Week 3 Assignment

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1. Let *Z* be a random variable with the following probability distribution *f*:
***f*(-2) = 0.3**
***f*(3) = 0.2**
***f*(5) = 0.5**

Compute the E(*Z*), Var(*Z*) and the standard deviation of *Z*.

$$\mu = E(Z) = \sum_{z} z f(z)$$

$$= (-2)(0.3) + (3)(0.2) + (5)(0.5) = \mathbf{2.5}$$

The expected value or mean of the discrete random variable *Z* is 2.5.

$$\sigma^2 = E[(Z-\mu)^2] = \sum_{z} (z-\mu)^2 f(z)$$

$$\sigma^2 = \sum_{x=-2}^{5} (z-2.5)^2 = (-2-2.5)^2(0.3) + (3-2.5)^2(0.2) + (5-2.5)^2(0.5) = \mathbf{9.25}$$

The variance of the discrete random variable *Z* is 9.25.

$$\sigma = \sqrt{9.25} \cong \mathbf{3.04}$$

The standard deviation of the discrete random variable Z is approx. 3.04.

2. Tossing a fair die is an experiment that can result in any integer number from 1 to 6 with equal probabilities. Let X be the number of dots on the top face of a die. Compute E(X) and Var(X).

The sample space for this experiment is as follows.

$S = \{1, 2, 3, 4, 5, 6\}$

Since the 6 sample points are all equally likely to occur, the probabilities for the discrete random variable X are as follows:

$$P(X=1) = \frac{1}{6}, P(X=2) = \frac{1}{6}, P(X=3) = \frac{1}{6}, P(X=4) = \frac{1}{6},$$
$$P(X=5) = \frac{1}{6}, P(X=6) = \frac{1}{6}$$

$$\mu = E(X) = \sum_x x f(x)$$

$$= (1)\left(\frac{1}{6}\right) + (2)\left(\frac{1}{6}\right) + (3)\left(\frac{1}{6}\right) + (4)\left(\frac{1}{6}\right) + (5)\left(\frac{1}{6}\right) + (6)\left(\frac{1}{6}\right) =$$
$$\frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = \frac{21}{6} = \mathbf{\frac{7}{2}} = \mathbf{3.5}$$

On average, one can see 3.5 points facing on the top of the die.

$$\sigma^2 = E[(X-\mu)^2] = \sum_x (x-\mu)^2 f(x)$$

$$\sigma^2 = \sum_{x=1}^{6} (X - 3.5)^2$$
$$= (1-3.5)^2\left(\frac{1}{6}\right) + (2-3.5)^2\left(\frac{1}{6}\right) + (3-3.5)^2\left(\frac{1}{6}\right) + (4-3.5)^2\left(\frac{1}{6}\right)$$
$$+ (5-3.5)^2\left(\frac{1}{6}\right) + (6-3.5)^2\left(\frac{1}{6}\right) \cong \mathbf{2.92}$$

The variance of the random variable X is about 2.92.

$$\sigma = \sqrt{2.92} \cong \mathbf{1.71}$$

The standard deviation of the discrete random variable X is approx. 1.71.

3. The number of home runs hit by a certain team in one game is a random variable with the following distribution:
$X = 0$, $P(X = 0) = 0.4$
$X = 1$, $P(X = 1) = 0.4$
$X = 2$, $P(X = 2) = 0.2$
The team plays 2 games. The number of home runs hit in one game, is independent of the number of home runs in the other game. Let Y be the total number of home runs. Find $E(Y)$ and $\mathrm{Var}(Y)$.

If the discrete random distribution *f(x)* applies to both games, we can expect the following outcomes for *f(y)*:

(0,0), (0,1), (1,0), (1,1), (0,2), (2,0), (1,2), (2,1), (2,2)
(for scoring 0 home runs in both games, 1 home run in either game, 1 home run in both games, two home runs in either game, 1 and 2 home runs in either game or two home runs in both games)

The associated probabilities for *f(y)* are as follows:

f(0,0) = (0.4)(0.4) = 0.16
f(0,1) = (0.4)(0.4) = 0.16
f(1,0) = (0.4)(0.4) = 0.16
f(1,1) = (0.4)(0.4) = 0.16
f(0,2) = (0.4)(0.2) = 0.08
f(2,0) = (0.2)(0.4) = 0.08
f(1,2) = (0.4)(0.2) = 0.08
f(2,1) = (0.2)(0.4) = 0.08
f(2,2) = (0.2)(0.2) = 0.04

$$\mu = E(Y) = \sum_y y f(y)$$
$$\begin{aligned}
&= (0)(0)(0.16) + (0)(1)(0.16) + (1)(0)(0.16) + (1)(1)(0.16) + (0)(2)(0.08) \\
&\quad + (2)(0)(0.08) + (1)(2)(0.08) + (2)(1)(0.08) + (2)(2)(0.04) \\
&= (1)(1)(0.16) + (1)(2)(0.08) + (2)(1)(0.08) + (2)(2)(0.04) \\
&= 0.16 + 0.16 + 0.16 + 0.16 = \mathbf{0.64}
\end{aligned}$$

