exercises where computing is used to help illustrate the notions of randomness and variability. Because they are freely available, we use the R statistical language with the *R Studio* interface. Information on downloading R and *R Studio* is may be found in the Labs at **openintro.org**. Nearly all examples and exercises can be adapted to either SAS, Stata or other software, but we have not done that.

Textbook overview

The chapters of this book are as follows:

1. **Introduction to data.** Data structures, variables, summaries, graphics, and basic data collection techniques.

2. **Probability (special topic).** The basic principles of probability. An understanding of this chapter is not required for the main content in Chapters **??**-**??**.

3. **Distributions of random variables.** Introduction to the normal model and other key distributions.

4. **Foundations for inference.** General ideas for statistical inference in the context of estimating the population mean.

5. **Inference for numerical data.** Inference for one or two sample means using the normal model and t distribution, and also comparisons of many means using ANOVA.

6. **Inference for categorical data.** Inference for proportions using the normal and chi-square distributions, as well as simulation and randomization techniques.

7. **Introduction to linear regression.** An introduction to regression with two variables. Most of this chapter could be covered after Chapter 1.

8. **Multiple and logistic regression.** An introduction to multiple regression and logistic regression for an accelerated course.

The remainder of this section requires revision

OpenIntro Statistics was written to allow flexibility in choosing and ordering course topics. The material is divided into two pieces: main text and special topics. The main text has been structured to bring statistical inference and modeling closer to the front of a course. Special topics, labeled in the table of contents and in section titles, may be added to a course as they arise naturally in the curriculum.

Examples, exercises, and appendices

Examples and within-chapter exercises throughout the textbook may be identified by their distinctive bullets:

● **Example 0.1** Large filled bullets signal the start of an example.

Full solutions to examples are provided and often include an accompanying table or figure.

⊙ **Guided Practice 0.2** Large empty bullets signal to readers that an exercise has been inserted into the text for additional practice and guidance. Students may find it useful to fill in the bullet after understanding or successfully completing the exercise. Solutions are provided for all within-chapter exercises in footnotes.[1]

There are exercises at the end of each chapter that are useful for practice or homework assignments. Many of these questions have multiple parts, and odd-numbered questions include solutions in Appendix A.

Probability tables for the normal, t, and chi-square distributions are in Appendix B, and PDF copies of these tables are also available from **openintro.org** for anyone to download, print, share, or modify.

[1] Full solutions are located down here in the footnote!

OpenIntro, online resources, and getting involved

OpenIntro is an organization focused on developing free and affordable education materials. *OpenIntro Statistics*, our first project, is intended for introductory statistics courses at the high school through university levels.

We encourage anyone learning or teaching statistics to visit **openintro.org** and get involved. We also provide many free online resources, including free course software. Data sets for this textbook are available on the website and through a companion R package.[2] All of these resources are free, and we want to be clear that anyone is welcome to use these online tools and resources with or without this textbook as a companion.

We value your feedback. If there is a particular component of the project you especially like or think needs improvement, we want to hear from you. You may find our contact information on the title page of this book or on the About section of **openintro.org**.

Acknowledgements

This project would not be possible without the dedication and volunteer hours of all those involved. No one has received any monetary compensation from this project, and we hope you will join us in extending a *thank you* to all those volunteers below.

The authors would like to thank Andrew Bray, Meenal Patel, Yongtao Guan, Filipp Brunshteyn, Rob Gould, and Chris Pope for their involvement and contributions. We are also very grateful to Dalene Stangl, Dave Harrington, Jan de Leeuw, Kevin Rader, and Philippe Rigollet for providing us with valuable feedback.

[2] Diez DM, Barr CD, Çetinkaya-Rundel M. 2012. `openintro`: OpenIntro data sets and supplement functions. http://cran.r-project.org/web/packages/openintro.

Chapter 1

Introduction to data

Scientists seek to answer questions using rigorous methods and careful observations. These observations – collected from the likes of field notes, surveys, and experiments – form the backbone of a statistical investigation and are called **data**. Statistics is the study of how best to collect, analyze, and draw conclusions from data. It is helpful to put statistics in the context of a general process of investigation:

1. Identify a question or problem.
2. Collect relevant data on the topic.
3. Analyze the data.
4. Form a conclusion.

Statistics as a subject focuses on making stages 2-4 objective, rigorous, and efficient. That is, statistics has three primary components: How best can we collect data? How should it be analyzed? And what can we infer from the analysis?

Many scientific investigations can be conducted with a small number of data collection techniques and analytic tools. This chapter provides a brief introduction to the basic principles of these areas that will be encountered later in the book, and illustrates the important role statistics plays in medicine and biology.

1.1 Case study: Preventing Peanut Allergies

Section 1.1 introduces an important problem in medicine: evaluating the effect of an intervention. Terms in this section, and indeed much of this chapter, will all be revisited later in more detail.

The proportion of young children with peanut allergies in Western countries has doubled in the last 10 years. Does the exposure to peanut products during the first 5 years of a child's life reduced the probability that a child will develop an allergy? This section describes an experiment (a clinical trial, in the terminology of medical research) designed to assess the effectiveness of exposing infants at risk for peanut allergy either to consume or avoid peanut products during the first 5 years of life. The study was called the "Learning Early about Peanut Allergy" (LEAP), enrolled children in the United Kingdom between 2006 and 2009, and was reported in the New England Journal of Medicine in 2015. .[1] Earlier research had suggested that infants predisposed to peanut allergies might develop resistance to the allergy with exposure to peanut products before the allergy appeared.

The study team selected 640 infants with either or both of excema and egg allergies and randomly assigned each child to peanut consumption (the treatment group) or avoidance (the control group) for five years. In this study, the control group provides a reference point for estimating the effect of peanut exposure in the treatment group. Each child was tested for a peanut allergy at age 5 using an oral food challenge (OFC); the main analysis was based on 530 children with a negative skin test at the time of study entry. Among these 530 children, 263 were assigned to 'Peanut Avoidance' and 267 to 'Peanut Consumption'. The outcome at 5 years was coded as either 'Fail OFC' (allergic reaction) or 'Pass OFC' (no allergic reaction. The dataset `LEAP` contains the treatment and outcome data the 530 children.

Table 1.1 shows the participant's study ID number, treatment assignment and outcome from the OFC for 5 children. All five of these children passed the food challenge.

Summary tables are generally more helpful than individual participant listings when

[1] Du Toit, George, et al. Randomized trial of peanut consumption in infants at risk for peanut allergy. New England Journal of Medicine 372.9 (2015): 803-813.

	participant.ID	treatment.group	overall.V60.outcome
1	LEAP_100522	Peanut Consumption	PASS OFC
2	LEAP_103358	Peanut Consumption	PASS OFC
3	LEAP_105069	Peanut Avoidance	PASS OFC
	⋮	⋮	⋮
639	LEAP_994047	Peanut Avoidance	PASS OFC
640	LEAP_997608	Peanut Consumption	PASS OFC

Table 1.1: Results for five children from the peanut study.

looking for patterns in data. Table 1.2 shows outcomes grouped by treatment group and the result of the OFC test.

	FAIL OFC	PASS OFC	Sum
Peanut Avoidance	36	227	263
Peanut Consumption	5	262	267
Sum	41	489	530

Table 1.2: LEAP Study Results

The table makes it possible to compute some simple summary statistics. A **summary statistic** is a single number summarizing a large amount of data.[2] In the Peanut Avoidance intervention, the proportion of participants failing the food challenge a 5 years of age was $36/263 = 0.137$ (13.7%); in the Peanut Consumption intervention, the proportion failing was $5/267 = 0.019$ (1.9%). The difference between these two proportions 11.8% is a single summary statistic showing the gap between the two proportions. A second summary statistic, the ratio of the two proportions, $0.137/0.019 = 7.31$, indicates that the proportion failing on the Avoidance group was more than 7 times that on the Consumption group. This ratio is called a **relative risk**.

The summary statistics for the LEAP study highlight an important point – the results of a study can sometimes be surprising. Someone unaware of early preliminary results about the potential value of exposure to peanut products (perhaps a parent of a child allergic to eggs) might be justifiably skeptical about the advisability of feeding peanut butter to his or her child. The LEAP study suggests that, at least in children similar to those in the study, the benefit might be substantial.

[2]Formally, a summary statistic is a value computed from the data. Some summary statistics are more useful than others.

There are important aspects of the study to be cautious about. This study was conducted in the United Kingdom at a single site of pediatric care, and it is not at all clear that results in children from that site can be generalized to other countries or cultures. Even if the study can be generalized, the results also raise an important statistical issue. Peanut consumption among infants susceptible to peanut allergies should be adopted only if the study results are definitive Does the study provide definitive evidence that peanut consumption is beneficial? In other words, is the 11.8% difference between the two groups larger than one would expect by chance variation alone?

Suppose a coin is flipped 100 times. While the chance a coin lands heads in any given coin flip is 50%, it is unlikely for exactly 50 heads to be observed. This type of fluctuation is part of almost experiment or study. It may well be possible that the 8% difference in the stent study is due to this natural variation. However, the larger the difference we observe (for a particular study size), the less credible it is that the difference is due to chance alone. If out of 100 flips, a coin landed heads up only 5 times, it would be reasonable to doubt that the outcome was due to chance; perhaps the coin is weighted so that tails are more likely to occur.

The material on hypothesis testing will provide the statistical tools to examine this issue. In LEAP, we will be able to show that the 11.8% difference was indeed larger that that expected by chance alone if the two interventions were equally effective at preventing subsequent allergies.

1.2 Data basics

Effective presentation and description of data is a first step in most analyses. This section introduces one structure for organizing data as well as some terminology that will be used throughout this book.

1.2.1 Observations, variables, and data matrices

This section describes data used in a study published in the *Journal of Evolutionary Biology* about maternal investment at differing altitudes, conducted in a frog species endemic to

the Tibetan Plateau (*Rana kukunoris*)[3]. Reproduction is a costly process for females, necessitating a trade-off between individual egg size and total number of eggs produced. Researchers collected measurements on egg clutches found at breeding ponds across 11 study sites; for 5 sites, they also collected data on individual female frogs.

	altitude	latitude	egg.size	clutch.size	clutch.volume	body.size
1	3,462.00	34.82	1.95	181.97	177.83	3.63
2	3,462.00	34.82	1.95	269.15	257.04	3.63
3	3,462.00	34.82	1.95	158.49	151.36	3.72
150	2,597.00	34.05	2.24	537.03	776.25	NA

Table 1.3: Frog Study Data Matrix

Table 1.3 displays rows 1, 2, 3, and 150 of the data from the 431 clutches. The complete set of observations will be referred to as the `frog` dataset. Each row in the table corresponds to a single clutch, indicating where the clutch was collected (`altitude` and `latitude`), `egg.size`, `clutch.size`, `clutch.volume`, and `body.size` of the mother when available. `NA` corresponds to a missing value; information on individual females was not collected for that particular site. The columns represent characteristics, called `variables`, for each clutch.

For example, the first row represents a clutch collected at altitude 3,462 meters above sea level, latitude 34.82 degrees; the clutch contained an estimated 182 eggs, with individual eggs averaging 1.95 mm in diameter, for a total volume of 177.8 mm^3. The eggs were laid by a female measuring 3.63 cm long. It is important to understand the definitions of variables, as they are not always obvious. For example, why has `clutch.size` not been recorded as whole numbers? This has to do with how the observations were collected. In a given clutch, researchers counted approximately 5 grams' worth of eggs and then estimated the total number of eggs based on the mass of the entire clutch. Definitions of the variables are given in Table 1.3.

JV:please create this table of defs, using the famuss table as a model. Also, we should add that the data discussed here are in the original scale, not transformed, as in the paper. Note that I have changed some variable names. Please see the R file oi_biosta_ch1.R

[3] Chen, W., et al. Maternal investment increases with altitude in a frog on the Tibetan Plateau. Journal of evolutionary biology 26.12 (2013): 2710-2715.

[variable definitions]

The data in Table 1.3 are organized as a **data matrix**. Each row of a data matrix corresponds to a unique observational unit, and each column corresponds to a variable. A data matrix for the LEAP study introduced in Section 1.1 is shown in Table 1.1 on page 10, in which the cases were patients and three variables were recorded for each patient. Data matrices are a convenient way to record and store data. If the data are collected for another individual, another row can easily be added; similarly, another column can be added for a new variable.

1.2.2 Types of variables

The functional polymorphisms Associated with Human Muscle Size and Strength study (FAMuSS) [4], funded by the National Institutes of Health (NIH), measured a variety of demographic, phenotypic, and genetic characteristics of about 1300 participants. Data from the study has been used in many subsequent populations [5], such as a study examining the relationship between muscle strength and a location on the gene *actn3* [6] Four rows of the `FAMuSS` dataset are shown in Table 1.4, and the variables are summarized in Table 1.5. Additional

	sex	age	race	height	weight	actn3.r577x	ndrm.ch
1	Female	27	Caucasian	65.0	199.0	CC	40.0
2	Male	36	Caucasian	71.7	189.0	CT	25.0
3	Female	24	Caucasian	65.0	134.0	CT	40.0
	⋮	⋮	⋮	⋮	⋮	⋮	
595	Female	30	Caucasian	64.0	134.0	CC	43.8

Table 1.4: Four rows from the `FAMuSS` data matrix.

The variables `age`, `height`, `weight`, and `ndrm.ch` are **numerical** variables. They can take on a wide range of numerical values, and it is possible to add, subtract, or take averages with these values. On the other hand, we would not classify a variable reporting telephone area codes as numerical since their average, sum, and difference have no clear

[4] Thompson PD, Moyna M, Seip, R, et al., 2004. Functional Polymorphisms Associated with Human Muscle Size and Strength. Medicine and Science in Sports and Exercise 36:1132 - 1139

[5] Pescatello L, et al. Highlights from the functional single nucleotide polymorphisms associated with human muscle size and strength or FAMuSS study, BioMed Research International 2013.

[6] Clarkson P, et al., Journal of Applied Physiology 99: 154163, 2005.

variable	description
`sex`	Sex of the participant
`age`	Age in years
`race`	Recorded as African AM (African American), Caucasian, Hispanic and Other.
`height`	Height in inches
`weight`	Weight in lbs
`actn3.r577x`	Genotype at the location r577x in the gene actn3. The four genotypes observed were CC, CT and TT
`ndrm.ch`	Percent change in strength in the non-dominant arm, comparing strength after to before training

Table 1.5: Variables and their descriptions for the `FAMuSS` data set.

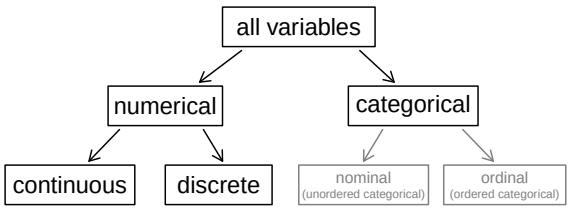


Figure 1.6: Breakdown of variables into their respective types.

meaning. Age measured in years is said to be **discrete**, since it can only take numerical values with jumps. On the other hand, percent change in strength in the non-dominant arm (`ndrm.ch`) is said to be **continuous**.

The variables `sex`, `race`, and `actn3.3577x` are **categorical** variables, [7] and the possible values are called the variable's **levels**. For example, the levels of `actn3.3577x` are the three possible genotypes at this particular locus: CC, CT, or TT. Categorical variables with levels that have a natural ordering can be more specifically referred to as **ordered categorical** variables. There are no ordered categorical variables in the `FAMuSS` data, but it would be easy to create one. Age of the participants grouped into 5 year intervals (15 - 20, 21 - 25, 26 - 30, etc) would be an ordered categorical variable. Statistical software such as R call categorical variables **factors**, and the possible values of factors are called **levels**.

Example 1.1 Suppose data were collected about students in a statistics course. Three variables were recorded for each student: number of siblings, student height, and whether the student had previously taken a statistics course. Classify each of the variables as continuous numerical, discrete numerical, or categorical.

[7] sometimes called **nominal** variables.

The number of siblings and student height represent numerical variables. Because the number of siblings is a count, it is discrete. Height varies continuously, so it is a continuous numerical variable. The last variable classifies students into two categories – those who have and those who have not taken a statistics course – which makes this variable categorical.

⊙ **Guided Practice 1.2** Characterize the data types for the variables `participant.ID` `treatment.group` and `overall.V60.outcome` from the LEAP study in Section 1.1.[8]

1.2.3 Relationships between variables

Many studies are motivated by a researcher examining a possible relationship between two or more variables. Statistical relationships between two variables occur when they tend to vary in a related way.

A **response variable** measures an outcome of interest, while an **explanatory variable** may be useful in predicting or understanding the response variable. There may be several possible explanatory variables for a single response variable in a given study.

Researchers were interested in using the `FAMuSS` data in order to answer the following question: is ACTN3 genotype associated with variation in muscle function? The ACTN3 gene codes for a protein involved in muscle function. A common polymorphism of ACTN3 at residue 577 that changes C to T produces a stop codon; TT individuals are unable to produce any ACTN3 protein in their muscle. The TT genotype does not cause any discernible phenotype changes, which suggests that the ACTN3 protein is not critical to muscle function. However, the ACTN3 gene is highly conserved, and may potentially influence variation in muscle function. *'conserved' is a technical term that needs a definition. Perhaps we can avoid it altogether. There are too many technical terms in this paragraph generally.*

The response variable in this study is `ndrm.ch`, the change in non-dominant arm strength, with strength gain being used as a way to measure muscle function. The ex-

[8] All these variables measure non-numerical quantities, and are categorical. The variables `treatment.group` and `outcome.V60.overall` have two values or levels, while `participant.ID` has many possible values.

planatory variable of interest is `actn3.r557x`, ACTN3 genotype at residue 577. Later in the text we will examine methods for characterizing a relationship numerically. *too vague*

⊙ **Guided Practice 1.3** Use the variables from the `FAMuSS` data set described in Table 1.5 to pose two questions about the relationships between these variables that are different from the question of interest to the research team. [9]

A Scatterplot is a graph used to explore the relationship between two numerical variables. Figure 1.7 shows the relationship between `height` and `weight` for participants in the FAMuSS study. Each point on the plot represents a participant. As expected, taller participants tend to be heavier. *plot should be fancified, and one of the heavy participants highlighted.*

The variables `height` and `weight` are said to be associated because the plot shows a discernible pattern. As expected, taller participants tend to be heavier. Because of the upward trend in the plot, the two variables are said to be **positively associated**. If two variables are not associated, then they are said to be **independent**. That is, two variables are independent if there is no evident relationship between the two.

