Question 1: Probability

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1 Part A

Visitors to your website are asked to answer a single survey question before they get access to the content on the page. Among all of the users, there are two categories: Random Clicker (RC), and Truthful Clicker (TC). There are two possible answers to the survey: yes and no. Random clickers would click either one with equal probability. You are also giving the information that the expected fraction of random clickers is 0.3. After a trial period, you get the following survey results: 65 percent said Yes and 35 percent said No. What fraction of people who are truthful clickers answered yes? Hint: use the rule of total probability.

P(RC) = 0.3
P(TC) = 1 - P(RC) = 0.7
P(Yes given RC) = 0.5
P(No given RC) = 0.5
P(Yes given TC) = ?
P(No given TC) = 1 - P(Yes given TC) = ?
Survey results:
P(Yes) = 0.65
P(No) = 0.35
P(Yes) = P(Yes given RC) * P(RC) + P(Yes given TC) * P(TC)
0.65 = 0.5 * 0.3 + P(Yes given TC) * 0.7
P(Yes given TC) = (0.65 - 0.5 * 0.3) / 0.7
P(Yes given TC) = (0.65 - 0.15) / 0.7
P(Yes given TC) = 0.5 / 0.7
ANSWER: P(Yes given TC) 0.7143 or 71.43 percent of people who are truthful clickers answered Yes.

2 Part B

Imagine a medical test for a disease with the following two attributes: The sensitivity is about 0.993. That is, if someone has the disease, there is a probability of 0.993 that they will test positive. The specificity is about 0.9999. This means that if someone doesn't have the disease, there is probability of 0.9999

that they will test negative. In the general population, incidence of the disease is reasonably rare: about 0.0025 percent of all people have it (or 0.000025 as a decimal probability). Suppose someone tests positive. What is the probability that they have the disease?

I will use Bayes theorem which states:

P(A given B) = (P(B given A) * P(A)) / P(B)

P(Disease given Pos Test) = (P(Pos Test given Disease) * P(Disease)) / P(Pos Test)

P(Disease given Positive Test) = ?

P(Positive Test given Disease) = 0.993

P(Disease) = 0.000025

Calcuation of P(Positive Test):

P(Positive Test) = P(Positive Test given Disease) * P(Disease) + P(Positive Test given No Disease) * P(No Disease)

P(Positive Test given No Disease) = 1 - Specificity = 1 - 0.9999 = 0.0001

P(No Disease) = 1 - P(Disease) = 1 - 0.000025 = 0.999975

P(Disease given Positive Test) = (0.993 * 0.000025) / P(Positive Test)

P(Positive Test) = (0.993 * 0.000025) + (0.0001 * (1 - 0.000025))

ANSWER: P(Disease given Positive Test) = 0.1988 or 19.88 percent chance if someone tests positive for the disease, they actually have it.