Math 523 HW 4

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Section 2.1

Use the rules for differentiation to find the derivatives of **(2)** and **(4)**.

(2) $z^2 + 10z$

(4) $[\cos\,(z^2)]^3$

For each function f listed in Exercises **(8)** and **(10)**, find an analytic function F with $F' = f$.

(8) $f(z) = z - 2$

(10) $f(z) = \sin\,z\,\cos\,z$

(14) Let $P(z) = A(z - z_1)...(z - z_n)$, where A and $z_1, ...z_n$ are complex numbers and $A \neq 0$. Show that

$$\frac{P'(z)}{P(z)} = \sum_{j=1}^{n} \frac{1}{z - z_j},\ \ z \neq z_1, ..., z_n.$$

(16) Find the derivative of the **linear fractional transformation** $T(z) = (az+b)/(cz+d), ad \neq bc$. In what way does the condition $ad - bc \neq 0$ enter? Conclude that $T'(z)$ is never zero, $z \neq -d/c$.

(18) Show that $h(z) = \bar{z}$ is not analytic on any domain. (**Hint:** check the Cauchy-Riemann equations.)

(20) Let $f = u + iv$ be analytic. In each of the following, find v given u.

(20a) $u = x^2 - y^2$

(20b) $u = \frac{x}{x^2+y^2}$

Section 2.2

In exercises **(2)** and **(4)**, use Theorem 2 or Example 4 to find the radius of convergence of the following power series.

(2) $\sum_{k=0}^{\infty} \frac{(k!)^2}{(2k)!}(z-2)^k$

(4) $\sum_{k=0}^{\infty}(-1)^k z^{2k}$

In exercises **(8)** and **(10)**, find the power series about the origin for the given function.

(8) $z^2 \cos z$

(10) $\frac{1+z}{1-z}$, $|z| < 1$

In exercises **(14)**, **(16)**, and **(18)**, find a "closed form" (that is, a simple expression) for each of the given power series.

(14) $\sum_{n=0}^{\infty} \frac{z^{2n}}{n!}$

(16) $\sum_{n=1}^{\infty} n(z-1)^{n-1}$

(18) $\sum_{n=2}^{\infty} n(n-1)z^n$

(22)

(22a) If $f(z) = \sum_{n=0}^{\infty} a_n(z-z_0)^n$ has radius of convergence $R > 0$ and if $f(z) = 0$ for all $z, |z-z_0| < r \leq R$, show that $a_0 = a_1 = ... = 0$.

(22b) If $F(z) = \sum_{n=0}^{\infty} a_n(z-z_0)^n$ and $G(z) = \sum_{n=0}^{\infty} b_n(z-z_0)^n$ are equal on some disc $|z-z_0| < r$, show that $a_n = b_n$ for all n.