The variables `height` and `weight` are said to be associated because the plot shows a discernible pattern. As expected, taller participants tend to be heavier. Because of the upward trend in the plot, the two variables are said to be **positively associated**. If a scatterplot shows a downward trend, the two variables in the plot are said to be **negatively associated**.

Because taller people tend, naturally, to be heavier, weight itself is not a good measure of whether someone is overweight. Body mass index (BMI) is a measure of weight that is less affected by a person's height. In the metric system, BMI is a person's weight in kilograms (kg) divided by his or her height in meters squared. If height and weight are measured in inches and pounds, then BMI is weight in pounds divided by height in inches squared, then multiplied by 703. The `famuss` dataset includes the variable `bmi` for each participant, and figure 1.8 shows the relationship between `height` and `bmi`. The strong upward trend in Figure 1.7 is no longer evident, indicating that `height` and `bmi` have a

[9] Two sample questions: (1) Do participants appear respond differently to training according to race? (2) Do male participants appear to respond differently to training than females.

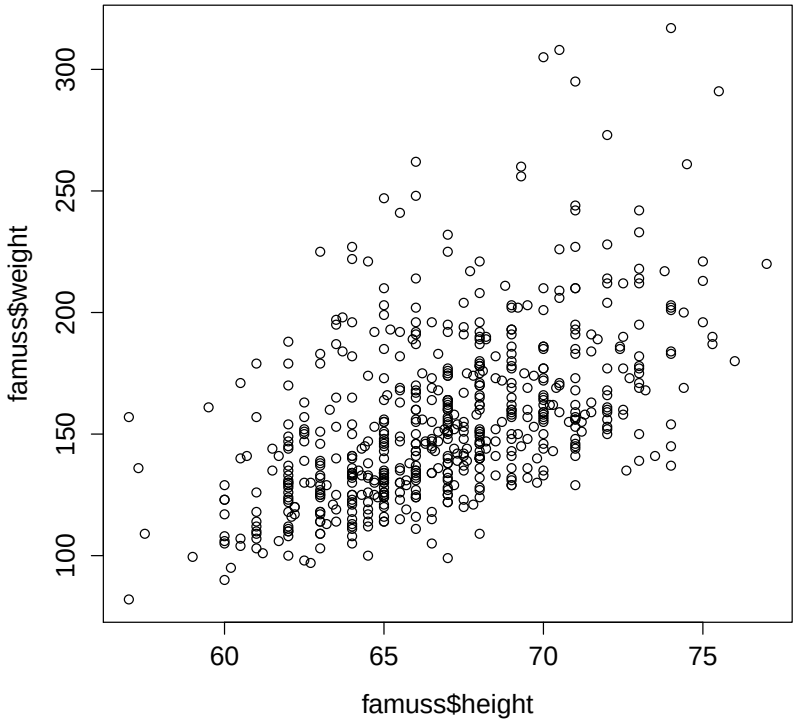


Figure 1.7: A scatterplot showing `height` (horizontal axis) vs. `weight` (vertical axis).

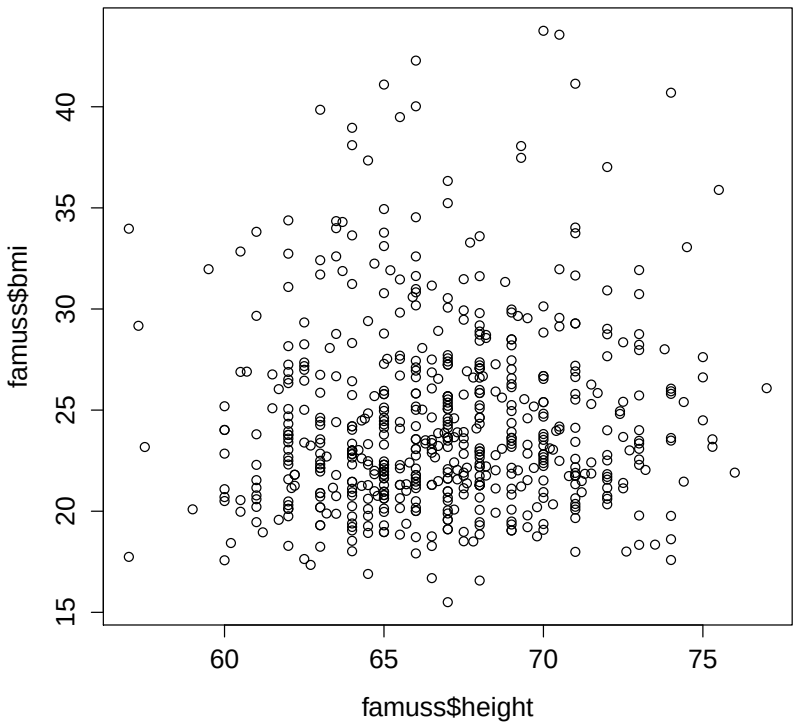


Figure 1.8: A scatterplot showing `height` (horizontal axis) vs. `bmi` (vertical axis).

much weaker association. For this reason, the US NIH, the World Health Organization and other health agencies use BMI rather that weight as a measure of obesity.

If two variables are not associated, then they are said to be **independent**. That is, two variables are independent if there is no evident relationship between the two. It is generally not easy to determine definitively from a scatterplot whether two variables are independent, even in 1.8.

Caution: association does not imply causation

Labeling variables as *explanatory* and *response* does not guarantee the relationship between the two is causal, even if there is an association identified between the two variables. We use these labels only to keep track of which variable we suspect influences the other. Taller people do tend to be heavier, a variety of genetic and environmental factors influence weight as well.

revisions to here 18jul2015, 14:57

1.3 Data collection principles

The first step in conducting research is to identify questions to investigate. A clearly articulated research question is essential in identifying which subjects should be studied, what variables are relevant, and how data should be measured. In order to obtain reliable data, it is also important to consider *how* data are collected.

1.3.1 Introducing experiments and observational studies

There are two primary types of data collection: experiments and observational studies.

When researchers want to investigate the possibility of a causal connection, they conduct an **experiment**. For instance, we may suspect that administering a certain drug will reduce mortality in heart attack patients. To find evidence for a causal connection between the explanatory and response variables, researchers will collect a sample of individuals and split them into groups. The individuals in each group are randomly assigned into one of two groups: the first group receives a **placebo** (fake treatment) and the second group receives the drug.

Researchers perform an **observational study** when they collect data in a way that does not directly interfere with how the data arise. For instance, researchers may collect information via surveys, review medical or company records, or follow a **cohort** of many similar individuals to study why certain diseases might develop. In each of these situations, researchers merely observe the data that arise. Observational studies can provide

evidence of an association between variables, but they cannot by themselves show a causal connection.

1.3.2 Experiments

Studies where the researchers assign treatments to cases are called **experiments**. Randomized experiments are generally built on three principles.

Controlling. Researchers assign treatments to cases, and they do their best to **control** for any other differences in the groups. For example, in the stent study, patients in both groups received medical management, which included medications, management of stroke risk factors, and counseling on lifestyle modification. Effectively controlling variables is essential for making meaningful comparisons between treatment and control groups.

Randomization. Researchers randomize patients into treatment groups to account for variables that cannot be controlled. For example, some patients may have been more susceptible to stroke than others due to dietary habits or genetic risk factors. Randomizing patients into the treatment or control group helps even out such differences. In situations where researchers suspect that variables other than the treatment influence the response, they may first group individuals into **blocks** and then, within each block, randomize cases to treatment groups; this technique is referred to as **blocking** or **stratification**. Methods for analyzing blocked data are relatively complicated and will not be covered in this book.

Replication. The more cases researchers observe, the more accurately they can estimate the effect of the explanatory variable on the response. In a single study, we **replicate** by collecting a sufficiently large sample.

1.3.3 Observational studies

Generally, data in observational studies are collected only by monitoring what occurs, while experiments require the primary explanatory variable in a study be assigned for each subject by the researchers.

Making causal conclusions based on experiments is often reasonable. However, making the same causal conclusions based on observational data can be treacherous and is not recommended. Thus, observational studies are generally only sufficient to show associations.

⊙ **Guided Practice 1.4** Figure **??** shows a negative association between the homeownership rate and the percentage of multi-unit structures in a county. However, it is unreasonable to conclude that there is a causal relationship between the two variables. Suggest one or more other variables that might explain the relationship visible in Figure **??**.[10]

Observational studies come in two forms: prospective and retrospective studies. A **prospective study** identifies individuals and collects information as events unfold. For instance, medical researchers may identify and follow a group of similar individuals over many years to assess the possible influences of behavior on cancer risk. One example of such a study is The Nurses' Health Study, started in 1976 and expanded in 1989.[11] This prospective study recruits registered nurses and then collects data from them using questionnaires. **Retrospective studies** collect data after events have taken place, e.g. researchers may review past events in medical records. Some data sets, such as `county`, may contain both prospectively- and retrospectively-collected variables. Local governments prospectively collect some variables as events unfolded (e.g. retails sales) while the federal government retrospectively collected others during the 2010 census (e.g. county population counts).

[**briefly discuss simple random sampling**]

1.4 Examining numerical data

some revisions from JV in this section

This section introduces techniques for exploring and summarizing numerical variables. The `frog_altitude` dataset from Section 1.2 provides rich opportunities for examples.

[10] Answers will vary. Population density may be important. If a county is very dense, then this may require a larger fraction of residents to live in multi-unit structures. Additionally, the high density may contribute to increases in property value, making homeownership infeasible for many residents.

[11] www.channing.harvard.edu/nhs

1.4.1 Measures of center: mean and median

The **mean**, sometimes called the average, is a common way to measure the center of a **distribution** of data. To find the average clutch volume for all observed egg clutches, we add up all the clutch volumes and divide by the total number of clutches. For computational convenience, the volumes are rounded to the first decimal.

$$\bar{x} = \frac{177.8 + 257.0 + \cdots + 933.3}{431} = 882.5 mm^3 \tag{1.5}$$

$\bar{x}$
sample mean

The sample mean is often labeled $\bar{x}$. The letter x is being used as a generic placeholder for the variable of interest, `clutch.volume`, and the bar over on the x communicates that the average volume of the 431 clutches was $882.5 mm^3$. It is useful to think of the mean as the balancing point of the distribution.

Mean

The sample mean of a numerical variable is computed as the sum of all of the observations divided by the number of observations:

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} \tag{1.6}$$

where $x_1, x_2, \ldots, x_n$ represent the n observed values.

n
sample size

Another measure of center is the **median**, which is the middle number in a distribution after the values have been ordered from smallest to largest. If the distribution contains an even number of observations, the median is the average of the middle two observations. There are 431 clutches in the dataset, so the median is the clutch volume of the 216^{th} observation in the `clutch.volume` variable: 831.8 mm^2.

1.4.2 Measures of spread: standard deviation and interquartile range

The standard deviation roughly describes how far away the typical observation is from the mean. The distance of an observation from its mean its **deviation**. Below are the deviations for the 1^{st}, 2^{nd}, 3^{rd}, and 431^{th} observations in the `clutch.volume` variable. For computational convenience, clutch volume is rounded to the first decimal.

$$\begin{aligned} x_1 - \bar{x} &= 177.8 - 882.5 = -704.7 \\ x_2 - \bar{x} &= 257.0 - 882.5 = -625.5 \\ x_3 - \bar{x} &= 151.4 - 882.5 = -731.1 \\ &\vdots \\ x_4 31 - \bar{x} &= 933.2 - 882.5 = 50.7 \end{aligned}$$

If we square these deviations and then take an average, the result is about equal to the sample **variance**, denoted by s^2:

s^2 sample variance

$$\begin{aligned} s^2 &= \frac{-704.7^2 + (-625.5)^2 + (-731.1)^2 + \cdots + 50.7^2}{431 - 1} \\ &= \frac{496,602.09 + 391,250.25 + 534,507.21 + \cdots + 2570.49}{430} \\ &= 143,680.9 \end{aligned}$$

We divide by $n-1$, rather than dividing by n, when computing the variance; you need not worry about this mathematical nuance for the material in this textbook.

The **standard deviation** is defined as the square root of the variance:

$$s = \sqrt{143,680.9} = 379.05$$

s sample standard deviation

The standard deviation of clutch volume for the egg clutches observed is about 380 mm^3.

insert tip box for SD formula

Variability can also be described by the **interquartile range** (IQR). To calculate the IQR, find the **first quartile** (the 25^{th} percentile, i.e. 25% of the data fall below this value) and the **third quartile** (the 75^{th} percentile). These are often labeled Q_1 and Q_3, respectively. The IQR is the difference: $Q_3 - Q_1$.

The IQR for `clutch.volume` is $1096.0 - 609.6 = 486.4_(mm)^3$.

1.4.3 Robust statistics

The median and IQR are called **robust estimates** because extreme observations have little effect on their values. The mean and standard deviation are much more affected by changes in extreme observations.

adapt this for the largest value in `clutch.size`

How are the sample statistics of the `num_char` data set affected by the observation, 64,401? What would have happened if this email wasn't observed? What would happen to these summary statistics if the observation at 64,401 had been even larger, say 150,000? These scenarios are plotted alongside the original data in Figure 1.9, and sample statistics are computed under each scenario in Table 1.10.

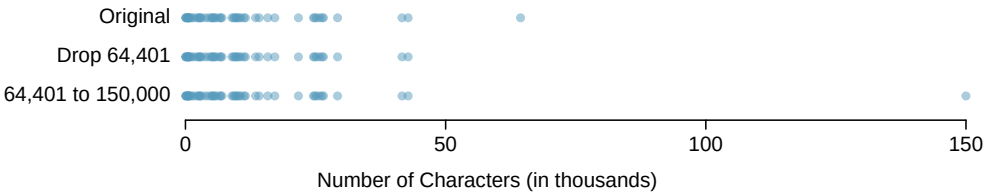


Figure 1.9: Dot plots of the original character count data and two modified data sets.

	robust		**not robust**	
scenario	median	IQR	$\bar{x}$	s
original `num_char` data	6,890	12,875	11,600	13,130
drop 66,924 observation	6,768	11,702	10,521	10,798
move 66,924 to 150,000	6,890	12,875	13,310	22,434

Table 1.10: A comparison of how the median, IQR, mean ($\bar{x}$), and standard deviation (s) change when extreme observations are present.

1.4.4 Visualizing distributions of data: dot plots and histograms

Graphical summaries are useful tools for visualizing how data are distributed. A **dot plot** provides the most basic of displays, representing data as points plotted on a single axis.

An example using the number of characters from 50 emails is shown in Figure 1.11. A stacked version of this dot plot is shown in Figure 1.12.

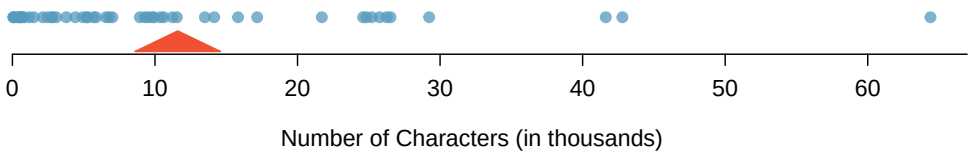


Figure 1.11: A dot plot of `num_char` for the `email50` data set.

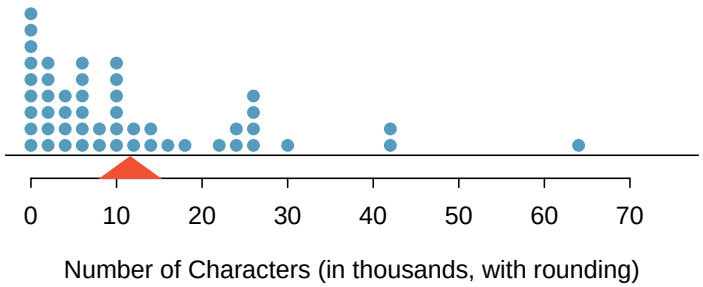


Figure 1.12: A stacked dot plot of `num_char` for the `email50` data set. The values have been rounded to the nearest 2,000 in this plot.

Dot plots show the exact value for each observation. This is useful for small data sets, but they can become hard to read with larger samples. Rather than showing the value of each observation, we prefer to think of the value as belonging to a *bin*. For example, in the `email50` data set, we create a table of counts for the number of cases with character counts between 0 and 5,000, then the number of cases between 5,000 and 10,000, and so on. Observations that fall on the boundary of a bin (e.g. 5,000) are allocated to the lower bin. This tabulation is shown in Table 1.13. These binned counts are plotted as bars in Figure 1.14 into what is called a **histogram**, which resembles the stacked dot plot shown in Figure 1.12.

Histograms provide a view of the **data density**. Higher bars represent where the data

Characters (in thousands)	0-5	5-10	10-15	15-20	20-25	25-30	···	55-60	60-65
Count	19	12	6	2	3	5	···	0	1

Table 1.13: The counts for the binned `num_char` data.

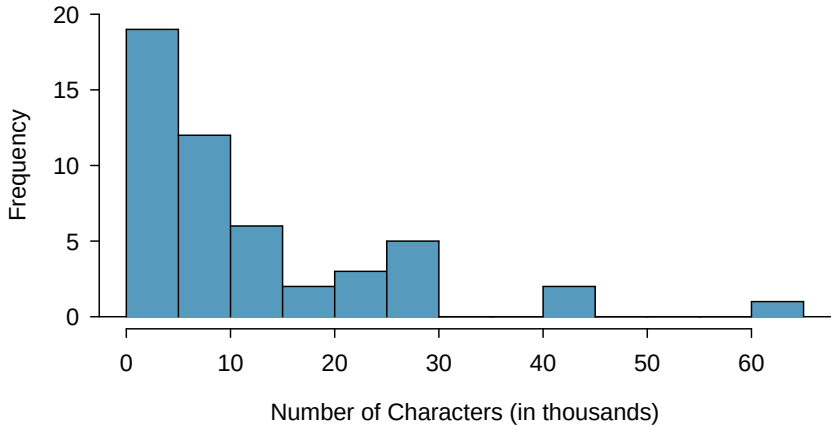


Figure 1.14: A histogram of `num_char`. This distribution is very strongly skewed to the right.

are relatively more common. For instance, there are many more emails with fewer than 20,000 characters than emails with at least 20,000 in the data set. The bars make it easy to see how the density of the data changes relative to the number of characters.

