Week 7 Assignment

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1. **An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of 40 hours. If a sample of 30 bulbs has an average life of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm.**

   Population standard deviation: $\sigma = 40$
   Sample size: $n = 30$
   Sample mean: $\bar{x} = 780$

   $\alpha = 1 - 0.96 = 0.04$
   $z_{0.02} = 2.054$

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> qnorm(1-0.02)
[1] 2.053749
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   The point estimate of μ is $\bar{x} = 780$. The *z*-value leaving an area of 0.02 to the right, and therefore an area of 0.98 to the left is, $z_{0.02} = 2.054$. Hence, the 96% confidence interval is

$$\bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

$$780 - (2.054)\frac{40}{\sqrt{30}} < \mu < 780 + (2.054)\frac{40}{\sqrt{30}}$$

$$\mathbf{765} < \mu < \mathbf{795}$$

   We can be 96% confident, that the average length of life of light bulbs manufactured by a firm lies between 765 and 795 hours.

2. **How large a sample is needed in the previous exercise if we wish to be 96% confident that our sample mean will be within 10 hours of the true mean?**

   e=10

$$n = (\frac{z_{\alpha/2}\sigma}{e})^2$$

$$n = (\frac{(2.054)(40)}{10})^2 = (\frac{82.16}{10})^2 = \mathbf{67.5}$$

$$\boldsymbol{n = 68}$$

   We can be 96% confident, that a random sample of size 68 will provide an estimate $\bar{x}$ differing from μ by less than 10 hours.

3. The following measurements were recorded for the drying time, in hours, of a certain brand of latex paint:

3.4, 2.5, 4.8, 2.9, 3.6, 2.8, 3.3, 5.6, 3.7, 2.8, 4.4, 4.0, 5.2, 3.0, 4.8.

Assuming that the measurements represent a random sample from a normal population, find a 95% *prediction* interval for the drying time for the next trial of the paint.

Unknown mean and standard deviation of population
Sample size: $n = 15$
Degrees of freedom: $v = 14$

Calculated sample mean: $\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n} \approx 3.787$
Calculated sample standard deviation:

$$s^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})}{n-1}$$

$$s = \sqrt{s} \approx 0.971$$

$\alpha = 1 - 0.95 = 0.05$
$t_{0.025} = 2.145$

For v=14 degrees of freedom, $t_{0.025}$= 2.145 (Table A4) or using R:

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> qt(0.025,14)
[1] -2.144787
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Hence, the 95% prediction interval is

$$\bar{x} - t_{\alpha/2} s\sqrt{1 + 1/n} < x_0 < \bar{x} + t_{\alpha/2} s\sqrt{1 + 1/n}$$

$$3.787 - (2.145)(0.971)\sqrt{1 + 1/15} < x_0 < 3.787 + (2.145)(0.971)\sqrt{1 + 1/15}$$

$$\mathbf{1.636} < x_0 < \mathbf{5.938}$$

Which reduces to the interval $(1.64\ h, 5.94\ h)$

We can be 95% certain, that a future observation of drying time of a latex paint lies between 1.64 and 5.94 hours.

4. Compute a 98% confidence interval, using method 1 in page 297 of your textbook, for the proportion of defective items in a process when it is found that a sample of size 100 yields 8 defectives.

Sample size: $n = 100$
Proportion of defective items: $\hat{p} = \frac{8}{100} = 0.08$
Proportion of all other items: $\hat{q} = 1 - \hat{p} = 1 - 0.08 = 0.92$

$\alpha = 1 - 0.98 = 0.02$
$z_{0.01} = 2.326$

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> qnorm(1-0.01)
[1] 2.326348
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The point estimate of p is $\hat{p} = 0.08$. Using R/Table A3, we find that $z_{0.01} = 2.326$. Therefore, using method 1, the 98% confidence interval for p is

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.08 - (2.326)\sqrt{\frac{(0.08)(0.92)}{100}} < p < 0.08 + (2.326)\sqrt{\frac{(0.08)(0.92)}{100}}$$

$$\mathbf{0.0169 < p < 0.1431}$$

We can be 98% confident, that a process yields defective items with a proportion between 1.7% and 14.3%.