

WEEK 4 ASSIGNMENT

Concepts of Statistics 2 – DATA-51200 | Spring 2 2020

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1. In your own words, describe (in less than one page) stage 3: Assumptions in Multiple Regression Analysis.

Stage 3 of the Multiple Regression Analysis process deals with assessing the assumptions of this multivariate technique. The following four areas need to be investigated before and after model estimation: 1. Linearity, 2. Homoscedasticity, 3. Normality of error term distribution and 4. Independence of error terms. These assumptions must be tested both for the dependent and independent variables as well as for the variate. Graphical visualization provides an effective means to inspect possible violations of the assumptions. The residual is the measure for prediction error and denotes the difference between the actual and the predicted values of the dependent variable. Thus, plotting the residuals versus the predicted dependent values leads to characteristic patterns which allow for simple inspection and judgement.

In the following the individual assumptions are briefly described and measures to correct for noted.

If linearity is met, we would expect to see a linear pattern of the residuals. Nonlinear relationships like curvilinear patterns can be corrected through data transformations (e.g. logarithm, square root, etc.), the incorporation of polynomial terms or the application of specific nonlinear regression techniques.

The occurrence of unequal variances in residuals can lead to an inaccurate estimation of the standard error and following decreased statistical power. Strategies to overcome this phenomenon of heteroscedasticity are the transformation of variables, the use of weighted least squares or special heteroscedasticity-consistent standard errors.

Normality should be assessed by using Normal probability plots, but histograms and visual inspection of the distribution are also applicable. If the sample size is large enough (>200) though, Multiple Regression shouldn't have issues with normality.

Addressing the independence of error terms is important to avoid groupings of residuals due to time series or hierarchically ordered data.

References:

[1] Multivariate Data Analysis by Joseph F. Hair Jr, William C. Black, Barry J. Babin and Rolphe E. Anderson, Pearson, 8th edition, 2019

2. (By Hand) For the dependent variable Y and the independent variables X1 and X2, the linear regression model is given by:

$Y = 0.08059 \cdot X_1 - 0.16109 \cdot X_2 + 5.26570$. Complete the following table:

Actual Y	x1	X2	Predicted Y	Residuals (Predication Error)
6	6.8	4.7	5.1	0.9
3.1	5.3	5.5	4.8	-1.7
5.8	4.5	6.2	4.6	1.2
4.5	8.8	7	4.8	-0.3
4.5	6.8	6.1	4.8	-0.3
3.7	8.5	5.1	5.1	-1.4
5.4	8.9	4.8	5.2	0.2
5.1	6.9	5.4	5.0	0.1
5.8	9.3	5.9	5.1	0.7
5.7	8.4	5.4	5.1	0.6

The predicted values for Y were calculated by plugging in the values for the independent variables X1 and X2 into the equation according to the table values. Values were rounded to one decimal place. The residuals were calculated by subtracting the predicted values of Y from the respective actual values of Y.

Is this a good model? Why? Why not?

To assess model fit, I would calculate the coefficient of determination (R^2) which is a common measure of predictive accuracy for regression models. It is calculated as the squared values from the correlations between the actual and predicted measures of the dependent variable. The values of R^2 range between 0 and 1, denoting bad and perfect prediction, respectively.

In order to calculate R^2 , I calculated the mean and the sum of squared errors (SS) of Y and Y predicted (\hat{Y}). Using this information, I calculated the correlation coefficient and subsequently R^2 .

$$M_Y = \frac{\sum Y_i}{n} = 4.96$$

$$SS_Y = \sum_{i=1}^n (Y_i - M_Y)^2 = 8.724$$

$$M_{\hat{Y}} = \frac{\sum \hat{Y}_i}{n} = 4.96$$

$$SS_{\hat{Y}} = \sum_{i=1}^n (\hat{Y}_i - M_{\hat{Y}})^2 = 0.344$$

$$\sum_{i=1}^n (Y_i - M_Y)(\hat{Y}_i - M_{\hat{Y}}) = 0.444$$

$$R = \sum_{i=1}^n \frac{(Y_i - M_Y)(\hat{Y}_i - M_{\hat{Y}})}{\sqrt{((SS_Y)(SS_{\hat{Y}}))}} = 0.2563$$

