WEEK 4 ASSIGNMENT

Concepts of Statistics 2 – DATA-51200 | Spring 2 2020 Christina Morgenstern

1. In your own words, describe (in less than one page) stage 3: Assumptions in Multiple Regression Analysis.

Stage 3 of the Multiple Regression Analysis process deals with assessing the assumptions of this multivariate technique. The following four areas need to be investigated before and after model estimation: 1. Linearity, 2. Homoscedasticity, 3. Normality of error term distribution and 4. Independence of error terms. These assumptions must be tested both for the dependent and independent variables as well as for the variate. Graphical visualization provides an effective means to inspect possible violations of the assumptions. The residual is the measure for prediction error and denotes the difference between the actual and the predicted values of the dependent variable. Thus, plotting the residuals versus the predicted dependent values leads to characteristic patterns which allow for simple inspection and judgement.

In the following the individual assumptions are briefly described and measures to correct for noted.

If linearity is met, we would expect to see a linear pattern of the residuals. Nonlinear relationships like curvilinear patterns can be corrected through data transformations (e.g. logarithm, square root, etc.), the incorporation of polynomial terms or the application of specific nonlinear regression techniques.

The occurrence of unequal variances in residuals can lead to an inaccurate estimation of the standard error and following decreased statistical power. Strategies to overcome this phenomenon of heteroscedasticity are the transformation of variables, the use of weighted least squares or special heteroscedasticity-consistent standard errors.

Normality should be assessed by using Normal probability plots, but histograms and visual inspection of the distribution are also applicable. If the sample size is large enough (>200) though, Multiple Regression shouldn't have issues with normality.

Addressing the independence of error terms is important to avoid groupings of residuals due to time series or hierarchically ordered data.

References:

[1] Multivariate Data Analysis by Joseph F. Hair Jr, William C. Black, Barry J. Babin and Rolphe E. Anderson, Pearson, 8th edition, 2019

2. (By Hand) For the dependent variable Y and the independent variables X1 and X2, the linear regression model is given by:

| Actual Y | | | Predicted Y | Residuals |
|----------|-----|-----|-------------|---------------------|
| | x1 | X2 | | (Predication Error) |
| 6 | 6.8 | 4.7 | 5.1 | 0.9 |
| 3.1 | 5.3 | 5.5 | 4.8 | -1.7 |
| 5.8 | 4.5 | 6.2 | 4.6 | 1.2 |
| 4.5 | 8.8 | 7 | 4.8 | -0.3 |
| 4.5 | 6.8 | 6.1 | 4.8 | -0.3 |
| 3.7 | 8.5 | 5.1 | 5.1 | -1.4 |
| 5.4 | 8.9 | 4.8 | 5.2 | 0.2 |
| 5.1 | 6.9 | 5.4 | 5.0 | 0.1 |
| 5.8 | 9.3 | 5.9 | 5.1 | 0.7 |
| 5.7 | 8.4 | 5.4 | 5.1 | 0.6 |

The predicted values for Y were calculated by plugging in the values for the independent variables X1 and X2 into the equation according to the table values. Values were rounded to one decimal place. The residuals were calculated by subtracting the predicted values of Y from the respective actual values of Y

Is this a good model? Why? Why not?

To assess model fit, I would calculate the coefficient of determination (R^2) which is a common measure of predictive accuracy for regression models. It is calculated as the squared values from the correlations between the actual and predicted measures of the dependent variable. The values of R^2 range between 0 and 1, denoting bad and perfect prediction, respectively.

In order to calculate R^2 , I calculated the mean and the sum of squared errors (SS) of Y and Y predicted (\hat{Y}) . Using this information, I calculated the correlation coefficient and subsequently R^2 .

$$M_Y = \frac{\sum Y_i}{n} = 4.96$$

$$SS_Y = \sum_{i=1}^{n} (Y_i - M_Y)^2 = 8.724$$

$$M_{\hat{Y}} = \frac{\sum \hat{Y}_i}{n} = 4.96$$

$$SS_{\hat{Y}} = \sum_{i=1}^{n} (\hat{Y}_i - M_{\hat{Y}})^2 = 0.344$$

$$\sum_{i=1}^{n} (Y_i - M_Y)(\hat{Y}_i - M_{\hat{Y}}) = 0.444$$

$$R = \sum_{i=1}^{n} \frac{(Y_i - M_Y)(\hat{Y}_i - M_{\hat{Y}})}{\sqrt{((SS_Y)(SS_{\hat{Y}}))}} = \mathbf{0.2563}$$

Although, there is a positive correlation denoted by the value of the correlation coefficient (R) of 0.2563, the correlation is very weak.

$$R^2 = 0.0657$$

The coefficient of determination, which is the squared value of the correlation coefficient is close to 0 indicating a bad model fit. The value of 0.0657 means that the model only explained 6.5% of the possible variation in the dependent variable.

3. For the data set associated with this homework (HBAT (you may use any software and programming language you feel comfortable dealing with. Make sure to include your codes, diagrams and results).

Using X19 as the dependent variable and (X6, X7, X9, X11, X12 and X16) as the independent variables:

Using SAS Studio, I performed a regression analysis with the dependent variable x19 and the independent variables x6, x7, x9, x11 and x16. This is the code that was generated from the analysis:

a. Find the parameters (coefficients) for the Linear Regression Model, then write down the equation of the model.

| | Parameter Estimates | | | | | | | | | | | | | |
|-----------|---------------------|----|-----------------------|-------------------|---------|---------|------------|--------------------------|--|-----------------------------------|---|------------------------------------|-----------|-----------------------|
| Variable | Label | DF | Parameter Estimate | Standard Error | t Value | Pr > t | Type I SS | Standardized Estimate | Squared Semi-partial Corr Type I | Squared Partial Corr Type I | Squared Semi-partial Corr Type II | Squared Partial Corr Type II | Tolerance | Variance Inflation |
| Intercept | Intercept | 1 | -1.26902 | 0.49935 | -2.54 | 0.0127 | 4785.87240 | 0 | | | | | | 0 |
| x6 | х6 | 1 | 0.36499 | 0.04676 | 7.81 | <.0001 | 33.26012 | 0.42760 | 0.23651 | 0.23651 | 0.13283 | 0.39587 | 0.72651 | 1.37644 |
| x7 | x7 | 1 | -0.43635 | 0.13103 | -3.33 | 0.0012 | 17.50215 | -0.25647 | 0.12446 | 0.16301 | 0.02417 | 0.10655 | 0.36752 | 2.72096 |
| x9 | x9 | 1 | 0.22577 | 0.08074 | 2.80 | 0.0063 | 35.93795 | 0.22891 | 0.25555 | 0.39991 | 0.01704 | 0.07756 | 0.32528 | 3.07424 |
| x11 | x11 | 1 | 0.17655 | 0.06034 | 2.93 | 0.0043 | 0.75326 | 0.19484 | 0.00536 | 0.01397 | 0.01866 | 0.08431 | 0.49166 | 2.03392 |
| x12 | x12 | 1 | 0.78167 | 0.08814 | 8.87 | <.0001 | 23.75201 | 0.70329 | 0.16890 | 0.44668 | 0.17145 | 0.45821 | 0.34663 | 2.88493 |
| x16 | x16 | 1 | 0.15911 | 0.09215 | 1.73 | 0.0875 | 0.91402 | 0.12400 | 0.00650 | 0.03107 | 0.00650 | 0.03107 | 0.42269 | 2.36581 |

Table 1. Parameter Estimates for Linear Regression model on HBAT data. The red box denotes the coefficients b_0 , b_1 , b_2 , b_3 , b_4 , b_5 and b_6 from top to bottom.

The coefficients for the model can be seen in the Parameter Estimates table (see Table 1). The values from the Parameter Estimate column for each variable was extracted and plugged into the regression equation.

$$\hat{Y} = b_0 + b_1 \times x6 + b_2 \times x7 + b_3 \times x9 + b_4 \times x11 + b_5 \times x12 + b_6 \times x16$$

$$\hat{Y} = -1.26902 + 0.36499 \times x6 - 0.43635 \times x7 + 0.22577 \times x9 + 0.17655 \times x11 + 0.78167 \times x12 + 0.15911 \times x16$$

b. Find the coefficient of determination and the standard error of the estimate. How accurate is the model?

| Root MSE | 0.55366 | R-Square | 0.7973 |
|----------------|---------|----------|--------|
| Dependent Mean | 6.91800 | Adj R-Sq | 0.7842 |
| Coeff Var | 8.00317 | | |

Table 2. Coefficient of determination (R-Square, R²) for Linear Regression model on variables x6, x7, x9, x11, x12, x16 and x19 for HBAT data set.

The value for the coefficient of determination (R^2) is 0.7973, meaning that the model explains 79.73% of the dependent variable. A value of R^2 close to 1 is considered to be a good fit. Also, the adjusted R^2 (see Table 2, Adj R-Sq) with a value 0.7842 doesn't drop to low and is close to R^2 , indicating a robust model.

c. If you are asked to remove two independent variables, which two variables would you choose and Why?

To assess which variables to delete, correlation analysis of the independent and dependent variables is performed. The following SAS Code was generated for the Correlation analysis:



Table 3. Result of correlation analysis performed on variables x6, x7, x9, x11, x12, x16 and x19 of HBAT data set.

Examining the correlation values of the independent variables (x6, x7, x9, x11, x12 and x16) with the dependent variable x19, shows that variable x7 has the lowest correlation with x19, followed by variable x6. If I had to remove two variables, I would choose x6 and x7 because of their relatively low correlation values. All other variables have correlation values of > 0.5 denoting strong positive correlations with x19.

