Stochastic Modeling of Oil and Gas Production

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Introduction

Montney Wells

- Formation
  - Most of reserves are in British Columbia and Alberta.
  - Covers a geographical region of approximately 130,000km, with thickness between 100m-300m.
  - Siltstone of Montney holds approximately 449 Tcf of marketable natural gas.
  - Multiple stack reserves.

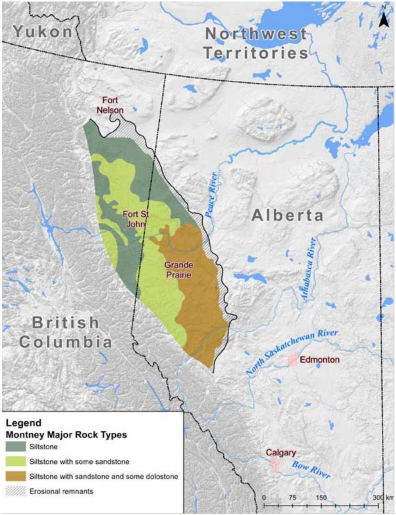


Figure: Montney Major Rock Types.

Introduction

Background

Many Challenges of the oil and gas industry subject to production uncertainty are as follows:

- Finding oil and gas reserves is highly unpredictable.
- Productivity of an individual well change in random ways.
- Very noisy data.
- Market moves unexpectedly.
- Future prices and interest rates are unknown.

Introduction

Motivation

- In Industries we are interested in the mechanism of how uncertainties are treated as workload moves to reserves production development.
- Basic ODE's cannot capture these uncertainties very well.
- Stochastic models offers a methodology to better capture the uncertainties of the process. by treating some unknown parameters as random variables.

Introduction

Goal

- To model the unknown parameters as stochastic processes.
- Solve the resulting stochastic differential equations.
- Comparing our results with data from actual well to witness the performance.

Introduction

Basic Definitions

- Flow rate (q): The mount of gas produced per time unit for several consecutive period.
- Cumulative gas production (Q): $\int_0^T qdt$
- Decline rate(D): $D = -\frac{1}{q}\frac{\Delta q}{\Delta t}$

Introduction

Decline Curve Analysis

- Use to estimate the production of a well, while original data is given.
- Use to forecast the future performance of wells.
- Production trends can be estimated using different differential equations, for example, Hyperbolic, Exponential etc.

Introduction

Production Delcine Curve

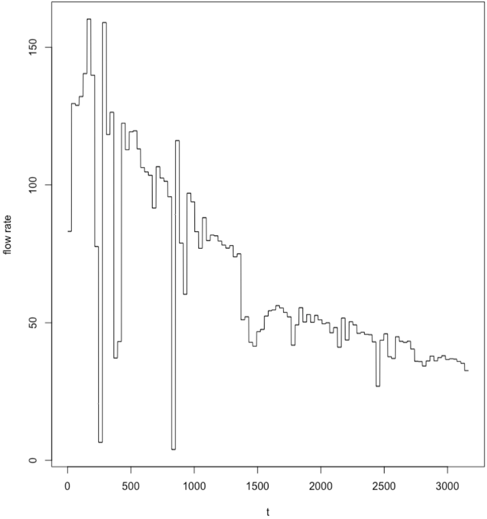


Figure: An example of decline curve of flow rate of gas.

Modeling and Estimation

Data Description

X_UWI_DISPLAY	PROD_DATE	GAS
00/01-01-059-21W5/4	2011-12-01	288.3
00/01-01-059-21W5/4	2012-01-01	2080.3
00/01-01-059-21W5/4	2012-03-01	4320.6
00/01-01-059-21W5/4	2012-04-01	3744.2
00/01-01-059-21W5/4	2012-05-01	3190.2
00/01-01-059-21W5/4	2012-06-01	1478.6
00/01-01-059-21W5/4	2012-07-01	2569.7
00/01-01-059-21W5/4	2012-08-01	2301.8
00/01-01-059-21W5/4	2012-09-01	2010.4
00/01-01-059-21W5/4	2012-10-01	1626.7
00/01-01-059-21W5/4	2012-11-01	1558.5

Figure: An snapshot of dataset.