Histograms are especially convenient for describing the shape of the data distribution. Figure 1.14 shows that most emails have a relatively small number of characters, while fewer emails have a very large number of characters. When data trail off to the right in this way and have a longer right tail, the shape is said to be **right skewed**.[12]

Data sets with the reverse characteristic – a long, thin tail to the left – are said to be **left skewed**. We also say that such a distribution has a long left tail. Data sets that show roughly equal trailing off in both directions are called **symmetric**.

In addition to looking at whether a distribution is skewed or symmetric, histograms can be used to identify modes. A **mode** is represented by a prominent peak in the distribution.[13] There is only one prominent peak in the histogram of `num_char`.

[12] Other ways to describe data that are skewed to the right: **skewed to the right**, **skewed to the high end**, or **skewed to the positive end**.

[13] Another definition of mode, which is not typically used in statistics, is the value with the most occurrences. It is common to have *no* observations with the same value in a data set, which makes this other definition useless for many real data sets.

Figure 1.15 shows histograms that have one, two, or three prominent peaks. Such distributions are called **unimodal**, **bimodal**, and **multimodal**, respectively. Any distribution with more than 2 prominent peaks is called multimodal. Notice that there was one prominent peak in the unimodal distribution with a second less prominent peak that was not counted since it only differs from its neighboring bins by a few observations.

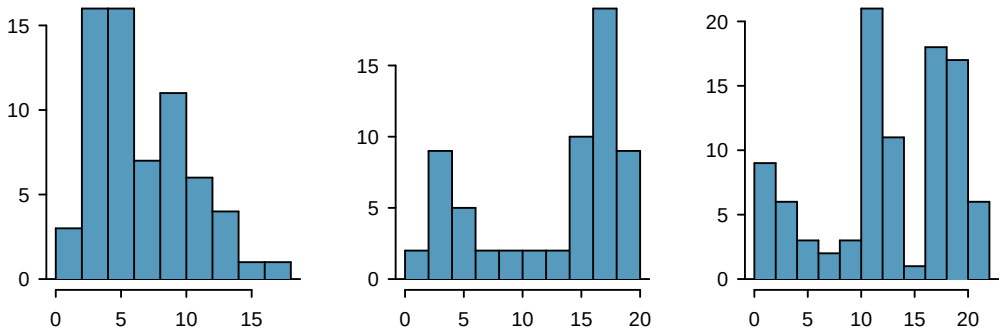


Figure 1.15: Counting only prominent peaks, the distributions are (left to right) unimodal, bimodal, and multimodal.

⊙ **Guided Practice 1.7** Figure 1.14 reveals only one prominent mode in the number of characters. Is the distribution unimodal, bimodal, or multimodal?[14]

1.4.5 Boxplots, quantiles, outliers

A **boxplot** summarizes a dataset using five statistics while also plotting unusual observations. Figure 1.16 provides a vertical dot plot alongside a box plot of the `num_char` variable from the `email50` data set.

The first step in building a box plot is drawing a dark line denoting the **median**, which splits the data in half. Figure 1.16 shows 50% of the data falling below the median (dashes) and other 50% falling above the median (open circles).

The second step in building a box plot is drawing a rectangle to represent the middle 50% of the data, which is the IQR.

Extending out from the box, the **whiskers** capture the data that fall between 1.5 ×

[14] Unimodal. Remember that *uni* stands for 1 (think *uni*cycles). Similarly, *bi* stands for 2 (think *bi*cycles). (We're hoping a *multicycle* will be invented to complete this analogy.)

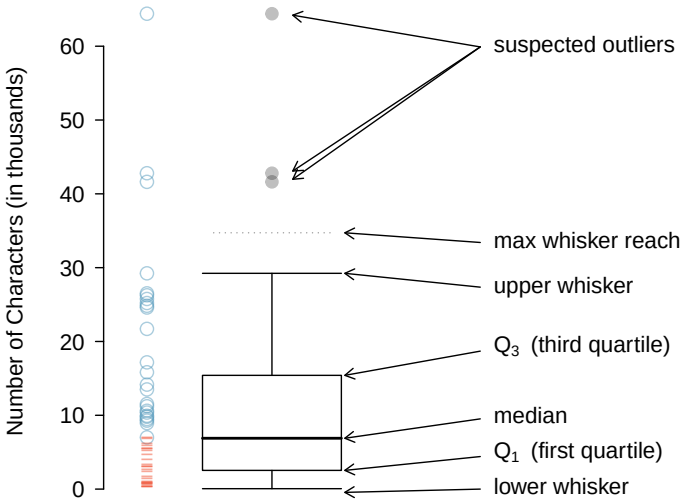


Figure 1.16: A vertical dot plot next to a labeled box plot for the number of characters in 50 emails. The median (6,890), splits the data into the bottom 50% and the top 50%, marked in the dot plot by horizontal dashes and open circles, respectively.

IQR.[15] In Figure 1.16, the upper whisker does not extend to the last three points, which is beyond $Q_3 + 1.5 \times IQR$, and so it extends only to the last point below this limit. The lower whisker stops at the lowest value, 33, since there is no additional data to reach; the lower whisker's limit is not shown in the figure because the plot does not extend down to $Q_1 - 1.5 \times IQR$. In a sense, the box is like the body of the box plot and the whiskers are like its arms trying to reach the rest of the data.

Any observation that lies beyond the whiskers is labeled with a dot. The purpose of labeling these points is to help identify any observations that appear to be unusually distant from the rest of the data. These observations are called outliers; An **outlier** is an observation that appears extreme relative to the rest of the data. In this case, it would be reasonable to classify the emails with character counts of 41,623, 42,793, and 64,401 as outliers since they are numerically distant from most of the data. Outliers can potentially provide insight into interesting properties of the data.

[15] While the choice of exactly 1.5 is arbitrary, it is the most commonly used value for box plots.

1.4.6 Scatterplots

A **scatterplot** provides a case-by-case view of data for two numerical variables. In Figure #, a scatterplot is used to examine the relationship between clutch volume and female body size in the `frog` dataset. In any scatterplot, each point represents a single case. Since body size was measured for 129 frogs, there are 129 points in Figure #.

The `clutch.volume` and `body.size` are said to be **associated** because the plot shows a discernible pattern. Since the points tend to lie in a straight line, the two variables are **linearly associated**.

Two variables are **positively associated** if increasing values of one tend to occur with increasing values of the other; similarly, variables are **negatively associated** if increasing values of one variable occurs with decreasing values of the other. Figure # shows an upward trend – larger frogs tend to produce clutches with larger volume. Frog embryos are surrounded by a gelatinous matrix that may protect developing embryos from temperature fluctuation or ultraviolet radiation; these observations suggest that larger females are capable of producing greater quantities of this material.

If two variables are not associated, they are said to be **independent**.

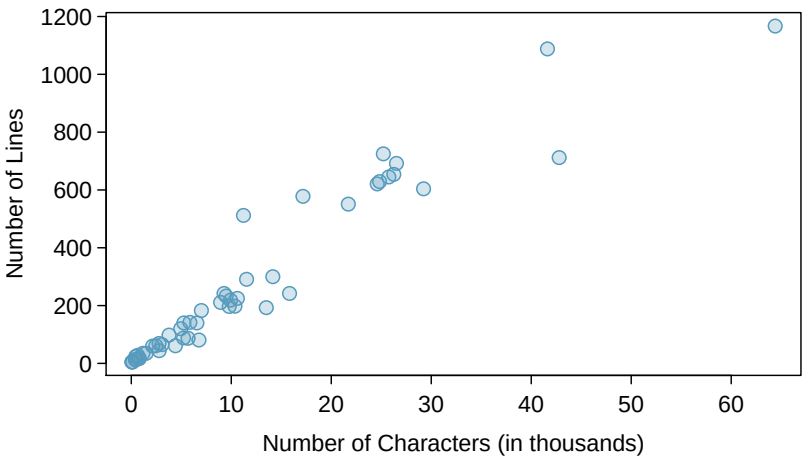


Figure 1.17: A scatterplot of `line_breaks` versus `num_char` for the `email50` data.

Example 1.8 Consider a new data set of 54 cars with two variables: vehicle price

and weight.[16] A scatterplot of vehicle price versus weight is shown in Figure **??**. What can be said about the relationship between these variables?

The relationship is evidently nonlinear, as highlighted by the dashed line. This is different from previous scatterplots we've seen, such as Figure **??** on page ?? and Figure 1.17, which show relationships that are very linear.

[16] Subset of data from www.amstat.org/publications/jse/v1n1/datasets.lock.html

1.5 Considering categorical data

Like numerical data, categorical data can also be organized and analyzed; however, numerical calculations cannot be done with categorical data. In this section, we will introduce tables and other basic tools for categorical data, using the `FAMuSS` dataset introduced in Section 1.2.2.

1.5.1 Contingency tables

A table for a single variable is called a **frequency table**. Table # is a frequency table for the `actn3.r577x` variable. Recall that `actn3.r577x` is a categorical variable that describes genotype at a particular locus on the ACTN3 gene: CC, CT, or TT. If we replaced the counts with percentages or proportions, the table would be called a **relative frequency table**.

[insert frequency table for `actn3.r577x`]

none	small	big	Total
549	2827	545	3921

Table 1.18: A frequency table for the `number` variable.

Table # summarizes two variables: `race` and `actn3.r577x`. A table that summarizes data for two categorical variables in this way is called a **contingency table**. Each value in the table represents the number of times a particular combination of variable outcomes occurred. For example, the first row of the table shows that of the African-American individuals, 16 are CC, 6 are CT, and 5 are TT.

Row and column totals, known collectively as **marginal totals**, are also included. The **row totals** provide the total counts across each row; **column totals** are the total counts down each column.

[insert race by genotype table]

Table # shows the row proportions for Table #. The **row proportions** are computed as the counts divided by their row totals. The value 16 at the intersection of `African American` and`CC` is replaced by $16/27 = 0.593$; i.e., 16 divided by the row total, 27. The

		number			
		none	small	big	Total
spam	spam	149	168	50	367
	not spam	400	2659	495	3554
	Total	549	2827	545	3921

Table 1.19: A contingency table for spam and number.

	none	small	big	Total
spam	$149/367 = 0.406$	$168/367 = 0.458$	$50/367 = 0.136$	1.000
not spam	$400/3554 = 0.113$	$2657/3554 = 0.748$	$495/3554 = 0.139$	1.000
Total	$549/3921 = 0.140$	$2827/3921 = 0.721$	$545/3921 = 0.139$	1.000

Table 1.20: A contingency table with row proportions for the spam and number variables.

value 0.593 corresponds to the proportion of African-Americans in the study of the CC genotype.

A contingency table of the column proportions is computed in a similar way, where each **column proportion** is computed as the count divided by the corresponding column total. Table # shows such a table, and here the value 0.092 indicates that 9.2% of CC individuals in the study are African-American.

	none	small	big	Total
spam	$149/549 = 0.271$	$168/2827 = 0.059$	$50/545 = 0.092$	$367/3921 = 0.094$
not spam	$400/549 = 0.729$	$2659/2827 = 0.941$	$495/545 = 0.908$	$3684/3921 = 0.906$
Total	1.000	1.000	1.000	1.000

Table 1.21: A contingency table with column proportions for the spam and number variables.

1.5.2 Bar plots

A bar plot is a common way to display a single categorical variable. The left panel of Figure # shows a **bar plot** for the `actn3.r577x` variable. In the right panel, the counts are converted into proportions (e.g. $173/595 = 0.291$ for `none`), showing the proportion of observations that are in each level (i.e. in each category).

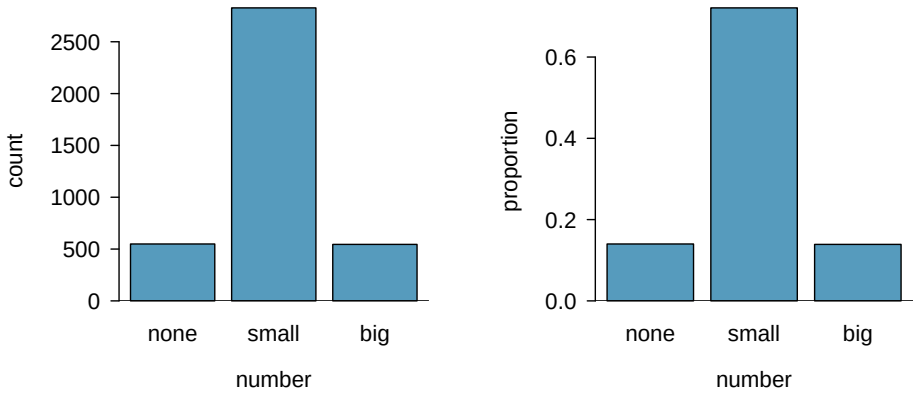


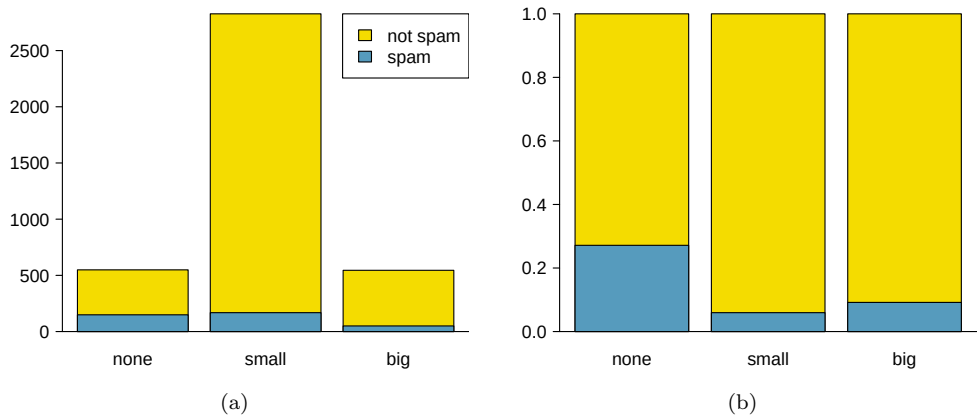
Figure 1.22: Two bar plots of `number`. The left panel shows the counts, and the right panel shows the proportions in each group.

Segmented bar plots provide a way to visualize the information in contingency tables. A **segmented bar plot** is a graphical display of contingency table information. For example, a segmented bar plot representing Table # is shown in #, where a bar plot was created using the `actn3.r577x` variable, with each group divided by the levels of `race`. The column proportions of Table # have been translated into a standardized segmented bar plot in Figure #, which is a helpful visualization of the races represented in each level of `actn3.r577x`.

1.5.3 Comparing numerical data across groups

Some of the more interesting investigations can be considered by examining numerical data across groups. In this section, two convenient methods are introduced: side-by-side box plots and hollow histograms.

The **side-by-side box plot** is a traditional tool for comparing across groups. Another useful plotting method uses **hollow histograms** to compare numerical data across groups.



(a) (b)

Figure 1.23: (a) Segmented bar plot for numbers found in emails, where the counts have been further broken down by `spam`. (b) Standardized version of Figure (a).

These are just the outlines of histograms of each group put on the same plot.

Recall the question introduced in Section 1.2.3: is ACTN3 genotype associated with variation in muscle function? To explore this question, genotype and variation in muscle function (measured by `ndrm.ch`) can be compared using side-by-side boxplots and hollow histograms. The histograms are useful for seeing distribution shape, skew, and groups of anomalies, while the side-by-side boxplots are especially useful for comparing centers and spreads. Comparison of median change in non-dominant arm strength between the two groups reveals that the TT genotype is associated with a greater increase in strength than CC or TT. In other words, the T allele appears to be associated with greater muscle function.

Not all data will show such apparent trends. For example, consider the question of interest in the `frog` dataset: how does maternal investment vary with altitude? Researchers collected data at 11 altitudes from 2,035 to 3,495 m above sea level, measuring attributes of egg clutches such as clutch volume. A side-by-side boxplot comparing clutch volume across altitudes is shown in Figure #. It seems that as a general rule, clutches found at higher altitudes have greater volume. However, more advanced statistical methods are required to thoroughly investigate the potential association between altitude and clutch size.

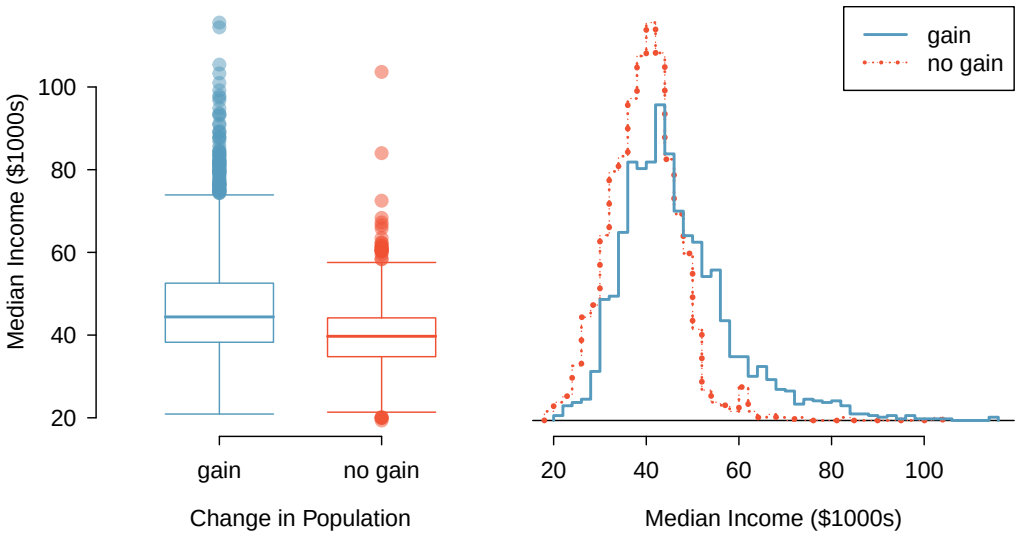


Figure 1.24: Side-by-side box plot (left panel) and hollow histograms (right panel) for `med_income`, where the counties are split by whether there was a population gain or loss from 2000 to 2010. The income data were collected between 2006 and 2010.

Appendix A

End of chapter exercise solutions

1 Introduction to data

1.1 (a) Treatment: $10/43 = 0.23 \rightarrow 23\%$. Control: $2/46 = 0.04 \rightarrow 4\%$. (b) There is a 19% difference between the pain reduction rates in the two groups. At first glance, it appears patients in the treatment group are more likely to experience pain reduction from the acupuncture treatment. (c) Answers may vary but should be sensible. Two possible answers: [1]Though the groups' difference is big, I'm skeptical the results show a real difference and think this might be due to chance. [2]The difference in these rates looks pretty big, so I suspect acupuncture is having a positive impact on pain.