Variable x7 refers to the E-Commerce Activities whereas variable x6 deals with the perceived quality of HBAT's products.

d. After removing the two variables found in part c, re-run parts a and b. Compare the results. Which model is more accurate and why?

I removed both x6 and x7 from the linear regression analysis set up and re-run the analysis without x6 and x7.

The following SAS code was generated:

| | Parameter Estimates | | | | | | | | | | | | | |
|-----------|---------------------|----|-----------------------|-------------------|---------|---------|------------|--------------------------|--|-----------------------------------|---|------------------------------------|-----------|-----------------------|
| Variable | Label | DF | Parameter Estimate | Standard Error | t Value | Pr > t | Type I SS | Standardized Estimate | Squared Semi-partial Corr Type I | Squared Partial Corr Type I | Squared Semi-partial Corr Type II | Squared Partial Corr Type II | Tolerance | Variance Inflation |
| Intercept | Intercept | 1 | 0.42420 | 0.52097 | 0.81 | 0.4175 | 4785.87240 | 0 | | | | | | 0 |
| x9 | x9 | 1 | 0.14901 | 0.10489 | 1.42 | 0.1587 | 51.17801 | 0.15108 | 0.36393 | 0.36393 | 0.00777 | 0.02080 | 0.34029 | 2.93870 |
| x11 | x11 | 1 | 0.39805 | 0.06991 | 5.69 | <.0001 | 9.21753 | 0.43927 | 0.06555 | 0.10305 | 0.12476 | 0.25440 | 0.64654 | 1.54668 |
| x12 | x12 | 1 | 0.51916 | 0.07296 | 7.12 | <.0001 | 27.80110 | 0.46710 | 0.19769 | 0.34651 | 0.19490 | 0.34770 | 0.89329 | 1.11946 |
| x16 | x16 | 1 | 0.16656 | 0.12187 | 1.37 | 0.1750 | 1.01096 | 0.12981 | 0.00719 | 0.01928 | 0.00719 | 0.01928 | 0.42665 | 2.34383 |

Table 4. Parameter Estimates for Linear Regression model on HBAT data. The red box denotes the coefficients b₀, b₁, b₂, b₃ and b₄ from top to bottom.

The amended regression equation is as follows:

$$\hat{Y} = b_0 + b_1 \times x9 + b_2 \times x11 + b_3 \times x12 + b_4 \times x16$$

$$\hat{Y} = 0.42420 + 0.14901 \times x9 + 0.39805 \times x11 + 0.51916 \times x12 + 0.16656 \times x16$$

| Root MSE | 0.73571 | R-Square | 0.6344 |
|----------------|----------|----------|--------|
| Dependent Mean | 6.91800 | Adj R-Sq | 0.6190 |
| Coeff Var | 10.63466 | | |

Table 5. Coefficient of determination (R-Square, R²) for Linear Regression model on variables x9, x11, x12, x16 and x19 for HBAT data set.

The coefficient of determination with a value of 0.6344 for the second model has a lower value of this metric than the previous model. In this case, the model is only able to explain 63.44% of the dependent variable as compared to almost 80% previously.

It seems that the factors concerning the perceived quality of the products (x6) and the user-friendliness of the website (x7) are an integral part to explain customer satisfaction (x19).

Model: MODEL1 Dependent Variable: x19 x19 Number of Observations Read 100

Number of Observations Read 100

Number of Observations Used 100

| | | Analysis of V | /ariance | | |
|-----------------|----|----------------|----------------|---------|--------|
| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
| Model | 6 | 112.11952 | 18.68659 | 60.96 | <.0001 |
| Error | 93 | 28.50808 | 0.30654 | | |
| Corrected Total | 99 | 140.62760 | | | |

| Root MSE | 0.55366 | R-Square | 0.7973 |
|----------------|---------|----------|--------|
| Dependent Mean | 6.91800 | Adj R-Sq | 0.7842 |
| Coeff Var | 8.00317 | | |

| | | | | | | | Para | meter Estimates | 3 | | | | | |
|-----------|-----------|----|-----------------------|-------------------|---------|---------|------------|--------------------------|--|-----------------------------------|---|------------------------------------|-----------|-----------------------|
| Variable | Label | DF | Parameter Estimate | Standard Error | t Value | Pr > t | Type I SS | Standardized Estimate | Squared Semi-partial Corr Type I | Squared Partial Corr Type I | Squared Semi-partial Corr Type II | Squared Partial Corr Type II | Tolerance | Variance Inflation |
| Intercept | Intercept | 1 | -1.26902 | 0.49935 | -2.54 | 0.0127 | 4785.87240 | 0 | | | | | | 0 |
| х6 | x6 | 1 | 0.36499 | 0.04676 | 7.81 | <.0001 | 33.26012 | 0.42760 | 0.23651 | 0.23651 | 0.13283 | 0.39587 | 0.72651 | 1.37644 |
| x7 | x7 | 1 | -0.43635 | 0.13103 | -3.33 | 0.0012 | 17.50215 | -0.25647 | 0.12446 | 0.16301 | 0.02417 | 0.10655 | 0.36752 | 2.72096 |
| х9 | x9 | 1 | 0.22577 | 0.08074 | 2.80 | 0.0063 | 35.93795 | 0.22891 | 0.25555 | 0.39991 | 0.01704 | 0.07756 | 0.32528 | 3.07424 |
| x11 | x11 | 1 | 0.17655 | 0.06034 | 2.93 | 0.0043 | 0.75326 | 0.19484 | 0.00536 | 0.01397 | 0.01866 | 0.08431 | 0.49166 | 2.03392 |
| x12 | x12 | 1 | 0.78167 | 0.08814 | 8.87 | <.0001 | 23.75201 | 0.70329 | 0.16890 | 0.44668 | 0.17145 | 0.45821 | 0.34663 | 2.88493 |
| x16 | x16 | 1 | 0.15911 | 0.09215 | 1.73 | 0.0875 | 0.91402 | 0.12400 | 0.00650 | 0.03107 | 0.00650 | 0.03107 | 0.42269 | 2.36581 |

| 1 0.1 | 5911 0.092 | 215 1.73 | 0.0875 | 0.91402 | 0.12400 | 0.00650 | 0.03107 | 0.00650 | 0.03107 | l | | |
|--------|-----------------------------------|----------|------------|------------|----------------|------------|------------|------------|------------|---|--|--|
| · | | | | | | | | | | | | |
| | | | | Collinear | ty Diagnostics | S | | | | | | |
| | Condition Proportion of Variation | | | | | | | | | | | |
| Number | Eigenvalue | Index | Intercept | х6 | х7 | х9 | x11 | x12 | x16 | | | |
| 1 | 6.83195 | 1.00000 | 0.00026035 | 0.00046381 | 0.00026670 | 0.00031740 | 0.00049481 | 0.00029964 | 0.00039476 | | | |
| 2 | 0.07535 | 9.52206 | 0.00066644 | 0.01874 | 0.04777 | 0.01173 | 0.07218 | 0.06240 | 0.00939 | | | |
| 3 | 0.04669 | 12.09656 | 0.01648 | 0.19278 | 0.00060397 | 0.08277 | 0.00967 | 0.00114 | 0.10228 | | | |
| 4 | 0.01944 | 18.74715 | 0.05670 | 0.13714 | 0.01637 | 0.00475 | 0.60208 | 0.02732 | 0.22365 | | | |
| 5 | 0.01008 | 26.03138 | 0.00034697 | 0.12412 | 0.18455 | 0.57303 | 0.19822 | 0.14258 | 0.43476 | | | |
| 6 | 0.00963 | 26.64144 | 0.80988 | 0.42344 | 0.00034541 | 0.03798 | 0.00048683 | 0.13714 | 0.13072 | | | |
| 7 | 0.00687 | 31.54647 | 0.11566 | 0.10332 | 0.75009 | 0.28942 | 0.11688 | 0.62912 | 0.09881 | | | |

