Math 521 HW 11

Morgan Gribbins

Exercise 3.3.9. Follow these steps to prove the final implication in Theorem 3.3.8.

Assume K satisfies (i) and (ii), and let $\{O_\lambda : \lambda \in \Lambda\}$ be an open cover for K. For contradiction, let's assume that no finite subcover exists. Let I_0 be a closed interval containing K.

3.3.9.a. Show that there exists a nested sequence of closed intervals $... \subseteq I_2 \subseteq I_1 \subseteq I_0$ with the property that, for each n, $I_n \cap K$ cannot be finitely covered and $\lim |I_n| = 0$.

3.3.9.b. Argue that there exists an $x \in K$ such that $x \in I_n$ for all n.

3.3.9.c. Because $x \in K$, there must exist an open set O_{λ_0} from the original collection that contains x as an element. Explain how this leads to the desired contradiction.