Week 6 Assignment

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Problem 1

1. The heights of 1000 students are approximately normally distributed with a mean of 174.5 cm and a standard deviation of 6.9 cm. Suppose 200 random samples of size 25 are drawn from this population and the means recorded to the nearest tenth of a centimeter. Determine:

a) the mean and standard deviation of the sampling distribution of $\bar{X}$;
b) the number of sample means that fall between 172.5 and 175.8 cm inclusive. (Hint: First compute the probability that a sample mean is between 172.5 and 175.8. Then multiply that number by the number of samples, 200).

a)

Population size: *1000*
Population mean: $\mu = 174.5$
Population standard deviation: $\sigma = 6.9$
Sample size: $n = 25$

Since the heights of the students in the population is approximately normally distributed, we can apply the Central Limit Theorem, when determining the mean and standard deviation of the sampling distribution of $\bar{X}$.

$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

$$\mu_{\bar{X}} = 174.5$$
$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{6.9}{\sqrt{25}} = \frac{6.9}{5} = 1.38$$

The sampling distribution of $\bar{X}$ has the same mean as the population ($\mu_{\bar{X}} = 174.5$) and a standard deviation of $\sigma_{\bar{X}} = 1.38$.

b)

$P(172.45 < \bar{X} < 175.85)$

Perform z-Transformation to convert normal distribution to standard normal distribution:

$$z_1 = \frac{172.45 - 174.5}{1.38} \approx -1.49$$

$$z_2 = \frac{175.85 - 174.5}{1.38} \approx 0.98$$

Make use of the table A.3 in the book and look up areas for the z-values:

$P(172.45 < \bar{X} < 175.85) = P(-1.49 < Z < 0.98) = 0.8365 - 0.0681 = 0.7684$

Calculation of the number of sample means, that fall between 172.5 and 175.8 inclusive:

$$(0.7684)(200) = \mathbf{153.68}$$

Approximately 154 sample means fall between 172.5 and 175.8 inclusive.

Verify using R:

```
> pnorm(0.98)-pnorm(-1.49)
[1] 0.7683448
```

```
> pnorm(175.85,174.5,1.38)-pnorm(172.45,174.5,1.38)
[1] 0.7673226
```

The latter calculation leads to more accurate results because the z-values were only rounded to 2 decimal places.

Problem 2

2. The amount of time that a drive-through bank teller spends on a customer is a random variable with a mean $\mu = 3.2$ minutes and a standard deviation $\sigma = 1.6$ minutes. If a random sample of 64 customers is observed, find the probability that their mean time at the teller's window is:

a) At most 2.7 minutes
b) more than 3.5 minutes

Population mean: $\mu = 3.2$
Population standard deviation: $\sigma = 1.6$
Sample size: $n = 64$

Since the sample size is sufficiently large (>30), we can apply the Central Limit Theorem, when determining properties about the sampling distribution of $\bar{X}$, because we can assume a normal distribution of the sampling distribution.

$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

$\mu_{\bar{X}} = 3.2$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{1.6}{\sqrt{64}} = \frac{1.6}{8} = 0.2$$

The sampling distribution of $\bar{X}$ has the same mean as the population ($\mu_{\bar{X}} = 3.2$) and a standard deviation of $\sigma_{\bar{X}} = 0.2$.

a)

$P(\bar{X} < 2.7)$

Perform z-Transformation to convert normal distribution to standard normal distribution:

$$Z = \frac{2.7 - 3.2}{0.2} = -2.5$$

$P(\bar{X} < 2.7) = P(Z < -2.5) = \mathbf{0.0062}$

The probability, that the mean time at the bank teller window is at most 2.7 minutes is 0.0062 or 0.62%.

Verify using R:

```
> pnorm(-2.5)
[1] 0.006209665

> pnorm(2.7, 3.2, 0.2)
[1] 0.006209665
```

b)

$P(\bar{X} > 3.5)$

Perform z-Transformation to convert normal distribution to standard normal distribution:

$$z = \frac{3.5 - 3.2}{0.2} = 1.5$$

$$P(\bar{X} > 3.5) = 1 - P(Z < 1.5) = 1 - 0.9332 = 0.0668$$

The probability, that the mean time at the bank teller window is more than 3.5 minutes is 0.00668 or 6.68%.

Verify using R:

```
> 1-pnorm(1.5)
[1] 0.0668072

> 1-pnorm(3.5,3.2,0.2)
[1] 0.0668072
```

Problem 3

3. If a certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms, what is the probability that a random sample of 36 of these resistors will have a combined resistance of more than 1458 ohms?
(Hint: What will the average resistance be if the combined resistance is more than 1458 ohms?)

Population mean: $\mu = 40$
Population standard deviation: $\sigma = 2$
Sample size: $n = 36$

Since the sample size is sufficiently large (>30), we can apply the Central Limit Theorem, when determining properties about the sampling distribution of $\bar{X}$, because we can assume a normal distribution of the sampling distribution.

$$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$$

$$\mu_{\bar{X}} = 40$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{36}} = \frac{2}{6} = \frac{1}{3} = 0.333$$

The sampling distribution of $\bar{X}$ has the same mean as the population ($\mu_{\bar{X}} = 40$) and a standard deviation of $\sigma_{\bar{X}} = 0.3$.

$$P\left(\bar{X} > \frac{1458}{36}\right) = P(\bar{X} > 40.5)$$

Perform z-Transformation to convert normal distribution to standard normal distribution:

$$z = \frac{40.5 - 40}{0.3} = 1.5$$

$$P\left(\sum_{i=1}^{36} X_i > 1458\right) = P(\bar{X} > 40.5) = P(Z > 1.5) = 1 - P(Z < 1.5) = 1 - 0.9332 = 0.0668$$

The probability, that a random sample of 36 will have a combined resistance greater than 1458 is 0.00668 or 6.68%.

Verify using R:

```
> 1-pnorm(1.5)
[1] 0.0668072

> 1-pnorm(40.5, 40, 1/3)
[1] 0.0668072
```

Problem 4:

4. A manufacturing firm claims that the batteries used in their electronic games will last an average of 30 hours. To maintain this average, 16 batteries are tested each month. If the computed t-value falls between $-t_{0.025}$ and $t_{0.025}$, the firm is satisfied with its claim. Should they be satisfied if they test a sample that has a mean of $\bar{x} = 27.5$ hours and a standard deviation of $s = 5$ hours? Assume the distribution of battery lives to be approximately normal.

Population mean: $\mu = 30$
Sample size: $n = 16$

Sample mean: $\mu_{\bar{X}} = 27.5$
Sample standard deviation: $s = 5$

If we have a random sample of n=15 from a normal distribution, the random variable *T* has a t-distribution with v=n-1=15 degrees of freedom.

From table A.4 we find that $t_{0.025} = 2.131$ for 15 degrees of freedom Therefore, the manufacturing firm can be satisfied with the sample, if the sample yields a t-value between -2.131 and 2.131:

$$t = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{27.5 - 30}{5/\sqrt{16}} = \frac{-2.5}{5/4} = \mathbf{-2}$$

The sample has a t-value of -2 and thus the firm will be satisfied.