Although, there is a positive correlation denoted by the value of the correlation coefficient (R) of 0.2563, the correlation is very weak.

$$R^2 = 0.0657$$

The coefficient of determination, which is the squared value of the correlation coefficient is close to 0 indicating a bad model fit. The value of 0.0657 means that the model only explained 6.5% of the possible variation in the dependent variable.

3. For the data set associated with this homework (HBAT (you may use any software and programming language you feel comfortable dealing with. Make sure to include your codes, diagrams and results)).

Using X19 as the dependent variable and (X6, X7, X9, X11, X12 and X16) as the independent variables:

Using SAS Studio, I performed a regression analysis with the dependent variable x19 and the independent variables x6, x7, x9, x11 and x16. This is the code that was generated from the analysis:

```
ods noproctitle;
ods graphics / imagemap=on;

proc reg data=WORK.IMPORT alpha=0.05 plots(only)=(diagnostics
residuals
observedbypredicted);
model x19=x6 x7 x9 x11 x12 x16 / stb ssl influence r p pcorr1
pcorr2 scorrl
scorr2 collin tol vif;
run;
quit;
```

a. Find the parameters (coefficients) for the Linear Regression Model, then write down the equation of the model.

Parameter Estimates													
Variable	Label	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Type III SS	Standardized Estimate	Squared Semi-partial Corr Type I	Squared Partial Corr Type I	Squared Semi-partial Corr Type II	Squared Partial Corr Type II	Variance Inflation
Intercept	Intercept	1	-1.26902	0.49935	-2.54	0.0127	4785.87240	0	0
x6	x6	1	0.36499	0.04676	7.81	<.0001	33.26012	0.42760	0.23651	0.23651	0.13283	0.39587	1.37644
x7	x7	1	-0.43635	0.13103	-3.33	0.0012	17.50215	-0.25647	0.12446	0.16301	0.02417	0.10655	2.72096
x9	x9	1	0.22577	0.08074	2.80	0.0063	35.93795	0.22891	0.25555	0.39991	0.01704	0.07756	3.07424
x11	x11	1	0.17655	0.06034	2.93	0.0043	0.75326	0.19484	0.00536	0.01397	0.01866	0.08431	2.03392
x12	x12	1	0.78167	0.08814	8.87	<.0001	23.75201	0.70329	0.16890	0.44668	0.17145	0.45821	2.88493
x16	x16	1	0.15911	0.09215	1.73	0.0875	0.91402	0.12400	0.00650	0.03107	0.00650	0.03107	2.36581

Table 1. Parameter Estimates for Linear Regression model on HBAT data. The red box denotes the coefficients b_0 , b_1 , b_2 , b_3 , b_4 , b_5 and b_6 from top to bottom.

The coefficients for the model can be seen in the Parameter Estimates table (see Table 1). The values from the Parameter Estimate column for each variable was extracted and plugged into the regression equation.

$$\hat{Y} = b_0 + b_1 \times x_6 + b_2 \times x_7 + b_3 \times x_9 + b_4 \times x_{11} + b_5 \times x_{12} + b_6 \times x_{16}$$

$$\hat{Y} = -1.26902 + 0.36499 \times x_6 - 0.43635 \times x_7 + 0.22577 \times x_9 + 0.17655 \times x_{11} + 0.78167 \times x_{12} + 0.15911 \times x_{16}$$

b. Find the coefficient of determination and the standard error of the estimate. How accurate is the model?

