Week 2 Assignment

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1. **Out of six computer chips, two are defective. If two of the chips are chosen randomly for testing, compute the probability that both of them are defective. List all the outcomes in the sample space, if we were checking all six computer chips.**

Let *D* stand for defective computer chips and *N* for non-defective computer chips.
The sample space for this experiment is:

$$S = \{DD, DN, ND, NN\}$$

The probability for examining 2 defective chips is:

$$P(DD) = \frac{2}{6} \times \frac{1}{5} = \mathbf{\frac{1}{15}} \cong \mathbf{0.067}$$

The probability, of choosing 2 defective computer chips out of six is $\frac{1}{15}$.

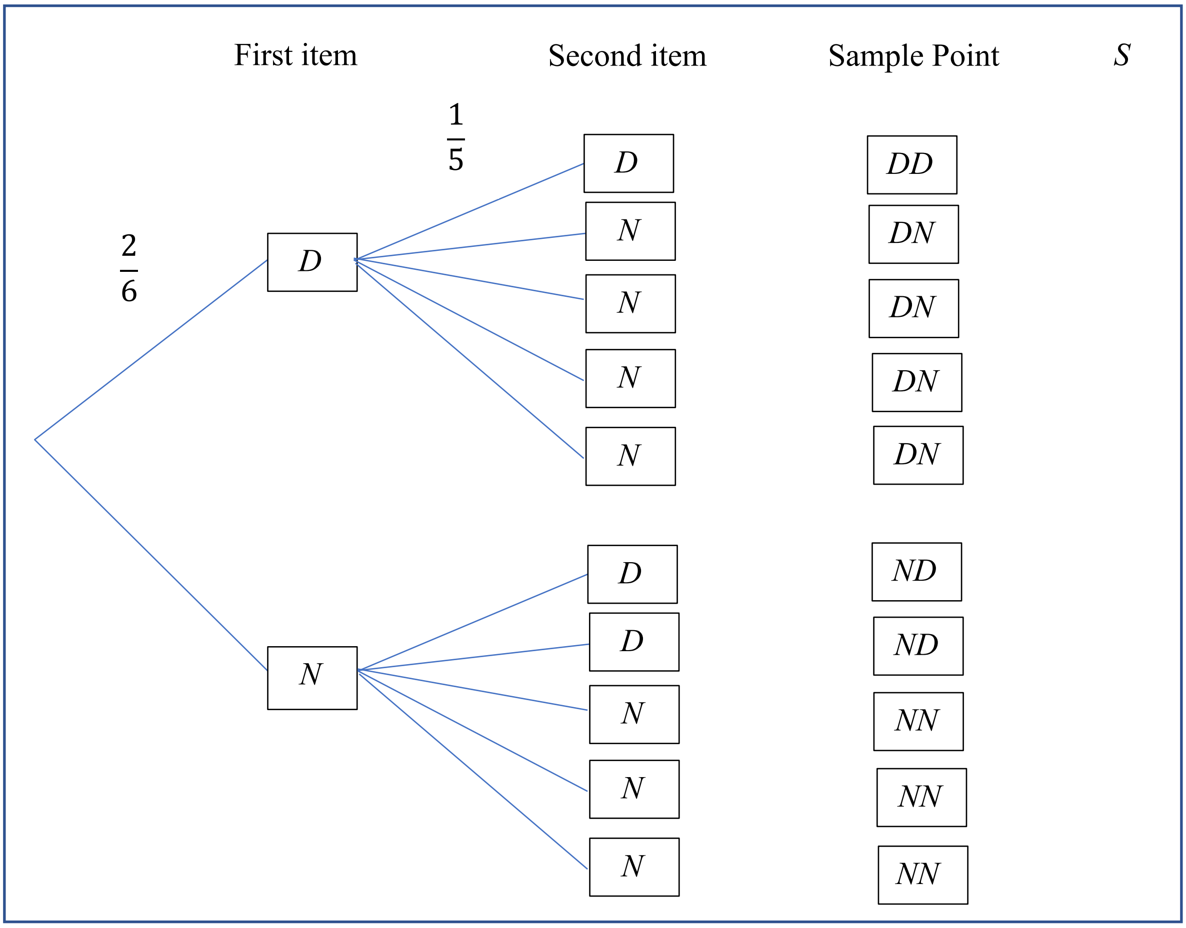

Figure 1. Tree diagram of exercise 1, showing the sample space and individual sample points of all possible outcomes.

2. A quiz consists of 6 multiple-choice questions. Each question has 4 possible answers. A student is unprepared, and he has no choice but to guess answers completely at random. He passes the quiz if he gets at least 3 questions correctly. What is the probability that he will pass?

For every question, the probability of guessing the correct answer is $\frac{1}{4}$.

The probability, that he will answer 3 questions correctly is:

$$P(c = 3) = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64} \cong 0.0156$$

Or he might guess 4 questions correctly:

$$P(c = 4) = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{256} \cong 0.0039$$

Or he might guess 5 questions correctly:

$$P(c = 5) = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{1024} \cong 0.00098$$

Or he might guess all 6 questions correctly:

$$P(c = 6) = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{4096} \cong 0.00024$$

As the events of passing with 3, 4, 5 or 6 correct questions are mutually exclusive events, the overall probability of passing the quiz can be calculated using the Additive rule.

$$P(correct > 3) = P(c = 3) + P(c = 4) + P(c = 5) + P(c = 6)$$
$$= \frac{1}{64} + \frac{1}{256} + \frac{1}{1024} + \frac{1}{4096} = \mathbf{0.02}$$

The probability, that he will pass the quiz with at least 3 correct answers, is about 0.02.

A computer program is tested by 5 *independent* tests. If there is an error, these tests will discover it with probabilities 0.1, 0.2, 0.3, 0.4, and 0.5 respectively. Suppose that the program contains an error.
What is the probability that it will be found

3. **by at least one test? (*Hint: this event is the complement of the event where the error is not found*)**
4. **by all five tests?**

3.

Let A be the event, where the error is not found by any test. Then A' is the event (complement), that picks up the error by at least one test.

$$P(A) + P(A') = 1$$

$$P(A') = 1 - P(A)$$

$$P(A) = 0.9 \times 0.8 \times 0.7 \times 0.6 \times 0.5 = 0.1512$$

$$P(A') = 1 - 0.1512 \cong \mathbf{0.85}$$

The probability, that at least one test picks up the error in the program is 0.85.

4.
To obtain the probability, that all five independent tests will pick up the error in the computer program, we can find the product of their individual probabilities.

Let A_{All} be the event, where all 5 tests pick up the error in the program independently.

Let $P(A_1)$, $P(A_2)$, $P(A_3)$, $P(A_4)$ and $P(A_5)$ be the probabilities of successful picking up an error in a computer program by five independent tests.

$$\begin{aligned} P(A_{All}) = P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5) &= P(A_1) \times P(A_2) \times P(A_3) \times P(A_4) \times P(A_5) \\ &= 0.1 \times 0.2 \times 0.3 \times 0.4 \times 0.5 = \mathbf{0.0012} \end{aligned}$$

The probability, that all five tests pick up the error in the program independently is 0.0012.