Model: MODEL1

| | | | | | | | | Dependent V | MODEL1 ariable: x19 | x19 | | | | | | | | | |
|----------|-----------------------|--------------------|--------------------------|----------------------|-----------------------|---------------------|------------|----------------|------------------------|------------------|------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|------------|
| | | | Std | | | | | Output | Statistics | | | | | | | OFBETAS | | | |
| Obs | Dependent Variable | Predicted Value | Error Mean Predict | Residual | Std Error Residual | Student Residual | -2-1 0 1 2 | Cook's D | RStudent | Hat Diag H | Cov Ratio | DFFITS | Intercept | х6 | х7 | х9 | x11 | x12 | x |
| 1 | 8.2 5.7 | 7.8144 6.6098 | 0.1211 0.2075 | 0.3856 | 0.540 0.513 | 0.714 | | 0.004 0.073 | 0.7119 | 0.0478 0.1404 | 1.0901 0.9866 | 0.1596 | -0.0514 -0.1454 | 0.0847 0.0859 | -0.0352 -0.0888 | 0.0134 | -0.1002 -0.1107 | 0.0536 0.3210 | 0.05 |
| 3 | 8.9 | 8.5690 | 0.1713 | 0.3310 | 0.527 | 0.629 | * | 0.006 | 0.6266 | 0.0957 | 1.1577 | 0.2038 | -0.0532 | 0.0166 | -0.1097 | -0.1271 | 0.0858 | 0.1243 | 0.11 |
| 5 | 7.1 | 5.4938 6.8637 | 0.1516 0.1081 | -0.6938 0.2363 | 0.533 0.543 | -1.303 0.435 | ** | 0.020 | -1.3079 0.4333 | 0.0749 0.0381 | 1.0249 1.1055 | -0.3723 0.0863 | -0.1869 -0.0053 | 0.1502 0.0277 | -0.0007 | 0.2868 -0.0556 | -0.0616 0.0082 | -0.0078 -0.0072 | 0.0 |
| 6 | 4.7 | 5.0315 | 0.1239 | -0.3315 | 0.540 | -0.614 | * | 0.003 | -0.6122 | 0.0500 | 1.1036 | -0.1405 | -0.1217 | 0.0450 | 0.0247 | 0.0250 | 0.0277 | 0.0336 | -0.0 |
| 8 | 5.7 6.3 | 5.1832 5.9440 | 0.1806 0.1302 | 0.5168 0.3560 | 0.523 0.538 | 0.988 0.662 | * | 0.017 | 0.9874 0.6596 | 0.1064 0.0553 | 1.1212 1.1047 | 0.3408 0.1596 | 0.1486 0.0871 | 0.0577 -0.0269 | -0.0296 -0.0604 | -0.0189 -0.0129 | -0.1612 -0.0854 | 0.0572 0.0324 | -0.0 |
| 9 | 7.0 5.5 | 7.0652 6.5948 | 0.1393 0.1253 | -0.0652 -1.0948 | 0.536 0.539 | -0.122 -2.030 | | 0.000 | -0.1210 -2.0654 | 0.0633 0.0512 | 1.1502 0.8278 | -0.0314 -0.4799 | -0.0103 -0.0166 | 0.0179 0.1796 | 0.0112 | -0.0137 -0.2206 | -0.0033 -0.0081 | -0.0102 0.1829 | 0.0 |
| 11 | 7.4 | 6.9947 | 0.1000 | 0.4053 | 0.545 | 0.744 | * | 0.003 | 0.7426 | 0.0312 | 1.0693 | 0.1363 | 0.0188 | 0.0026 | -0.2029 | -0.2200 | 0.0811 | 0.0289 | -0.0 |
| 12 13 | 6.0 8.4 | 5.8684 8.5014 | 0.1544 0.2057 | 0.1316 -0.1014 | 0.532 0.514 | 0.248 -0.197 | | 0.001 | 0.2463 | 0.0777 | 1.1642 | 0.0715 -0.0785 | 0.0100 0.0563 | -0.0144 -0.0295 | 0.0328 | -0.0054 -0.0202 | -0.0062 0.0079 | -0.0043 0.0300 | -0.0 |
| 14 | 7.6 | 7.5799 | 0.1510 | 0.0201 | 0.533 | 0.038 | 1 1 1 | 0.000 | 0.0375 | 0.0744 | 1.1652 | 0.0106 | -0.0031 | -0.0007 | 0.0031 | -0.0042 | 0.0082 | -0.0019 | 0.0 |
| 15 16 | 8.0 6.6 | 7.2008 7.0770 | 0.1374 0.1480 | 0.7992 -0.4770 | 0.536 0.534 | 1.490 -0.894 | | 0.021 | 1.5000 -0.8930 | 0.0616 0.0714 | 0.9706 1.0935 | 0.3843 | -0.0309 -0.0008 | -0.2115 0.0034 | 0.1630 | 0.1223 -0.1304 | -0.0592 | -0.0877 0.1229 | -0.0 0. |
| 17 18 | 6.4 7.4 | 6.0465 7.0570 | 0.1236 0.1415 | 0.3535 0.3430 | 0.540 0.535 | 0.655 0.641 | * | 0.003 | 0.6530 0.6387 | 0.0498 0.0653 | 1.0990 | 0.1495 0.1689 | 0.0571 0.0073 | -0.0959 -0.1018 | 0.0569 0.0356 | 0.0565 0.0258 | 0.0071 0.0219 | -0.0599 -0.0373 | -0.0 |
| 19 | 6.8 | 6.9156 | 0.1381 | -0.1156 | 0.536 | -0.216 | | 0.000 | -0.2144 | 0.0622 | 1.1461 | -0.0552 | -0.0220 | 0.0365 | 0.0277 | -0.0016 | -0.0073 | -0.0216 | + |
| 20 21 | 7.6 5.4 | 8.7354 5.3374 | 0.1603 0.1454 | -1.1354 0.0626 | 0.530 0.534 | -2.142 0.117 | **** | 0.060 | -2.1855 0.1165 | 0.0838 | 0.8261 1.1574 | -0.6611 0.0317 | 0.3535 0.0191 | -0.3905 -0.0166 | 0.0619 | -0.2622 0.0143 | 0.3099 | -0.2862 -0.0084 | -0.0 |
| 22 | 9.9 | 9.3263 | 0.2304 | 0.5737 | 0.503 | 1.139 | ** | 0.039 | 1.1413 | 0.1732 | 1.1823 | 0.5223 | -0.3441 | 0.0721 | 0.1830 | -0.0865 | 0.2551 | 0.1004 | -0.0 |
| 23 | 7.0 8.6 | 7.1769 8.5585 | 0.1249 | -0.1769 0.0415 | 0.539 | -0.328 0.082 | | 0.001 | -0.3264 0.0811 | 0.0509 0.1535 | 1.1273 1.2735 | -0.0756 0.0345 | -0.0036 | 0.0092 0.0054 | -0.0051 -0.0202 | -0.0067 | -0.0489 -0.0014 | 0.0046 | 0.0 |
| 25 26 | 4.8 6.6 | 6.0194 6.5957 | 0.1345 0.1632 | -1.2194 0.004285 | 0.537 0.529 | -2.271 0.008 | **** | 0.046 | -2.3236 0.008056 | 0.0590 0.0869 | 0.7690 1.1813 | -0.5820 0.0025 | -0.1553 0.0004 | 0.3434 | -0.1340 -0.0003 | 0.3602 0.0001 | -0.1657 -0.0015 | 0.0816 | -0.3 |
| 27 | 6.3 | 6.9111 | 0.1425 | -0.6111 | 0.535 | -1.142 | ** | 0.013 | -1.1441 | 0.0662 | 1.0463 | -0.3047 | -0.0179 | 0.0548 | -0.0008 | 0.1321 | -0.1441 | 0.0899 | - |
| 28 29 | 5.4 6.3 | 5.9024 7.0199 | 0.1022 0.1576 | -0.5024 -0.7199 | 0.544 | -0.923 -1.356 | * | 0.004 | -0.9225 -1.3626 | 0.0341 | 1.0470 | -0.1733 -0.4047 | -0.0877 -0.0053 | 0.0181 | -0.0289 0.0019 | -0.0618 0.1804 | -0.1558 | 0.0913 | + |
| 30 | 5.4 | 5.9222 | 0.1524 | -0.5222 | 0.532 | -0.981 | * | 0.011 | -0.9808 | 0.0758 | 1.0851 | -0.2809 | -0.0486 | -0.0260 | -0.0265 | 0.1668 | 0.0809 | 0.0489 | + |
| 31 | 6.1 | 6.1291 5.9606 | 0.1274 0.1308 | -0.0291 0.4394 | 0.539 0.538 | -0.054 0.817 | | 0.000 | -0.0538 0.8153 | 0.0529 0.0558 | 1.1386 | -0.0127 0.1983 | -0.0057 0.1201 | 0.0076 0.0276 | -0.0888 | 0.0077 0.0475 | -0.0094 | -0.0014 0.0129 | + |
| 33 34 | 5.4 7.3 | 6.0460 6.7317 | 0.0997 0.1131 | -0.6460 0.5683 | 0.545 0.542 | -1.186 1.049 | ** | 0.007 | -1.1888 1.0492 | 0.0325 0.0418 | 1.0020 1.0357 | -0.2177 0.2190 | -0.1057 0.0289 | 0.0646 0.0840 | -0.0519 -0.0300 | -0.1129 0.0623 | 0.0364 | 0.0657 -0.0255 | 0.0 |
| 35 | 6.3 | 6.9781 | 0.1931 | -0.6781 | 0.519 | -1.307 | ** | 0.034 | -1.3117 | 0.1216 | 1.0785 | -0.4880 | -0.1004 | 0.1272 | 0.1892 | 0.1008 | -0.1715 | -0.3292 | - |
| 36 37 | 5.4 7.1 | 5.3180 6.7682 | 0.1672 0.0875 | 0.0820 0.3318 | 0.528 0.547 | 0.155 0.607 | | 0.000 | 0.1546 0.6048 | 0.0912 | 1.1847 | 0.0490 | 0.0127 0.0010 | 0.0051 | 0.0264 | 0.0043 | 0.0034 | -0.0378 -0.0126 | + |
| 38 | 8.7 | 8.5800 | 0.1277 | 0.1200 | 0.539 | 0.223 | 1 1 1 | 0.000 | 0.2216 | 0.0532 | 1.1350 | 0.0525 | -0.0361 | 0.0219 | 0.0158 | 0.0225 | 0.0031 | -0.0088 | -0. |
| 39 40 | 7.6 6.0 | 7.0741 6.3103 | 0.1143 0.1468 | -0.3103 | 0.542 0.534 | 0.971 -0.581 | | 0.006 | -0.5791 | 0.0426 0.0703 | 1.0491 | 0.2048 -0.1592 | -0.0150 0.0031 | 0.0953 0.0272 | -0.0738 | 0.1162 0.0340 | -0.1649 0.0479 | -0.0419 0.0208 | + |
| 41 42 | 7.0 7.6 | 7.1273 7.3531 | 0.1127 0.0928 | -0.1273 0.2469 | 0.542 0.546 | -0.235 0.452 | | 0.000 | -0.2337 0.4505 | 0.0414 0.0281 | 1.1206 1.0928 | -0.0486 0.0766 | -0.0004 -0.0307 | -0.0242 0.0452 | 0.0161 0.0282 | -0.0072 0.0063 | 0.0354 0.0023 | -0.0103 -0.0211 | -0.0 |
| 43 | 8.9 | 8.2569 | 0.0928 | 0.6431 | 0.526 | 1.223 | ** | 0.001 | 1.2259 | 0.0201 | 1.0669 | 0.4028 | -0.2627 | 0.0432 | 0.0282 | 0.1869 | -0.0736 | -0.1330 | -0. |
| 44 45 | 7.6 5.5 | 7.7549 6.5517 | 0.1821 | -0.1549 -1.0517 | 0.523 0.522 | -0.296 -2.015 | | 0.002 | -0.2949 -2.0494 | 0.1081 | 1.2014 0.8881 | -0.1027 -0.7257 | 0.0106 -0.2019 | 0.0483 0.1282 | -0.0044 -0.0088 | -0.0224 -0.4058 | -0.0050 -0.1136 | -0.0348 0.3095 | 0.0 |
| 46 | 7.4 | 6.6840 | 0.1366 | 0.7160 | 0.537 | 1.335 | ** | 0.016 | 1.3402 | 0.0609 | 1.0031 | 0.3412 | 0.0513 | -0.2535 | 0.0522 | -0.1312 | 0.1645 | -0.0258 | + |
