Probability and Statistics:
Homework 1, Lesson 1 and 2 Continued

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1 Homework 1 Notes

- Problem 1
  - Gambler makes four bets:
  - Chance of success:
    - 1st: $A: \ P = 0.7$
    - 2nd: $B: \ P = 0.4$
  - Chance at least one fails:
    - $A^c \cup B^c: \ P = 0.75$
  - What is the probability that at least one of her bets is successful? $P(A \cup B)$
  - Need to find: $A \cap B$ in order to know $P(A \cup B)$
- Problem 2
  - $P(A) = 0.2, P(B) = 0.4, P(A|B) + P(B|A) = 0.75$
  - Find $P(A \cap B)$
  - Given that B occurs, what is the probability that A occurs? $P(A|B)$
  - Given that B occurs, what is the probability that A does not occur? $P(A^c|B)$

$$P(A|B) + P(B|A) = 0.75$$
$$\frac{P(A \cap B)}{P(B)} + \frac{P(A \cap B)}{P(A)} = 0.75$$

- Problem 3
  - Pick a number in $[1, 100]$
  - If in $[21, 44]$ answer Q_1 (Do you text while driving?) else, answer Q_2 (Is the month of your birthday even?).
  - 37.5% of all responses are "Yes."
  - What is the percentage of drivers who answer "Yes" to Q_1? $P(Y|T)$
  - T is the event that they answer Q_1.

- Y is the event that they answer "Yes."

$$\begin{aligned} P(Y) &= 0.375 \\ &= P(Y|T)P(T) + P(Y|T^c)P(T^c) \\ &= P(T \cap Y) + P(T^c \cap Y) \end{aligned}$$

- Problem 4
  - Coin equally likely to be in 5 boxes - if in box and you look in box, 0.8 chance to see coin, 0.2 chance to not see coin.
  - If you look inside the first box, and do not see the coin, what is the probability that the coin is in the second box?
  - $P(\text{“2”}|\text{didn't find in first box})$
  - B is the event that the coin is found in the first box
  - $P(\text{“2”}|B^c) = \frac{P(\text{“2”} \cap B^c)}{P(B^c)}$
- Problem 5
  - With a pair of fair six-sided dice rolled until a sum of 8 is obtained, what is the probability that a sum of 7 is not obtained before an 8?
  - (no 7 before 8) $= \{8,\ x8,\ xx8,\ \dots\}$ where x is not a sum of 7 or 8
  - $S = \{8,\ x8,\ xx8,\ \dots\}$ where x is not a sum of 8
  - $P(\text{“8”}) = \frac{5}{36}$
  - $P(\text{“7”}) = \frac{6}{36}$
  - $P(\text{“7”}^c \cap \text{“8”}^c) = 1 - P(\text{“7”} \cup \text{“8”}) = \frac{25}{36} = p$
  - $P(\text{no 7 before 8}) = \frac{5}{36} + \frac{5}{36}p + \frac{5}{36}p^2 + \dots$

2 Lesson 1 Cont.

2.1 Counting

2.2 Definitions

1. Factorial: $n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$
2. Permutation: $nPr = \frac{n!}{(n-r)!}$

2.2.1 Examples

- Cards
  - $P(\text{black or ace}) = P(B \cup A) = P(B) + P(A) - P(B \cap A)$
  - $P(\text{black or ace}) = \frac{26+4-2}{52} = \frac{28}{52}$
- 3 Fair dice
  - $P(\text{all 5 or 6}) = \frac{2 \times 2 \times 2}{6 \times 6 \times 6} = \frac{8}{216}$

- Probilility that all kids sit together when 3 kids and 2 parents are arranged randomly in a row of 5 seats?
  - $P(\text{all kids together}) = \frac{3! \times 3!}{5!} = \frac{6}{120} = \frac{1}{20}$
- In a room of 25 people, what is the probability that at least two people have the same birthday?
  - $P(\text{at least two people have the same birthday}) = 1 - P(\text{all birthdays are different})$
  - $P(\text{all birthdays are different}) = \frac{365 \times 364 \times \cdots \times 341}{365^{25}}$
  - $P(\text{at least two people have the same birthday}) = 1 - \frac{365 \times 364 \times \cdots \times 341}{365^{25}}$
  - $P(\text{at least two people have the same birthday}) = 1 - \frac{365P25}{365^{25}} \approx 0.43$

3 Lesson 2 Cont.

3.1 Definitions

1. Independent Events: for events A and B, A and B are independent if $P(A|B) = P(A)$
   $\therefore P(A \cap B) = P(A)P(B)$

3.2 Examples

1. A and B are independent events. $P(A) = 0.3, P(B) = 0.8$. Find $P(A \cup B)$

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= P(A) + P(B) - P(A)P(B) \\ &= 0.3 + 0.8 - 0.3 \times 0.8 \\ &= 0.3 + 0.8 - 0.24 \\ &= 0.86 \end{aligned}$$

2. A lightbulb has a 2% chance of failure, independently of other lightbulbs. What is the probability a chain of 15 lightbulbs will work?

$$\begin{aligned} P(\text{all 15 lightbulbs work}) &= P(\text{lightbulb works})^{15} \\ &= (1 - 0.02)^{15} \\ &= 0.98^{15} \\ &\approx 0.74 \end{aligned}$$