1.3 (a) 143,196 eligible study subjects born in Southern California between 1989 and 1993. (b) Measurements of carbon monoxide, nitrogen dioxide, ozone, and particulate matter less than $10\mu g/m^3$ (PM_{10}) collected at air-quality-monitoring stations as well as length of gestation. Continuous numerical variables. (c) "Is there an association between air pollution exposure and preterm births?"

1.5 (a) 160 children. (b) Age (numerical, continuous), sex (categorical), whether they were an only child or not (categorical). (c) Research question: "Does explicitly telling children not to cheat affect their likelihood to cheat?"

1.7 (a) $50 \times 3 = 150$. (b) Four continuous numerical variables: sepal length, sepal width, petal length, and petal width. (c) One categorical variable, species, with three levels: *setosa*, *versicolor*, and *virginica*.

1.9 (a) Population: all births, sample: 143,196 births between 1989 and 1993 in Southern California. (b) If births in this time span at the geography can be considered to be representative of all births, then the results are generalizable to the population of Southern California. However, since the study is observational the findings cannot be used to establish causal relationships.

1.11 (a) Population: all asthma patients aged 18-69 who rely on medication for asthma treatment. Sample: 600 such patients. (b) If the patients in this sample, who are likely not randomly sampled, can be considered to be representative of all asthma patients aged 18-69 who rely on medication for asthma treatment, then the results are generalizable to the population defined above. Additionally, since the study is experimental, the findings can be used to establish causal relationships.

1.13 (a) Observation. (b) Variable. (c) Sample statistic (mean). (d) Population parameter (mean).

1.15 (a) Explanatory: number of study hours per week. Response: GPA. (b) Somewhat weak positive relationship with data becoming more sparse as the number of study hours increases. One responded reported a GPA above 4.0, which is clearly a data error. There are a few respondents who reported unusually high study hours (60 and 70 hours/week). Variability in GPA is much higher for students who study less than those who study more, which might be due to the fact that there aren't many respondents who reported studying higher hours. (c) Observational. (d) Since observational, cannot infer causation.

1.17 (a) Observational. (b) Use stratified sampling to randomly sample a fixed number of students, say 10, from each section for a total sample size of 40 students.

1.19 (a) Positive, non-linear, somewhat strong. Countries in which a higher percentage of the population have access to the internet also tend to have higher average life expectancies, however rise in life expectancy trails off before around 80 years old. (b) Observational. (c) Wealth: countries with individuals who can widely afford the internet can probably also afford basic medical care. (Note: Answers may vary.)

1.21 (a) Simple random sampling is okay. In fact, it's rare for simple random sampling to not be a reasonable sampling method! (b) The student opinions may vary by field of study, so the stratifying by this variable makes sense and would be reasonable. (c) Students of similar ages are probably going to have more similar opinions, and we want clusters to be diverse with respect to the outcome of interest, so this would **not** be a good approach. (Additional thought: the clusters in this case may also have very different numbers of people, which can also create unexpected sample sizes.)

1.23 (a) The cases are 200 randomly sampled men and women. (b) The response variable is attitude towards a fictional microwave oven. (c) The explanatory variable is dispositional attitude. (d) Yes, the cases are sampled randomly. (e) This is an observational study since there is no random assignment to treatments. (f) No, we cannot establish a causal link between the explanatory and response variables since the study is observational. (g) Yes, the results of the study can be generalized to the population at large since the sample is random.

1.25 (a) Non-responders may have a different response to this question, e.g. parents who returned the surveys likely don't have difficulty spending time with their children. (b) It is unlikely that the women who were reached at the same address 3 years later are a random sample. These missing responders are probably renters (as opposed to homeowners) which means that they might be in a lower socio- economic status than the respondents. (c) There is no control group in this study, this is an observational study, and there may be confounding variables, e.g. these people may go running because they are generally healthier and/or do other exercises.

1.27 (a) Simple random sample. Non-response bias, if only those people who have strong opinions about the survey responds his sample may not be representative of the population. (b) Convenience sample. Under coverage bias, his sample may not be representative of the population since it consists only of his friends. It is also possible that the study will have non-response bias if some choose to not bring back the survey. (c) Convenience sample. This will have a similar issues to handing out surveys to friends. (d) Multi-stage sampling. If the classes are similar to each other with respect to student composition this approach should not introduce bias, other than potential non-response bias.

1.29 No, students were not randomly sampled (voluntary sample) and the sample only contains college students at a university in Ontario.

1.31 (a) Exam performance. (b) Light level: fluorescent overhead lighting, yellow overhead lighting, no overhead lighting (only desk lamps). (c) Sex: man, woman.

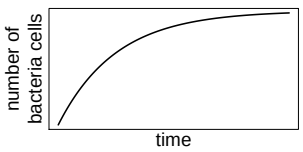
1.33 (a) Exam performance. (b) Light level (overhead lighting, yellow overhead lighting, no overhead lighting) and noise level (no noise, construction noise, and human chatter noise). (c) Since the researchers want to ensure equal gender representation, sex will be a blocking variable.

1.35 Need randomization and blinding. One possible outline: (1) Prepare two cups for each participant, one containing regular Coke and the other containing Diet Coke. Make sure the cups are identical and contain equal amounts of soda. Label the cups A (regular) and B (diet). (Be sure to randomize A and B for each trial!) (2) Give each participant the two cups, one cup at a time, in random order, and ask the participant to record a value that indicates how much she liked the beverage. Be sure that neither the participant nor the person handing out the cups knows the identity of the beverage to make this a double- blind experiment. (Answers may vary.)

1.37 (a) Experiment. (b) Treatment: 25 grams of chia seeds twice a day, control: placebo. (c) Yes, gender. (d) Yes, single blind since the patients were blinded to the treatment they received. (e) Since this is an experiment, we can make a causal statement. However, since the sample is not random, the causal statement cannot be generalized to the population at large.

1.39 (a) 1: linear. 3: nonlinear.
(b) 4: linear. (c) 2.

1.41



1.43 (a) Population mean, $\mu_{2007} = 52$; sample mean, $\bar{x}_{2008} = 58$. (b) Population mean, $\mu_{2001} = 3.37$; sample mean, $\bar{x}_{2012} = 3.59$.

1.45 Any 10 employees whose average number of days off is between the minimum and the mean number of days off for the entire workforce at this plant.

1.47 (a) Dist 2 has a higher mean since $20 > 13$, and a higher standard deviation since 20 is further from the rest of the data than 13. (b) Dist 1 has a higher mean since $-20 > -40$, and Dist 2 has a higher standard deviation since -40 is farther away from the rest of the data than -20. (c) Dist 2 has a higher mean since all values in this distribution are higher than those in Dist 1, but both distribution have the same standard deviation since they are equally variable around their respective means. (d) Both distributions have the same mean since they're both centered at 300, but Dist 2 has a higher standard deviation since the observations are farther from the mean than in Dist 1.

1.49 (a) Q1 $\approx$ 5, median $\approx$ 15, Q3 $\approx$ 35 (b) Since the distribution is right skewed, we would expect the mean to be higher than the median.

1.51 (a) About 30. (b) Since the distribution is right skewed the mean is higher than the median. (c) Q1: between 15 and 20, Q3: between 35 and 40, IQR: about 20. (d) Values that are considered to be unusually low or high lie more than 1.5×IQR away from the quartiles. Upper fence: Q3 + 1.5 × IQR = 37.5 + 1.5 × 20 = 67.5; Lower fence: Q1 - 1.5 × IQR = 17.5 + 1.5 × 20 = −12.5; The lowest AQI recorded is not lower than 5 and the highest AQI recorded is not higher than 65, which are both within the fences. Therefore none of the days in this sample would be considered to have an unusually low or high AQI.

1.53 The histogram shows that the distribution is bimodal, which is not apparent in the box plot. The box plot makes it easy to identify more precise values of observations outside of the whiskers.

1.55 (a) The distribution of number of pets per household is likely right skewed as there is a natural boundary at 0 and only a few people have many pets. Therefore the center would be best described by the median, and variability would be best described by the IQR. (b) The distribution of number of distance to work is likely right skewed as there is a natural boundary at 0 and only a few people live a very long distance from work. Therefore the center would be best described by the median, and variability would be best described by the IQR. (c) The distribution of heights of males is likely symmetric. Therefore the center would be best described by the mean, and variability would be best described by the standard deviation.

1.57 No, we would expect this distribution to be right skewed. There are two reasons for this: (1) there is a natural boundary at 0 (it is not possible to watch less than 0 hours of TV), (2) the standard deviation of the distribution is very large compared to the mean.

1.59 The statement "50% of Facebook users have over 100 friends" means that the median number of friends is 100, which is lower than the mean number of friends (190), which suggests a right skewed distribution for the number of friends of Facebook users.

1.61 (a) The median is a much better measure of the typical amount earned by these 42 people. The mean is much higher than the income of 40 of the 42 people. This is because the mean is an arithmetic average and gets affected by the two extreme observations. The median does not get effected as much since it is robust to outliers. (b) The IQR is a much better measure of variability in the amounts earned by nearly all of the 42 people. The standard deviation gets affected greatly by the two high salaries, but the IQR is robust to these extreme observations.

1.63 (a) The distribution is unimodal and symmetric with a mean of about 25 minutes and a standard deviation of about 5 minutes. There does not appear to be any counties with unusually high or low mean travel times. Since the distribution is already unimodal and symmetric, a log transformation is not necessary. (b) Answers will vary. There are pockets of longer travel time around DC, Southeastern NY, Chicago, Minneapolis, Los Angeles, and many other big cities. There is also a large section of shorter average commute times that overlap with farmland in the Midwest. Many farmers' homes are adjacent to their farmland, so their commute would be brief, which may explain why the average commute time for these counties is relatively low.

1.65 (a) We see the order of the categories and the relative frequencies in the bar plot. (b) There are no features that are apparent in the pie chart but not in the bar plot. (c) We usually prefer to use a bar plot as we can also see the relative frequencies of the categories in this graph.

1.67 The vertical locations at which the ideological groups break into the Yes, No, and Not Sure categories differ, which indicates that likelihood of supporting the DREAM act varies by political ideology. This suggests that the two variables may be dependent.

1.69 (a) (i) False. Instead of comparing counts, we should compare percentages of people in each group who suffered cardiovascular problems. (ii) True. (iii) False. Association does not imply causation. We cannot infer a causal relationship based on an observational study. The difference from part (ii) is subtle. (iv) True.
(b) Proportion of all patients who had cardiovascular problems: $\frac{7,979}{227,571} \approx 0.035$
(c) The expected number of heart attacks in the rosiglitazone group, if having cardiovascular problems and treatment were independent, can be calculated as the number of patients in that group multiplied by the overall cardiovascular problem rate in the study: $67,593 * \frac{7,979}{227,571} \approx 2370$.
(d) (i) H_0: The treatment and cardiovascular problems are independent. They have no relationship, and the difference in incidence rates between the rosiglitazone and pioglitazone groups is due to chance. H_A: The treatment and cardiovascular problems are not independent. The difference in the incidence rates between the rosiglitazone and pioglitazone groups is not due to chance and rosiglitazone is associated with an increased risk of serious cardiovascular problems. (ii) A higher number of patients with cardiovascular problems than expected under the assumption of independence would provide support for the alternative hypothesis as this would suggest that rosiglitazone increases the risk of such problems. (iii) In the actual study, we observed 2,593 cardiovascular events in the rosiglitazone group. In the 1,000 simulations under the independence model, we observed somewhat less than 2,593 in every single simulation, which suggests that the actual results did not come from the independence model. That is, the variables do not appear to be independent, and we reject the independence model in favor of the alternative. The study's results provide convincing evidence that rosiglitazone is associated with an increased risk of cardiovascular problems.

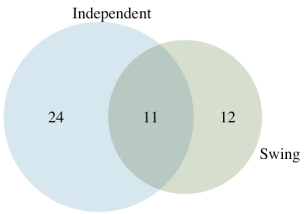
2 Probability

2.1 (a) False. These are independent trials. (b) False. There are red face cards. (c) True. A card cannot be both a face card and an ace.

2.3 (a) 10 tosses. Fewer tosses mean more variability in the sample fraction of heads, meaning there's a better chance of getting at least 60% heads. (b) 100 tosses. More flips means the observed proportion of heads would often be closer to the average, 0.50, and therefore also above 0.40. (c) 100 tosses. With more flips, the observed proportion of heads would often be closer to the average, 0.50. (d) 10 tosses. Fewer flips would increase variability in the fraction of tosses that are heads.

2.5 (a) $0.5^{10} = 0.00098$. (b) $0.5^{10} = 0.00098$. (c) $P(\text{at least one tails}) = 1 - P(\text{no tails}) = 1 - (0.5^{10}) \approx 1 - 0.001 = 0.999$.

2.7 (a) No, there are voters who are both independent and swing voters.
(b)



(c) Each Independent voter is either a swing voter or not. Since 35% of voters are Independents and 11% are both Independent and swing voters, the other 24% must not be swing voters. (d) 0.47. (e) 0.53. (f) P(Independent) × P(swing) = 0.35 × 0.23 = 0.08, which does not equal P(Independent and swing) = 0.11, so the events are dependent.

2.9 (a) If the class is not graded on a curve, they are independent. If graded on a curve, then neither independent nor disjoint – unless the instructor will only give one A, which is a situation we will ignore in parts (b) and (c). (b) They are probably not independent: if you study together, your study habits would be related, which suggests your course performances are also related. (c) No. See the answer to part (a) when the course is not graded on a curve. More generally: if two things are unrelated (independent), then one occurring does not preclude the other from occurring.

2.11 (a) $0.16 + 0.09 = 0.25$. (b) $0.17 + 0.09 = 0.26$. (c) Assuming that the education level of the husband and wife are independent: $0.25 \times 0.26 = 0.065$. You might also notice we actually made a second assumption: that the decision to get married is unrelated to education level. (d) The husband/wife independence assumption is probably not reasonable, because people often marry another person with a comparable level of education. We will leave it to you to think about whether the second assumption noted in part (c) is reasonable.

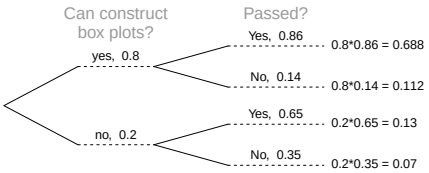
2.13 (a) Invalid. Sum is greater than 1. (b) Valid. Probabilities are between 0 and 1, and they sum to 1. In this class, every student gets a C. (c) Invalid. Sum is less than 1. (d) Invalid. There is a negative probability. (e) Valid. Probabilities are between 0 and 1, and they sum to 1. (f) Invalid. There is a negative probability.

2.15 (a) No, but we could if A and B are independent. (b-i) 0.21. (b-ii) 0.79. (b-iii) 0.3. (c) No, because $0.1 \neq 0.21$, where 0.21 was the value computed under independence from part (a). (d) 0.143.

2.17 (a) No, 0.18 of respondents fall into this combination. (b) $0.60 + 0.20 - 0.18 = 0.62$. (c) $0.18/0.20 = 0.9$. (d) $0.11/0.33 \approx 0.33$. (e) No, otherwise the answers to (c) and (d) would be the same. (f) $0.06/0.34 \approx 0.18$.

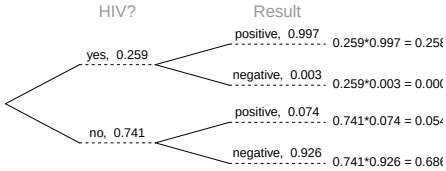
2.19 (a) No. There are 6 females who like Five Guys Burgers. (b) $162/248 = 0.65$. (c) $181/252 = 0.72$. (d) Under the assumption of a dating choices being independent of hamburger preference, which on the surface seems reasonable: $0.65 \times 0.72 = 0.468$. (e) $(252 + 6 - 1)/500 = 0.514$.

2.21 (a)

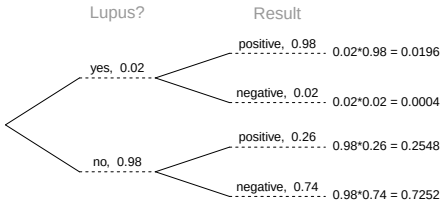


(b) 0.84

2.23 0.8247.



2.25 0.0714. Even when a patient tests positive for lupus, there is only a 7.14% chance that he actually has lupus. House may be right.



2.27 (a) 0.3. (b) 0.3. (c) 0.3. (d) $0.3 \times 0.3 = 0.09$. (e) Yes, the population that is being sampled from is identical in each draw.

2.29 (a) $2/9 \approx 0.22$. (b) $3/9 \approx 0.33$. (c) $\frac{3}{10} \times \frac{2}{9} \approx 0.067$. (d) No, e.g. in this exercise, removing one marble meaningfully changes the probability of what might be drawn next.

2.31 $P(^1\text{leggings}, {}^2\text{jeans}, {}^3\text{jeans}) = \frac{5}{24} \times \frac{7}{23} \times \frac{6}{23} = 0.0173$. However, the person with leggings could have come 2nd or 3rd, and these each have this same probability, so $3 \times 0.0173 = 0.519$.

2.33 (a) 13. (b) No, these 27 students are not a random sample from the university's student population. For example, it might be argued that the proportion of smokers among students who go to the gym at 9 am on a Saturday morning would be lower than the proportion of smokers in the university as a whole.

2.35 (a) E(X) = 3.59. SD(X) = 3.37. (b) E(X) = -1.41. SD(X) = 3.37. (c) No, the expected net profit is negative, so on average you expect to lose money.

2.37 5% increase in value.

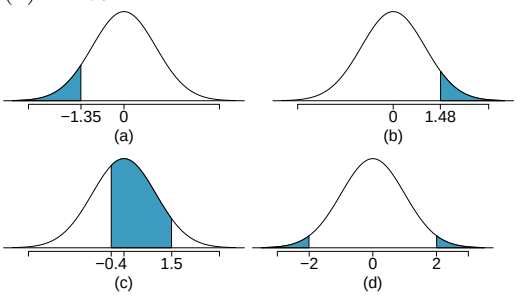
2.39 E = -0.0526. SD = 0.9986.

2.41 (a) E = \$3.90. SD = \$0.34. (b) E = \$27.30. SD = \$0.89.