Modeling and Estimation

Common PDE models in Delcine Analysis

Arps:

$$\frac{dq}{dt} = -\left(\frac{D_i}{q_1^b}\right) q^{b+1}$$

Power Law Exponential:

$$\frac{dq}{dt} = -\left(D_\infty + D_1 t^{-(1-n)}\right) q$$

Stretched Exponential:

$$\frac{dq}{dt} = -\left(\frac{n}{t}\left(\frac{t}{\tau}\right)^n\right) q$$

Logistic Growth:

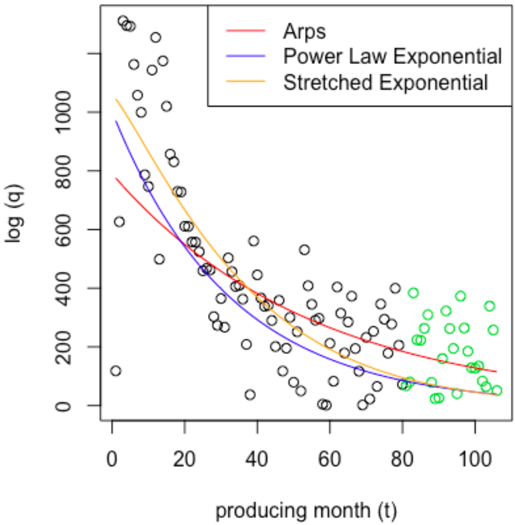
$$\frac{dq}{dt} = -\left(\frac{a - an + (1+n)t^n}{t\left(a + t^n\right)}\right) q$$

Duong:

$$\frac{dq}{dt} = -\left(mt^{-1} - at^{-m}\right) q$$

Modeling and Estimation

Parameter Estimation of Classic PDE Models by LS



Modeling and Estimation

Improved Power Law Exponential Model NO.1

- Power law exponential model:

$$log(q) = log(q_0) - D_\infty t - Dt^n \tag{1}$$

- First, we assume $log(q_0) \sim m + \varepsilon N(0,1)$
- To generate the uncertainty in the model, we also add a Brownian Motion item with a scale parameter λ: λB_t
- Therefore, the proposed model is:

$$log(q) = m + \varepsilon N(0,1) - D_\infty - Dt^n + \lambda B_t \tag{2}$$

Modeling and Estimation

Improved Power Law Exponential Model NO.2

- Since the $B_t \sim N(0, \sqrt{t})$ and the well is always run for a long time, the fluctuation of model NO.1 is too large at the late time of well.
- We weight B_t with $\dfrac{1}{1+t}$
- Therefore, the model change to:

$$log(q) = m + \varepsilon N(0,1) - D_\infty - Dt^n + \frac{\lambda B_t}{1+t} \tag{3}$$

Modeling and Estimation

Estimation of Model NO.2

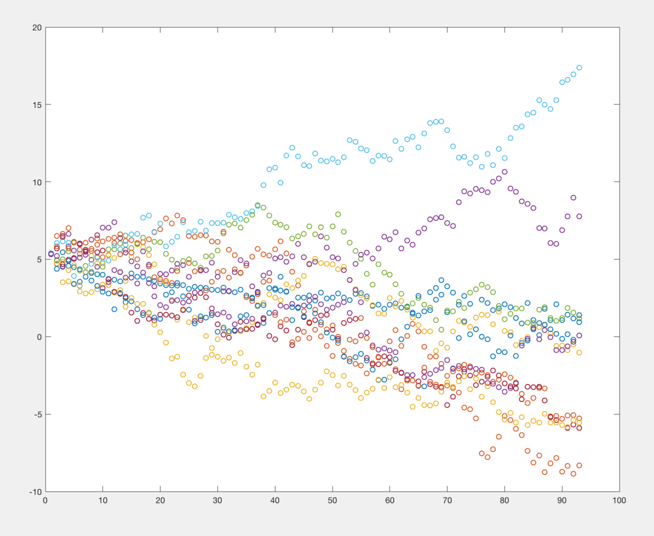
- Then we estimate the parameters $(m, \varepsilon, D_\infty, D, n, \lambda)$ in the model (by Maximum Likelihood) minimizing

$$\frac{1}{2} log(\varepsilon^2 2\pi) + \frac{(log(q_0) - m_0)^2}{2\pi\varepsilon^2}$$

$$+\sum_{i=1}^{N} \left(\tfrac{1}{2} log \left(\tfrac{\lambda^2 2\pi}{(1+t_{i-1})^2} \right) + \frac{(log(q_{t_i}) - log(q_{t_{i-1}}) + D_\infty + Dt_i^n)^2}{\frac{2\pi\lambda^2}{(1+t_{i-1})^2}} \right)$$