Root MSE	0.55366	R-Square	0.7973
Dependent Mean	6.91800	Adj R-Sq	0.7842
Coeff Var	8.00317		

Table 2. Coefficient of determination (R-Square, R^2) for Linear Regression model on variables x_6 , x_7 , x_9 , x_{11} , x_{12} , x_{16} and x_{19} for HBAAT data set.

The value for the coefficient of determination (R^2) is 0.7973, meaning that the model explains 79.73% of the dependent variable. A value of R^2 close to 1 is considered to be a good fit. Also, the adjusted R^2 (see Table 2, Adj R-Sq) with a value 0.7842 doesn't drop to low and is close to R^2 , indicating a robust model.

c. If you are asked to remove two independent variables, which two variables would you choose and Why?

To assess which variables to delete, correlation analysis of the independent and dependent variables is performed. The following SAS Code was generated for the Correlation analysis:

```
ods noproctitle;
ods graphics / imagemap=on;

proc corr data=WORK.IMPORT pearson nosimple noprob plots=none;
    var x6 x7 x9 x11 x12 x16 x19;
run;
```

7 Variables:

x6 x7 x9 x11 x12 x16 x19

Pearson Correlation Coefficients, N = 100							
	x6	x7	x9	x11	x12	x16	x19
x6 x6	1.00000	-0.13716	0.10637	0.47749	-0.15181	0.10430	0.48632
x7 x7	-0.13716	1.00000	0.14018	-0.05269	0.79154	0.15615	0.28275
x9 x9	0.10637	0.14018	1.00000	0.56142	0.22975	0.75687	0.60326
x11 x11	0.47749	-0.05269	0.56142	1.00000	-0.06132	0.42441	0.55055
x12 x12	-0.15181	0.79154	0.22975	-0.06132	1.00000	0.19513	0.50021
x16 x16	0.10430	0.15615	0.75687	0.42441	0.19513	1.00000	0.52173
x19 x19	0.48632	0.28275	0.60326	0.55055	0.50021	0.52173	1.00000

Table 3. Result of correlation analysis performed on variables x6, x7, x9, x11, x12, x16 and x19 of HBAT data set.

Examining the correlation values of the independent variables (x6, x7, x9, x11, x12 and x16) with the dependent variable x19, shows that variable x7 has the lowest correlation with x19, followed by variable x6. If I had to remove two variables, I would choose x6 and x7 because of their relatively low correlation values. All other variables have correlation values of > 0.5 denoting strong positive correlations with x19.

Variable x7 refers to the E-Commerce Activities whereas variable x6 deals with the perceived quality of HBAT's products.

d. After removing the two variables found in part c, re-run parts a and b. Compare the results. Which model is more accurate and why?

I removed both x6 and x7 from the linear regression analysis set up and re-run the analysis without x6 and x7.

The following SAS code was generated:

```
ods noproctitle;
ods graphics / imagemap=on;

proc reg data=WORK.IMPORT alpha=0.05 plots(only)=(diagnostics
residuals
      observedbypredicted);
      model x19=x9 x11 x12 x16 / stb ss1 influence r p pcorr1
pcorr2 scorrl scorrl2
      collin tol vif;
run;
quit;
```

Parameter Estimates														
Variable	Label	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Type I SS	Standardized Estimate	Squared Semi-partial Corr Type I	Squared Partial Corr Type I	Squared Semi-partial Corr Type II	Squared Partial Corr Type II	Tolerance	Variance Inflation
Intercept	Intercept	1	0.42420	0.52097	0.81	0.4175	4785.87240	0	0
x9	x9	1	0.14901	0.10489	1.42	0.1587	51.17801	0.15108	0.36393	0.36393	0.00777	0.02080	0.34029	2.93870
x11	x11	1	0.39805	0.06991	5.69	<.0001	9.21753	0.43927	0.06555	0.10305	0.12476	0.25440	0.64654	1.54668
x12	x12	1	0.51916	0.07296	7.12	<.0001	27.80110	0.46710	0.19769	0.34651	0.19490	0.34770	0.89329	1.11946
x16	x16	1	0.16656	0.12187	1.37	0.1750	1.01096	0.12981	0.00719	0.01928	0.00719	0.01928	0.42665	2.34383

Table 4. Parameter Estimates for Linear Regression model on HBAT data. The red box denotes the coefficients b_0 , b_1 , b_2 , b_3 and b_4 from top to bottom.

