

# Stochastic Modeling of Oil and Gas Production

Aiden Huffman, Ben Sloman, Jinniao Qiu, Syeda Ali, Tony  
Ware, Wenning Wei, Zuming Sun and Yilan Luo  
(Project Coordinator: Michael Morgan)

August 30, 2018

## 1.Introduction

## 2.Modeling and Estimation

## 3. Summary

# Introduction

## Montney Wells

- Oil and gas operators in both British Columbia and Alberta are currently pursuing development of the Montney formation.
- It is currently one of the top plays in North America, in terms of number of wells drilled per year, investment dollars spent and overall hydrocarbon production.
- It covers a geographical region of approximately 130,000km, with thickness between 100m-300m.
- Due to its thickness, much of the Montney can supported multiple layers of stack horizontal wells.
- The Montney holds approximately 449 Tcf of marketable natural gas.

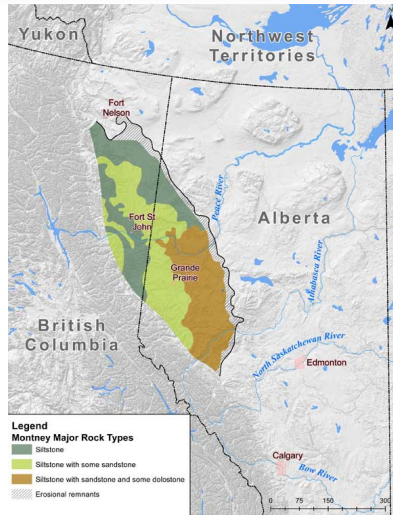


Figure: Montney Major Rock Types.

# Introduction

## Background

The oil and gas industry is subject to production uncertainty in many ways, including:

- Finding oil and gas reserves is highly unpredictable.
- Productivity of an individual well can be forecast only approximately and even the best priors are often only weakly informative.
- Production data can be very noisy.
- Market prices for commodities move unexpectedly.
- Economic factors, such as interest rates, are unknown.

# Introduction

## Motivation

- The industry is interested in how uncertainties are treated, especially how technical uncertainties can best be expressed during the evaluation of investment decisions.
- Basic ODEs cannot capture these uncertainties very well.
- Stochastic models offer a methodology to better capture the uncertainties of the process by treating some unknown parameters as random variables.

# Introduction

## Goal

- To model the unknown parameters as stochastic processes.
- Solve the resulting stochastic differential equations.
- Compare the probabilistic results with data from actual wells to asses model performance.

# Introduction

## Basic Definitions

- Flow rate ( $q$ ): The mount of gas produced per time unit for several consecutive period.
- Cumulative gas production ( $Q$ ):  $\int_0^T q dt$
- Decline rate(D):  $D = -\frac{1}{q} \frac{\Delta q}{\Delta t}$



# Introduction

## Decline Curve Analysis

- Decline curve analysis is the systematic evaluation of trends in historic production rates from oil and gas wells, most typically how the rate of production decreases (declines) over time.
- These trends can be extrapolated forward and thus used to forecast the future performance of wells.
- Most commonly, production trends are represented using simple differential equations, for example, hyperbolic, exponential etc.

# Introduction

## Production Decline Curve

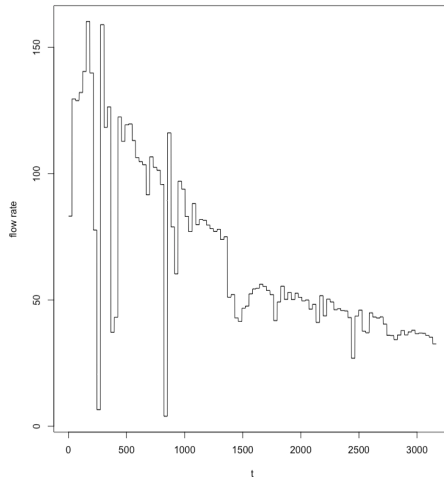


Figure: An example of the flow rate of gas from a gas well.

