Math 534 HW 7

Morgan Gribbins

(1) An isomorphism from a group to itself, i.e. an isomorphism $\alpha : G \to G$, is called an automorphism. Suppose that G is a finite abelian group which has no elements of order 2. Show that the function $\alpha(g) = g^2$ is an automorphism of G. Show by example that the result doesn't hold in the case that G is infinite.

(2) Consider the group $G = (\mathbb{Z}, +)$.

(2a) Show that G is isomorphic to the proper subgroup $H = \{2k : k \in \mathbb{Z}\}$ of even elements.

(2b) Show that there are in fact infinitely many subgroups of G to which it is isomorphic.

(3) Consider the group $\mathbb{R}^\times = (\mathbb{R} \setminus \{0\}, \times)$ of non-zero real numbers under multiplication. Prove that this group is not isomorphic to $(\mathbb{R}, +)$.

(4) Explain $\mathfrak{S}_8$ contains subgroups isomorphic to $\mathbb{Z}/15, (\mathbb{Z}/16)^\times$, and D_8. Here, D_8 denotes the group of symmetries of a regular convex octagon.