Modeling and Estimation

Estimation Result



Modeling and Estimation

Improved Power Law Exponential Model NO.3

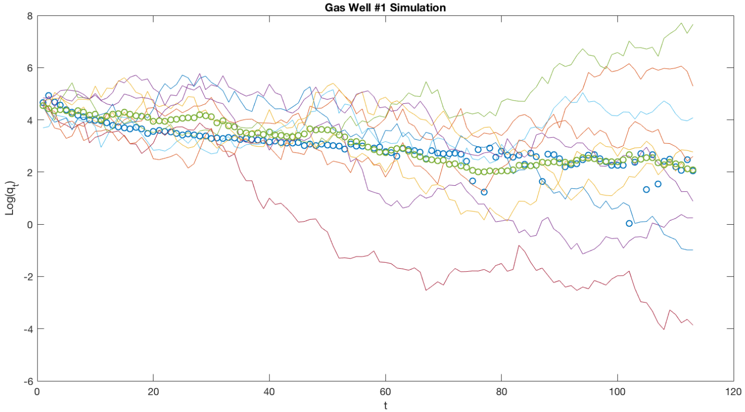
- We improve the model
- Then the log-likelihood: find $(m_0, \varepsilon, \lambda, m, D, D_\infty)$ minimizing

$$\frac{1}{2} log(\varepsilon^2 2\pi) + \frac{(log(q_0) - m_0)^2}{2\pi\varepsilon^2}$$

$$+\sum_{i=1}^{N} \left(\frac{1}{2} log(\tfrac{\lambda^2 2\pi}{(1+t_{i-1}^m)^2}) + \frac{(log(q_{t_i}) - log(q_{t_{i-1}}) + D_\infty + Dt_{i-1}^n)^2}{\frac{2\pi\lambda^2}{(1+t_{i-1}^m)^2}} \right)$$

Modeling and Estimation

Improved Power Law Exponential Model NO.3



Modeling and Estimation

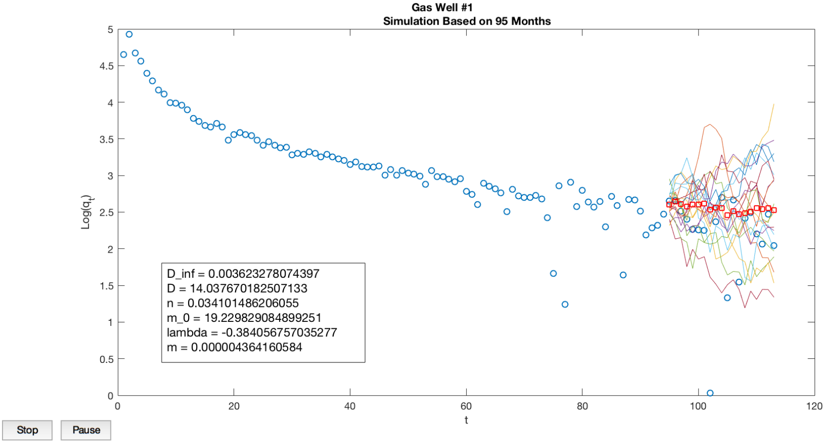
Improved Power Law Exponential Model NO.3

- Improve the model
- Modify the log-likelihood: find $(m_0, \varepsilon = 0, \lambda, m, D, D_\infty)$ minimizing

$$\sum_{i=1}^{N} log(\frac{2\pi\lambda^2}{(1+t_i^m)^2})$$
$$+\sum_{i=1}^{N}(log(q_{t_i} - log(q_{t_{i-1}}) + D_\infty + Dnt_i^{n-1})^2 \frac{(1+t_{i-1}^m)^2}{2\pi\lambda^2}$$
$$+\kappa\sum_{i=1}^{N}|m_0 - D_\infty t_i - Dt_i^n - log(q_i^{obs})|$$

Modeling and Estimation

Improved Power Law Exponential Model NO.4



Modeling and Estimation

Improved Power Law Exponential Model NO.4

- Except Brownian Motion, we also consider the Poisson process to describe the uncertainty in the reality.
- Simply, we replace the Brownian Motion by a Poisson Process N_t^{α}:

