Week 7 Homework

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1.

$$f(x, y) = x^3 + y$$

$$f(2{,}3) = 2^3 + 3 = 8 + 3 = \underline{\mathbf{11}}$$

$$f(x_0, y_0) = 4$$

$$\boldsymbol{x_0 = 1}$$
$$\boldsymbol{y_0 = 3}$$

$$f(x, y) = 1^3 + 3 = \underline{\mathbf{4}}$$

2.

Function	Graph	Contour Plot
1. $f(x, y) = -x^2 - y^2$	Graph 3	Graph B
2. $g(x, y) = -3x^2 - y^2$	Graph 2	Graph A
3. $h(x, y) = -x^2 - 3y^2$	Graph 1	Graph C

3.

$\vec{a}(2, 3, 1)$
$\vec{b}(-1, 2, 2)$

Equation for a line: $\vec{L}(t) = \vec{p} + t\vec{d}$

Set $\vec{a}$ as the point on the line and calculate the direction $\overrightarrow{ab}$:

$\vec{p} = \vec{a} = (2{,}3{,}1)$
$\vec{d} = \overrightarrow{ab} = (x_b - x_a, y_b - y_a, z_b - z_a)$ = (-1-2, 2-3, 2-1) = (-3, -1, 1)

The equation for the line between $\vec{a}$ and $\vec{b}$ is:

$\vec{L}(t) = \vec{p} + t\vec{d}$

$$\boldsymbol{\vec{L}(t)} = \begin{pmatrix} \mathbf{2} \\ \mathbf{3} \\ \mathbf{1} \end{pmatrix} + \boldsymbol{t} \begin{pmatrix} \mathbf{-3} \\ \mathbf{-1} \\ \mathbf{1} \end{pmatrix}$$

Validate equation:

Verify, that $\vec{a}$ lies on the line:
$t = 0$

$$\vec{L}(0) = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + 0 \begin{pmatrix} -3 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

Verify that $\vec{b}$ lies on the line:
$t = 1$

$$\vec{L}(1) = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + 1 \begin{pmatrix} -3 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \begin{pmatrix} -3 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$$

4.

Given $\vec{p}(1, -1, 1)$ and $\vec{L}(t) = \vec{p} + t \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ find the equation of the plane.

a. Find a point on the plane:
$\vec{p}(1, -1, 1)$ is a point on the plane

b. Find a vector perpendicular to the plane.
$\vec{L}(t) = \vec{p} + t \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ is perpendicular to $\vec{p}$.

$$\vec{L}(1) = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + 1 \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

c. Given a point $\vec{p}(1, -1, 1)$ and a vector $\vec{n} \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$ perpendicular to the plane, a point (vector) $\vec{v} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ is on the plane if…

$$\vec{n} \cdot (\vec{v} - \vec{p}) = 0$$

$$2(x - 1) + 0\big(y - (-1)\big) + 0(z - 1) = 0$$

$2(x - 1) = 0$
$\boldsymbol{f(x, y) = 2x - 2}$ **Equation of the plane**

d. Make sure, $\vec{p}$ satisfies equation:
$f(x, y) = 2x - 2$
$f(x, y) = 2(1) - 2 = 0$

5.

I used graphical representation (using the Wolframalpha.com widget) to find out, if the functions have a limit at (0,0) and if they are continuous at (0,0).

a)

$f(x, y) = \ln(xy)$

The function $f(x, y)$ is discontinuous because it is not a smooth picture and contains gaps (Fig.1).