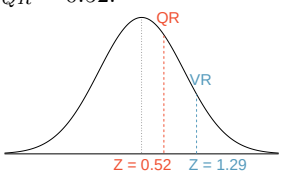
2.43 Approximate answers are OK. (a) $(29 + 32)/144 = 0.42$. (b) $21/144 = 0.15$. (c) $(26 + 12 + 15)/144 = 0.37$.

3 Distributions of random variables

3.1 (a) 8.85%. (b) 6.94%. (c) 58.86%. (d) 4.56%.



3.3 (a) Verbal: $N(\mu = 151, \sigma = 7)$, Quant: $N(\mu = 153, \sigma = 7.67)$. (b) $Z_{VR} = 1.29$, $Z_{QR} = 0.52$.



(c) She scored 1.29 standard deviations above the mean on the Verbal Reasoning section and 0.52 standard deviations above the mean on the Quantitative Reasoning section. (d) She did better on the Verbal Reasoning section since her Z-score on that section was higher. (e) $Perc_{VR} = 0.9007 \approx 90\%$, $Perc_{QR} = 0.6990 \approx 70\%$. (f) $100\% - 90\% = 10\%$ did better than her on VR, and $100\% - 70\% = 30\%$ did better than her on QR. (g) We cannot compare the raw scores since they are on different scales. Comparing her percentile scores is more appropriate when comparing her performance to others. (h) Answer to part (b) would not change as Z-scores can be calculated for distributions that are not normal. However, we could not answer parts (d)-(f) since we cannot use the normal probability table to calculate probabilities and percentiles without a normal model.

3.5 (a) $Z = 0.84$, which corresponds to approximately 160 on QR. (b) $Z = -0.52$, which corresponds to approximately 147 on VR.

3.7 (a) $Z = 1.2 \rightarrow 0.1151$.
(b) $Z = -1.28 \rightarrow 70.6°$F or colder.

3.9 (a) $N(25, 2.78)$. (b) $Z = 1.08 \rightarrow 0.1401$. (c) The answers are very close because only the units were changed. (The only reason why they differ at all is because 28°C is 82.4°F, not precisely 83°F.) (d) Since $IQR = Q3 - Q1$, we first need to find $Q3$ and $Q1$ and take the difference between the two. Remember that $Q3$ is the 75^{th} and $Q1$ is the 25^{th} percentile of a distribution. Q1 = 23.13, Q3 = 26.86, IQR = 26. 86 - 23.13 = 3.73.

3.11 (a) $Z = 0.67$. (b) $\mu = \$1650$, $x = \$1800$. (c) $0.67 = \frac{1800-1650}{\sigma} \rightarrow \sigma = \$223.88$.

3.13 $Z = 1.56 \rightarrow 0.0594$, i.e. 6%.

3.15 (a) $Z = 0.73 \rightarrow 0.2327$. (b) If you are bidding on only one auction and set a low maximum bid price, someone will probably outbid you. If you set a high maximum bid price, you may win the auction but pay more than is necessary. If bidding on more than one auction, and you set your maximum bid price very low, you probably won't win any of the auctions. However, if the maximum bid price is even modestly high, you are likely to win multiple auctions. (c) An answer roughly equal to the 10th percentile would be reasonable. Regrettably, no percentile cutoff point guarantees beyond any possible event that you win at least one auction. However, you may pick a higher percentile if you want to be more sure of winning an auction. (d) Answers will vary a little but should correspond to the answer in part (c). We use the 10^{th} percentile: $Z = -1.28 \rightarrow \$69.80$.

3.17 (a) 70% of the data are within 1 standard deviation of the mean, 95% are within 2 and 100% are within 3 standard deviations of the mean. Therefore, we can say that the data approximately follow the 68-95-99.7% Rule. (b) The distribution is unimodal and symmetric. The superimposed normal curve seems to approximate the distribution pretty well. The points on the normal probability plot also seem to follow a straight line. There is one possible outlier on the lower end that is apparent in both graphs, but it is not too extreme. We can say that the distribution is nearly normal.

3.19 (a) No. The cards are not independent. For example, if the first card is an ace of clubs, that implies the second card cannot be an ace of clubs. Additionally, there are many possible categories, which would need to be simplified. (b) No. There are six events under consideration. The Bernoulli distribution allows for only two events or categories. Note that rolling a die could be a Bernoulli trial if we simply to two events, e.g. rolling a 6 and not rolling a 6, though specifying such details would be necessary.

3.21 (a) $(1 - 0.471)^2 \times 0.471 = 0.1318$. (b) $0.471^3 = 0.1045$. (c) $\mu = 1/0.471 = 2.12$, $\sigma = \sqrt{2.38} = 1.54$. (d) $\mu = 1/0.30 = 3.33$, $\sigma = 2.79$. (e) When p is smaller, the event is rarer, meaning the expected number of trials before a success and the standard deviation of the waiting time are higher.

3.23 (a) $0.875^2 \times 0.125 = 0.096$.
(b) $\mu = 8$, $\sigma = 7.48$.

3.25 (a) Binomial conditions are met: (1) Independent trials: In a random sample, whether or not one 18-20 year old has consumed alcohol does not depend on whether or not another one has. (2) Fixed number of trials: $n = 10$. (3) Only two outcomes at each trial: Consumed or did not consume alcohol. (4) Probability of a success is the same for each trial: $p = 0.697$. (b) 0.203. (c) 0.203. (d) 0.167. (e) 0.997.

3.27 (a) $\mu = 34.85$, $\sigma = 3.25$ (b) $Z = \frac{45-34.85}{3.25} = 3.12$. 45 is more than 3 standard deviations away from the mean, we can assume that it is an unusual observation. Therefore yes, we would be surprised. (c) Using the normal approximation, 0.0009. With 0.5 correction, 0.0015.

3.29 Want to find the probability that there will be 1,786 or more enrollees. Using the normal approximation: 0.0582. With a 0.5 correction: 0.0559.

3.31 (a) $1 - 0.75^3 = 0.5781$. (b) 0.1406. (c) 0.4219. (d) $1 - 0.25^3 = 0.9844$.

3.33 (a) Geometric distribution: 0.109. (b) Binomial: 0.219. (c) Binomial: 0.137. (d) $1 - 0.875^6 = 0.551$. (e) Geometric: 0.084. (f) Using a binomial distribution with $n = 6$ and $p = 0.75$, we see that $\mu = 4.5$, $\sigma = 1.06$, and $Z =?2.36$. Since this is not within 2 SD, it may be considered unusual.

3.35 0 wins (-$3): 0.1458. 1 win (-$1): 0.3936. 2 wins (+$1): 0.3543. 3 wins (+$3): 0.1063.

3.37 (a) $\overset{Anna}{1/5} \times \overset{Ben}{1/4} \times \overset{Carl}{1/3} \times \overset{Damian}{1/2} \times \overset{Eddy}{1/1} = 1/5! = 1/120$. (b) Since the probabilities must add to 1, there must be $5! = 120$ possible orderings. (c) $8! = 40{,}320$.

3.39 (a) 0.0804. (b) 0.0322. (c) 0.0193.

3.41 (a) Negative binomial with $n = 4$ and $p = 0.55$, where a success is defined here as a female student. The negative binomial setting is appropriate since the last trial is fixed but the order of the first 3 trials is unknown. (b) 0.1838. (c) $\binom{3}{1} = 3$. (d) In the binomial model there are no restrictions on the outcome of the last trial. In the negative binomial model the last trial is fixed. Therefore we are interested in the number of ways of orderings of the other $k-1$ successes in the first $n-1$ trials.

3.43 (a) Poisson with $\lambda = 75$. (b) $\mu = \lambda = 75$, $\sigma = \sqrt{\lambda} = 8.66$. (c) $Z = -1.73$. Since 60 is within 2 standard deviations of the mean, it would not generally be considered unusual. Note that we often use this rule of thumb even when the normal model does not apply. (d) Using Poisson with $\lambda = 75$: 0.0402.

4 Foundations for inference

4.1 (a) Mean. Each student reports a numerical value: a number of hours. (b) Mean. Each student reports a number, which is a percentage, and we can average over these percentages. (c) Proportion. Each student reports Yes or No, so this is a categorical variable and we use a proportion. (d) Mean. Each student reports a number, which is a percentage like in part (b). (e) Proportion. Each student reports whether or not s/he expects to get a job, so this is a categorical variable and we use a proportion.

4.3 (a) Mean: 13.65. Median: 14. (b) SD: 1.91. IQR: $15 - 13 = 2$. (c) $Z_{16} = 1.23$, which is not unusual since it is within 2 SD of the mean. $Z_{18} = 2.23$, which is generally considered unusual. (d) No. Point estimates that are based on samples only approximate the population parameter, and they vary from one sample to another. (e) We use the SE, which is $1.91/\sqrt{100} = 0.191$ for this sample's mean.

4.5 (a) We are building a distribution of sample statistics, in this case the sample mean. Such a distribution is called a sampling distribution. (b) Because we are dealing with the distribution of sample means, we need to check to see if the Central Limit Theorem applies. Our sample size is greater than 30, and we are told that random sampling is employed. With these conditions met, we expect that the distribution of the sample mean will be nearly normal and therefore symmetric. (c) Because we are dealing with a sampling distribution, we measure its variability with the standard error. $SE = 18.2/\sqrt{45} = 2.713$. (d) The sample means will be more variable with the smaller sample size.

4.7 Recall that the general formula is

$$\text{point estimate} \pm Z^\star \times SE$$

First, identify the three different values. The point estimate is 45%, $Z^\star = 1.96$ for a 95% confidence level, and $SE = 1.2\%$. Then, plug the values into the formula:

$$45\% \pm 1.96 \times 1.2\% \quad \rightarrow \quad (42.6\%, 47.4\%)$$

We are 95% confident that the proportion of US adults who live with one or more chronic conditions is between 42.6% and 47.4%.

4.9 (a) False. Confidence intervals provide a range of plausible values, and sometimes the truth is missed. A 95% confidence interval "misses" about 5% of the time. (b) True. Notice that the description focuses on the true population value. (c) True. If we examine the 95% confidence interval computed in Exercise **??**, we can see that 50% is not included in this interval. This means that in a hypothesis test, we would reject the null hypothesis that the proportion is 0.5. (d) False. The standard error describes the uncertainty in the overall estimate from natural fluctuations due to randomness, not the uncertainty corresponding to individuals' responses.

4.11 (a) We are 95% confident that Americans spend an average of 1.38 to 1.92 hours per day relaxing or pursuing activities they enjoy. (b) Their confidence level must be higher as the width of the confidence interval increases as the

confidence level increases. (c) The new margin of error will be smaller since as the sample size increases the standard error decreases, which will decrease the margin of error.

4.13 (a) False. Provided the data distribution is not very strongly skewed ($n = 64$ in this sample, so we can be slightly lenient with the skew), the sample mean will be nearly normal, allowing for the method normal approximation described. (b) False. Inference is made on the population parameter, not the point estimate. The point estimate is always in the confidence interval. (c) True. (d) False. The confidence interval is not about a sample mean. (e) False. To be more confident that we capture the parameter, we need a wider interval. Think about needing a bigger net to be more sure of catching a fish in a murky lake. (f) True. Optional explanation: This is true since the normal model was used to model the sample mean. The margin of error is half the width of the interval, and the sample mean is the midpoint of the interval. (g) False. In the calculation of the standard error, we divide the standard deviation by the square root of the sample size. To cut the SE (or margin of error) in half, we would need to sample $2^2 = 4$ times the number of people in the initial sample.

4.15 Independence: sample from $< 10\%$ of population, and it is a random sample. We can assume that the students in this sample are independent of each other with respect to number of exclusive relationships they have been in. Notice that there are no students who have had no exclusive relationships in the sample, which suggests some student responses are likely missing (perhaps only positive values were reported). The sample size is at least 30. The skew is strong, but the sample is very large so this is not a concern. 90% CI: (2.97, 3.43). We are 90% confident that undergraduate students have been in 2.97 to 3.43 exclusive relationships, on average.

4.17 (a) $H_0 : \mu = 8$ (On average, New Yorkers sleep 8 hours a night.)
$H_A : \mu < 8$ (On average, New Yorkers sleep less than 8 hours a night.)
(b) $H_0 : \mu = 15$ (The average amount of company time each employee spends not working is 15 minutes for March Madness.)
$H_A : \mu > 15$ (The average amount of company time each employee spends not working is greater than 15 minutes for March Madness.)

4.19 The hypotheses should be about the population mean (μ), not the sample mean. The null hypothesis should have an equal sign and the alternative hypothesis should be about the null hypothesized value, not the observed sample mean. Correction:

$$H_0 : \mu = 10 \; hours$$
$$H_A : \mu > 10 \; hours$$

The one-sided test indicates that we are only interested in showing that 10 is an underestimate. Here the interest is in only one direction, so a one-sided test seems most appropriate. If we would also be interested if the data showed strong evidence that 10 was an overestimate, then the test should be two-sided.

4.21 (a) This claim does is not supported since 3 hours (180 minutes) is not in the interval. (b) 2.2 hours (132 minutes) is in the 95% confidence interval, so we do not have evidence to say she is wrong. However, it would be more appropriate to use the point estimate of the sample. (c) A 99% confidence interval will be wider than a 95% confidence interval, meaning it would enclose this smaller interval. This means 132 minutes would be in the wider interval, and we would not reject her claim based on a 99% confidence level.

4.23 $H_0 : \mu = 130$. $H_A : \mu \neq 130$. $Z = 1.39 \rightarrow$ p-value $= 0.1646$, which is larger than $\alpha = 0.05$. The data do not provide convincing evidence that the true average calorie content in bags of potato chips is different than 130 calories.

4.25 (a) Independence: The sample is random and 64 patients would almost certainly make up less than 10% of the ER residents. The sample size is at least 30. No information is provided about the skew. In practice, we would ask to see the data to check this condition, but here we will make the assumption that the skew is not very strong. (b) $H_0 : \mu = 127$. $H_A : \mu \neq 127$. $Z = 2.15 \rightarrow$ p-value $= 0.0316$. Since the p-value is less than $\alpha = 0.05$, we reject H_0. The data provide convincing evidence that the average ER wait time has increased over the last year. (c) Yes, it would change. The p-value is greater than 0.01, meaning we would fail to reject H_0 at $\alpha = 0.01$.

4.27 $Z = 1.65 = \frac{\bar{x} - 30}{10/\sqrt{70}} \rightarrow \bar{x} = 31.97$.

4.29 (a) H_0: Anti-depressants do not help symptoms of Fibromyalgia. H_A: Anti- depressants do treat symptoms of Fibromyalgia. Remark: Diana might also have taken special note if her symptoms got much worse, so a more scientific approach would have been to use a two-sided test. If you proposed a two-sided approach, your answers in (b) and (c) will be different. (b) Concluding that anti-depressants work for the treatment of Fibromyalgia symptoms when they actually do not. (c) Concluding that anti-depressants do not work for the treatment of Fibromyalgia symptoms when they actually do.

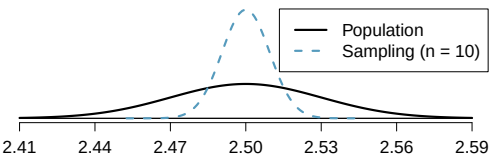
4.31 (a) Scenario I is higher. Recall that a sample mean based on less data tends to be less accurate and have larger standard errors. (b) Scenario I is higher. The higher the confidence level, the higher the corresponding margin of error. (c) They are equal. The sample size does not affect the calculation of the p- value for a given Z-score. (d) Scenario I is higher. If the null hypothesis is harder to reject (lower α), then we are more likely to make a Type 2 Error when the alternative hypothesis is true.

4.33 (a) The distribution is unimodal and strongly right skewed with a median between 5 and 10 years old. Ages range from 0 to slightly over 50 years old, and the middle 50% of the distribution is roughly between 5 and 15 years old. There are potential outliers on the higher end. (b) When the sample size is small, the sampling distribution is right skewed, just like the population distribution. As the sample size increases, the sampling distribution gets more unimodal, symmetric, and approaches normality. The variability also decreases. This is consistent with the Central Limit Theorem. (c) n = 5: $\mu_{\bar{x}} = 10.44$, $\sigma_{\bar{x}} = 4.11$; n = 30: $\mu_{\bar{x}} = 10.44$, $\sigma_{\bar{x}} = 1.68$; n = 100: $\mu_{\bar{x}} = 10.44$, $\sigma_{\bar{x}} = 0.92$. The centers of the sampling distributions shown in part (b) appear to be around 10. It is difficult to estimate the standard deviation for the sampling distribution when $n = 5$ from the histogram (since the distribution is somewhat skewed). If 1.68 is a plausible estimate for the standard deviation of the sampling distribution when $n = 30$, then using the 68-95-99.7% Rule, we would expect the values to range roughly between $10.44 \pm 3 * 1.68 = (5.4, 15.48)$, which seems to be the case. Similarly, when $n = 100$, we would expect the values to range roughly between $10.44 \pm 3 * 0.92 = (7.68, 13.2)$, which also seems to be the case.

4.35 (a) Right skewed. There is a long tail on the higher end of the distribution but a much shorter tail on the lower end. (b) Less than, as the median would be less than the mean in a right skewed distribution. (c) We should not. (d) Even though the population distribution is not normal, the conditions for inference are reasonably satisfied, with the possible exception of skew. If the skew isn't very strong (we should ask to see the data), then we can use the Central Limit Theorem to estimate this probability. For now, we'll assume the skew isn't very strong, though the description suggests it is at least moderate to strong. Use $N(1.3, SD_{\bar{x}} = 0.3/\sqrt{60})$: $Z = 2.58 \rightarrow 0.0049$. (e) It would decrease it by a factor of $1/\sqrt{2}$.

4.37 The centers are the same in each plot, and each data set is from a nearly normal distribution, though the histograms may not look very normal since each represents only 100 data points. The only way to tell which plot corresponds to which scenario is to examine the variability of each distribution. Plot B is the most variable, followed by Plot A, then Plot C. This means Plot B will correspond to the original data, Plot A to the sample means with size 5, and Plot C to the sample means with size 25.

4.39 (a) $Z = -3.33 \rightarrow 0.0004$. (b) The population SD is known and the data are nearly normal, so the sample mean will be nearly normal with distribution $N(\mu, \sigma/\sqrt{n})$, i.e. $N(2.5, 0.0095)$. (c) $Z = -10.54 \rightarrow \approx 0$. (d) See below:



(e) We could not estimate (a) without a nearly normal population distribution. We also could not estimate (c) since the sample size is not sufficient to yield a nearly normal sampling distribution if the population distribution is not nearly normal.