The amended regression equation is as follows:

$$\hat{Y} = b_0 + b_1 \times x_9 + b_2 \times x_{11} + b_3 \times x_{12} + b_4 \times x_{16}$$

$$\hat{Y} = 0.42420 + 0.14901 \times x_9 + 0.39805 \times x_{11} + 0.51916 \times x_{12} + 0.16656 \times x_{16}$$

Root MSE	0.73571	R-Square	0.6344
Dependent Mean	6.91800	Adj R-Sq	0.6190
Coeff Var	10.63466		

Table 5. Coefficient of determination (R-Square, R^2) for Linear Regression model on variables x9, x11, x12, x16 and x19 for HBAT data set.

The coefficient of determination with a value of 0.6344 for the second model has a lower value of this metric than the previous model. In this case, the model is only able to explain 63.44% of the dependent variable as compared to almost 80% previously.

It seems that the factors concerning the perceived quality of the products (x6) and the user-friendliness of the website (x7) are an integral part to explain customer satisfaction (x19).

Model: MODEL1		
Dependent Variable: x19 x19		
Number of Observations Read	100	
Number of Observations Used	100	

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	6	112.11952	18.68659	60.96	<.0001
Error	93	28.50808	0.30654		
Corrected Total	99	140.62760			

Root MSE	0.55366	R-Square	0.7973
Dependent Mean	6.91800	Adj R-Sq	0.7842
Coeff Var	8.00317		

Parameter Estimates												
Variable	Label	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Type I SS	Standardized Estimate	Squared Semi-partial Corr Type I	Squared Partial Corr Type I	Squared Semi-partial Corr Type II	Squared Partial Corr Type II
Intercept	Intercept	1	-1.26902	0.49935	-2.54	0.0127	4785.87240	0				
x6	x6	1	0.36499	0.04676	7.81	<.0001	33.26012	0.42760	0.23651	0.23651	0.13283	0.39587
x7	x7	1	-0.43635	0.13103	-3.33	0.0012	17.50215	-0.25647	0.12446	0.16301	0.02417	0.10655
x9	x9	1	0.22577	0.08074	2.80	0.0063	35.93795	0.22891	0.25555	0.39991	0.01704	0.07756
x11	x11	1	0.17655	0.06034	2.93	0.0043	0.75326	0.19484	0.00536	0.01397	0.01866	0.08431
x12	x12	1	0.78167	0.08814	8.87	<.0001	23.75201	0.70329	0.16890	0.44668	0.17145	0.45821
x16	x16	1	0.15911	0.09215	1.73	0.0875	0.91402	0.12400	0.00650	0.03107	0.00650	0.03107

Collinearity Diagnostics									
Number	Eigenvalue	Condition Index	Proportion of Variance						
			Intercept	x6	x7	x9	x11	x12	x16
1	6.83195	1.00000	0.00026035	0.00046381	0.00026670	0.00031740	0.00049481	0.00029964	0.00039476
2	0.07535	9.52206	0.00066644	0.01874	0.004777	0.001173	0.07218	0.02620	0.00939
3	0.04669	12.09656	0.01648	0.19278	0.0060397	0.08277	0.00967	0.00114	0.10228
4	0.01944	18.74715	0.05670	0.13714	0.01637	0.00475	0.60208	0.02732	0.22365
5	0.01008	26.03138	0.00034697	0.12412	0.18455	0.57303	0.19822	0.14258	0.43476
6	0.00963	26.64144	0.80988	0.42344	0.00034541	0.03798	0.00048683	0.13714	0.13072
7	0.00687	31.54647	0.11566	0.10332	0.75009	0.28942	0.11688	0.62912	0.08881

