Week 4 Homework

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1. Find the dot product of the vectors

$\vec{a} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$

$\begin{bmatrix} 3 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -1 \end{bmatrix} = 3 * 4 + 2 * -1 = 12 - 2 = \mathbf{10}$

2. The projection of $\vec{b}$ onto $\vec{a}$

$proj_{\vec{a}}\vec{b} = \frac{\vec{a} * \vec{b}}{|\vec{a}|^2}\vec{a} = \frac{10}{(\sqrt{3^2+2^2})^2}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \frac{10}{13}\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

Fraction of $\frac{10}{13}$ multiplies every component of the vector $\vec{a}$ resulting in a shrinked vector.

3.

The two vectors are perpendicular, in an angle of 90°, to each other.

Example:

$\vec{a} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$

$\begin{bmatrix} 4 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 4 \end{bmatrix} = -4 + 4 = 0$

4.

$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

Matrix A is a scaling matrix. It scales the vector by 2.

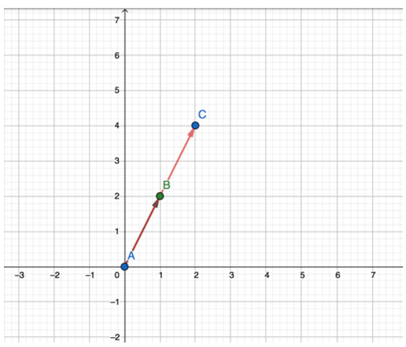

Fig. 1 Result of applying matrix multiplication with matrix A. Green = original vector. Red = resulting vector

$$B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$

Matrix B is a reflection matrix.

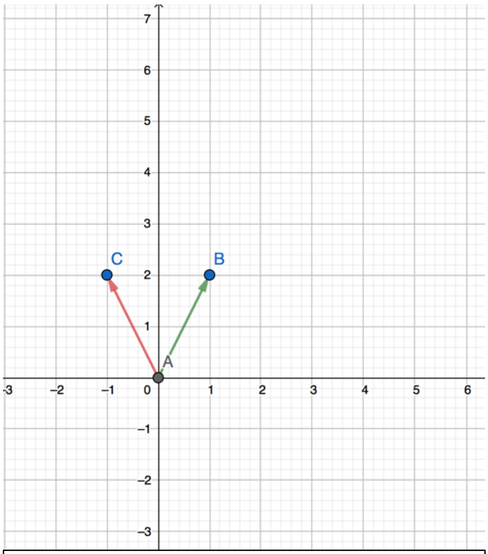

Fig. 2 Result of applying matrix multiplication with matrix B. Green = original vector. Red = resulting vector

$$C = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$$

Matrix C is a rotation matrix.

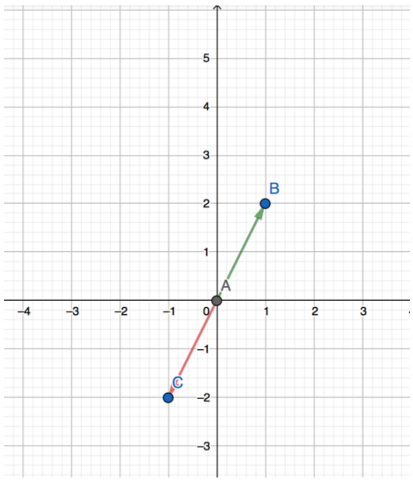

Fig. 3 Result of applying matrix multiplication with matrix C. Green = original vector. Red = resulting vector

5.

a)

A= $\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}2\\1\end{bmatrix}$

A= $\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-1\\3\end{bmatrix}$

A= $\begin{bmatrix}4\\6\end{bmatrix} = \begin{bmatrix}2 & -1\\1 & 3\end{bmatrix}\begin{bmatrix}4\\6\end{bmatrix} = \begin{bmatrix}2*4+-1*6\\1*4+3*6\end{bmatrix} = \begin{bmatrix}8-6\\4+18\end{bmatrix} = \begin{bmatrix}2\\22\end{bmatrix}$

b)

B= $\begin{bmatrix}2\\2\end{bmatrix} = \begin{bmatrix}3\\2\end{bmatrix}$

B= $\begin{bmatrix}0\\2\end{bmatrix} = \begin{bmatrix}-2\\2\end{bmatrix}$

B= $\begin{bmatrix}4\\6\end{bmatrix} = \begin{bmatrix}3 & -2\\2 & 2\end{bmatrix}\begin{bmatrix}4\\6\end{bmatrix} = \begin{bmatrix}3*4+-2*6\\2*4+2*6\end{bmatrix} = \begin{bmatrix}12-12\\8+12\end{bmatrix} = \begin{bmatrix}0\\20\end{bmatrix}$

c)

I graphed the vectors and could see that in this case the matrix transformation leads to different transformations, suggesting that B was not the same matrix in both cases.

d)

The same unaltered transformation matrix must be applied to each vector.
The number of columns in the transformation matrix must equal the number of rows in the vector column matrix.