4.41 (a) We cannot use the normal model for this calculation, but we can use the histogram. About 500 songs are shown to be longer than 5 minutes, so the probability is about $500/3000 = 0.167$. (b) Two different answers are reasonable. $^{Option\ 1}$Since the population distribution is only slightly skewed to the right, even a small sample size will yield a nearly normal sampling distribution. We also know that the songs are sampled randomly and the sample size is less than 10% of the population, so the length of one song in the sample is independent of another. We are looking for the probability that the total length of 15 songs is more than 60 minutes, which means that the average song should last at least $60/15 = 4$ minutes. Using $SD_{\bar{x}} = 1.63/\sqrt{15}$, $Z = 1.31 \rightarrow 0.0951$. $^{Option\ 2}$Since the population distribution is not normal, a small sample size may not be sufficient to yield a nearly normal sampling distribution. Therefore, we cannot estimate the probability using the tools we have learned so far. (c) We can now be confident that the conditions are satisfied. $Z = 0.92 \rightarrow 0.1788$.

4.43 (a) $H_0 : \mu_{2009} = \mu_{2004}$. $H_A : \mu_{2009} \neq \mu_{2004}$. (b) $\bar{x}_{2009} - \bar{x}_{2004} = -3.6$ spam emails per day. (c) The null hypothesis was not rejected, and the data do not provide convincing evidence that the true average number of spam emails per day in years 2004 and 2009 are different. The observed difference is about what we might expect from sampling variability alone. (d) Yes, since the hypothesis of no difference was not rejected in part (c).

4.45 (a) $H_0 : p_{2009} = p_{2004}$. $H_A : p_{2009} \neq p_{2004}$. (b) -7%. (c) The null hypothesis was rejected. The data provide strong evidence that the true proportion of those who once a month or less frequently delete their spam email was higher in 2004 than in 2009. The difference is so large that it cannot easily be explained as being due to chance. (d) No, since the null difference, 0, was rejected in part (c).

4.47 True. If the sample size is large, then the standard error will be small, meaning even relatively small differences between the null value and point estimate can be statistically significant.

5 Inference for numerical data

5.1 (a) $df = 6 - 1 = 5$, $t^\star_5 = 2.02$ (column with two tails of 0.10, row with $df = 5$). (b) $df = 21 - 1 = 20$, $t^\star_{20} = 2.53$ (column with two tails of 0.02, row with $df = 20$). (c) $df = 28$, $t^\star_{28} = 2.05$. (d) $df = 11$, $t^\star_{11} = 3.11$.

5.3 (a) between 0.025 and 0.05 (b) less than 0.005 (c) greater than 0.2 (d) between 0.01 and 0.025

5.5 The mean is the midpoint: $\bar{x} = 20$. Identify the margin of error: $ME = 1.015$, then use $t^\star_{35} = 2.03$ and $SE = s/\sqrt{n}$ in the formula for margin of error to identify $s = 3$.

5.7 (a) H_0: $\mu = 8$ (New Yorkers sleep 8 hrs per night on average.) H_A: $\mu < 8$ (New Yorkers sleep less than 8 hrs per night on average.) (b) Independence: The sample is random and from less than 10% of New Yorkers. The sample is small, so we will use a t-distribution. For this size sample, slight skew is acceptable, and the min/max suggest there is not much skew in the data. $T = -1.75$. $df = 25 - 1 = 24$. (c) $0.025 <$ p-value < 0.05. If in fact the true population mean of the amount New Yorkers sleep per night was 8 hours, the probability of getting a random sample of 25 New Yorkers where the average amount of sleep is 7.73 hrs per night or less is between 0.025 and 0.05. (d) Since p-value < 0.05, reject H_0. The data provide strong evidence that New Yorkers sleep less than 8 hours per night on average. (e) No, as we rejected H_0.

5.9 $t^\star_{19}$ is 1.73 for a one-tail. We want the lower tail, so set -1.73 equal to the T-score, then solve for $\bar{x}$: 56.91.

5.11 (a) We will conduct a 1-sample t-test. H_0: $\mu = 5$. H_A: $\mu < 5$. We'll use $\alpha = 0.05$. This is a random sample, so the observations are independent. To proceed, we assume the distribution of years of piano lessons is approximately normal. $SE = 2.2/\sqrt{20} = 0.4919$. The test statistic is $T = (4.6 - 5)/SE = -0.81$. $df = 20 - 1 = 19$. The one-tail p-value is about 0.21, which is bigger than $\alpha = 0.05$, so we do not reject H_0. That is, we do not have sufficiently strong evidence to reject Georgianna's claim.
(b) Using $SE = 0.4919$ and $t^\star_{df=19} = 2.093$, the confidence interval is (3.57, 5.63). We are 95% confident that the average number of years a child takes piano lessons in this city is 3.57 to 5.63 years.
(c) They agree, since we did not reject the null hypothesis and the null value of 5 was in the t-interval.

5.13 If the sample is large, then the margin of error will be about $1.96 \times 100/\sqrt{n}$. We want this value to be less than 10, which leads to $n \geq 384.16$, meaning we need a sample size of at least 385 (round up for sample size calculations!).

5.15 (a) Two-sided, we are evaluating a difference, not in a particular direction. (b) Paired, data are recorded in the same cities at two different time points. The temperature in a city at one point is not independent of the temperature in the same city at another time point. (c) t-test, sample is small and population standard deviation is unknown.

5.17 (a) Since it's the same students at the beginning and the end of the semester, there is a pairing between the datasets, for a given student their beginning and end of semester grades are dependent. (b) Since the subjects were sampled randomly, each observation in the men's group does not have a special correspondence with exactly one observation in the other (women's) group. (c) Since it's the same subjects at the beginning and the end of the study, there is a pairing between the datasets, for a subject student their beginning and end of semester artery thickness are dependent. (d) Since it's the same subjects at the beginning and the end of the study, there is a pairing between the datasets, for a subject student their beginning and end of semester weights are dependent.

5.19 (a) For each observation in one data set, there is exactly one specially-corresponding observation in the other data set for the same geographic location. The data are paired. (b) H_0 : $\mu_{diff} = 0$ (There is no difference in average daily high temperature between January 1, 1968 and January 1, 2008 in the continental US.) H_A : $\mu_{diff} > 0$ (Average daily high temperature in January 1, 1968 was lower than average daily high temperature in January, 2008 in the continental US.) If you chose a two-sided test, that would also be acceptable. If this is the case, note that your p-value will be a little bigger than what is reported here in part (d). (c) Independence: locations are random and represent less than 10% of all possible locations in the US. The sample size is at least 30. We are not given the distribution to check the skew. In practice, we would ask to see the data to check this condition, but here we will move forward under the assumption that it is not strongly skewed. (d) $Z = 1.60 \rightarrow$ p-value $= 0.0548$. (e) Since the p-value $> \alpha$ (since not given use 0.05), fail to reject H_0. The data do not provide strong evidence of temperature warming in the continental US. However it should be noted that the p-value is very close to 0.05. (f) Type 2 Error, since we may have incorrectly failed to reject H_0. There may be an increase, but we were unable to detect it. (g) Yes, since we failed to reject H_0, which had a null value of 0.

5.21 (a) (-0.03, 2.23). (b) We are 90% confident that the average daily high on January 1, 2008 in the continental US was 0.03 degrees lower to 2.23 degrees higher than the average daily high on January 1, 1968. (c) No, since 0 is included in the interval.

5.23 (a) Each of the 36 mothers is related to exactly one of the 36 fathers (and vice-versa), so there is a special correspondence between the mothers and fathers. (b) $H_0 : \mu_{diff} = 0$. $H_A : \mu_{diff} \neq 0$. Independence: random sample from less than 10% of population. Sample size of at least 30. The skew of the differences is, at worst, slight. $Z = 2.72 \rightarrow$ p-value $= 0.0066$. Since p-value < 0.05, reject H_0. The data provide strong evidence that the average IQ scores of mothers and fathers of gifted children are different, and the data indicate that mothers' scores are higher than fathers' scores for the parents of gifted children.

5.25 No, he should not move forward with the test since the distributions of total personal income are very strongly skewed. When sample sizes are large, we can be a bit lenient with skew. However, such strong skew observed in this exercise would require somewhat large sample sizes, somewhat higher than 30.

5.27 (a) These data are paired. For example, the Friday the 13th in say, September 1991, would probably be more similar to the Friday the 6th in September 1991 than to Friday the 6th in another month or year. (b) Let $\mu_{diff} = \mu_{sixth} - \mu_{thirteenth}$. $H_0 : \mu_{diff} = 0$. $H_A : \mu_{diff} \neq 0$. (c) Independence: The months selected are not random. However, if we think these dates are roughly equivalent to a simple random sample of all such Friday 6th/13th date pairs, then independence is reasonable. To proceed, we must make this strong assumption, though we should note this assumption in any reported results. With fewer than 10 observations, we would need to use the t-distribution to model the sample mean. The normal probability plot of the differences shows an approximately straight line. There isn't a clear reason why this distribution would be skewed, and since the normal quantile plot looks reasonable, we can mark this condition as reasonably satisfied. (d) $T = 4.94$ for $df = 10 - 1 = 9 \rightarrow$ p-value < 0.01. (e) Since p-value < 0.05, reject H_0. The data provide strong evidence that the average number of cars at the intersection is higher on Friday the 6^{th} than on Friday the 13^{th}. (We might believe this intersection is representative of all roads, i.e. there is higher traffic on Friday the 6^{th} relative to Friday the 13^{th}. However, we should be cautious of the required assumption for such a generalization.) (f) If the average number of cars passing the intersection actually was the same on Friday the 6^{th} and 13^{th}, then the probability that we would observe a test statistic so far from zero is less than 0.01. (g) We might have made a Type 1 Error, i.e. incorrectly rejected the null hypothesis.

5.29 (a) $H_0 : \mu_{diff} = 0$. $H_A : \mu_{diff} \neq 0$. $T = -2.71$. $df = 5$. $0.02 <$ p-value < 0.05. Since p-value < 0.05, reject H_0. The data provide strong evidence that the average number of traffic accident related emergency room admissions are different between Friday the 6^{th} and Friday the 13^{th}. Furthermore, the data indicate that the direction of that difference is that accidents are lower on Friday the 6^{th} relative to Friday the 13^{th}. (b) (-6.49, -0.17). (c) This is an observational study, not an experiment, so we cannot so easily infer a causal intervention implied by this statement. It is true that there is a difference. However, for example, this does not mean that a responsible adult going out on Friday the 13^{th} has a higher chance of harm than on any other night.

5.31 (a) Chicken fed linseed weighed an average of 218.75 grams while those fed horsebean weighed an average of 160.20 grams. Both distributions are relatively symmetric with no apparent outliers. There is more variability in the weights of chicken fed linseed. (b) $H_0 : \mu_{ls} = \mu_{hb}$. $H_A : \mu_{ls} \neq \mu_{hb}$. We leave the conditions to you to consider. $T = 3.02$, $df = min(11, 9) = 9$ $\rightarrow 0.01 <$ p-value < 0.02. Since p-value < 0.05, reject H_0. The data provide strong evidence that there is a significant difference between the average weights of chickens that were fed linseed and horsebean. (c) Type 1 Error, since we rejected H_0. (d) Yes, since p-value > 0.01, we would have failed to reject H_0.

5.33 $H_0 : \mu_C = \mu_S$. $H_A : \mu_C \neq \mu_S$. $T = 3.27$, $df = 11 \rightarrow$ p-value < 0.01. Since p-value < 0.05, reject H_0. The data provide strong evidence that the average weight of chickens that were fed casein is different than the average weight of chickens that were fed soybean (with weights from casein being higher). Since this is a randomized experiment, the observed difference can be attributed to the diet.

5.35 $H_0 : \mu_T = \mu_C$. $H_A : \mu_T \neq \mu_C$. $T = 2.24$, $df = 21 \rightarrow 0.02 <$ p-value < 0.05. Since p-value < 0.05, reject H_0. The data provide strong evidence that the average food consumption by the patients in the treatment and control groups are different. Furthermore, the data indicate patients in the distracted eating (treatment) group consume more food than patients in the control group.

5.37 Let $\mu_{diff} = \mu_{pre} - \mu_{post}$. $H_0 : \mu_{diff} = 0$: Treatment has no effect. $H_A : \mu_{diff} > 0$: Treatment is effective in reducing P.D.T. scores, the average pre-treatment score is higher than the average post-treatment score. Note that the reported values are pre minus post, so we are looking for a positive difference, which would correspond to a reduction in the P.D.T. score. Conditions are checked as follows. Independence: The subjects are randomly assigned to treatments, so the patients in each group are independent. All three sample sizes are smaller than 30, so we use t-tests. Distributions of differences are somewhat skewed. The sample sizes are small, so we cannot reliably relax this assumption. (We will proceed, but we would not report the results of this specific analysis, at least for treatment group 1.) For all three groups: $df = 13$. $T_1 = 1.89$ ($0.025 <$ p-value < 0.05), $T_2 = 1.35$ (p-value $= 0.10$), $T_3 = -1.40$ (p-value > 0.10). The only significant test reduction is found in Treatment 1, however, we had earlier noted that this result might not be reliable due to the skew in the distribution. Note that the calculation of the p-value for Treatment 3 was unnecessary: the sample mean indicated a increase in P.D.T. scores under this treatment (as opposed to a decrease, which was the result of interest). That is, we could tell without formally completing the hypothesis test that the p-value would be large for this treatment group.

5.39 Difference we care about: 40. Single tail of 90%: $1.28 \times SE$. Rejection region bounds: $\pm 1.96 \times SE$ (if 5% significance level). Setting $3.24 \times SE = 40$, subbing in $SE = \sqrt{\frac{94^2}{n} + \frac{94^2}{n}}$, and solving for the sample size n gives 116 plots of land for each fertilizer.

5.41 Alternative.

5.43 H_0: $\mu_1 = \mu_2 = \cdots = \mu_6$. H_A: The average weight varies across some (or all) groups. Independence: Chicks are randomly assigned to feed types (presumably kept separate from one another), therefore independence of observations is reasonable. Approx. normal: the distributions of weights within each feed type appear to be fairly symmetric. Constant variance: Based on the side-by-side box plots, the constant variance assumption appears to be reasonable. There are differences in the actual computed standard deviations, but these might be due to chance as these are quite small samples. $F_{5,65} = 15.36$ and the p-value is approximately 0. With such a small p-value, we reject H_0. The data provide convincing evidence that the average weight of chicks varies across some (or all) feed supplement groups.

5.45 (a) H_0: The population mean of MET for each group is equal to the others. H_A: At least one pair of means is different. (b) Independence: We don't have any information on how the data were collected, so we cannot assess independence. To proceed, we must assume the subjects in each group are independent. In practice, we would inquire for more details. Approx. normal: The data are bound below by zero and the standard deviations are larger than the means, indicating very strong skew. However, since the sample sizes are extremely large, even extreme skew is acceptable. Constant variance: This condition is sufficiently met, as the standard deviations are reasonably consistent across groups. (c) See below, with the last column omitted:

	Df	Sum Sq	Mean Sq	F value
coffee	4	10508	2627	5.2
Residuals	50734	25564819	504	
Total	50738	25575327		

(d) Since p-value is very small, reject H_0. The data provide convincing evidence that the average MET differs between at least one pair of groups.

5.47 (a) H_0: Average GPA is the same for all majors. H_A: At least one pair of means are different. (b) Since p-value > 0.05, fail to reject H_0. The data do not provide convincing evidence of a difference between the average GPAs across three groups of majors. (c) The total degrees of freedom is $195 + 2 = 197$, so the sample size is $197 + 1 = 198$.

5.49 (a) False. As the number of groups increases, so does the number of comparisons and hence the modified significance level decreases. (b) True. (c) True. (d) False. We need observations to be independent regardless of sample size.

5.51 (a) H_0: Average score difference is the same for all treatments. H_A: At least one pair of means are different. (b) We should check conditions. If we look back to the earlier exercise, we will see that the patients were randomized, so independence is satisfied. There are some minor concerns about skew, especially with the third group, though this may be acceptable. The standard deviations across the groups are reasonably similar. Since the p-value is less than 0.05, reject H_0. The data provide convincing evidence of a difference between the average reduction in score among treatments. (c) We determined that at least two means are different in part (b), so we now conduct $K = 3 \times 2/2 = 3$ pairwise t-tests that each use $\alpha = 0.05/3 = 0.0167$ for a significance level. Use the following hypotheses for each pairwise test. H_0: The two means are equal. H_A: The two means are different. The sample sizes are equal and we use the pooled SD, so we can compute $SE = 3.7$ with the pooled $df = 39$. The p-value only for Trmt 1 vs. Trmt 3 may be statistically significant: $0.01 <$ p-value < 0.02. Since we cannot tell, we should use a computer to get the p-value, 0.015, which is statistically significant for the adjusted significance level. That is, we have identified Treatment 1 and Treatment 3 as having different effects. Checking the other two comparisons, the differences are not statistically significant.

6 Inference for categorical data

6.1 (a) False. Doesn't satisfy success-failure condition. (b) True. The success-failure condition is not satisfied. In most samples we would expect $\hat{p}$ to be close to 0.08, the true population proportion. While $\hat{p}$ can be much above 0.08, it is bound below by 0, suggesting it would take on a right skewed shape. Plotting the sampling distribution would confirm this suspicion. (c) False. $SE_{\hat{p}} = 0.0243$, and $\hat{p} = 0.12$ is only $\frac{0.12-0.08}{0.0243} = 1.65$ SEs away from the mean, which would not be considered unusual. (d) True. $\hat{p} = 0.12$ is 2.32 standard errors away from the mean, which is often considered unusual. (e) False. Decreases the SE by a factor of $1/\sqrt{2}$.

6.3 (a) True. See the reasoning of 6.1(b). (b) True. We take the square root of the sample size in the SE formula. (c) True. The independence and success-failure conditions are satisfied. (d) True. The independence and success-failure conditions are satisfied.

6.5 (a) False. A confidence interval is constructed to estimate the population proportion, not the sample proportion. (b) True. 95% CI: 70% $\pm$ 8%. (c) True. By the definition of the confidence level. (d) True. Quadrupling the sample size decreases the SE and ME by a factor of $1/\sqrt{4}$. (e) True. The 95% CI is entirely above 50%.

