4.8.7

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$$

80%

$f_{yes}(x) = Pr(X \mid Y = yes)$ $\quad \bar{X} = 10$ $\quad$ var $\hat{\sigma}^2 = 36$

$f_{no}(x) = Pr(X \mid Y = no)$ $\quad \bar{X} = 6$

20%

$X = 4$ $\quad Pr(Y = yes \mid X = 4)$

Bayes' Theorem: $p_k(x) = \frac{\pi_k f_k(x)}{\sum_{l=1}^{2} \pi_l f_l(x)}$

$$p_{yes}(x) = \frac{p_{yes} f_{yes}(x)}{\sum_{l=1}^{2} \pi_l f_l(x)} = \frac{p_{yes} f_{yes}(x)}{p_{yes} f_{yes}(x) + p_{no} f_{no}(x)}$$

$$p_{yes}(4) = \frac{\pi_{yes} f_{yes}(4)}{\pi_{yes} f_{yes}(4) + \pi_{no} f_{no}(4)}$$

$$= \frac{0.8\left(\frac{1}{\sqrt{2\pi 36}} e^{-(4-\sqrt{36})^2/2\cdot 36}\right)}{0.8\frac{1}{\sqrt{2\pi 36}} e^{\frac{-(4-\sqrt{36})^2}{2\cdot 36}} + 0.2\frac{1}{\sqrt{2\pi 36}} e^{-(4-\sqrt{36})^2/2\cdot 36}}$$

$$= \frac{0.8}{0.8 + 0.2} \cdot e^{\frac{-(4-\sqrt{36})^2}{2\cdot 36}} = 0.8 \cdot e^{-4/72}$$

$$= 0.8 \cdot 0.945959$$

$$\approx 0.75677$$