# Modeling and Estimation

## Data Description

X_UWI_DISPLAY	PROD_DATE	GAS
00/01-01-059-21W5/4	2011-12-01	288.3
00/01-01-059-21W5/4	2012-01-01	2080.3
00/01-01-059-21W5/4	2012-03-01	4320.6
00/01-01-059-21W5/4	2012-04-01	3744.2
00/01-01-059-21W5/4	2012-05-01	3190.2
00/01-01-059-21W5/4	2012-06-01	1478.6
00/01-01-059-21W5/4	2012-07-01	2569.7
00/01-01-059-21W5/4	2012-08-01	2301.8
00/01-01-059-21W5/4	2012-09-01	2010.4
00/01-01-059-21W5/4	2012-10-01	1626.7
00/01-01-059-21W5/4	2012-11-01	1558.5

Figure: An snapshot of the production data set.

# Modeling and Estimation

## Common PDE models in Decline Analysis

Arps:

$$\frac{dq}{dt} = - \left( \frac{D_i}{q_1^b} \right) q^{b+1}$$

Power Law Exponential:

$$\frac{dq}{dt} = - \left( D_\infty + D_1 t^{-(1-n)} \right) q$$

Stretched Exponential:

$$\frac{dq}{dt} = - \left( \frac{n}{t} \left( \frac{t}{\tau} \right)^n \right) q$$

Logistic Growth:

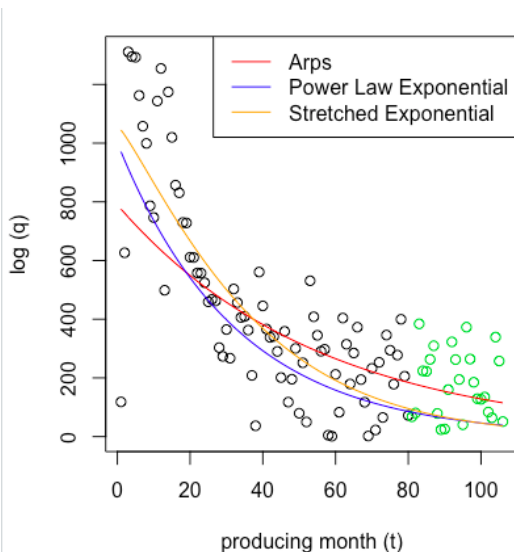
$$\frac{dq}{dt} = - \left( \frac{a - an + (1+n)t^n}{t(a+t^n)} \right) q$$

Duong:

$$\frac{dq}{dt} = - \left( mt^{-1} - at^{-m} \right) q$$

# Modeling and Estimation

## Parameter Estimation of Classic PDE Models by LS



# Modeling and Estimation

## Improved Power Law Exponential Model NO.1

- Power law exponential model:

$$\log(q) = \log(q_0) - D_\infty t - Dt^n \quad (1)$$

- First, we assume  $\log(q_0) \sim m + \varepsilon N(0,1)$
- To generate the uncertainty in the model, we add a Brownian Motion term with a scale parameter  $\lambda$ :  $\lambda B_t$ . This could correspond to the assumption that uncertainty is caused by normal errors in the measurement of production data.
- Therefore, the proposed model is:

$$\log(q) = m + \varepsilon N(0,1) - D_\infty t - Dt^n + \lambda B_t \quad (2)$$

# Modeling and Estimation

## Improved Power Law Exponential Model NO.2

- Since the  $B_t \sim N(0, \sqrt{t})$  and the well is always run for a long time, the fluctuation of model NO.1 is too large at the late time of well.
- We weight  $B_t$  with  $\frac{1}{1+t}$
- Therefore, the model change to:

$$\log(q) = m + \varepsilon N(0, 1) - D_\infty - Dt^n + \frac{\lambda B_t}{1+t} \quad (3)$$