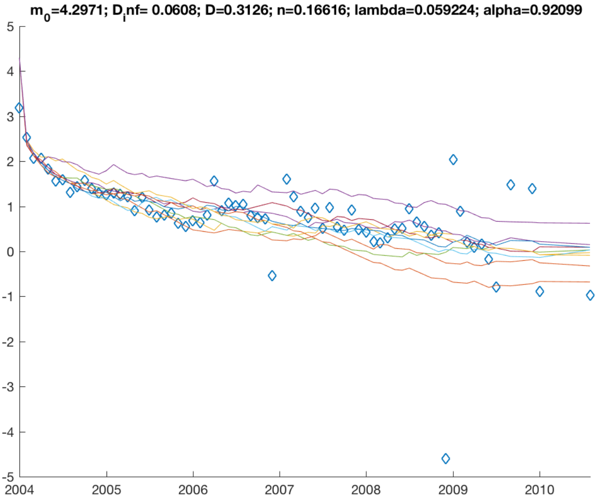
$$log(q) = m + \varepsilon N(0,1) - D_{\infty} - Dt^n + \lambda N_t^{\alpha} \tag{4}$$

- Then the least square norm: $(\varepsilon = 0)$: find $(m_0, \lambda, m, D, D_{\infty}, \alpha)$ minimizing

$$E \sum_{i=1}^{N} \left(\Delta log(q_{t_i}^{mod}) - \Delta log(q_{t_i}^{obs})\right)^2$$
$$+\kappa \sum_{i=0}^{N} \left(Elog(q_{t_i}^{mod}) - log(q_{t_i}^{obs})\right)^2$$

Modeling and Estimation

Improved Power Law Exponential Model NO.4



Modeling and Estimation

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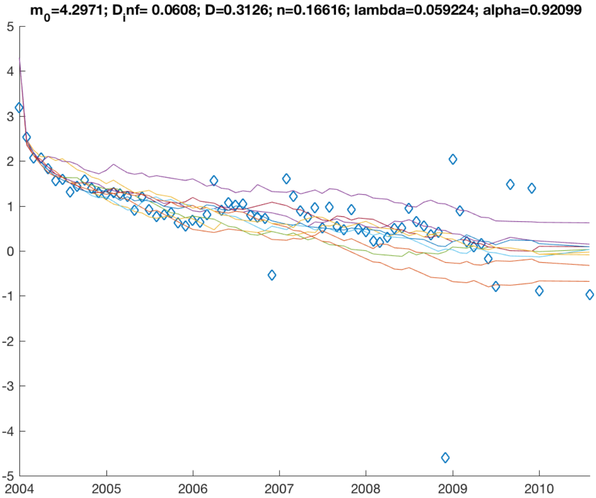

Modeling and Estimation

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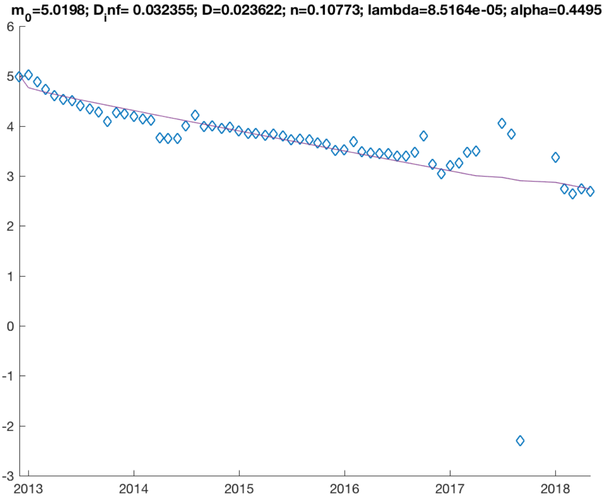
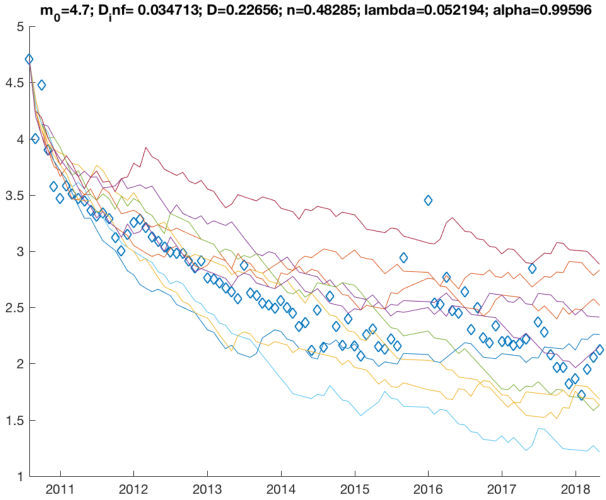


