Week 1 Homework

Math 510 Christina Morgenstern

1.

Graph 1 has a limit at x = 1, because if the value 1 is approached from both sides, it approximates to the same value. A reasonable estimate of the limit as x approaches 1 is 10 ($).

$$\lim_{x \to 1} f(x) = 10$$

If x approaches 1 from the left side, values for *f(x)* are increasing towards 10.

x	*f(x)*
0.9	Approx. 5.9
0.99	Approx. 7.8
0.999	Approx. 9.8

As x approaches 1 from the right side, values for *f(x)* are increasing towards 10.

x	*f(x)*
1.5	Approx. 2
1.9	Approx. 7
1.99	Approx. 9.8

Graph 2 has a limit at x = 1 and it´s about 0.9 ($). As the value 1 is approached from both sides, in both cases, it approximates to the value of 0.9.

$$\lim_{x \to 1} f(x) = 0.9$$

Graph 3 has NO limit at x = 1, because the function contains jumps. As x approaches 1.0 from the left, values of *f(x)* are increasing, as x approaches 1.0 from the right, values of *f(x)* are decreasing. Since both attempts of getting infinitely close to 1.0 do not approach the same value, there is no limit.

$$\lim_{x \to 1} f(x) = Does\ not\ exist$$

2.

A limit does not exist in a function, that is unbounded towards infinity i.e. if the limit cannot be approached from both sides and leading to different values from the right- and left-hand sides, respectively.

$$f(x) = \frac{1}{x}$$

$$\lim_{x \to 0} f\left(\frac{1}{x}\right) = Does\ not\ exist$$

3.
The function in graph 1 is discontinuous, leading to infinity.

The function in graph 2 is continuous, it is smooth and has no holes, jumps or gaps.

The function in graph 3 is discontinuous, because it contains a hole at x = 2.0, meaning the function is not defined at this point.

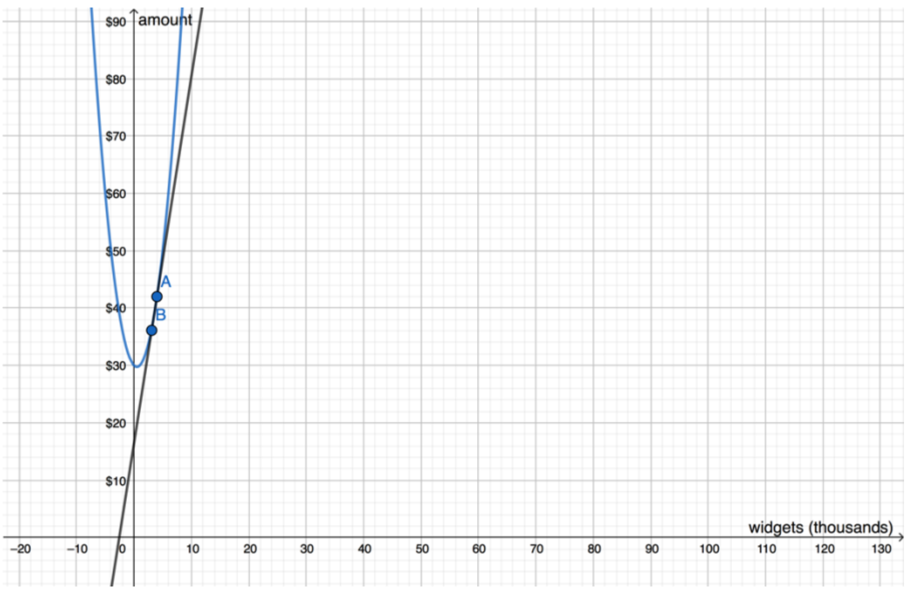
4.

- False. Only true for continuous functions not for discontinuous ones.
- True.
- False. Function can be undefined at some point, but limit can still exist.
- False. If there is no limit, there can still a function value be defined at some point.
- True.
- True.

5.
The average rate of change of f between x = 3 and x = 4 is calculated as follows:

$$\frac{(42-36)}{(4-3)} = 6 \;\; amount\ of\ revenue(\$)\ per\ widgets\ (thousands)$$

On average, the revenue of the company is 6$ per 1000 widgets produced.



Graph Problem 5 (graphed using GeoGebra)
$f(x) = x^2 - x + 30$

6.
The instantaneous rate of change cannot be determined, when the limit cannot be determined.

A natural class of examples would be paths of Brownian motion. These are continuous but non-differentiable everywhere.

7.
Instantaneous velocity when kicking a ball, riding a bike, skiing down the hill: the velocity of the ball, the bike, skier can be found at any time *x*.

Height of a person changes with time. Finding the instantaneous height at a certain age.

8.
Graph 1and Graph B: If the quadratic function is differentiated, the result is a linear curve. If *f(x)* is decreasing, the slope of the tangent line is negative. If *f(x)* is increasing, the slope of the tangent line is positive.
Graph 2 and Graph C: The differentiation of the cubic functions leads to a quadratic curve (parabola).
Graph 3 and Graph A: when the quadratic curve is differentiated, it becomes a linear curve.

9.
The function $f'(x) = 6$ describes the instantaneous rate of change of the function f at $x = 6$.
In lay terms, it states, the time of a claim to be processed per number of workers at the company if the company employs 6000 workers.
If $f'(x) > 0$ is increasing, the slope of the tangent line is getting more positive. The time of a claim to be processed decreases as the number of workers increases.
If $f'(x) < 0$ is decreasing, the slope of the tangent line is negative.
The time of a claim to be processed increases as the number of workers decreases.

10.
A function is not differentiable at a point, if there is a discontinuity or a sharp corner.
Examples might be the fluctuations of some stock exchange index, which first rises to some point over time and then unexpectedly crashes.