# Modeling and Estimation

## Estimation of Model NO.2

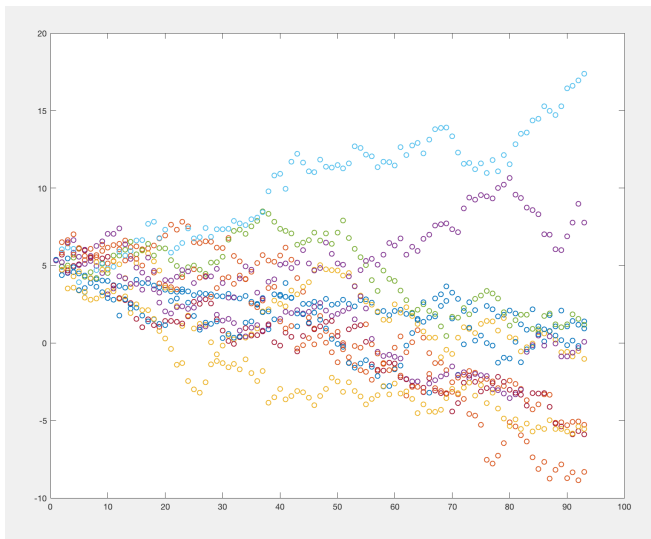
- Then we estimate the parameters  $(m, \varepsilon, D_\infty, D, n, \lambda)$  in the model (by Maximum Likelihood) minimizing

$$\frac{1}{2} \log(\varepsilon^2 2\pi) + \frac{(\log(q_0) - m_0)^2}{2\pi\varepsilon^2} + \sum_{i=1}^N \left( \frac{1}{2} \log \left( \frac{\lambda^2 2\pi}{(1+t_{i-1})^2} \right) + \frac{(\log(q_{t_i}) - \log(q_{t_{i-1}}) + D_\infty + Dt_i^n)^2}{\frac{2\pi\lambda^2}{(1+t_{i-1})^2}} \right)$$



# Modeling and Estimation

## Estimation Result



# Modeling and Estimation

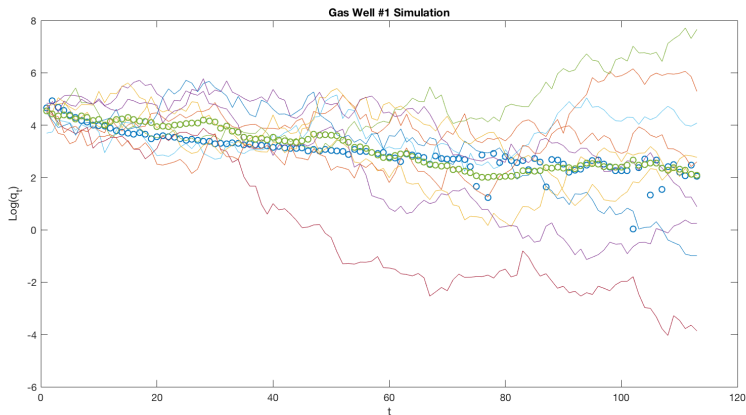
## Improved Power Law Exponential Model NO.3

- We improve the model
- Then the log-likelihood: find  $(m_0, \varepsilon, \lambda, m, D, D_\infty)$  minimizing

$$\frac{1}{2} \log(\varepsilon^2 2\pi) + \frac{(\log(q_0) - m_0)^2}{2\pi\varepsilon^2} \\ + \sum_{i=1}^N \left( \frac{1}{2} \log\left(\frac{\lambda^2 2\pi}{(1+t_{i-1}^m)^2}\right) + \frac{(\log(q_{t_i}) - \log(q_{t_{i-1}}) + D_\infty + Dt_{i-1}^n)^2}{\frac{2\pi\lambda^2}{(1+t_{i-1}^m)^2}} \right)$$

# Modeling and Estimation

## Improved Power Law Exponential Model NO.3



# Modeling and Estimation

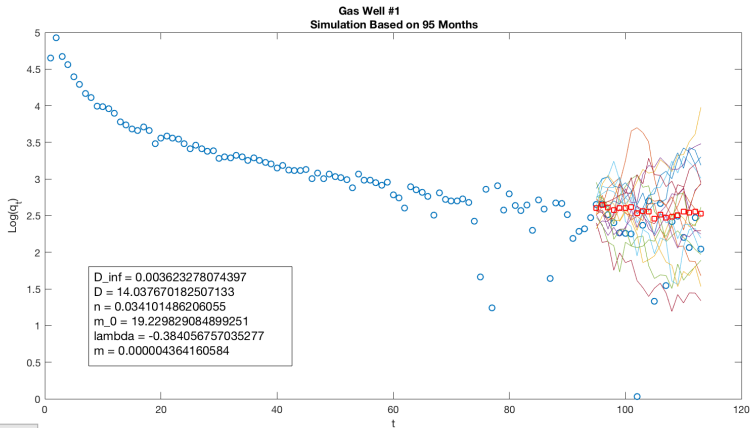
## Improved Power Law Exponential Model NO.3