In both games, the teams scores on average 0.64 home runs.

$$\sigma^2 = E[(Y-\mu)^2] = \sum_y (y-\mu)^2 f(y)$$

$$\sigma^2 = \sum_{x,y=0}^{2} (Y - 0.64)^2$$

$$= \big((0)(0) - 0.64\big)^2(0.16) + \big((0)(1) - 0.64\big)^2(0.16) + \big((1)(0) - 0.64\big)^2(0.16) + \big((1)(1) - 0.64\big)^2(0.16) + \big((0)(2) - 0.64\big)^2(0.08) + \big((2)(0) - 0.64\big)^2(0.08) + \big((1)(2) - 0.64\big)^2(0.08) + \big((2)(1) - 0.64\big)^2(0.08) + \big((2)(2) - 0.64\big)^2(0.04) =$$

$$= 0.065536 + 0.065536 + 0.065536 + 0.020736 + 0.032768 + 0.032768 + 0.147968 + 0.147968 + 0.451584 = \mathbf{1.0304}$$

The variance of the discrete random variable *Y* is about 1.0304.

$$\sigma = \sqrt{1.0304} \cong \mathbf{1.02}$$

The standard deviation of the discrete random variable *Y* is approx. 1.02.

4. Consider the random variable *Z* from problem 1, and the random variable *X* from problem 2.
Also let *f*(*X*,*Z*)represent the joint probability distribution of *X* and *Z*. *f* is defined as follows:
***f*(1,-2) = 1/6**
***f*(2,-2) = 2/15**
***f*(3,-2) = 0**
***f*(4,-2) = 0**
***f*(5,-2) = 0**
***f*(6,-2) = 0**
***f*(1,3) = 0**
***f*(2,3) = 1/30**
***f*(3,3) = 1/6**
***f*(4,3) = 0**
***f*(5,3) = 0**
***f*(6,3) = 0**
***f*(1,5) = 0**
***f*(2,5) = 0**
***f*(3,5) = 0**
***f*(4,5) = 1/6**
***f*(5,5) = 1/6**
***f*(6,5) = 1/6**

Compute the covariance of *X* and *Z*.

$$\sigma_{XZ} = E(XZ) - \mu_x\mu_z$$

$$\mu_z = 2.5$$
$$\mu_x = 3.5$$

$$\begin{aligned} E(XZ) &= (1)(-2)\left(\frac{1}{6}\right) + (2)(-2)\left(\frac{2}{15}\right) + (3)(-2)(0) + (4)(-2)(0) + (5)(-2)(0) \\ &\quad + (6)(-2)(0) + (1)(3)(0) + (2)(3)\left(\frac{1}{30}\right) + (3)(3)\left(\frac{1}{6}\right) + (4)(3)(0) \\ &\quad + (5)(3)(0) + (6)(3)(0) + (1)(5)(0) + (2)(5)(0) + (3)(5)(0) \\ &\quad + (4)(5)\left(\frac{1}{6}\right) + (5)(5)\left(\frac{1}{6}\right) + (6)(5)\left(\frac{1}{6}\right) \\ &= (1)(-2)\left(\frac{1}{6}\right) + (2)(-2)\left(\frac{2}{15}\right) + (2)(3)\left(\frac{1}{30}\right) + (3)(3)\left(\frac{1}{6}\right) + (4)(5)\left(\frac{1}{6}\right) \\ &\quad + (5)(5)\left(\frac{1}{6}\right) + (6)(5)\left(\frac{1}{6}\right) \\ &= \left(-\frac{2}{6}\right) + \left(-\frac{8}{15}\right) + \left(\frac{6}{30}\right) + \left(\frac{9}{6}\right) + \left(\frac{20}{6}\right) + \left(\frac{25}{6}\right) + \left(\frac{30}{6}\right) = \frac{40}{3} \cong \mathbf{13.33} \end{aligned}$$

$$\sigma_{XZ} = 13.33 - (3.5)(2.5) = \cong \mathbf{4.58}$$

The covariance of the two random variables *X* and *Z* returns a positive value (4.58), suggesting, that the two variables are positively correlated i.e. if *X* increases, *Z* increases and if *X* decreases, *Z* decreases.

Then, compute the correlation coefficient of *X* and *Z*. (Note: You will need values that you computed in problems 1 and 2.)

$$\rho = \frac{\sigma_{XZ}}{\sigma_X \sigma_Z} = \frac{4.58}{(1.71)(3.04)} \cong \mathbf{0.88}$$

The correlation coefficient of *X* an *Z* is 0.88.