Model: MODEL1
Dependent Variable: x19 x19

Output Statistics																			
	Dependent Variable	Predicted Value	Std Error Mean Predict	Residual	Std Error Residual	Student Residual	-2-1 0 1 2	Cook's D	RStudent	Hat Diag H	Cov Ratio	DFFITS	Intercept	x6	x7	x9	x11	x12	x16
Obs																			
1	8.2	7.8144	0.1211	0.3856	0.540	0.714	*	0.004	0.7119	0.0478	1.0901	0.1596	-0.0514	0.0847	-0.0352	0.0134	-0.1002	0.0536	0.0520
2	5.7	6.6098	0.2075	-0.9098	0.513	-1.772	***	0.073	-1.7934	0.1404	0.9866	-0.7249	-0.1454	0.0859	-0.0888	-0.4661	-0.1107	0.3210	0.0465
3	8.9	8.5690	0.1713	0.3310	0.527	0.629	*	0.006	0.6266	0.0957	1.1577	0.2038	-0.0532	0.0166	-0.1097	-0.1271	0.0858	0.1243	0.1181
4	4.8	5.4938	0.1516	-0.6938	0.533	-1.303	**	0.020	-1.3079	0.0749	1.0249	-0.3723	-0.1869	0.1502	0.0551	0.2868	-0.0616	-0.0078	-0.0248
5	7.1	6.8637	0.1081	0.2363	0.543	0.435		0.001	0.4333	0.0381	1.1055	0.0863	-0.0053	0.0277	-0.0007	-0.0556	0.0082	-0.0072	0.0511
6	4.7	5.0315	0.1239	-0.3315	0.540	-0.614	*	0.003	-0.6122	0.0500	1.1036	-0.1405	-0.1217	0.0450	0.0247	0.0250	0.0277	0.0336	-0.0175
7	5.7	5.1832	0.1806	0.5168	0.523	0.988	*	0.017	0.9874	0.1064	1.1212	0.3408	0.1486	0.0577	-0.0296	-0.0189	-0.1612	0.0572	-0.0923
8	6.3	5.9440	0.1302	0.3560	0.538	0.662	*	0.004	0.6596	0.0553	1.1047	0.1596	0.0871	-0.0269	-0.0604	-0.0129	-0.0854	0.0324	0.0511
9	7.0	7.0652	0.1393	-0.0652	0.536	-0.122		0.000	-0.1210	0.0633	1.1502	-0.0314	-0.0103	0.0179	0.0112	-0.0137	-0.0033	-0.0102	0.0130
10	5.5	6.5948	0.1253	-1.0948	0.539	-2.030	****	0.032	-2.0654	0.0512	0.8278	-0.4799	-0.0166	0.1796	-0.2829	-0.2206	-0.0081	0.1829	0.2117
11	7.4	6.9947	0.1000	0.4053	0.545	0.744	*	0.003	0.7426	0.0326	1.0693	0.1363	0.0188	0.0026	-0.0411	-0.0550	0.0811	0.0289	-0.0094
12	6.0	5.8684	0.1544	0.1316	0.532	0.248		0.001	0.2463	0.0777	1.1642	0.0715	0.0100	-0.0144	0.0328	-0.0054	-0.0062	-0.0043	-0.0194
13	8.4	8.5014	0.2057	-0.1014	0.514	-0.197		0.001	-0.1962	0.1380	1.2476	-0.0785	0.0563	-0.0295	-0.0606	-0.0202	0.0079	0.0300	0.0081
14	7.6	7.5799	0.1510	0.0201	0.533	0.038		0.000	0.0375	0.0744	1.1652	0.0106	-0.0031	-0.0007	0.0031	-0.0042	0.0082	-0.0019	0.0000
15	8.0	7.2008	0.1374	0.7992	0.536	1.490	**	0.021	1.5000	0.0616	0.9706	0.3943	-0.3039	-0.2115	0.1630	0.1223	0.1258	-0.0877	-0.0897
16	6.6	7.0770	0.1480	-0.4770	0.534	-0.894	*	0.009	-0.8930	0.0714	1.0935	-0.2477	-0.0008	0.0034	-0.0622	-0.1304	-0.0592	0.1229	0.1161
17	6.4	6.0465	0.1236	0.3535	0.540	0.655	*	0.003	0.6530	0.0498	1.0990	0.1495	0.0571	-0.0959	0.0569	0.0565	0.0071	-0.0599	-0.0442
18	7.4	7.0570	0.1415	0.3430	0.535	0.641	*	0.004	0.6387	0.0653	1.1188	0.1689	0.0073	-0.1018	0.0356	0.0258	0.0219	-0.0373	0.0454