- Improve the model
- Modify the log-likelihood: find  $(m_0, \varepsilon = 0, \lambda, m, D, D_\infty)$  minimizing

$$\begin{aligned} & \sum_{i=1}^N \log\left(\frac{2\pi\lambda^2}{(1+t_i^m)^2}\right) \\ & + \sum_{i=1}^N \left(\log(q_{t_i}) - \log(q_{t_{i-1}}) + D_\infty + Dnt_i^{n-1}\right)^2 \frac{(1+t_{i-1}^m)^2}{2\pi\lambda^2} \\ & + \kappa \sum_{i=1}^N |m_0 - D_\infty t_i - Dt_i^n - \log(q_i^{obs})| \end{aligned}$$

# Modeling and Estimation

## Improved Power Law Exponential Model NO.3



Stop

Pause

# Modeling and Estimation

## Improved Power Law Exponential Model NO.4

- Instead of Brownian Motion we could consider the case that a Poisson process describes the physical processes underlying the uncertainty. This could correspond to an assumption of random changes in pumping rate or back-pressure occurring at random time intervals.

- Simply, we replace the Brownian Motion by a Poisson Process  $N_t^\alpha$ :

$$\log(q) = m + \varepsilon N(0, 1) - D_\infty - Dt^n + \lambda N_t^\alpha \quad (4)$$

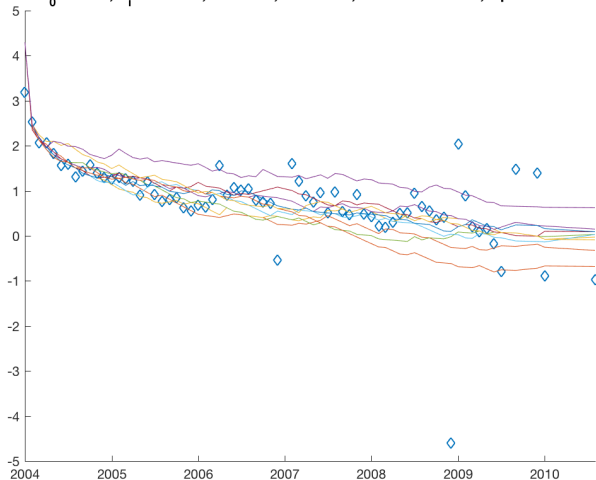
- Then the least square norm: ( $\varepsilon = 0$ ): find  $(m_0, \lambda, m, D, D_\infty, \alpha)$  minimizing

$$\begin{aligned} & E \sum_{i=1}^N (\Delta \log(q_{t_i}^{mod}) - \Delta \log(q_{t_i}^{obs}))^2 \\ & + \kappa \sum_{i=0}^N (E \log(q_{t_i}^{mod}) - \log(q_{t_i}^{obs}))^2 \end{aligned}$$

# Modeling and Estimation

## Improved Power Law Exponential Model NO.4

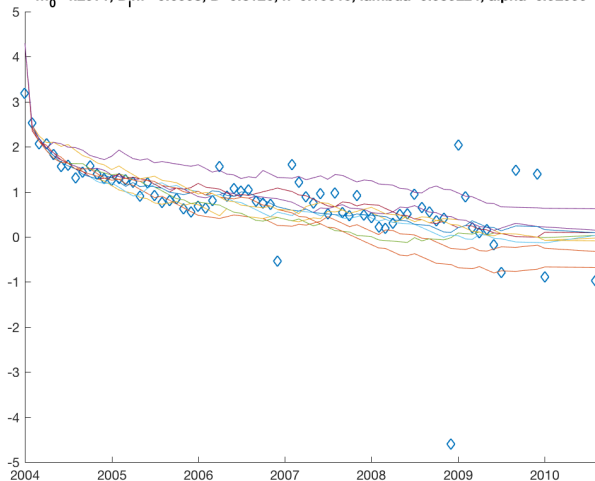
$m_0=4.2971$ ;  $D_{inf}=0.0608$ ;  $D=0.3126$ ;  $n=0.16616$ ;  $\lambda=0.059224$ ;  $\alpha=0.92099$