| 48 | 7.1 7.6 | 7.1934 7.5772 | 0.2074 0.1818 | -0.0934 0.0228 | 0.513 0.523 | -0.182 0.044 | | 0.001 | -0.1810 0.0434 | 0.1403 0.1079 | 1.2515 | -0.0731 0.0151 | 0.0292 0.0006 | -0.0447 -0.0062 | -0.0474 -0.0037 | -0.0269 -0.0000 | -0.0016 | 0.0473 | -0.0 |
| 49 50 | 8.7 8.6 | 9.2962 7.6264 | 0.1726 0.0962 | -0.5962 0.9736 | 0.526 0.545 | -1.133 1.786 | ** | 0.020 0.014 | -1.1350 1.8073 | 0.0972 0.0302 | 1.0839 0.8712 | -0.3724 0.3190 | 0.1597 -0.0451 | -0.1561 -0.0667 | 0.2151 -0.0517 | 0.0142 0.0023 | -0.0461 0.1910 | -0.2963 0.1308 | 0.0 -0. |
| 51 | 5.4 | 5.7721 | 0.0988 | -0.3721 | 0.545 | -0.683 | * | 0.002 | -0.6810 | 0.0302 | 1.0756 | -0.1235 | -0.0682 | 0.0456 | -0.0053 | 0.0705 | -0.0008 | 0.0137 | -0.0 |
| 52 53 | 5.7 8.7 | 6.7504 8.0806 | 0.1889 0.1497 | -1.0504 0.6194 | 0.520 0.533 | -2.018 1.162 | **** | 0.077 | -2.0529 1.1643 | 0.1164 0.0731 | 0.8922 1.0504 | -0.7451 0.3269 | -0.1294 0.0036 | 0.0966 0.0652 | -0.0895 -0.2539 | -0.4442 -0.0223 | -0.1002 0.0710 | 0.3670 0.1712 | 0.0 |
| 54 | 6.1 | 6.0722 | 0.1284 | 0.0278 | 0.539 | 0.052 | | 0.000 | 0.0514 | 0.0538 | 1.1397 | 0.0123 | 0.0060 | -0.0069 | -0.0012 | -0.0036 | 0.0086 | 0.0009 | -0.0 |
| 55 56 | 7.3 7.7 | 6.7861 7.8356 | 0.1203 0.0926 | 0.5139 -0.1356 | 0.540 0.546 | 0.951 -0.249 | * | 0.006 | 0.9504 | 0.0472 0.0280 | 1.0572 1.1045 | 0.2115 -0.0420 | 0.0207 0.0190 | 0.0772 -0.0262 | -0.0283 0.0018 | 0.0491 -0.0101 | -0.1616 0.0018 | -0.0248 -0.0095 | 0.0 |
| 57 58 | 9.0 | 8.5166 7.3617 | 0.1576 | 0.4834 0.8383 | 0.531 0.548 | 0.911 1.530 | * | 0.010 | 0.9099 | 0.0810 | 1.1024 0.9215 | 0.2702 | -0.1533 0.0107 | -0.0413 -0.0840 | 0.1200 | 0.0706 | 0.0422 | -0.0189 0.0956 | 0.0 |
| 59 | 7.1 | 6.3995 | 0.1858 | 0.7005 | 0.522 | 1.343 | ** | 0.033 | 1.3490 | 0.1127 | 1.0598 | 0.4807 | -0.1300 | 0.2475 | 0.3076 | -0.1387 | -0.0621 | -0.2418 | 0.0 |
| 60 61 | 7.9 6.6 | 7.3313 7.0940 | 0.1915 | 0.5687 -0.4940 | 0.520 0.535 | 1.095 -0.923 | ** | 0.023 | 1.0959 -0.9219 | 0.1196 0.0650 | 1.1188 | 0.4039 | -0.0543 0.0155 | 0.2229 | 0.0133 | 0.1496 0.0798 | -0.2420 -0.1096 | -0.1550 0.0976 | -0. |
| 62 | 8.0 | 7.1389 | 0.1621 | 0.8611 | 0.529 | 1.627 | *** | 0.035 | 1.6414 | 0.0857 | 0.9640 | 0.5027 | 0.0105 | 0.1938 | -0.0723 | 0.2612 | -0.1775 | 0.1457 | -0.3 |
| 63 64 | 6.3 | 6.6745 5.6100 | 0.1228 0.1433 | -0.3745 0.3900 | 0.540 0.535 | -0.694 0.729 | * | 0.004 | -0.6917 0.7275 | 0.0492 0.0670 | 1.0939 | -0.1573 0.1950 | 0.0289 0.1388 | -0.0956 0.0104 | -0.0693 -0.0981 | -0.0176 0.0321 | 0.1056 -0.0577 | 0.0705 0.0017 | -0.0 |
| 65 66 | 5.4 7.6 | 5.6334 7.1102 | 0.0940 | -0.2334 0.4898 | 0.546 0.538 | -0.428 0.911 | | 0.001 | -0.4259 0.9097 | 0.0288 | 1.0954 | -0.0734 0.2213 | -0.0501 0.0596 | 0.0315 | -0.0054 -0.0657 | 0.0270 0.0783 | -0.0044 0.0110 | 0.0088 | -0.0 |
| 67 | 6.4 | 6.2641 | 0.1273 | 0.1359 | 0.539 | 0.252 | 1 1 1 | 0.001 | 0.2510 | 0.0528 | 1.1333 | 0.0593 | 0.0161 | -0.0360 | 0.0208 | 0.0138 | -0.0035 | -0.0249 | 0.0 |
| 68 69 | 6.1 5.2 | 5.8250 5.6092 | 0.1176 0.1358 | 0.2750 -0.4092 | 0.541 0.537 | 0.508 -0.762 | * | 0.002 | 0.5062 -0.7607 | 0.0451 | 1.1078 | -0.1925 | -0.0619 | -0.0887 -0.0110 | 0.0126 | -0.0162 0.1211 | 0.0197 | -0.0249 0.0284 | -0. |
| 70 | 6.6 | 6.2719 | 0.1213 | 0.3281 | 0.540 | 0.607 | * | 0.003 | 0.6053 | 0.0480 | 1.1019 | 0.1360 | 0.0489 | -0.0158 | -0.0252 | -0.0006 | -0.0826 | 0.0100 | 0.0 |
| 71 72 | 7.6 5.8 | 8.7657 5.7067 | 0.1658 0.1596 | -1.1657 0.0933 | 0.528 0.530 | -2.207 0.176 | **** | 0.068 | -2.2545 0.1751 | 0.0896 0.0831 | 0.8131 1.1737 | -0.7074 0.0527 | 0.4009 0.0285 | -0.3562 -0.0292 | 0.1153 | 0.1375 | 0.2337 | -0.3490 0.0075 | -0. -0. |
| 73 74 | 7.9 8.6 | 7.2514 8.9164 | 0.1104 0.2019 | 0.6486 | 0.543 0.516 | 1.195 -0.614 | ** | 0.008 | 1.1982 -0.6118 | 0.0397 0.1330 | 1.0079 1.2093 | 0.2437 | -0.0092 0.0714 | -0.1239 -0.0644 | 0.0576 0.1311 | 0.0933 0.1127 | 0.0530 | -0.0047 -0.1904 | -0. -0. |
| 75 | 8.2 | 7.6286 | 0.1064 | 0.5714 | 0.543 | 1.052 | ** | 0.006 | 1.0522 | 0.0369 | 1.0300 | 0.2060 | -0.0561 | 0.1188 | -0.0503 | 0.0287 | -0.1206 | 0.0879 | 0. |
| 76 77 | 7.1 6.4 | 7.4511 6.4091 | 0.1507 0.1017 | -0.3511 -0.009102 | 0.533 0.544 | -0.659 -0.017 | * | 0.005 | -0.6570 -0.0166 | 0.0741 | 1.1273 1.1162 | -0.1858 -0.0031 | 0.0031 -0.0006 | -0.1103 -0.0004 | 0.0698 | -0.0578 0.0015 | 0.0330 | -0.0557 -0.0000 | -0. |
| 78 | 7.6 | 7.5655 | 0.1087 | 0.0345 | 0.543 | 0.064 | | 0.000 | 0.0632 | 0.0386 | 1.1215 | 0.0127 | -0.0052 | 0.0076 | 0.0047 | 0.0070 | -0.0031 | -0.0045 | -0. |
| 79 80 | 8.9 5.7 | 9.0774 6.6728 | 0.1463 0.1593 | -0.1774 -0.9728 | 0.534 0.530 | -0.332 -1.835 | *** | 0.001 | -0.3307 -1.8587 | 0.0699 0.0828 | 1.1500 0.9085 | -0.0906 -0.5584 | 0.0386 -0.2067 | -0.0283 0.0771 | 0.0401 | -0.0401 0.0272 | -0.0009 -0.0949 | -0.0421 -0.3702 | 0. |
| 81 82 | 7.1 7.4 | 7.4219 7.0968 | 0.1421 0.1020 | -0.3219 0.3032 | 0.535 0.544 | -0.602 0.557 | * | 0.004 | -0.5995 0.5551 | 0.0659 0.0339 | 1.1236 1.0906 | -0.1592 0.1040 | 0.0028 0.0070 | -0.0965 0.0011 | 0.0665 | -0.0275 -0.0643 | 0.0214 0.0578 | -0.0550 0.0219 | 0. |
| 82 | 6.6 | 6.0300 | 0.1020 | 0.3032 | 0.544 | 1.057 | ** | 0.002 | 1.0576 | 0.0339 | 1.0906 | 0.1040 | 0.0070 | 0.0011 | -0.0337 | -0.0643 | 0.0578 | -0.0011 | -0. |
| 84 85 | 5.0 8.2 | 5.2220 7.2772 | 0.1815 0.1337 | -0.2220 0.9228 | 0.523 0.537 | -0.424 1.718 | | 0.003 0.026 | -0.4225 1.7361 | 0.1075 0.0583 | 1.1923 0.9140 | -0.1466 0.4319 | -0.0876 0.1366 | -0.0006 -0.1267 | 0.0374 | -0.0450 0.0851 | 0.0620 0.1350 | -0.0271 0.1008 | 0. -0. |
| 86 | 5.2 | 5.7395 | 0.1079 | -0.5395 | 0.543 | -0.994 | * | 0.006 | -0.9935 | 0.0380 | 1.0405 | -0.1975 | -0.0864 | -0.0452 | -0.0429 | -0.0590 | 0.1270 | 0.0733 | 0. |
| 87 88 | 5.2 8.2 | 4.6160 7.3634 | 0.1733 0.1356 | 0.5840 0.8366 | 0.526 0.537 | 1.111 1.558 | ** | 0.019 | 1.1121 1.5707 | 0.0980 | 1.0891 0.9534 | 0.3665 0.3967 | 0.2683 0.1071 | -0.2345 -0.1159 | -0.0086 -0.2434 | -0.1200 0.0286 | 0.1106 0.1136 | 0.0156 | -0. 0. |
| 89 | 7.3 | 7.1229 | 0.0880 | 0.1771 | 0.547 | 0.324 | | 0.000 | 0.3224 | | 1.0978 | 0.0519 | -0.0145 | 0.0270 | 0.0148 | -0.0087 | -0.0058 | | - |
| 90 91 | 7.4 | 8.2101 7.3970 | 0.1934 0.1475 | -0.0101 0.003039 | 0.519 0.534 | -0.019 0.006 | | 0.000 | -0.0193 0.005664 | 0.1220 0.0710 | 1.2284 1.1611 | -0.0072 0.0016 | -0.0003 | 0.0028 | -0.0011 -0.0003 | -0.0002 -0.0013 | -0.0007 0.0006 | -0.0022 0.0005 | - |
| 92 93 | 4.8 7.6 | 4.4951 7.5189 | 0.1927 0.1570 | 0.3049 0.0811 | 0.519 0.531 | 0.587 0.153 | * | 0.007 | 0.5853 0.1520 | | 1.1958 1.1708 | 0.2173 0.0449 | 0.0701 | 0.0151 | 0.1233 0.0124 | -0.0053 -0.0200 | -0.0519 0.0356 | -0.1029 -0.0069 | - |
| 94 | 8.9 | 9.0589 | 0.1673 | -0.1589 | 0.528 | -0.301 | | 0.001 | -0.2997 | 0.0913 | 1.1789 | -0.0950 | 0.0332 | -0.0286 | 0.0338 | -0.0599 | 0.0039 | -0.0357 | 0.0 |
| 95 96 | 7.7 7.3 | 7.5077 7.2665 | 0.1105 0.1418 | 0.1923 0.0335 | 0.543 0.535 | 0.354 0.063 | | 0.001 | 0.3527 0.0623 | 0.0398 0.0656 | | 0.0718 0.0165 | -0.0187 -0.0082 | 0.0289 | -0.0072 0.0120 | -0.0232 -0.0049 | 0.0234 | 0.0250 | - |
| 07 | | | | | | | | + | 1 | | - | | | | | | | | + |