19	6.8	6.9156	0.1381	-0.1156	0.536	-0.216		0.000	-0.2144	0.0622	1.1461	-0.0552	-0.0220	0.0365	0.0277	-0.0016	-0.0073	-0.0216	-0.0099
20	7.6	8.7354	0.1603	-1.1354	0.530	-2.142	****	0.060	-2.1855	0.0838	0.8261	-0.6611	0.3535	-0.3905	0.0619	-0.2622	0.3099	-0.2862	0.1510
21	5.4	5.3374	0.1454	0.0626	0.534	0.117		0.000	0.1165	0.0690	1.1574	0.0317	0.0191	-0.0166	0.0068	0.0143	-0.0058	-0.0084	-0.0163
22	9.9	9.3263	0.2304	0.5737	0.503	1.139	**	0.039	1.1413	0.1732	1.1823	0.5223	-0.3441	0.0721	0.1830	-0.0865	0.2551	0.1004	-0.0818
23	7.0	7.1769	0.1249	-0.1769	0.539	-0.328		0.001	-0.3264	0.0509	1.1273	-0.0756	0.0093	0.0092	-0.0051	0.0532	-0.0489	-0.0398	0.0046
24	8.6	8.5585	0.2169	0.0415	0.509	0.082		0.000	0.0811	0.1535	1.2735	0.0345	-0.0036	0.0054	-0.0202	-0.0067	-0.0014	0.0107	0.0220
25	4.8	6.0194	0.1345	-1.2194	0.537	-2.271	****	0.046	-2.3236	0.0590	0.7690	-0.5820	-0.1553	0.3434	-0.1340	0.3602	-0.1657	0.0816	-0.3392
26	6.6	6.9597	0.1632	0.004285	0.529	0.008		0.000	0.008056	0.0869	1.1813	0.0025	0.0004	-0.0001	-0.0003	0.0001	-0.0015	-0.0000	0.0013
27	6.3	6.9111	0.1425	-0.6111	0.535	-1.142	**	0.013	-1.1441	0.0662	1.0463	-0.3047	-0.0179	0.0548	-0.0008	0.1321	-0.1441	0.0899	-0.1660
28	5.4	5.9024	0.1022	-0.5024	0.544	-0.923	*	0.004	-0.9225	0.0341	1.0470	-0.1733	-0.0877	0.0181	-0.0289	-0.0618	0.0911	0.0913	-0.0062
29	6.3	7.0199	0.1576	-0.7199	0.531	-1.356	**	0.023	-1.3626	0.0811	1.0206	-0.4047	-0.0053	0.0632	0.0019	0.1804	-0.1558	0.1130	-0.2602
30	5.4	5.9222	0.1524	-0.5222	0.532	-0.981	*	0.011	-0.9808	0.0758	1.0851	-0.2809	-0.0486	-0.0260	-0.0265	0.1668	0.0809	0.0489	-0.2100
31	6.1	6.1291	0.1274	-0.0291	0.539	-0.054		0.000	-0.0538	0.0529	1.1386	-0.0127	-0.0057	0.0076	0.0018	0.0077	-0.0094	-0.0014	-0.0013
32	6.4	5.9606	0.1308	0.4394	0.538	0.817	*	0.006	0.8153	0.0558	1.0863	0.1983	0.1201	0.0276	-0.0888	0.0475	-0.0379	0.0129	-0.0916
33	5.4	6.0460	0.0997	-0.6460	0.545	-1.186	**	0.007	-1.1888	0.0325	1.0020	-0.2177	-0.1057	0.0646	-0.0519	-0.1129	0.0364	0.0657	0.1386
34	7.3	6.7317	0.1131	0.5683	0.542	1.049	**	0.007	1.0492	0.0418	1.0357	0.2190	0.0289	0.0840	-0.0300	0.0623	-0.1715	-0.0255	0.0044
35	6.3	6.9781	0.1931	-0.6781	0.519	-1.307	**	0.034	-1.3117	0.1216	1.0785	-0.4880	-0.1004	0.1272	0.1892	0.1008	-0.1895	-0.3292	0.1917
36	5.4	5.3180	0.1672	0.0820	0.528	0.155		0.000	0.1546	0.0912	1.1847	0.0490	0.0127	0.0051	0.0264	0.0043	0.0034	-0.0378	-0.0152
37	7.1	6.7682	0.0875	0.3318	0.547	0.607	*	0.001	0.6048	0.0250	1.0760	0.0968	0.0010	0.0414	0.0043	-0.0362	0.0100	-0.0126	0.0097
38	8.7	8.5800	0.1277	0.1200	0.539	0.223		0.000	0.2216	0.0532	1.1350	0.0525	-0.0361	0.0219	0.0158	0.0225	0.0031	-0.0088	-0.0084
