Week 2 Homework

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1. The derivatives of the given functions are:

- $f(x) = 12x^3 \quad > \quad f'(x) = \mathbf{36x^2}$
- $g(x) = 3x^2e^x \quad > \quad g'(x) = 6xe^x + 3x^2e^x = \mathbf{3x(x+2)e^x}$

2.

Calculate the function values and derivatives at x=3:

f(3)	*f´(3)*	*g(3)*	*g´(3)*
324	324	542	904

*Values of *g(3)* and *g´(3)* rounded to unit position

$$\boldsymbol{\frac{d}{dx}(fg)(x)}$$

Apply the product rule as follows:

$$\frac{d}{dx} = (fg)(3) = f'(3)g(3) + f(3)g'(3)$$

$$\frac{d}{dx} = (fg)(3) = \mathbf{468504}$$

$$\boldsymbol{\frac{d}{dx}(-2f)(3)}$$

Apply the constant rule as follows:

$$\frac{d}{dx}(-2f)(3) = 3 \times (-2 \times 36x^2) = \mathbf{-1944}$$

3.

I can imagine a fever curve, where a person´s body temperature is rising due to an infection above 37°C (local maximum). After taking some medication, the temperature drops to around normal again. However, it´s a serious infection and upon fading of the medicine, the body temperature rises again, and this time it climbs up really high until the body´s threshold (global maximum) is reached and the person might eventually die.

The discrepancy between the two maxima arises due to function values rising and decreasing at different rates.

Another example would be the adaptive immune response (see Figure 1). After first encounter with a foreign antigen the concentration of antibodies in the blood rises after a lag phase (local maximum; primary immune response). Upon continued exposure to the same antigen, the concentration of antibody in the blood rises to a higher level in order to be able to combat the foreign antigen (global maximum; secondary immune response).

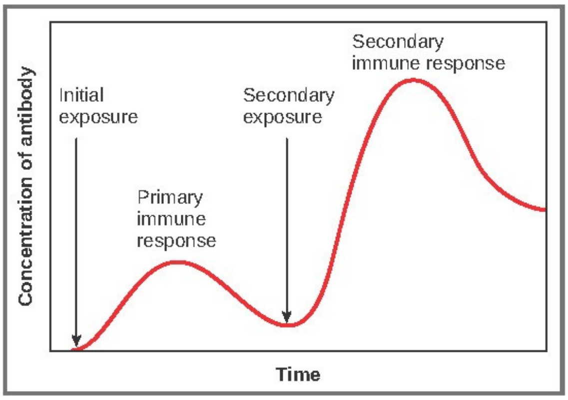

Figure 1. Primary and secondary immune response upon exposure to an antigen. The antibody concentration in the blood is plotted against time. (Fig. taken from: https://courses.lumenlearning.com/suny-ap2/chapter/the-adaptive-immune-response-b-lymphocytes-and-antibodies/

4.

The derivative represents the slope of the tangent line, which is 0 at points of maxima and minima. It represents kind of a turning point, where the function values are rising/decreasing before and are decreasing/rising afterwards and where the instantaneous rate of change is 0.

5.

Linear approximation would probably be a bad estimate for functions with only few data points and huge data gaps.

Or if the behavior of the function is somewhat chaotic and doesn´t follow a defined path.

6.

Given *f(2)* =5 and *f´(2)* = 3 estimate *f(2.5), f(1.5)*

I can approximate the function values at *f(2.5)* and *f(1.5)* by using the instantaneous rate (*f´(2)*) of change and my current position (*f(2)*).

$$f(2.5) \approx f(2) + f'(2) \times (2.5 - 2) \approx 5 + 3 \times 0.5 \approx \mathbf{6.5}$$

$$f(1.5) \approx f(2) - f'(2) \times (2 - 1.5) \approx \mathbf{3.5}$$