The limit at (0,0) does not exist because there is a disagreement on what the function value should be, if (0,0) is approached from different directions.

$$\lim_{(x,y)\to(x_0,y_0)} f(x, y) = L$$

$$\lim_{(x,y)\to(0,0)} f(x, y) = Limit\ does\ not\ exist$$

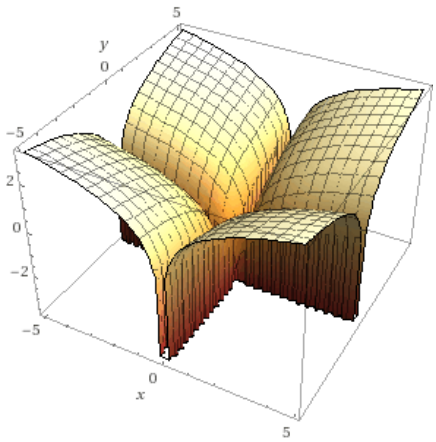

Fig. 1. Graph of *f(x,y)*. Generated with the help of www.wolframalpha.com.

b)

$$g(x, y) = \frac{x^3 + x + xy - x^2y - y - y^2}{x - y}$$

The function $g(x, y)$ is continuous because the graphical representation shows a smooth picture without gaps, holes or jumps (Fig. 2).

The function $g(x, y)$ has a limit at (0,0) because there is an agreement on the function value at (0,0) when the value is approached from different sides.

$$\lim_{(x,y)\to(0,0)} g(x, y) = 1$$

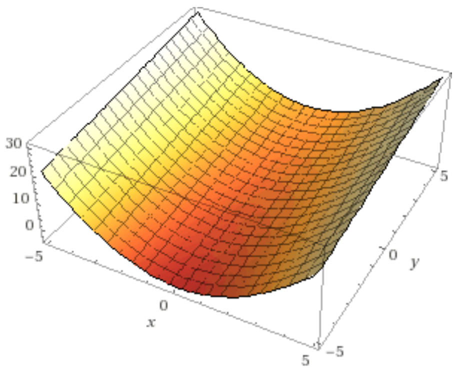

Fig. 2. Graph of *g(x,y)* Generated with the help of www.wolframalpha.com

c)

$$h(x,y) = \frac{xy}{\cos\,(x+y)}$$

The function $h(x,y)$ is discontinuous because it is not a smooth picture and contains gaps holes and jumps (Fig.3).

The limit at (0,0) does not exist because there is a disagreement on what the function value should be if (0,0) is approached from different directions.

$$\lim_{(x,y)\to(0,0)} h(x,y) = Limit\ does\ not\ exist$$

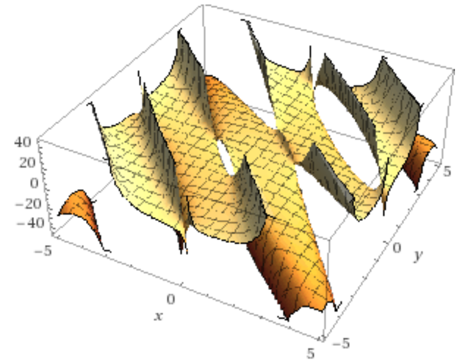

Fig. 3. Graph of *h(x,y)*
Generated with the help of
www.wolframalpha.com

d)

$$k(x,y) = x^2y + \ y^2x$$

The function $k(x,y)$ is continuous, because the graphical representation does not show any gaps, jumps or holes (Fig.4).

The function $k(x,y)$ has a limit at (0,0) because there is an agreement on the function value at (0,0) when the value is approached from different sides.

$$\lim_{(x,y)\to(0,0)} k(x,y) = 0$$

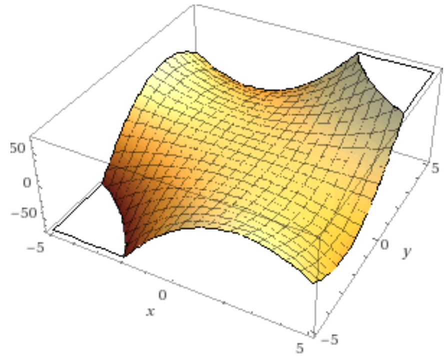

Fig. 4. Graph of *k(x,y)*
Generated with the help of
www.wolframalpha.com