# Modeling and Estimation

## Improved Power Law Exponential Model NO.4

$m_0=4.2971$ ;  $D_{inf}=0.0608$ ;  $D=0.3126$ ;  $n=0.16616$ ;  $\lambda=0.059224$ ;  $\alpha=0.92099$

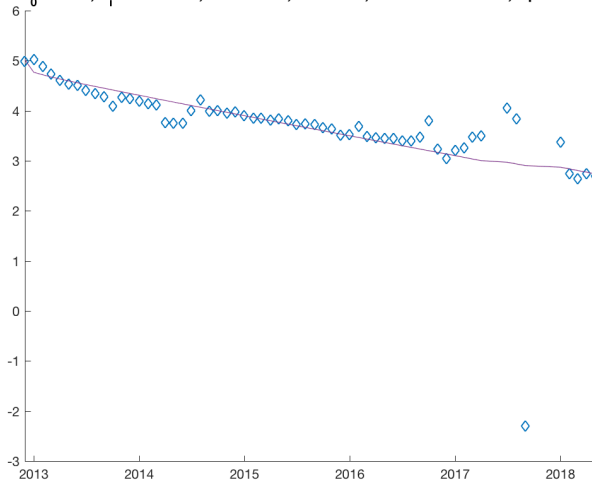




# Modeling and Estimation

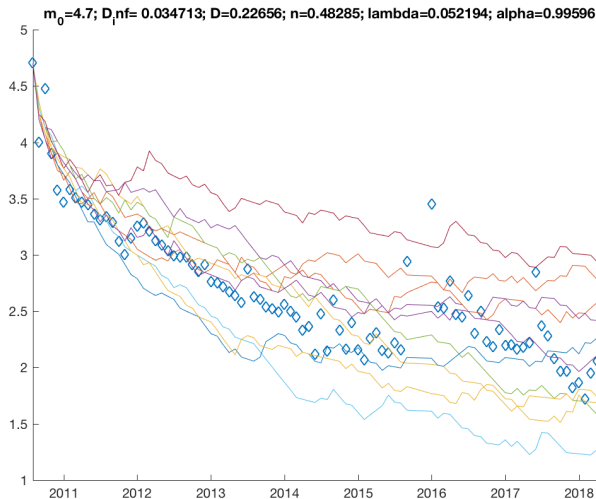
## Improved Power Law Exponential Model NO.4

$m_0=5.0198$ ;  $D_{inf}=0.032355$ ;  $D=0.023622$ ;  $n=0.10773$ ;  $\lambda=8.5164e-05$ ;  $\alpha=0.4495$



# Modeling and Estimation

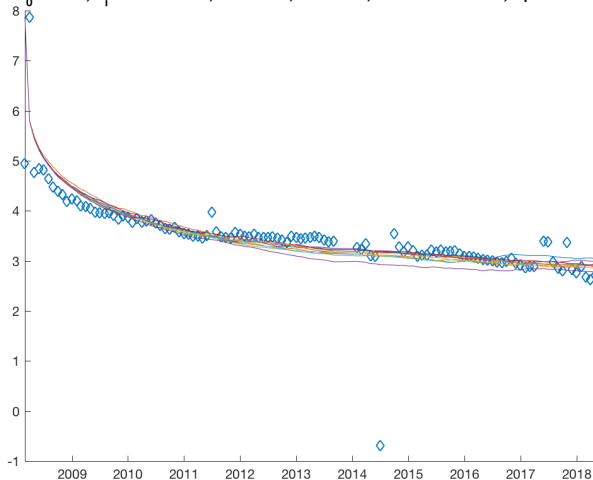
## Improved Power Law Exponential Model NO.4



