Week 3 Homework

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1.
x=4, y=5, z=-1 is a solution to the given system of linear equations. I just plugged in the corresponding values for *x, y* and *z* into the equations and calculated the left sides. In all three cases, the left sides yielded the values on the right side. All three equations are true, and thus the system of linear equations has a solution with the given values of *x, y* and *z*.

$$\begin{aligned} 1x + 3y + 0z &= 19 \\ 0x + 1y + 3z &= 2 \\ -2x + -6y + 1z &= -39 \end{aligned}$$

Solving the equations:

$$1 \times 4 + 3 \times 5 + 0 \times -1 = 19$$
$$4 + 15 + 0 = 19$$

$$0 \times 4 + 1 \times 5 + 3 \times -1 = 2$$
$$0 + 5 - 3 = 2$$

$$-2 \times 4 + -6 \times 5 + 1 \times -1 = -39$$
$$-8 - 30 - 1 = -39$$

2. Solve the system of linear equations:

$$\begin{aligned} Row\ 1: &-2x + (-1)y + (-1)z = -4 \\ Row\ 2: &\ 3x + 1y + 0z = 5 \\ Row\ 3: &\ 3x + 1y + 1z = 6 \end{aligned}$$

1. Multiply row 1 with **-0.5**:

$$\begin{aligned} Row\ 1: &\ 1x + 0.5y + 0.5z = 2 \\ Row\ 2: &\ 3x + 1y + 0z = 5 \\ Row\ 3: &\ 3x + 1y + 1z = 6 \end{aligned}$$

2. Subtract row 2 from row 3:

$$\begin{aligned} Row\ 1: &\ 1x + 0.5y + 0.5z = 2 \\ Row\ 2: &\ 3x + 1y + 0z = 5 \\ Row\ 3: &\ 0x + 0y + 1z = 1 \end{aligned}$$

3. Subtract 3 times row 1 from row 2:

$$\begin{aligned} Row\ 1: &\ 1x + 0.5\,y + 0.5z = 2 \\ Row\ 2: &\ 0x + -0.5y + -1.5z = -1 \\ Row\ 3: &\ 0x + 0y + 1z = 1 \end{aligned}$$

4. Add row 2 to row 1:

$$\begin{array}{lrcrcrcr} Row\ 1: & 1x & + & 0\,y & + & -1z & = & 1 \\ Row\ 2: & 0x & + & -0.5y & + & -1.5z & = & -1 \\ Row\ 3: & 0x & + & 0y & + & 1z & = & 1 \end{array}$$

5. Add row 3 to row 1:

$$\begin{array}{lrcrcrcr} Row\ 1: & 1x & + & 0\,y & + & 0z & = & 2 \\ Row\ 2: & 0x & + & -0.5y & + & -0.5z & = & -1 \\ Row\ 3: & 0x & + & 0y & + & 1z & = & 1 \end{array}$$

6. Divide row 2 by **-0.5**:

$$\begin{array}{lrcrcrcr} Row\ 1: & 1x & + & 0y & + & 0z & = & 2 \\ Row\ 2: & 0x & + & 1y & + & 3z & = & 2 \\ Row\ 3: & 0x & + & 0y & + & 1z & = & 1 \end{array}$$

7. Subtract 3 times row 3 from row 2:

$$\begin{array}{lrcrcrcr} Row\ 1: & 1x & + & 0y & + & 0z & = & 2 \\ Row\ 2: & 0x & + & 1y & + & 0z & = & -1 \\ Row\ 3: & 0x & + & 0y & + & 1z & = & 1 \end{array}$$

The solution for the given system of linear equations is: ***x*=2, *y*=-1, *z*=1**

3. Matrix multiplication

$$\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} (3\times 4)+(-1\times 2) & (3\times -3)+(-1\times 3) \\ (1\times 4)+(2\times 2) & (1\times -3)+(2\times 3) \end{bmatrix} = \begin{bmatrix} \mathbf{10} & \mathbf{-12} \\ \mathbf{8} & \mathbf{3} \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 4 \\ 0 & 4 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \\ 4 & 1 & 2 \\ 3 & -1 & 6 \end{bmatrix} =$$

$$\begin{bmatrix} (1\times 2)+(-3\times 4)+(4\times 3) & (1\times 3)+(-3\times 1)+(4\times -1) & (1\times 4)+(-3\times 2)+(4\times 6) \\ (0\times 2)+(4\times 4)+(1\times 3) & (0\times 3)+(4\times 1)+(1\times -1) & (0\times 4)+(4\times 2)+(1\times 6) \end{bmatrix}$$
$$= \begin{bmatrix} \mathbf{2} & \mathbf{-4} & \mathbf{22} \\ \mathbf{19} & \mathbf{3} & \mathbf{14} \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & -2 \\ 0 & 4 & 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \\ 4 & 1 & 2 \end{bmatrix} =$$ cannot be solved, because the number of columns of the first matrix (which is 3) is NOT the same as the number of rows of the second matrix (which is 2).