Math 521 HW 6

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Exercise 2.4.1

2.4.1.a. Prove that the sequence defined by $x_1 = 3$ and

$$x_{n+1} = \frac{1}{4 - x_n}$$

converges.

2.4.1.b. Now that we know that $\lim x_n$ exists, explain why $\lim x_{n+1}$ must also exist and equal the same value.

2.4.1.c. Take the limit of each side of the recursive equation in part (a) to explicitly compute $\lim x_n$.

Exercise 2.4.2.

2.4.2.a. Consider the recursively defined equation $y_1 = 1$,

$$y_{n+1} = 3 - y_n,$$

and set $y = \lim y_n$. Because (y_n) and (y_{n+1}) have the same limit, taking the limit across the recursive equation gives $y = 3 - y$. Solving for y, we conclude $\lim y_n = 3/2$.
What is wrong with this argument?

2.4.2.b. This time set $y_1 = 1$ and $y_{n+1} = 3 - 1/y_n$. Can the strategy in (a) be applied to compute the limit of this sequence?

Exercise 2.4.5. (Calculating Square Roots) Let $x_1 = 2$ and define

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right).$$

2.4.5.a. Show that x_n^2 is always greater than or equal to 2, and then use this to prove that $x_n - x_{n+1} \geq 0$. Conclude that $\lim x_n = \sqrt{2}$.

2.4.5.b. Modify the sequence (x_n) so that it converges to $\sqrt{c}$.