4.8.3

If the normal density is:

$$f_k(x) = \frac{1}{\sqrt{2\pi}\,\sigma_k} \exp\left(\frac{-1}{2\sigma_k^2}(x-\mu_k)^2\right)$$

and Bayes' theorem states:

$$p_k(x) = \frac{\pi_k f_k(x)}{\sum_{l=1}^{K} \pi_l f_l(x)},$$

we can substitute $f_k(x)$ above to get:

$$\frac{\frac{1}{\sqrt{2\pi}\,\sigma_k} \exp\left(\frac{-1}{2\sigma_k^2}(x-\mu_k)^2\right) \pi_k}{\sum_{l=1}^{K} \pi_l \frac{1}{\sqrt{2\pi}\,\sigma_l} \exp\left(\frac{-1}{2\sigma_l^2}(x-\mu_l)^2\right)}$$

from here, we want to find the class k to maximize the numerator:

$$\arg_k \max p_k(x) = \arg_k \max \pi_k \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(\frac{-1}{2\sigma^2}(x-\mu_k)^2\right)$$

$$= \arg_k \max \log(\pi_k) - \log\left(\sqrt{2\pi}\,\sigma_k\right) - \underbrace{\log \exp\left(\frac{1}{2\sigma^2}(x-\mu_k)^2\right)}_{= \frac{1}{2\sigma^2}(x-\mu_k)^2 \underbrace{\cancel{\log e}}_{1}}$$

using log rules:

$$\arg_k \max \log(\pi_k) - \frac{1}{2\sigma^2}(x^2 - 2x\mu_k + \mu_k^2)$$

$$= \arg_k \max \left[\frac{-1}{2\sigma^2} x^2 + \frac{\mu_k}{\sigma_k^2} x - \frac{\mu_k^2}{2\sigma^2} + \log(\pi_k) \right.$$

since this equation is quadratic w.r.t. x, you can see that Bayes classifier is quadratic in nature.