Math 521 HW 9

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Exercise 2.7.1. Proving the Alternating Series Test amounts to showing that the sequence of partial sums

$$s_n = a_1 - a_2 + ... \pm a_n$$

converges.

2.7.1.a. Prove the Alternating Series Test by showing that (s_n) is a Cauchy sequence.

2.7.1.b. Supply another proof for this result using the Nested Interval Property.

2.7.1.c. Consider the subsequences (s_{2n}) and (s_{2n+1}), and show how the Monotone Convergence Theorem leads to a third proof for the Alternating Series Test.

Exercise 2.7.4. Give an example to show that it is possible for both $\sum x_n$ and $\sum y_n$ to diverge but for $\sum x_n y_n$ to converge.

Exercise 3.2.3. Decide whether the following sets are open, closed, or neither. If a set is not open, find a point in the set for which there is no ϵ-neighborhood contained in the set. If a set is not closed, find a limit point that is not contained in the set.

3.2.3.a. $\mathbb{Q}$

3.2.3.b. $\mathbb{N}$

3.2.3.c. $\{x \in \mathbb{R} : x > 0\}$

3.2.3.d. $(0, 1] = \{x \in \mathbb{R} : 0 < x \leq 1\}$

3.2.3.e. $\{1 + 1/4 + 1/9 + ... + 1/n^2 : n \in \mathbb{N}\}$