39	7.6	7.0741	0.1143	0.5259	0.542	0.971	*	0.006	0.9704	0.0426	1.0491	0.2048	-0.0150	0.0953	0.0134	0.1162	-0.1649	-0.0419	-0.0057
40	6.0	6.3103	0.1468	-0.3103	0.534	-0.581	*	0.004	-0.5791	0.0703	1.1309	-0.1592	0.0031	0.0272	-0.0738	0.0340	0.0479	0.0208	-0.0019
41	7.0	7.1273	0.1127	-0.1273	0.542	-0.235		0.000	-0.2337	0.0414	1.1206	-0.0486	-0.0004	-0.0242	0.0161	-0.0072	0.0354	-0.0103	-0.0117
42	7.6	7.3531	0.0928	0.2469	0.546	0.452		0.001	0.4505	0.0281	1.0928	0.0766	-0.0307	0.0282	0.0062	0.0063	0.0023	-0.0159	-0.0159
43	8.9	8.2569	0.1728	0.6431	0.526	1.223	**	0.023	1.2259	0.0974	1.0669	0.4028	-0.2627	0.1729	0.2739	0.1869	-0.0736	-0.1330	-0.0340
44	7.6	7.7549	0.1821	-0.1549	0.523	-0.296		0.002	-0.2949	0.1081	1.2014	-0.1027	0.0106	0.0483	-0.0044	-0.0224	-0.0050	-0.0348	0.0120
45	5.5	6.5517	0.1848	-1.0517	0.522	-2.015	****	0.073	-2.0494	0.1114	0.8881	-0.7257	-0.2019	0.1282	-0.0088	-0.4058	-0.1136	0.3095	0.1660
46	7.4	6.6840	0.1366	0.7160	0.537	1.335	**	0.016	1.3402	0.0609	1.0031	0.3412	0.0513	-0.2535	0.0522	-0.1312	0.1645	-0.0258	0.1551
47	7.1	7.1934	0.2074	-0.0934	0.513	-0.182		0.001	-0.1810	0.1403	1.2515	-0.0731	0.0292	-0.0447	-0.0474	-0.0269	0.0418	0.0473	-0.0019
48	7.6	7.5772	0.1818	0.0228	0.523	0.044		0.000	0.0434	0.1079	1.2088	0.0151	0.0006	-0.0062	-0.0037	-0.0000	-0.0016	0.0029	0.0083
49	8.7	9.2962	0.1726	-0.5962	0.526	-1.133	**	0.020	-1.1350	0.0972	1.0839	-0.3724	0.1597	-0.1561	0.2151	0.0142	-0.0481	-0.2963	0.0274
50	8.6	7.2624	0.0962	0.9736	0.545	1.786	****	0.014	1.8073	0.0302	0.8712	0.1390	-0.0451	-0.0667	-0.0517	0.0023	0.1910	0.1308	-0.1144
51	5.4	5.7721	0.0988	-0.3721	0.545	-0.683	*	0.002	-0.6810	0.0319	1.0756	-0.1235	-0.0682	0.0456	-0.0053	0.0705	-0.0008	0.0137	-0.0227
52	5.7	6.7504	0.1889	-1.0504	0.520	-2.018	****	0.077	-2.0529	0.1164	0.8922	-0.7451	-0.1294	0.0966	-0.0895	-0.4442	-0.1002	0.3670	0.3168
53	8.7	8.0806	0.1497	0.6194	0.533	1.162	**	0.015	1.1643	0.0731	1.0504	0.3269	0.0036	0.0652	-0.2539	-0.0223	0.0710	0.1712	0.0276
54	6.1	6.0722	0.1284	0.0278	0.539	0.052		0.000	0.0514	0.0538	1.1397	0.0123	0.0060	-0.0069	-0.0012	-0.0036	0.0086	0.0009	-0.0037
55	7.3	6.7681	0.1203	0.5139	0.540	0.951	*	0.006	0.9504	0.0472	1.0572	0.2115	0.0207	0.0772	-0.0283	0.0491	-0.1616	-0.0248	0.0678
56	7.7	7.8356	0.0926	-0.1356	0.546	-0.249		0.000	-0.2472	0.0280	1.1045	-0.0400	0.0190	-0.0262	0.0018	-0.0101	0.0018	-0.0095	0.0150
57	9.0	8.5166	0.1576	0.4834	0.531	0.911	*	0.010	0.9099	0.0810	1.1024	0.2702	-0.1533	-0.0413	0.1200	0.0706	0.0422	-0.0189	0.0047
58	8.2	7.3617	0.0800	0.8383	0.548	1.530	****	0.007	1.5414	0.0209	0.9215	0.2250	0.0107	-0.0840	-0.0689				