0.000

0.007

0.052

0.000

Sum of Residuals

0.0468

0.6299

-2.6934

-0.1693

0.0078

0.2197

-0.6248

-0.0431

0.0269 | 1.1083

0.0610 1.1461

0

0.6682

0.1084

0.0511

0.0012

0.0019

-0.1086

-0.0011

0.0953

-0.1253

0.0177

0.0006

-0.0893

0.0858

-0.0213

-0.0052

0.0634

0.0402

0.0274

-0.0005

-0.1321

-0.2324

-0.0089

0.0032

0.0605

0.2873

-0.0155 -0.0145

0.0015

0.0091

-0.0145

0.0184

97

98

99

100

6.3

5.4

6.4

6.4

6.2743

5.0696

7.8062

6.4913

0.0909

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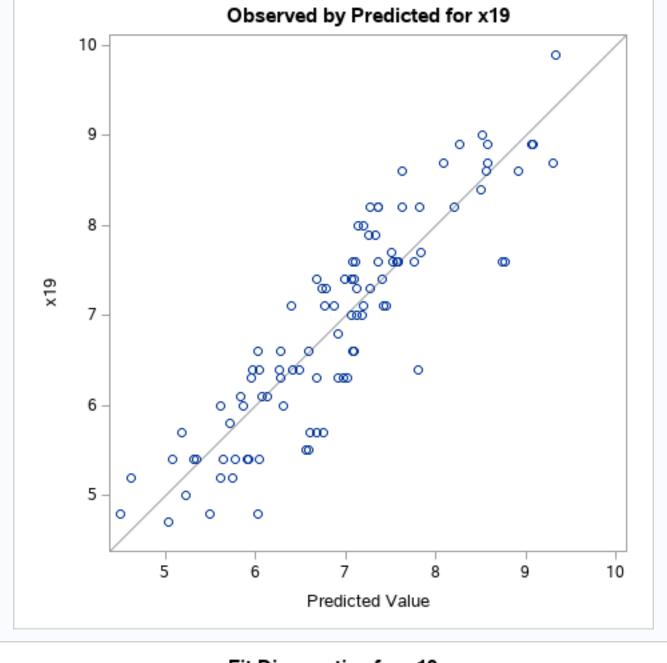
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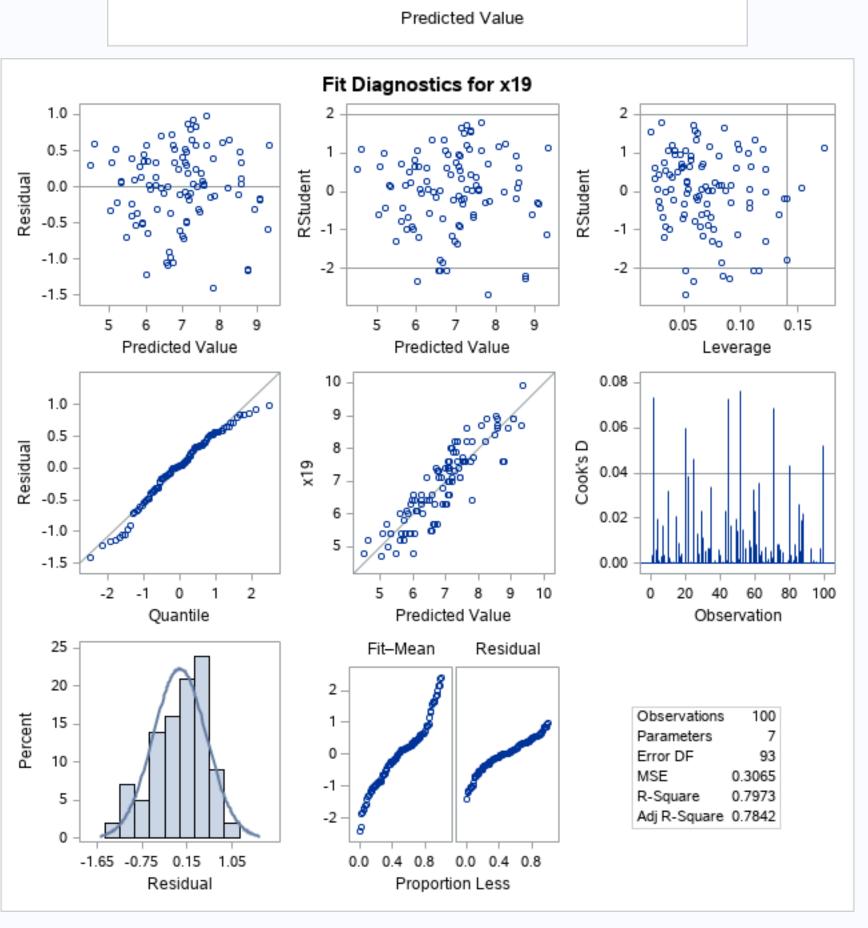
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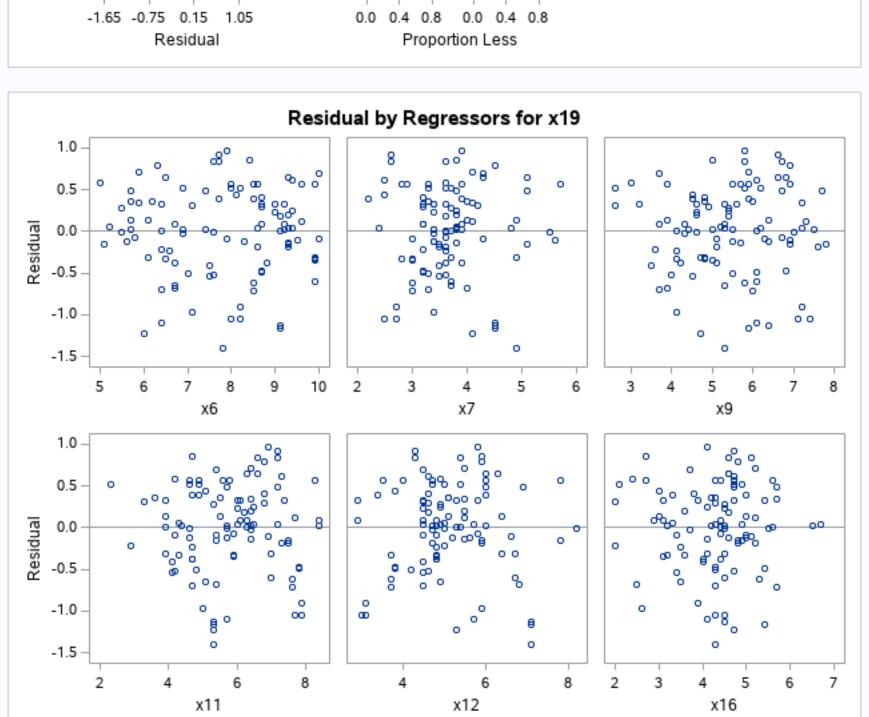
0.523

