Math 521 HW 11

Morgan Gribbins

Exercise 3.3.9. Follow these steps to prove the final implication in Theorem 3.3.8.

Assume K satisfies (i) and (ii), and let $\{O_\lambda : \lambda \in \Lambda\}$ be an open cover for K. For contradiction, let's assume that no finite subcover exists. Let I_0 be a closed interval containing K.

3.3.9.a. Show that there exists a nested sequence of closed intervals $... \subseteq I_2 \subseteq I_1 \subseteq I_0$ with the property that, for each n, $I_n \cap K$ cannot be finitely covered and $\lim |I_n| = 0$.

3.3.9.b. Argue that there exists an $x \in K$ such that $x \in I_n$ for all n.

By the Nested Interval Property, there must exists one $x \in K$ such that $x \in \bigcap_{n\in\mathbb{N}} I_n$, because of the established definitions of each I_n.

3.3.9.c. Because $x \in K$, there must exist an open set O_{λ_0} from the original collection that contains x as an element. Explain how this leads to the desired contradiction.

Exercise 4.2.2. For each stated limit, find the largest possible δ-neighborhood that is a proper response to the given ϵ challenge.

4.2.2.a. $\lim_{x\to 3}(5x - 6) = 9$, where $\epsilon = 1$.

4.2.2.b. $\lim_{x\to 4} \sqrt{x} = 2$, where $\epsilon = 1$.

4.2.2.c. $\lim_{x\to \pi}[[x]] = 3$, where $\epsilon = 1$.

4.2.2.d. $\lim_{x\to \pi}[[x]] = 3$, where $\epsilon = .01$.

Exercise 4.2.5. Use Definition 4.2.1 to supply a proper proof for the following limit statements.

4.2.5.a. $\lim_{x\to 2}(3x + 4) = 10$.

4.2.5.b. $\lim_{x\to 0} x^3 = 0$.

4.2.5.c. $\lim_{x\to 2}(x^2 + x - 1) = 5$.

4.2.5.d. $\lim_{x\to 3} 1/x = 1/3$.

Exercise 4.3.5. Show using Definition 4.3.1 that if c is an isolated point of $A \subseteq \mathbb{R}$, then $f : A \to \mathbb{R}$ is continuous at c.

Exercise 4.3.6. Provide an example of each or explain why the request is impossible.

4.3.6.a. Two functions f and g, neither of which is continuous at 0 but such that $f(x)g(x)$ and $f(x) + g(x)$ are continuous at 0.

4.3.6.b. A function $f(x)$ continuous at 0 and $g(x)$ not continuous at 0 such that $f(x) + g(x)$ is continuous at 0.

4.3.6.c. A function $f(x)$ continuous at 0 and $g(x)$ not continuous at 0 such that $f(x)g(x)$ is continuous at 0.

4.3.6.d. A function $f(x)$ not continuous at 0 such that $f(x) + 1/f(x)$ is continuous at 0.

4.3.6.e. A function $f(x)$ not continuous at 0 such that $[f(x)]^3$ is continuous at 0.

Exercise 4.3.9. Assume $h : \mathbb{R} \to \mathbb{R}$ is continuous on $\mathbb{R}$ and let $K = \{x : h(x) = 0\}$. Show that K is a closed set.