Math 534 HW 4

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(1) Let G be a cyclic group that has exactly 3 subgroups: G itself, the trivial subgroup, and a proper subgroup of order 7. Find $|G|$, making sure to justify your answer.

The cyclic group of order 49 is the only group with such properties. The only divisors of 49 are $1, 7$, and 49, which correspond to the trivial subgroup, the proper subgroup of order 7, and G, respectively.

(2) Let G be an abelian group with $|G| = 35$, and suppose that every element $g \in G$ satisfies the equality $g^{35} = e$. Prove that G is cyclic. (hint: I claim the result follows once you have your hands on an element of order 5 and an element of order 7. Think of the possible of orders of elements in the group, and deduce that you must have elements as above. Finally, at some point you might use the fact that neither 4 nor 6 divide 34.)

(3) Let G be a group.

(3a) Let $H \leq G$ and $K \leq G$ be subgroups. Show that $H \cap K \subseteq G$ is a subgroup of G.

(3b) Let $a, b \in G$ such that $|a|$ and $|b|$ are finite and relatively prime. Show that $\langle a \rangle \cap \langle b \rangle = \{e\}$.

(4) Let $G = \mathbb{Z}/30$.

(4a) How many distinct subgroups of G are there?

(4b) Draw the lattice of subgroups of G.