0.539

0.537







Model: MODEL1 Dependent Variable: x19 x19

Number of Observations Read 100

Number of Observations Used 100

Corrected Total 99 140.62760

| Analysis of Variance | | | | | | | | | | | | | |
|----------------------|----|----------------|----------------|---------|--------|--|--|--|--|--|--|--|--|
| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F | | | | | | | | |
| Model | 4 | 89.20760 | 22.30190 | 41.20 | <.0001 | | | | | | | | |
| Error | 95 | 51.42000 | 0.54126 | | | | | | | | | | |

| Root MSE | 0.73571 | R-Square | 0.6344 |
|----------------|----------|----------|--------|
| Dependent Mean | 6.91800 | Adj R-Sq | 0.6190 |
| Coeff Var | 10.63466 | | |

| | Parameter Estimates | | | | | | | | | | | | | | |
|-----------|---------------------|----|-----------------------|-------------------|---------|---------|------------|--------------------------|--|-----------------------------------|---|------------------------------------|-----------|-----------------------|--|
| Variable | Label | DF | Parameter Estimate | Standard Error | t Value | Pr > t | Type I SS | Standardized Estimate | Squared Semi-partial Corr Type I | Squared Partial Corr Type I | Squared Semi-partial Corr Type II | Squared Partial Corr Type II | Tolerance | Variance Inflation | |
| Intercept | Intercept | 1 | 0.42420 | 0.52097 | 0.81 | 0.4175 | 4785.87240 | 0 | | | | | | 0 | |
| х9 | x9 | 1 | 0.14901 | 0.10489 | 1.42 | 0.1587 | 51.17801 | 0.15108 | 0.36393 | 0.36393 | 0.00777 | 0.02080 | 0.34029 | 2.93870 | |
| x11 | x11 | 1 | 0.39805 | 0.06991 | 5.69 | <.0001 | 9.21753 | 0.43927 | 0.06555 | 0.10305 | 0.12476 | 0.25440 | 0.64654 | 1.54668 | |
| x12 | x12 | 1 | 0.51916 | 0.07296 | 7.12 | <.0001 | 27.80110 | 0.46710 | 0.19769 | 0.34651 | 0.19490 | 0.34770 | 0.89329 | 1.11946 | |
| x16 | x16 | 1 | 0.16656 | 0.12187 | 1.37 | 0.1750 | 1.01096 | 0.12981 | 0.00719 | 0.01928 | 0.00719 | 0.01928 | 0.42665 | 2.34383 | |

| Collinearity Diagnostics | | | | | | | | | | | | | | |
|--------------------------|------------|-----------|------------|------------|-------------|------------|------------|--|--|--|--|--|--|--|
| | | Condition | | Propo | rtion of Va | riation | | | | | | | | |
| Number | Eigenvalue | Index | Intercept | х9 | x11 | x12 | x16 | | | | | | | |
| 1 | 4.89343 | 1.00000 | 0.00082009 | 0.00065189 | 0.00127 | 0.00150 | 0.00078123 | | | | | | | |
| 2 | 0.05237 | 9.66650 | 0.01838 | 0.02176 | 0.15325 | 0.41255 | 0.01502 | | | | | | | |
| 3 | 0.02939 | 12.90451 | 0.09051 | 0.08601 | 0.39647 | 0.00003909 | 0.21652 | | | | | | | |
| 4 | 0.01545 | 17.79819 | 0.65239 | 0.10269 | 0.19191 | 0.44517 | 0.14104 | | | | | | | |
| 5 | 0.00937 | 22.84926 | 0.23790 | 0.78888 | 0.25710 | 0.14075 | 0.62665 | | | | | | | |