6.7 With a random sample from $<$ 10% of the population, independence is satisfied. The success-failure condition is also satisfied. $ME = z^\star \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.96\sqrt{\frac{0.56\times 0.44}{600}} = 0.0397 \approx 4\%$

6.9 (a) Proportion of graduates from this university who found a job within one year of graduating. $\hat{p} = 348/400 = 0.87$. (b) This is a random sample from less than 10% of the population, so the observations are independent. Success-failure condition is satisfied: 348 successes, 52 failures, both well above 10. (c) (0.8371, 0.9029). We are 95% confident that approximately 84% to 90% of graduates from this university found a job within one year of completing their undergraduate degree. (d) 95% of such random samples would produce a 95% confidence interval that includes the true proportion of students at this university who found a job within one year of graduating from college. (e) (0.8267, 0.9133). Similar interpretation as before. (f) 99% CI is wider, as we are more confident that the true proportion is within the interval and so need to cover a wider range.

6.11 (a) No. The sample only represents students who took the SAT, and this was also an online survey. (b) (0.5289, 0.5711). We are 90% confident that 53% to 57% of high school seniors who took the SAT are fairly certain that they will participate in a study abroad program in college. (c) 90% of such random samples would produce a 90% confidence interval that includes the true proportion. (d) Yes. The interval lies entirely above 50%.

6.13 (a) This is an appropriate setting for a hypothesis test. $H_0 : p = 0.50$. $H_A : p > 0.50$. Both independence and the success-failure condition are satisfied. $Z = 1.12 \rightarrow$ p-value $= 0.1314$. Since the p-value $> \alpha = 0.05$, we fail to reject H_0. The data do not provide strong evidence in favor of the claim. (b) Yes, since we did not reject H_0 in part (a).

6.15 (a) $H_0 : p = 0.38$. $H_A : p \neq 0.38$. Independence (random sample, $< 10\%$ of population) and the success-failure condition are satisfied. $Z = -20.5 \rightarrow$ p-value ≈ 0. Since the p-value is very small, we reject H_0. The data provide strong evidence that the proportion of Americans who only use their cell phones to access the internet is different than the Chinese proportion of 38%, and the data indicate that the proportion is lower in the US. (b) If in fact 38% of Americans used their cell phones as a primary access point to the internet, the probability of obtaining a random sample of 2,254 Americans where 17% or less or 59% or more use their only their cell phones to access the internet would be approximately 0. (c) (0.1545, 0.1855). We are 95% confident that approximately 15.5% to 18.6% of all Americans primarily use their cell phones to browse the internet.

6.17 (a) $H_0 : p = 0.5$. $H_A : p > 0.5$. Independence (random sample, $< 10\%$ of population) is satisfied, as is the success-failure conditions (using $p_0 = 0.5$, we expect 40 successes and 40 failures). $Z = 2.91 \rightarrow$ p-value $= 0.0018$. Since the p-value < 0.05, we reject the null hypothesis. The data provide strong evidence that the rate of correctly identifying a soda for these people is significantly better than just by random guessing. (b) If in fact people cannot tell the difference between diet and regular soda and they randomly guess, the probability of getting a random sample of 80 people where 53 or more identify a soda correctly would be 0.0018.

6.19 (a) Independence is satisfied (random sample from $< 10\%$ of the population), as is the success-failure condition (40 smokers, 160 non-smokers). The 95% CI: (0.145, 0.255). We are 95% confident that 14.5% to 25.5% of all students at this university smoke. (b) We want $z^\star SE$ to be no larger than 0.02 for a 95% confidence level. We use $z^\star = 1.96$ and plug in the point estimate $\hat{p} = 0.2$ within the SE formula: $1.96\sqrt{0.2(1-0.2)/n} \leq 0.02$. The sample size n should be at least 1,537.

6.21 The margin of error, which is computed as $z^\star SE$, must be smaller than 0.01 for a 90% confidence level. We use $z^\star = 1.65$ for a 90% confidence level, and we can use the point estimate $\hat{p} = 0.52$ in the formula for SE. $1.65\sqrt{0.52(1-0.52)/n} \leq 0.01$. Therefore, the sample size n must be at least 6,796.

6.23 This is not a randomized experiment, and it is unclear whether people would be affected by the behavior of their peers. That is, independence may not hold. Additionally, there are only 5 interventions under the provocative scenario, so the success-failure condition does not hold. Even if we consider a hypothesis test where we pool the proportions, the success-failure condition will not be satisfied. Since one condition is questionable and the other is not satisfied, the difference in sample proportions will not follow a nearly normal distribution.

6.25 (a) False. The entire confidence interval is above 0. (b) True. (c) True. (d) True. (e) False. It is simply the negated and reordered values: (-0.06,-0.02).

6.27 (a) (0.23, 0.33). We are 95% confident that the proportion of Democrats who support the plan is 23% to 33% higher than the proportion of Independents who do. (b) True.

6.29 (a) College grads: 23.7%. Non-college grads: 33.7%. (b) Let p_{CG} and p_{NCG} represent the proportion of college graduates and non-college graduates who responded "do not know". $H_0 : p_{CG} = p_{NCG}$. $H_A : p_{CG} \neq p_{NCG}$. Independence is satisfied (random sample, $< 10\%$ of the population), and the success-failure condition, which we would check using the pooled proportion ($\hat{p} = 235/827 = 0.284$), is also satisfied. $Z = -3.18 \rightarrow$ p-value $= 0.0014$. Since the p-value is very small, we reject H_0. The data provide strong evidence that the proportion of college graduates who do not have an opinion on this issue is different than that of non-college graduates. The data also indicate that fewer college grads say they "do not know" than non-college grads (i.e. the data indicate the direction after we reject H_0).

6.31 (a) College grads: 35.2%. Non-college grads: 33.9%. (b) Let p_{CG} and p_{NCG} represent the proportion of college graduates and non-college grads who support offshore drilling. $H_0 : p_{CG} = p_{NCG}$. $H_A : p_{CG} \neq p_{NCG}$. Independence is satisfied (random sample, $< 10\%$ of the population), and the success-failure condition, which we would check using the pooled proportion ($\hat{p} = 286/827 = 0.346$), is also satisfied. $Z = 0.39 \rightarrow$ p-value $= 0.6966$. Since the p-value $> \alpha$ (0.05), we fail to reject H_0. The data do not provide strong evidence of a difference between the proportions of college graduates and non-college graduates who support offshore drilling in California.

6.33 Subscript $_C$ means control group. Subscript $_T$ means truck drivers. $H_0 : p_C = p_T$. $H_A : p_C \neq p_T$. Independence is satisfied (random samples, $< 10\%$ of the population), as is the success-failure condition, which we would check using the pooled proportion ($\hat{p} = 70/495 = 0.141$). $Z = -1.58 \rightarrow$ p-value $= 0.1164$. Since the p-value is high, we fail to reject H_0. The data do not provide strong evidence that the rates of sleep deprivation are different for non-transportation workers and truck drivers.

6.35 (a) Summary of the study:

		Virol. failure		
		Yes	No	Total
Treatment	Nevaripine	26	94	120
	Lopinavir	10	110	120
	Total	36	204	240

(b) $H_0 : p_N = p_L$. There is no difference in virologic failure rates between the Nevaripine and Lopinavir groups. $H_A : p_N \neq p_L$. There is some difference in virologic failure rates between the Nevaripine and Lopinavir groups. (c) Random assignment was used, so the observations in each group are independent. If the patients in the study are representative of those in the general population (something impossible to check with the given information), then we can also confidently generalize the findings to the population. The success-failure condition, which we would check using the pooled proportion ($\hat{p} = 36/240 = 0.15$), is satisfied. $Z = 3.04 \rightarrow$ p-value $= 0.0024$. Since the p-value is low, we reject H_0. There is strong evidence of a difference in virologic failure rates between the Nevaripine and Lopinavir groups do not appear to be independent.

6.37 No. The samples at the beginning and at the end of the semester are not independent since the survey is conducted on the same students.

6.39 (a) False. The chi-square distribution has one parameter called degrees of freedom. (b) True. (c) True. (d) False. As the degrees of freedom increases, the shape of the chi-square distribution becomes more symmetric.

6.41 (a) H_0: The distribution of the format of the book used by the students follows the professor's predictions. H_A: The distribution of the format of the book used by the students does not follow the professor's predictions. (b) $E_{hard\ copy} = 126 \times 0.60 = 75.6$. $E_{print} = 126 \times 0.25 = 31.5$. $E_{online} = 126 \times 0.15 = 18.9$. (c) Independence: The sample is not random. However, if the professor has reason to believe that the proportions are stable from one term to the next and students are not affecting each other's study habits, independence is probably reasonable. Sample size: All expected counts are at least 5. (d) $\chi^2 = 2.32$, $df = 2$, p-value > 0.3. (e) Since the p-value is large, we fail to reject H_0. The data do not provide strong evidence indicating the professor's predictions were statistically inaccurate.

6.43 Use a chi-squared goodness of fit test. H_0: Each option is equally likely. H_A: Some options are preferred over others. Total sample size: 99. Expected counts: (1/3) * 99 = 33 for each option. These are all above 5, so conditions are satisfied. $df = 3 - 1 = 2$ and $\chi^2 = \frac{(43-33)^2}{33} + \frac{(21-33)^2}{33} + \frac{(35-33)^2}{33} = 7.52 \rightarrow 0.02 <$ p-value < 0.05. Since the p-value is less than 5%, we reject H_0. The data provide convincing evidence that some options are preferred over others.

6.45 (a). Two-way table:

	Quit		
Treatment	Yes	No	Total
Patch + support group	40	110	150
Only patch	30	120	150
Total	70	230	300

(b-i) $E_{row_1,col_1} = \frac{(row\ 1\ total) \times (col\ 1\ total)}{table\ total} = 35$. This is lower than the observed value.
(b-ii) $E_{row_2,col_2} = \frac{(row\ 2\ total) \times (col\ 2\ total)}{table\ total} = 115$. This is lower than the observed value.

6.47 H_0: The opinion of college grads and non-grads is not different on the topic of drilling for oil and natural gas off the coast of California. H_A: Opinions regarding the drilling for oil and natural gas off the coast of California has an association with earning a college degree.

$$E_{row\ 1,col\ 1} = 151.5 \quad E_{row\ 1,col\ 2} = 134.5$$
$$E_{row\ 2,col\ 1} = 162.1 \quad E_{row\ 2,col\ 2} = 143.9$$
$$E_{row\ 3,col\ 1} = 124.5 \quad E_{row\ 3,col\ 2} = 110.5$$

Independence: The samples are both random, unrelated, and from less than 10% of the population, so independence between observations is reasonable. Sample size: All expected counts are at least 5. $\chi^2 = 11.47$, $df = 2 \rightarrow 0.001 <$ p-value < 0.005. Since the p-value $< \alpha$, we reject H_0. There is strong evidence that there is an association between support for off-shore drilling and having a college degree.

6.49 (a) H_0: The age of Los Angeles residents is independent of shipping carrier preference variable. H_A: The age of Los Angeles residents is associated with the shipping carrier preference variable. (b) The conditions are not satisfied since some expected counts are below 5.

6.51 No. For a confidence interval, we check the success-failure condition using the data, and there are only 9 respondents who said bullying is no problem at all.

6.53 (a) $H_0 : p = 0.69$. $H_A : p \neq 0.69$. (b) $\hat{p} = \frac{17}{30} = 0.57$. (c) The success-failure condition is not satisfied; note that it is appropriate to use the null value ($p_0 = 0.69$) to compute the expected number of successes and failures. (d) Answers may vary. Each student can be represented with a card. Take 100 cards, 69 black cards representing those who follow the news about Egypt and 31 red cards representing those who do not. Shuffle the cards and draw with replacement (shuffling each time in between draws) 30 cards representing the 30 high school students. Calculate the proportion of black cards in this sample, $\hat{p}_{sim}$, i.e. the proportion of those who follow the news in the simulation. Repeat this many times (e.g. 10,000 times) and plot the resulting sample proportions. The p-value will be two times the proportion of simulations where $\hat{p}_{sim} \leq 0.57$. (Note: we would generally use a computer to perform these simulations.) (e) The p-value is about $0.001 + 0.005 + 0.020 + 0.035 + 0.075 = 0.136$, meaning the two-sided p-value is about 0.272. Your p-value may vary slightly since it is based on a visual estimate. Since the p-value is greater than 0.05, we fail to reject H_0. The data do not provide strong evidence that the proportion of high school students who followed the news about Egypt is different than the proportion of American adults who did.

6.55 The subscript $_{pr}$ corresponds to provocative and $_{con}$ to conservative. (a) $H_0 : p_{pr} = p_{con}$. $H_A : p_{pr} \neq p_{con}$. (b) -0.35. (c) The left tail for the p-value is calculated by adding up the two left bins: $0.005 + 0.015 = 0.02$. Doubling the one tail, the p-value is 0.04. (Students may have approximate results, and a small number of students may have a p-value of about 0.05.) Since the p-value is low, we reject H_0. The data provide strong evidence that people react differently under the two scenarios.

7 Introduction to linear regression

7.1 (a) The residual plot will show randomly distributed residuals around 0. The variance is also approximately constant. (b) The residuals will show a fan shape, with higher variability for smaller x. There will also be many points on the right above the line. There is trouble with the model being fit here.

7.3 (a) Strong relationship, but a straight line would not fit the data. (b) Strong relationship, and a linear fit would be reasonable. (c) Weak relationship, and trying a linear fit would be reasonable. (d) Moderate relationship, but a straight line would not fit the data. (e) Strong relationship, and a linear fit would be reasonable. (f) Weak relationship, and trying a linear fit would be reasonable.

7.5 (a) Exam 2 since there is less of a scatter in the plot of final exam grade versus exam 2. Notice that the relationship between Exam 1 and the Final Exam appears to be slightly nonlinear. (b) Exam 2 and the final are relatively close to each other chronologically, or Exam 2 may be cumulative so has greater similarities in material to the final exam. Answers may vary for part (b).

7.7 (a) $r = -0.7 \rightarrow$ (4). (b) $r = 0.45 \rightarrow$ (3). (c) $r = 0.06 \rightarrow$ (1). (d) $r = 0.92 \rightarrow$ (2).

7.9 (a) True. (b) False, correlation is a measure of the linear association between any two numerical variables.

7.11 (a) The relationship is positive, weak, and possibly linear. However, there do appear to be some anomalous observations along the left where several students have the same height that is notably far from the cloud of the other points. Additionally, there are many students who appear not to have driven a car, and they are represented by a set of points along the bottom of the scatterplot. (b) There is no obvious explanation why simply being tall should lead a person to drive faster. However, one confounding factor is gender. Males tend to be taller than females on average, and personal experiences (anecdotal) may suggest they drive faster. If we were to follow-up on this suspicion, we would find that sociological studies confirm this suspicion. (c) Males are taller on average and they drive faster. The gender variable is indeed an important confounding variable.

7.13 (a) There is a somewhat weak, positive, possibly linear relationship between the distance traveled and travel time. There is clustering near the lower left corner that we should take special note of. (b) Changing the units will not change the form, direction or strength of the relationship between the two variables. If longer distances measured in miles are associated with longer travel time measured in minutes, longer distances measured in kilometers will be associated with longer travel time measured in hours. (c) Changing units doesn't affect correlation: $r = 0.636$.

7.15 (a) There is a moderate, positive, and linear relationship between shoulder girth and height. (b) Changing the units, even if just for one of the variables, will not change the form, direction or strength of the relationship between the two variables.

7.17 In each part, we can write the husband ages as a linear function of the wife ages.
(a) $age_H = age_W + 3$.
(b) $age_H = age_W - 2$.
(c) $age_H = 2 \times age_W$.
Since the slopes are positive and these are perfect linear relationships, the correlation will be exactly 1 in all three parts. An alternative way to gain insight into this solution is to create a mock data set, e.g. 5 women aged 26, 27, 28, 29, and 30, then find the husband ages for each wife in each part and create a scatterplot.

7.19 Correlation: no units. Intercept: kg. Slope: kg/cm.

7.21 Over-estimate. Since the residual is calculated as *observed* − *predicted*, a negative residual means that the predicted value is higher than the observed value.

7.23 (a) There is a positive, very strong, linear association between the number of tourists and spending. (b) Explanatory: number of tourists (in thousands). Response: spending (in millions of US dollars). (c) We can predict spending for a given number of tourists using a regression line. This may be useful information for determining how much the country may want to spend in advertising abroad, or to forecast expected revenues from tourism. (d) Even though the relationship appears linear in the scatterplot, the residual plot actually shows a nonlinear relationship. This is not a contradiction: residual plots can show divergences from linearity that can be difficult to see in a scatterplot. A simple linear model is inadequate for modeling these data. It is also important to consider that these data are observed sequentially, which means there may be a hidden structure not evident in the current plots but that is important to consider.

7.25 (a) First calculate the slope: $b_1 = R \times s_y/s_x = 0.636 \times 113/99 = 0.726$. Next, make use of the fact that the regression line passes through the point $(\bar{x}, \bar{y})$: $\bar{y} = b_0 + b_1 \times \bar{x}$. Plug in $\bar{x}$, $\bar{y}$, and b_1, and solve for b_0: 51. Solution: $\widehat{travel\ time} = 51 + 0.726 \times distance$. (b) b_1: For each additional mile in distance, the model predicts an additional 0.726 minutes in travel time. b_0: When the distance traveled is 0 miles, the travel time is expected to be 51 minutes. It does not make sense to have a travel distance of 0 miles in this context. Here, the y-intercept serves only to adjust the height of the line and is meaningless by itself. (c) $R^2 = 0.636^2 = 0.40$. About 40% of the variability in travel time is accounted for by the model, i.e. explained by the distance traveled. (d) $\widehat{travel\ time} = 51 + 0.726 \times distance = 51 + 0.726 \times 103 \approx 126$ minutes. (Note: we should be cautious in our predictions with this model since we have not yet evaluated whether it is a well-fit model.) (e) $e_i = y_i - \hat{y}_i = 168 - 126 = 42$ minutes. A positive residual means that the model underestimates the travel time. (f) No, this calculation would require extrapolation.

7.27 There is an upwards trend. However, the variability is higher for higher calorie counts, and it looks like there might be two clusters of observations above and below the line on the right, so we should be cautious about fitting a linear model to these data.

