Week 5 Homework

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1.

$A = \begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$

$\det(A) = ad - bc$

$\det(A) = 3 * 2 - (-1) * 4 = 6 + 4 = 10$

The determinant of the matrix *A* is 10.

2.

$A = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$

$A = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

$\det(A) = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} = 3 * 2 - 0 * 0 = 6$

The determinant of the matrix *A* is 6.

3.

The determinant of the matrix *A* is 0:

$\mathrm{A} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$\det(A) = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = 2 * 2 - 2 * 2 = 4 - 4 = 0$

$A\overrightarrow{e_1}$ and $A\overrightarrow{e_1}$ is $non - zero$:

$\overrightarrow{e_1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
$\overrightarrow{e_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$A\overrightarrow{e_1} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 * 0 + 2 * 1 \\ 2 * 0 + 2 * 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$A\overrightarrow{e_2} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2*1+2*1 \\ 2*1+2*1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

$\vec{v}$ gets send to zero by A:

$\vec{v} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

$A\vec{v} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2*-2+2*2 \\ 2*-2+2*2 \end{bmatrix} = \begin{bmatrix} -4+4 \\ -4+4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

4.

$A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$

$A - \lambda I_n = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 4-\lambda & 2-0 \\ 1-0 & 3-\lambda \end{bmatrix} = (4-\lambda)(3-\lambda) - 2$

$= 12 - 3\,\lambda - 4\lambda + \lambda^2 - 2 = \lambda^2 - 7\lambda + 10 = 0$
$= (\lambda - 2)(\lambda - 5) = 0$

$So\ \lambda = 2\,,5$

$For\ \lambda = 2, \qquad A - \lambda I_n = \begin{bmatrix} 4-2 & 2 \\ 1 & 3-2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$

$Doing\ the\ matrix\ muliplication\ you\ get\ the\ system\ of\ linear\ equations$:
$2x + y = 0$
$2x + y = 0$

x=1 and y=-2 is a possible solution.

$So \begin{bmatrix} 1 \\ -2 \end{bmatrix}\ is\ an\ eigenvector\ for\ the\ eigenvalue\ \lambda = 2.$

$For\ \lambda = 5, \qquad A - \lambda I_n = \begin{bmatrix} 4-5 & 2 \\ 1 & 3-5 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix}$

$Doing\ the\ matrix\ muliplication\ you\ get\ the\ system\ of\ linear\ equations$:
$-x + y = 0$
$2x - 2y = 0$

x=1 and y=1 is a possible solution.

$So \begin{bmatrix} 1 \\ 1 \end{bmatrix}\ is\ an\ eigenvector\ for\ the\ eigenvalue\ \lambda = 5.$

Mapping eigenvectors with matrix A:

$$A\overrightarrow{e_1} = \lambda\overrightarrow{e_1} = 2\begin{bmatrix}1\\-2\end{bmatrix} = \begin{bmatrix}2\\-4\end{bmatrix}$$

$$A\overrightarrow{e_2} = \lambda\overrightarrow{e_2} = 5\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}5\\5\end{bmatrix}$$

Through transformation by A, eigenvectors experience only changes in magnitude or sign, the orientation is the same as that of the original vector. The eigenvalue λ is the amount of "stretch" or "shrink" to which the eigenvector is subjected when transformed by A.

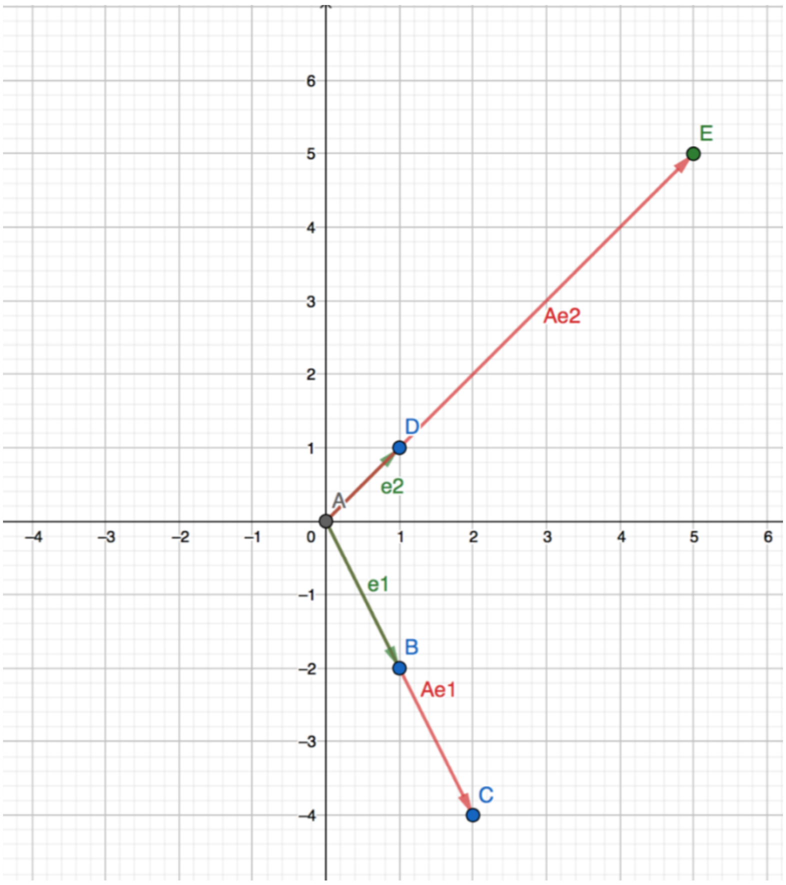

Fig. 1 Graph of the eigenvectors $\overrightarrow{e_1}$ and $\overrightarrow{e_2}$ (green) and mapping of $\overrightarrow{e_1}$ and $\overrightarrow{e_2}$ with A (red).

5.

$\overrightarrow{e_1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
$\lambda = 3$

$\overrightarrow{e_2} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$
$\lambda = 6$

If $\overrightarrow{e_1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is an eigenvector of *A* corresponding to eigenvalue λ=3, then $\overrightarrow{e_2} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ is an eigenvector of *A* as well because $\overrightarrow{e_2}$ is a multiple of $\overrightarrow{e_1}$.

$\overrightarrow{e_2}$ is an eigenvector of *A* corresponding to the eigenvalue λ=6, if and only if $\overrightarrow{e_2}$ and λ satisfy $(A-\lambda I_n)\,\overrightarrow{e_2} = 0$.