Week 5 Assignment

MATH 511 Christina Morgenstern

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1. A bus arrives every 10 minutes at a bus stop. It is assumed that the waiting time for a particular individual is a random variable with a continuous variable with a *continuous uniform distribution*.

a) What is the probability that the individual waits more than 7 minutes?

The density function of the continuous uniform random variable X on the interval [A, B] is:

$$f(x, A, B) = \begin{cases} \frac{1}{B-A}, & A \leq x \leq B \\ 0, & elsewhere \end{cases}$$

In the example, the interval [A, B] corresponds to the time interval every 10 minutes [0, 10].

The probability that an individual waits more than 7 minutes is given by the continuous uniform random variable *X:* $P(X \geq 7)$

In order to find the probability $P(X \geq 7)$, we need to calculate the area under the rectangle for the interval [7,10].

$$P(X \geq 7) = \frac{(c-d)}{(B-A)} = \frac{10-7}{10-0} = \mathbf{0.3}$$

The probability, that a person will wait more than 7 minutes is 0.3 (or 30%).

R command:

```
> 1-punif(7,0,10)
[1] 0.3
```

b) What is the probability that the individual waits between 2 and 7 minutes?

$$P(2 \leq X \leq 7) = \frac{(c-d)}{(B-A)} = \frac{7-2}{10-0} = \mathbf{0.5}$$

The probability, that a person will wait between 2 and 7 minutes is 0.5 (or 50%).

R command:

```
> punif(7,0,10)-punif(2,0,10)
[1] 0.5
```

2. **The loaves of rye bread distributed to local stores by a certain bakery have an average length of 30 centimeters and a standard deviation of 2 centimeters. Assuming that the lengths are *normally distributed*, what percentage of the loaves are:**

a) longer than 31.7 centimeters?

X is a normal random variable, which measures the length of rye bread loaves.

$\mu = 30\ cm$
$\sigma = 2\ cm$
$\sigma^2 = 4\ cm$

We should find the area under the curve that lies to right of 31.7%.

Transform the random variable *X* with given μ=30 cm and σ^2=4 cm, to the random variable *Z* with μ=30 cm and σ^2=2.

$$z = \frac{X-\mu}{\sigma} = \frac{31.7-30}{2} = 0.85$$

Hence,

$$P(X > 31.7) = P(Z > 0.85) = 1 - P(Z < 0.85) = 1 - 0.8023 = \mathbf{0.1977}$$

About 20% of rye bread loaves are longer than 31.7 cm.

R command:

```
> 1-pnorm(0.85, 0, 1)
[1] 0.1976625
```

b) between 29.3 and 33.5 centimeters in length?

We should find the area under the curve that lies between 29.3% and 33.5%.
Calculate z_1 and z_2 values from x_1 (29.3) and x_2 (33.5) values to make use of the standard normal distribution.

$$z_1 = \frac{x_1-\mu}{\sigma} = \frac{29.3-30}{2} = -0.35$$

$$z_2 = \frac{x_2-\mu}{\sigma} = \frac{33.5-30}{2} = 1.75$$

Hence,

$$P(29.3 < X < 33.5) = P(-0.35 < Z < 1.75) = P(1.75) - P(-0.35) = 0.9599 - 0.3632 = \mathbf{0.5967}$$

About 60% of rye bread loaves lie between 29.3 cm and 33.5 cm of length.

R command:

```
> pnorm(1.75,0,1)-pnorm(-0.35,0,1)
[1] 0.5967715
```

3. In the November 1990 issue of *Chemical Engineering Progress*, a study discussed the percent purity of oxygen from a certain supplier. Assume that the mean was 99.61 with a standard deviation of 0.08. Assume that the distribution of percent purity was approximately *normal*.

a) What percentage of the purity values would you expect to be between 99.5 and 99.7?

X is a normal random variable, which measures percent purity of oxygen.

$\mu = 99.61$
$\sigma = 0.08$
$\sigma^2 = 0.0064$

We should find the area under the curve that lies between 99.5 and 99.7.
Calculate z_1 and z_2 values from x_1 (99.5) and x_2 (99.7) values to make use of the standard normal distribution.

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{99.5 - 99.61}{0.08} = -1.375$$

$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{99.7 - 99.61}{0.08} = 1.125$$

Hence,

$P(99.5 < x < 99.7) = P(-1.375 < z < 1.125) = P(z < 1.125) - P(z < 1.375) = \mathbf{0.7851}$

R command:

```
> pnorm(1.125,0,1)-pnorm(-1.375,0,1)
[1] 0.7851398
```

About 78.51% of oxygen purity values lie between 99.5 and 99.7.

b) What purity value would you expect to exceed 5% of the population?

Given the normal distribution of oxygen purities with μ=99.61 and σ=0.08, find the value of x, that has 5% of the area to the right.

$$z = \frac{x - \mu}{\sigma}$$

We require a z value that leaves 0.05 of the area to the right. From book´s table A.3, we find the z value for the area of 0.0505 to be -1.604. Hence,

$$x = \sigma z + \mu = (0.08)(-1.604) + 99.61 = \mathbf{99.48168}$$

The oxygen purity value exceeding 5% of the population is about 99.48.

4. A certain type of device has an advertised failure rate of 0.01 per hour. The failure rate is constant and the *exponential distribution* applies.

a) What is the mean time to failure?

$1/\beta = 0.01$

$\mu = \beta = 100$

The mean time to failure is 100 hours.

b) What is the probability that 200 hours will pass before a failure is observed?

Given $f(x) = \begin{cases} 0.01e^{-0.01x}, x > 0 \\ 0, \quad elsewhere \end{cases}$

$$P(X > 200) = \int 0.01\, e^{-0.01x} dx = -e^{-0.01x}| = e^{-2}$$

R command:

```
> 1-pexp(200,0.01)
[1] 0.1353353
```

The probability, that 200 hours will pass before a failure is observed is approx. 0.1353 (or 13.53%).