Model: MODEL1
Dependent Variable: x19 x19

Number of Observations Read	100
Number of Observations Used	100

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	89.20760	22.30190	41.20	<.0001
Error	95	51.42000	0.54126		
Corrected Total	99	140.62760			

Root MSE	0.73571	R-Square	0.6344
Dependent Mean	6.91800	Adj R-Sq	0.6190
Coeff Var	10.63466		

Parameter Estimates														
Variable	Label	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Type I SS	Standardized Estimate	Squared Semi-partial Corr Type I	Squared Partial Corr Type I	Squared Semi-partial Corr Type II	Squared Partial Corr Type II	Tolerance	Variance Inflation
Intercept	Intercept	1	0.42420	0.52097	0.81	0.4175	4785.87240	0	0
x9	x9	1	0.14901	0.10489	1.42	0.1587	51.17801	0.15108	0.36393	0.36393	0.00777	0.02080	0.34029	2.93870
x11	x11	1	0.39805	0.06991	5.69	<.0001	9.21753	0.43927	0.06555	0.10305	0.12476	0.25440	0.64654	1.54668
x12	x12	1	0.51916	0.07296	7.12	<.0001	27.80110	0.46710	0.19769	0.34651	0.19490	0.34770	0.89329	1.11946
x16	x16	1	0.16656	0.12187	1.37	0.1750	1.01096	0.12981	0.00719	0.01928	0.00719	0.01928	0.42865	2.34383