Modeling and Estimation

Improved Power Law Exponential Model NO.4



Modeling and Estimation

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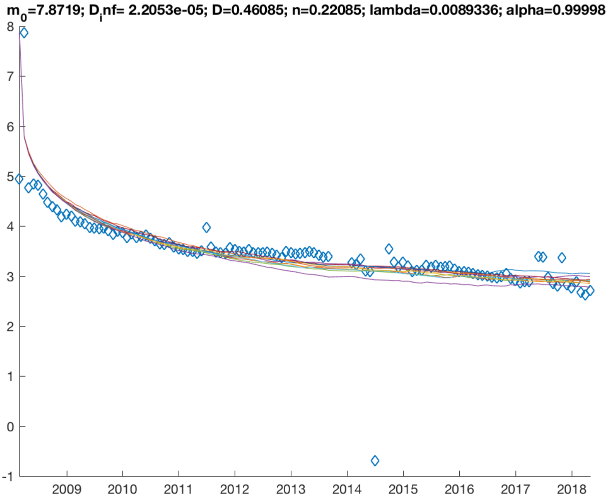

Modeling and Estimation

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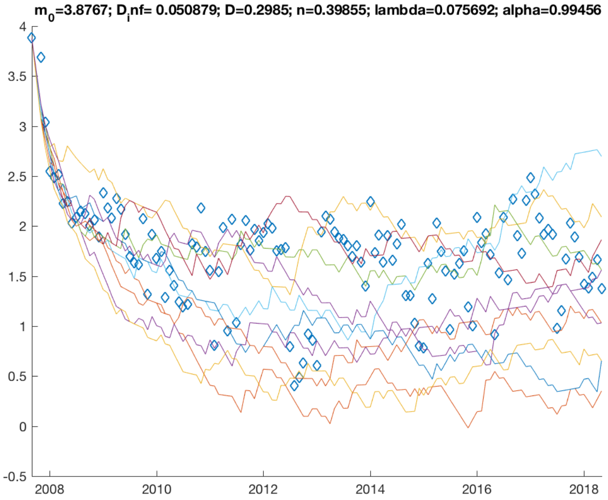

Modeling and Estimation

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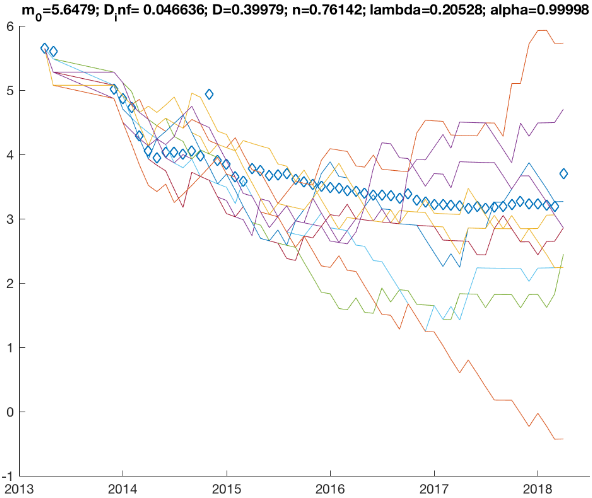
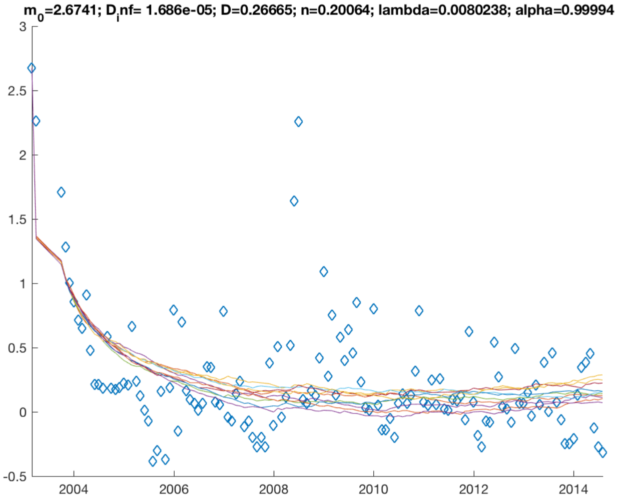


Modeling and Estimation

Improved Power Law Exponential Model NO.4



Summary

Future Work

- Improve the fit of uncertainty in the model for instance by combining Poisson and Brownian Motions.
- Find the spatial correlation between the gas production of wells in a region of interest, and use the correlation to forecast the performance of a new site based on the old wells around it.
- Relate the gas production forecast to the energy market forecast to calculate the profit in the future.

Thank You!