| | | | | | | | | | el: MODEL1 t Variable: x | 19 x19 | | | | | | | | |
|----------|-----------------------|--------------------|--------------------------|--------------------|-----------------------|---------------------|------------|------------|-----------------------------|--------------------|------------------|------------------|--------------------|--------------------|--------------------|--------------------|--------------------|-------------------------------|
| | | | Std | | | | (| Outp | out Statistics | | | | | | | DFBETAS | | |
| Obs | Dependent Variable | Predicted Value | Error Mean Predict | Residual | Std Error Residual | Student Residual | -2-1 0 1 2 | | Cook's D | RStudent | Hat Diag | Cov Ratio | DFFITS | Intercept | x9 | x11 | x12 | x16 |
| 1 2 | 8.2 5.7 | 7.2016 6.9006 | 0.1305 0.2715 | 0.9984 | 0.724 0.684 | 1.379 -1.756 | | 1 | 0.012 | 1.3856 -1.7757 | 0.0314 0.1362 | 0.9839 | 0.2497 | -0.0354 -0.1533 | 0.0059 | -0.1334 -0.0818 | 0.0683 0.3922 | 0.1036 0.4292 |
| 3 | 8.9 | 8.1148 5.8988 | 0.1902 | 0.7852 | 0.711 | 1.105 | *** | - <u>-</u> | 0.017 | 1.1062 | 0.0669 | 1.0591 | 0.2961 | -0.1620 -0.1353 | -0.2086 0.3074 | 0.1790 | 0.1051 | 0.1917 |
| 5 | 7.1 | 6.5837 | 0.1360 | 0.5163 | 0.723 | 0.714 | * | | 0.004 | 0.7122 | 0.0342 | 1.0627 | 0.1340 | 0.0201 | -0.1010 | 0.0406 | -0.0238 | 0.0862 |
| 7 | 4.7 5.7 | 5.2673 4.8804 | 0.1537 0.2355 | -0.5673 0.8196 | 0.719 0.697 | -0.788 1.176 | * | | 0.006 | -0.7869 1.1783 | 0.0436 0.1024 | 1.0668 | -0.1681 0.3980 | -0.1477 0.2614 | 0.0190 | -0.1833 | 0.1046 0.0599 | -0.0172 -0.1102 |
| 9 | 6.3 7.0 | 5.9364 7.5151 | 0.1581 0.1394 | 0.3636 -0.5151 | 0.719 0.722 | 0.506 -0.713 | * | | 0.002 | 0.5041 -0.7112 | 0.0462 0.0359 | 1.0906 1.0647 | 0.1109 | 0.0546 0.0183 | -0.0011 -0.1068 | -0.0879 0.0387 | -0.0166 -0.0217 | 0.0343 |
| 10 | 5.5 7.4 | 7.2442 6.8673 | 0.1165 0.1266 | -1.7442 0.5327 | 0.726 0.725 | -2.401 0.735 | **** | | 0.030 | -2.4643 0.7333 | 0.0251 0.0296 | 0.7908 1.0560 | -0.3953 0.1281 | 0.0102 0.0118 | -0.2635 -0.0511 | 0.0927 | -0.0891 -0.0053 | 0.2347 |
| 12 13 | 6.0 8.4 | 6.3800 8.4748 | 0.1764 0.1469 | -0.3800 -0.0748 | 0.714 0.721 | -0.532 -0.104 | * | | 0.003 | -0.5301 -0.1033 | 0.0575 0.0399 | 1.1021 1.0975 | -0.1309 -0.0211 | -0.0294 0.0153 | 0.0146 | 0.0293 | -0.0755 -0.0128 | 0.0370 0.0009 |
| 14 15 | 7.6 8.0 | 7.8289 8.0216 | 0.1910 0.1283 | -0.2289 -0.0216 | 0.710 0.724 | -0.322 -0.030 | | | 0.001 | -0.3206 -0.0297 | 0.0674 0.0304 | 1.1244 1.0873 | -0.0862 -0.0053 | 0.0293 0.0026 | 0.0388 | -0.0781 -0.0006 | -0.0077 -0.0016 | -0.0102 0.0017 |
| 16 17 | 6.6 6.4 | 7.2313 6.8548 | 0.1903 0.1062 | -0.6313 -0.4548 | 0.711 0.728 | -0.888 -0.625 | * | | 0.011 | -0.8873 -0.6227 | 0.0669 | 1.0838 1.0548 | -0.2376 -0.0908 | -0.0195 -0.0235 | -0.1256 -0.0638 | -0.0681 | 0.1177 0.0162 | 0.1126 0.0414 |
| 18 | 7.4 | 7.8494 7.3869 | 0.1431 | -0.4494 | 0.722 | -0.623 -0.806 | * | | 0.003 | -0.6207 -0.8048 | 0.0378 | 1.0736 | -0.1231 | 0.0476 | -0.0383 | 0.0288 | 0.0071 | -0.0420 |
| 19 20 | 7.6 | 7.9231 | 0.1699 | -0.5869 -0.3231 | 0.728 | -0.451 | * | | 0.003 | -0.4495 | 0.0213 | 1.0409 | -0.1187 -0.1067 | 0.0248 | -0.0414 -0.0419 | 0.0284 | -0.0712 | 0.0281 |
| 21 | 5.4 9.9 | 6.0561 9.3728 | 0.1586 0.2843 | -0.6561 0.5272 | 0.718 0.679 | -0.913 0.777 | * | | 0.008 | -0.9125 0.7753 | 0.0464 0.1494 | 1.0580 1.2006 | -0.2014 0.3249 | -0.1163 -0.2186 | -0.1296 -0.0820 | 0.1212 | 0.0309 0.2584 | 0.1285 |
| 23 24 | 7.0 8.6 | 7.3295 7.8152 | 0.1643 0.2274 | -0.3295 0.7848 | 0.717 0.700 | -0.459 1.122 | | | 0.002 0.027 | -0.4575 1.1232 | 0.0498 0.0955 | 1.0974 | -0.1048 0.3651 | 0.0231 -0.1031 | 0.0748 | -0.0717 0.0069 | 0.0004 | -0.0561 0.2774 |
| 25 26 | 4.8 6.6 | 6.7686 6.5699 | 0.1367 0.2149 | -1.9686 0.0301 | 0.723 0.704 | -2.723 0.043 | **** | | 0.053 0.000 | -2.8212 0.0426 | 0.0345 0.0853 | 0.7273 1.1525 | -0.5334 0.0130 | -0.0044 0.0018 | 0.3886 0.0010 | -0.0051 -0.0098 | -0.0763 -0.0024 | -0.4029 0.0070 |
| 27 28 | 6.3 5.4 | 7.1173 6.0511 | 0.1862 0.1331 | -0.8173 -0.6511 | 0.712 0.724 | -1.148 -0.900 | ** | | 0.018 0.005 | -1.1503 -0.8989 | 0.0641 0.0327 | 1.0505 1.0443 | -0.3010 -0.1653 | 0.0152 -0.1060 | 0.1262 -0.0614 | -0.1353 0.1104 | 0.1398 0.1060 | -0.1650 -0.0018 |
| 29 | 6.3 5.4 | 7.2137 5.8630 | 0.2069 | -0.9137 -0.4630 | 0.706 0.708 | -1.294 -0.654 | ** | | 0.029 | -1.2989 -0.6522 | 0.0791 | 1.0475 | -0.3807 -0.1851 | 0.0361 | 0.1650 0.1184 | -0.1365 0.0513 | 0.1705 0.0313 | -0.2517 -0.1425 |
| 31 | 6.1 | 6.6688 5.5973 | 0.1348 0.1529 | -0.5688 0.8027 | 0.723 0.720 | -0.786 1.115 | * | | 0.004 | -0.7848 1.1169 | 0.0336 0.0432 | 1.0559 | -0.1463 0.2373 | -0.0210 0.1937 | 0.0914 | -0.0944 -0.0433 | -0.0077 -0.1267 | -0.1423 -0.0112 -0.1336 |
| 33 | 5.4 | 6.3709 | 0.1220 | -0.9709 | 0.726 | -1.338 | ** | | 0.010 | -1.3438 | 0.0275 | 0.9858 | -0.2261 | -0.1207 | -0.1360 | 0.0851 | 0.0398 | 0.1555 |
| 35 | 7.3 6.3 | 6.3275 7.1015 | 0.1368 | 0.9725 -0.8015 | 0.723 | 1.345 -1.146 | | | 0.013 | 1.3512 -1.1479 | 0.0346 0.0962 | 0.9920 1.0881 | 0.2558 -0.3745 | 0.1093 | 0.0674 0.0507 | -0.1939 | -0.1091 -0.2590 | 0.0654 |
| 36 37 | 5.4 7.1 | 5.5149 6.4838 | 0.1865 0.1051 | -0.1149 0.6162 | 0.712 0.728 | -0.162 0.846 | | | 0.000 | -0.1607 0.8450 | 0.0643 0.0204 | 1.1252 1.0363 | -0.0421 0.1220 | -0.0292 0.0439 | -0.0005 -0.0623 | -0.0080 0.0484 | 0.0286 | 0.0131 |
| 38 | 8.7 7.6 | 8.2985 6.6772 | 0.1468 0.1343 | 0.4015 0.9228 | 0.721 0.723 | 0.557 1.276 | * | | 0.003 0.011 | 0.5550 1.2801 | 0.0398 0.0333 | 1.0802 | 0.1130 0.2377 | -0.0633 0.0680 | 0.0432 0.1321 | 0.0410 | 0.0101 | -0.0152 -0.0017 |
| 40 41 | 6.0 7.0 | 6.7640 6.5406 | 0.1686 0.1185 | -0.7640 0.4594 | 0.716 0.726 | -1.067 0.633 | ** | | 0.013 0.002 | -1.0676 0.6307 | 0.0525 0.0259 | 1.0477 1.0598 | -0.2513 0.1029 | -0.0035 0.0321 | 0.0706 0.0134 | 0.1222 | -0.1122 -0.0147 | -0.1041 0.0309 |
| 42 | 7.6 8.9 | 7.0430 8.1015 | 0.0904 0.1417 | 0.5570 0.7985 | 0.730 0.722 | 0.763 1.106 | * | | 0.002 | 0.7612 1.1074 | 0.0151 0.0371 | 1.0381 1.0262 | 0.0943 0.2173 | 0.0031 | -0.0079 0.1128 | 0.0488 | -0.0032 0.1028 | -0.0193 -0.0909 |
| 44 | 7.6 5.5 | 8.4674 6.8208 | 0.2129 | -0.8674 -1.3208 | 0.704 0.695 | -1.232 -1.901 | ** | | 0.028 | -1.2352 -1.9278 | 0.0838 | 1.0617 0.9736 | -0.3735 -0.6704 | 0.1890 | -0.1274 -0.4097 | 0.0866 | -0.2686 0.4468 | 0.0576 |
| 46 | 7.4 | 7.5724 | 0.1165 | -0.1724 | 0.726 | -0.237 | l I | | 0.000 | -0.2361 | 0.0251 | 1.0783 | -0.0379 | 0.0162 | 0.0166 | -0.0086 | -0.0075 | -0.0257 |