# Modeling and Estimation

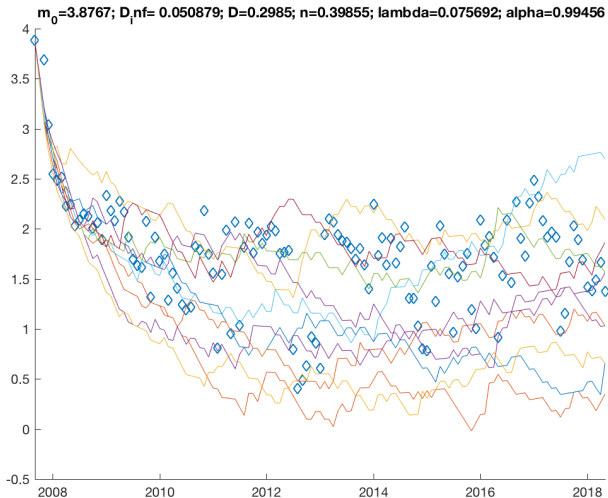
## Improved Power Law Exponential Model NO.4

$m_0=7.8719$ ;  $D_{inf}=2.2053e-05$ ;  $D=0.46085$ ;  $n=0.22085$ ;  $\lambda=0.0089336$ ;  $\alpha=0.99998$



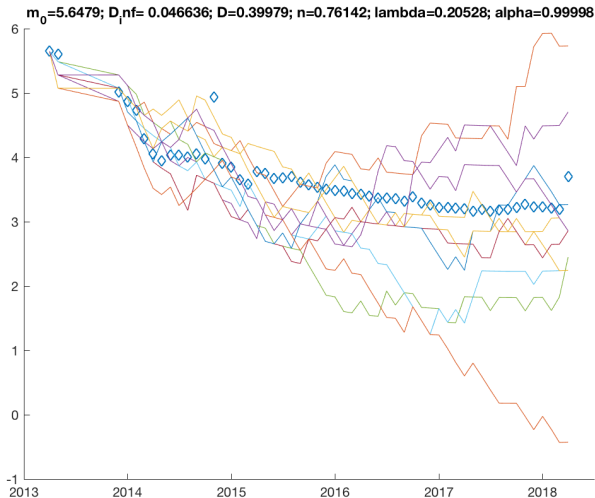
# Modeling and Estimation

## Improved Power Law Exponential Model NO.4



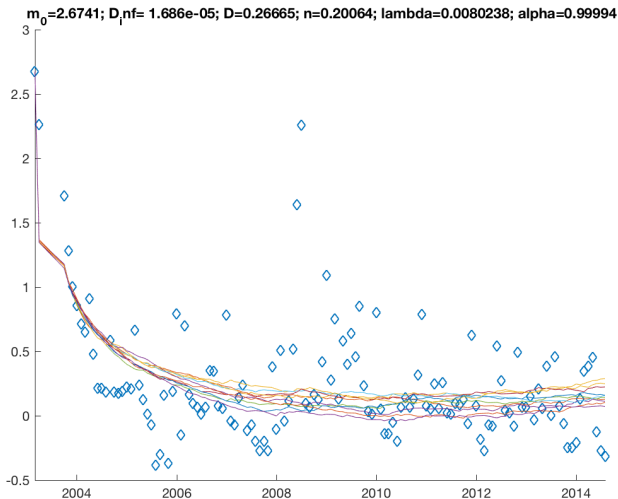
# Modeling and Estimation

## Improved Power Law Exponential Model NO.4



# Modeling and Estimation

## Improved Power Law Exponential Model NO.4



## Summary

## Future Work

- It may be possible to improve the fit of uncertainty in the model for instance by combining Poisson and Brownian Motions.
- Find the spatial correlation between the gas production of wells in a region of interest, and use the correlation to forecast the performance of a new site based on the old wells around it.
- Relate the gas production forecast to the energy market forecast to calculate the profit in the future.

# Thank You!