7.29 (a) $\widehat{murder} = -29.901 + 2.559 \times poverty\%$ (b) Expected murder rate in metropolitan areas with no poverty is -29.901 per million. This is obviously not a meaningful value, it just serves to adjust the height of the regression line. (c) For each additional percentage increase in poverty, we expect murders per million to be lower on average by 2.559. (d) Poverty level explains 70.52% of the variability in murder rates in metropolitan areas. (e) $\sqrt{0.7052} = 0.8398$

7.31 (a) There is an outlier in the bottom right. Since it is far from the center of the data, it is a point with high leverage. It is also an influential point since, without that observation, the regression line would have a very different slope.
(b) There is an outlier in the bottom right. Since it is far from the center of the data, it is a point with high leverage. However, it does not appear to be affecting the line much, so it is not an influential point.
(c) The observation is in the center of the data (in the x-axis direction), so this point does *not* have high leverage. This means the point won't have much effect on the slope of the line and so is not an influential point.

7.33 (a) There is a negative, moderate-to-strong, somewhat linear relationship between percent of families who own their home and the percent of the population living in urban areas in 2010. There is one outlier: a state where 100% of the population is urban. The variability in the percent of homeownership also increases as we move from left to right in the plot. (b) The outlier is located in the bottom right corner, horizontally far from the center of the other points, so it is a point with high leverage. It is an influential point since excluding this point from the analysis would greatly affect the slope of the regression line.

7.35 (a) The relationship is positive, moderate-to-strong, and linear. There are a few outliers but no points that appear to be influential. (b) $\widehat{weight} = -105.0113 + 1.0176 \times height$. Slope: For each additional centimeter in height, the model predicts the average weight to be 1.0176 additional kilograms (about 2.2 pounds). Intercept: People who are 0 centimeters tall are expected to weigh -105.0113 kilograms. This is obviously not possible. Here, the y-intercept serves only to adjust the height of the line and is meaningless by itself. (c) H_0: The true slope coefficient of height is zero ($\beta_1 = 0$). H_0: The true slope coefficient of height is greater than zero ($\beta_1 > 0$). A two-sided test would also be acceptable for this application. The p-value for the two-sided alternative hypothesis ($\beta_1 \neq 0$) is incredibly small, so the p-value for the one-sided hypothesis will be even smaller. That is, we reject H_0. The data provide convincing evidence that height and weight are positively correlated. The true slope parameter is indeed greater than 0. (d) $R^2 = 0.72^2 = 0.52$. Approximately 52% of the variability in weight can be explained by the height of individuals.

7.37 (a) H_0: $\beta_1 = 0$. H_A: $\beta_1 > 0$. A two-sided test would also be acceptable for this application. The p-value, as reported in the table, is incredibly small. Thus, for a one-sided test, the p-value will also be incredibly small, and we reject H_0. The data provide convincing evidence that wives' and husbands' heights are positively correlated. (b) $\widehat{height}_W = 43.5755 + 0.2863 \times height_H$. (c) Slope: For each additional inch in husband's height, the average wife's height is expected to be an additional 0.2863 inches on average. Intercept: Men who are 0 inches tall are expected to have wives who are, on average, 43.5755 inches tall. The intercept here is meaningless, and it serves only to adjust the height of the line. (d) The slope is positive, so r must also be positive. $r = \sqrt{0.09} = 0.30$. (e) 63.2612. Since R^2 is low, the prediction based on this regression model is not very reliable. (f) No, we should avoid extrapolating.

7.39 (a) $r = \sqrt{0.28} \approx -0.53$. We know the correlation is negative due to the negative association shown in the scatterplot. (b) The residuals appear to be fan shaped, indicating non-constant variance. Therefore a simple least squares fit is not appropriate for these data.

7.41 (a) $H_0 : \beta_1 = 0; H_A : \beta_1 \neq 0$ (b) The p-value for this test is approximately 0, therefore we reject H_0. The data provide convincing evidence that poverty percentage is a significant predictor of murder rate. (c) $n = 20, df = 18, T^*_{18} = 2.10$; $2.559 \pm 2.10 \times 0.390 = (1.74, 3.378)$; For each percentage point poverty is higher, murder rate is expected to be higher on average by 1.74 to 3.378 per million. (d) Yes, we rejected H_0 and the confidence interval does not include 0.

7.43 This is a one-sided test, so the p-value should be half of the p-value given in the regression table, which will be approximately 0. Therefore the data provide convincing evidence that poverty percentage is positively associated with murder rate.

8 Multiple and logistic regression

8.1 (a) $\widehat{baby\_weight} = 123.05 - 8.94 \times smoke$ (b) The estimated body weight of babies born to smoking mothers is 8.94 ounces lower than babies born to non-smoking mothers. Smoker: $123.05 - 8.94 \times 1 = 114.11$ ounces. Non-smoker: $123.05 - 8.94 \times 0 = 123.05$ ounces. (c) H_0: $\beta_1 = 0$. H_A: $\beta_1 \neq 0$. $T = -8.65$, and the p-value is approximately 0. Since the p-value is very small, we reject H_0. The data provide strong evidence that the true slope parameter is different than 0 and that there is an association between birth weight and smoking. Furthermore, having rejected H_0, we can conclude that smoking is associated with lower birth weights.

8.3 (a) $\widehat{baby\_weight} = -80.41 + 0.44 \times gestation - 3.33 \times parity - 0.01 \times age + 1.15 \times height + 0.05 \times weight - 8.40 \times smoke$. (b) $\beta_{gestation}$: The model predicts a 0.44 ounce increase in the birth weight of the baby for each additional day of pregnancy, all else held constant. β_{age}: The model predicts a 0.01 ounce decrease in the birth weight of the baby for each additional year in mother's age, all else held con-

stant. (c) Parity might be correlated with one of the other variables in the model, which complicates model estimation. (d) $\widehat{baby\_weight} = 120.58$. $e = 120 - 120.58 = -0.58$. The model over-predicts this baby's birth weight. (e) $R^2 = 0.2504$. $R^2_{adj} = 0.2468$.

8.5 (a) (-0.32, 0.16). We are 95% confident that male students on average have GPAs 0.32 points lower to 0.16 points higher than females when controlling for the other variables in the model. (b) Yes, since the p-value is larger than 0.05 in all cases (not including the intercept).

8.7 Remove age.

8.9 Based on the p-value alone, either gestation or smoke should be added to the model first. However, since the adjusted R^2 for the model with gestation is higher, it would be preferable to add gestation in the first step of the forward-selection algorithm. (Other explanations are possible. For instance, it would be reasonable to only use the adjusted R^2.)

8.11 She should use p-value selection since she is interested in finding out about significant predictors, not just optimizing predictions.

8.13 Nearly normal residuals: The normal probability plot shows a nearly normal distribution of the residuals, however, there are some minor irregularities at the tails. With a data set so large, these would not be a concern.
Constant variability of residuals: The scatterplot of the residuals versus the fitted values does not show any overall structure. However, values that have very low or very high fitted values appear to also have somewhat larger outliers. In addition, the residuals do appear to have constant variability between the two parity and smoking status groups, though these items are relatively minor.
Independent residuals: The scatterplot of residuals versus the order of data collection shows a random scatter, suggesting that there is no apparent structures related to the order the data were collected.
Linear relationships between the response variable and numerical explanatory variables: The residuals vs. height and weight of mother are randomly distributed around 0. The residuals vs. length of gestation plot also does not show any clear or strong remaining structures, with the possible exception of very short or long gestations. The rest of the residuals do appear to be randomly distributed around 0.
All concerns raised here are relatively mild. There are some outliers, but there is so much data that the influence of such observations will be minor.

8.15 (a) There are a few potential outliers, e.g. on the left in the `total_length` variable, but nothing that will be of serious concern in a data set this large. (b) When coefficient estimates are sensitive to which variables are included in the model, this typically indicates that some variables are collinear. For example, a possum's gender may be related to its head length, which would explain why the coefficient (and p-value) for `sex_male` changed when we removed the `head_length` variable. Likewise, a possum's skull width is likely to be related to its head length, probably even much more closely related than the head length was to gender.

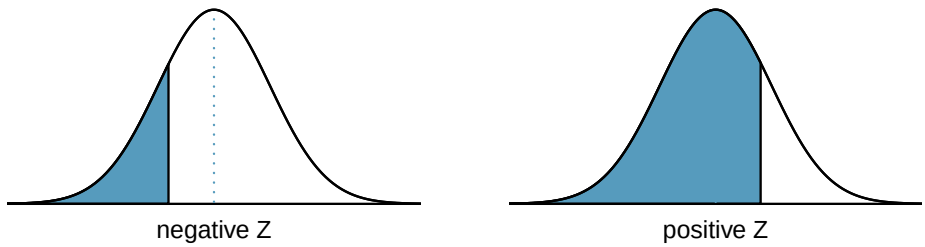
8.17 (a) The logistic model relating $\hat{p}_i$ to the predictors may be written as $\log\left(\frac{\hat{p}_i}{1-\hat{p}_i}\right) = 33.5095 - 1.4207 \times sex\_male_i - 0.2787 \times skull\_width_i + 0.5687 \times total\_length_i - 1.8057 \times tail\_length_i$. Only `total_length` has a positive association with a possum being from Victoria. (b) $\hat{p} = 0.0062$. While the probability is very near zero, we have not run diagnostics on the model. We might also be a little skeptical that the model will remain accurate for a possum found in a US zoo. For example, perhaps the zoo selected a possum with specific characteristics but only looked in one region. On the other hand, it is encouraging that the possum was caught in the wild. (Answers regarding the reliability of the model probability will vary.)

Appendix B

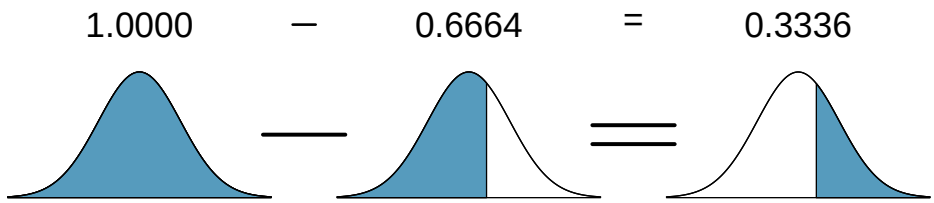
Distribution tables

B.1 Normal Probability Table

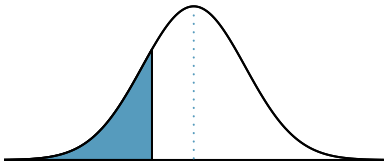
The area to the left of Z represents the percentile of the observation. The normal probability table always lists percentiles.



To find the area to the right, calculate 1 minus the area to the left.

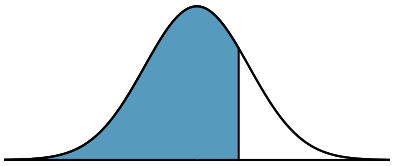


For additional details about working with the normal distribution and the normal probability table, see Section **??**, which starts on page **??**.



Second decimal place of Z										
0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	Z
0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	−3.4
0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	−3.3
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	−3.2
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	−3.1
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	−3.0
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	−2.9
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	−2.8
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	−2.7
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	−2.6
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	−2.5
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	−2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	−2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	−2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	−2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	−2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	−1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	−1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	−1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	−1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	−1.5
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	−1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	−1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	−1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	−1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	−1.0
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	−0.9
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	−0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	−0.7
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	−0.6
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	−0.5
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	−0.4
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	−0.3
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	−0.2
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	−0.1
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	−0.0

*For $Z \leq -3.50$, the probability is less than or equal to 0.0002.



	Second decimal place of Z									
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

*For $Z \geq 3.50$, the probability is greater than or equal to 0.9998.

B.2 t-Probability Table

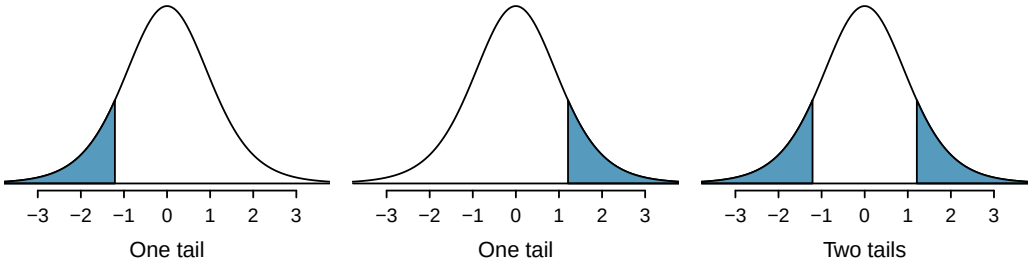


Figure B.1: Tails for the t-distribution.

one tail		0.100	0.050	0.025	0.010	0.005
two tails		0.200	0.100	0.050	0.020	0.010
df	1	3.08	6.31	12.71	31.82	63.66
	2	1.89	2.92	4.30	6.96	9.92
	3	1.64	2.35	3.18	4.54	5.84
	4	1.53	2.13	2.78	3.75	4.60
	5	1.48	2.02	2.57	3.36	4.03
	6	1.44	1.94	2.45	3.14	3.71
	7	1.41	1.89	2.36	3.00	3.50
	8	1.40	1.86	2.31	2.90	3.36
	9	1.38	1.83	2.26	2.82	3.25
	10	1.37	1.81	2.23	2.76	3.17
	11	1.36	1.80	2.20	2.72	3.11
	12	1.36	1.78	2.18	2.68	3.05
	13	1.35	1.77	2.16	2.65	3.01
	14	1.35	1.76	2.14	2.62	2.98
	15	1.34	1.75	2.13	2.60	2.95
	16	1.34	1.75	2.12	2.58	2.92
	17	1.33	1.74	2.11	2.57	2.90
	18	1.33	1.73	2.10	2.55	2.88
	19	1.33	1.73	2.09	2.54	2.86
	20	1.33	1.72	2.09	2.53	2.85
	21	1.32	1.72	2.08	2.52	2.83
	22	1.32	1.72	2.07	2.51	2.82
	23	1.32	1.71	2.07	2.50	2.81
	24	1.32	1.71	2.06	2.49	2.80
	25	1.32	1.71	2.06	2.49	2.79
	26	1.31	1.71	2.06	2.48	2.78
	27	1.31	1.70	2.05	2.47	2.77
	28	1.31	1.70	2.05	2.47	2.76
	29	1.31	1.70	2.05	2.46	2.76
	30	1.31	1.70	2.04	2.46	2.75

one tail		0.100	0.050	0.025	0.010	0.005
two tails		0.200	0.100	0.050	0.020	0.010
df	31	1.31	1.70	2.04	2.45	2.74
	32	1.31	1.69	2.04	2.45	2.74
	33	1.31	1.69	2.03	2.44	2.73
	34	1.31	1.69	2.03	2.44	2.73
	35	1.31	1.69	2.03	2.44	2.72
	36	1.31	1.69	2.03	2.43	2.72
	37	1.30	1.69	2.03	2.43	2.72
	38	1.30	1.69	2.02	2.43	2.71
	39	1.30	1.68	2.02	2.43	2.71
	40	1.30	1.68	2.02	2.42	2.70
	41	1.30	1.68	2.02	2.42	2.70
	42	1.30	1.68	2.02	2.42	2.70
	43	1.30	1.68	2.02	2.42	2.70
	44	1.30	1.68	2.02	2.41	2.69
	45	1.30	1.68	2.01	2.41	2.69
	46	1.30	1.68	2.01	2.41	2.69
	47	1.30	1.68	2.01	2.41	2.68
	48	1.30	1.68	2.01	2.41	2.68
	49	1.30	1.68	2.01	2.40	2.68
	50	1.30	1.68	2.01	2.40	2.68
	60	1.30	1.67	2.00	2.39	2.66
	70	1.29	1.67	1.99	2.38	2.65
	80	1.29	1.66	1.99	2.37	2.64
	90	1.29	1.66	1.99	2.37	2.63
	100	1.29	1.66	1.98	2.36	2.63
	150	1.29	1.66	1.98	2.35	2.61
	200	1.29	1.65	1.97	2.35	2.60
	300	1.28	1.65	1.97	2.34	2.59
	400	1.28	1.65	1.97	2.34	2.59
	500	1.28	1.65	1.96	2.33	2.59
	∞	1.28	1.65	1.96	2.33	2.58

B.3 Chi-Square Probability Table

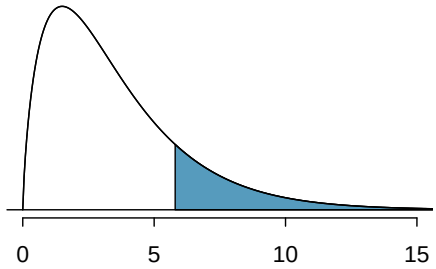


Figure B.2: Areas in the chi-square table always refer to the right tail.

Upper tail		0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001
df	1	1.07	1.64	2.71	3.84	5.41	6.63	7.88	10.83
	2	2.41	3.22	4.61	5.99	7.82	9.21	10.60	13.82
	3	3.66	4.64	6.25	7.81	9.84	11.34	12.84	16.27
	4	4.88	5.99	7.78	9.49	11.67	13.28	14.86	18.47
	5	6.06	7.29	9.24	11.07	13.39	15.09	16.75	20.52
	6	7.23	8.56	10.64	12.59	15.03	16.81	18.55	22.46
	7	8.38	9.80	12.02	14.07	16.62	18.48	20.28	24.32
	8	9.52	11.03	13.36	15.51	18.17	20.09	21.95	26.12
	9	10.66	12.24	14.68	16.92	19.68	21.67	23.59	27.88
	10	11.78	13.44	15.99	18.31	21.16	23.21	25.19	29.59
	11	12.90	14.63	17.28	19.68	22.62	24.72	26.76	31.26
	12	14.01	15.81	18.55	21.03	24.05	26.22	28.30	32.91
	13	15.12	16.98	19.81	22.36	25.47	27.69	29.82	34.53
	14	16.22	18.15	21.06	23.68	26.87	29.14	31.32	36.12
	15	17.32	19.31	22.31	25.00	28.26	30.58	32.80	37.70
	16	18.42	20.47	23.54	26.30	29.63	32.00	34.27	39.25
	17	19.51	21.61	24.77	27.59	31.00	33.41	35.72	40.79
	18	20.60	22.76	25.99	28.87	32.35	34.81	37.16	42.31
	19	21.69	23.90	27.20	30.14	33.69	36.19	38.58	43.82
	20	22.77	25.04	28.41	31.41	35.02	37.57	40.00	45.31
	25	28.17	30.68	34.38	37.65	41.57	44.31	46.93	52.62
	30	33.53	36.25	40.26	43.77	47.96	50.89	53.67	59.70
	40	44.16	47.27	51.81	55.76	60.44	63.69	66.77	73.40
	50	54.72	58.16	63.17	67.50	72.61	76.15	79.49	86.66