| 48 | 7.1 | 6.6815 8.0083 | 0.1358 | -0.4083 | 0.723 | -0.580 | | | 0.002 | -0.5768 | 0.0341 | 1.0724 | 0.1083 | 0.0297 | -0.0200 | -0.0630 | -0.0594 -0.0054 | -0.1021 |
| 49 50 | 8.7 8.6 | 8.3474 7.7290 | 0.1565 0.1235 | 0.3526 0.8710 | 0.719 0.725 | 0.490 1.201 | ** | | 0.002 | 0.4885 1.2037 | 0.0452 | 1.0904 | 0.1063 0.2050 | -0.0699 -0.0807 | -0.0069 0.0134 | 0.0547 | 0.0809 | -0.0160 -0.0814 |
| 51 52 | 5.4 5.7 | 6.0791 7.0304 | 0.1218 0.2469 | -0.6791 -1.3304 | 0.726 0.693 | -0.936 -1.920 | * | | 0.005 0.093 | -0.9353 -1.9476 | 0.0274 0.1126 | 1.0350 0.9749 | -0.1570 -0.6937 | -0.0780 -0.1208 | 0.0888 | -0.0614 | 0.0163 0.4442 | -0.0703 0.2984 |
| 53 54 | 8.7 6.1 | 7.4099 6.6004 | 0.1161 0.1408 | 1.2901 -0.5004 | 0.726 0.722 | 1.776 -0.693 | ** | | 0.016 0.004 | 1.7965 -0.6911 | 0.0249 0.0366 | 0.9133 1.0670 | 0.2870 -0.1347 | -0.0535 -0.0331 | -0.0067 0.0316 | 0.1608 | -0.0743 -0.0067 | 0.0145 0.0551 |
| 55 56 | 7.3 7.7 | 6.3757 7.3594 | 0.1468 0.0958 | 0.9243 0.3406 | 0.721 0.729 | 1.282 0.467 | ** | | 0.014 | 1.2866 0.4651 | 0.0398 0.0170 | 1.0063 1.0603 | 0.2620 0.0611 | 0.0953 -0.0123 | 0.0540 0.0112 | -0.1936 0.0232 | -0.1096 0.0206 | 0.0935 |
| 57 58 | 9.0 8.2 | 8.9691 7.4852 | 0.1840 0.0905 | 0.0309 0.7148 | 0.712 0.730 | 0.043 0.979 | | | 0.000 | 0.0431 0.9788 | 0.0626 0.0151 | 1.1246 1.0176 | 0.0111 0.1213 | -0.0086 -0.0469 | 0.0031 | 0.0014 0.0586 | 0.0058 0.0321 | 0.0006 0.0175 |
| 59 60 | 7.1 7.9 | 6.0775 6.5109 | 0.1473 0.2122 | 1.0225 1.3891 | 0.721 0.704 | 1.419 1.972 | ** | | 0.017 | 1.4263 2.0030 | 0.0401 | 0.9869 | 0.2915 0.6032 | 0.1124 0.1625 | -0.2247 0.2016 | 0.0813 | -0.0207 -0.4460 | 0.1057 0.1790 |
| 61 62 | 6.6 | 7.3102 6.5529 | 0.1838 | -0.7102 1.4471 | 0.712 | -0.997 | * | | 0.013 | -0.9969 2.0757 | 0.0624 | 1.0669 | -0.2571 0.5727 | 0.0298 | 0.0873 | -0.1198 | 0.1162 | -0.1300 -0.4762 |
| 63 | 6.3 | 6.2798 | 0.1108 | 0.0202 | 0.727 | 0.028 | l I | | 0.000 | 0.0276 | 0.0227 | 1.0788 | 0.0042 | 0.0021 | -0.0003 | -0.0026 | -0.0013 | 0.0015 |
| 64 65 | 6.0 5.4 | 5.2804 5.9809 | 0.1639 | 0.7196 -0.5809 | 0.717 | 1.003 -0.799 | | | 0.011 | 1.0034 -0.7974 | 0.0496 0.0233 | 1.0518 | 0.2293 | 0.2063 -0.0815 | 0.0581 | -0.0882 | -0.1660 0.0091 | -0.0676 -0.0007 |
| 66 67 | 7.6 6.4 | 7.5893 7.0477 | 0.1316 0.1179 | 0.0107 -0.6477 | 0.724 0.726 | 0.015 -0.892 | | | 0.000 | 0.0148 -0.8909 | 0.0320 0.0257 | 1.0892 | -0.1446 | -0.0006 -0.0087 | 0.0018 -0.0630 | -0.0010 | 0.0007 | -0.0008 -0.0258 |
| 68 69 | 6.1 5.2 | 6.6319 5.5802 | 0.0894 0.1799 | -0.5319 -0.3802 | 0.730 0.713 | -0.728 -0.533 | * | | 0.002 0.004 | -0.7266 -0.5309 | 0.0147 0.0598 | 1.0406 1.1047 | -0.0889 -0.1339 | -0.0301 -0.0661 | 0.0036 0.0888 | 0.0372 | 0.0245 0.0213 | -0.0307 -0.0779 |
| 70 71 | 6.6 7.6 | 6.3007 7.9985 | 0.1575 0.1858 | 0.2993 -0.3985 | 0.719 0.712 | 0.416 -0.560 | | | 0.002 0.004 | 0.4146 -0.5578 | 0.0458 0.0637 | 1.0949 1.1077 | 0.0909 -0.1455 | 0.0293 0.0735 | 0.0035 0.0462 | -0.0715 0.0181 | -0.0099 -0.0947 | 0.0357 |
| 72 73 | 5.8 7.9 | 6.3025 7.7965 | 0.1733 0.1203 | -0.5025 0.1035 | 0.715 0.726 | -0.703 0.143 | * | | 0.006 0.000 | -0.7009 0.1418 | 0.0555 0.0267 | 1.0875 1.0822 | -0.1698 0.0235 | -0.0499 -0.0092 | 0.0743 0.0130 | -0.1027 -0.0007 | -0.0076 0.0087 | 0.0437 |
| 74 75 | 8.6 8.2 | 8.0722 7.0420 | 0.2102 | 0.5278 1.1580 | 0.705 0.728 | 0.749 1.591 | * | | 0.010 | 0.7469 1.6046 | 0.0816 0.0219 | 1.1145 0.9417 | 0.2226 | -0.1071 -0.0085 | -0.1319 0.0220 | 0.1534 | 0.1613 0.1041 | 0.0273 |
| 76 77 | 7.1 | 6.5261 6.2860 | 0.1392 0.1338 | 0.5739 0.1140 | 0.722 0.723 | 0.794 0.158 | * | | 0.005 | 0.7929 0.1568 | 0.0358 | 1.0577 | 0.1528 0.0290 | 0.0586 | 0.0568 | 0.0245 | -0.0078 -0.0023 | -0.1227 0.0188 |
| 78 | 7.6 | 7.1955 | 0.1044 | 0.4045 | 0.728 | 0.555 | * | | 0.001 | 0.5534 | 0.0201 | 1.0586 | 0.0793 | 0.0021 | 0.0456 | 0.0076 | -0.0167 | -0.0382 |
| 79 80 | 5.7 | 8.4712 6.5215 | 0.1618 | -0.8215 | 0.718 | -1.150 | | | 0.004 | -1.1516 | 0.0484 | 1.0873 | 0.1343 | -0.0764 -0.0645 | -0.0133 | -0.0322 | -0.1299 | -0.0346 0.1804 |
| 81 82 | 7.1 7.4 | 6.5129 6.9654 | 0.1242 0.1281 | 0.5871 0.4346 | 0.725 0.724 | 0.810 0.600 | * | | 0.004 | 0.8081 0.5978 | 0.0285 0.0303 | 1.0483 1.0669 | 0.1384 0.1057 | 0.0547 -0.0018 | 0.0252 -0.0666 | 0.0350 | -0.0034 -0.0079 | -0.0954 0.0342 |
| 83 84 | 6.6 5.0 | 5.7608 5.0439 | 0.1601 0.2332 | 0.8392 -0.0439 | 0.718 0.698 | 1.169 -0.063 | ** | | 0.014 0.000 | 1.1710 -0.0626 | 0.0473 0.1005 | 1.0294 1.1719 | 0.2611 | 0.1694 -0.0147 | -0.0153 -0.0074 | 0.0623 0.0107 | -0.0717 0.0005 | -0.1326 0.0132 |
| 85 86 | 8.2 5.2 | 7.2889 5.6293 | 0.1333 0.1364 | 0.9111 | 0.724 0.723 | 1.259 -0.594 | ** | | 0.011 0.003 | 1.2632 -0.5918 | 0.0328 0.0344 | 1.0021 1.0718 | 0.2327 | -0.0018 -0.0927 | 0.1004 | 0.0524 0.0705 | -0.1132 0.0399 | -0.0496 0.0316 |
| 87 88 | 5.2 | 5.4867 7.3704 | 0.1769 | -0.2867 0.8296 | 0.714 | -0.401 1.147 | ** | | 0.002 | -0.3997 1.1488 | 0.0578 | 1.1096 | -0.0990 | -0.0617 -0.0190 | 0.0284 | 0.0015 | -0.0112 -0.1092 | 0.0315 |
| 89 | 7.3 | 6.8511 8.9094 | 0.0951 | 0.4489 | 0.730 0.698 | 0.615 | ** | | 0.003 | 0.6133 | 0.0167 0.1006 | | 0.0800 | 0.0101 0.2225 | -0.0290 -0.0275 | 0.0170 | -0.0269 -0.2660 | 0.0364 |
| 90 | 7.4 | 7.0792 | 0.1888 | 0.3208 | 0.711 | 0.451 | ** | | 0.003 | 0.4493 | 0.0658 | 1.1166 | 0.1193 | -0.0198 | -0.1075 | 0.0636 | 0.0367 | 0.0682 |
| 92 | 7.6 | 4.7945 7.7824 | 0.2106 | -0.1824 | 0.705 | -0.258 | | | 0.000 | -0.2563 | 0.0819 | | -0.0722 | 0.0018 | -0.0003 0.0358 | -0.0006 -0.0660 | -0.0001 -0.0080 | -0.0008 |
| 94 95 | 8.9 7.7 | 8.4344 7.1122 | 0.1952 0.1333 | 0.4656 0.5878 | 0.709 0.724 | 0.656 0.812 | * | | 0.007 | 0.6544 0.8110 | 0.0704 0.0328 | 1.1087 1.0528 | 0.1801 0.1494 | -0.0729 -0.0140 | 0.1301 | 0.0218 | 0.0271 0.0663 | -0.0963 -0.0279 |
| 96 97 | 7.3 6.3 | 7.3776 6.1122 | 0.1238 0.1179 | -0.0776 0.1878 | 0.725 0.726 | -0.107 0.259 | | | 0.000 | -0.1065 0.2573 | 0.0283 0.0257 | 1.0844 1.0783 | -0.0182 0.0418 | 0.0061 0.0248 | 0.0118 0.0028 | -0.0043 -0.0296 | -0.0067 -0.0129 | -0.0124 0.0081 |

Sum of Squared Residuals51.42000Predicted Residual SS (PRESS)57.58256

Sum of Residuals

0.000

0.034

0.0693

-1.8708

0

0.0880 1.1558

0.0476 0.9221

0.0215

-0.4184

0.0120

0.1720

-0.0112 | 0.0085 | -0.0100

0.0842 | -0.0137 | -0.3622 | -0.0208

0.0081

98

6.4

7.7259 0.1606

5.9142 0.1523

0.2183

5.3510

0.0490

-1.3259

0.703

0.718

0.070

-1.847

