Week 8 Assignment

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1. A fabric manufacturer believes that the proportion of orders for raw material arriving late is *p* = 0.6. If a random sample of 10 orders shows that 3 or fewer arrived late, the hypothesis that *p* = 0.6 should be rejected in favor of the alternative *p* < 0.6. Use the binomial distribution.

n=10

H_0: $p = 0.6$
H_1: $p < 0.6$

The test statistic *X* is the number of orders for raw materials arriving late. Any score smaller than 3 is in the critical region. The value 3 is the critical value.
This is a 1-tailed test.

a) Find the probability of committing a type I error if the true proportion is *p* = 0.6.

$$\alpha = P(type\ I\ error) = P(X \leq 3\ when\ p = 0.6)$$

$$= \sum_{x=0}^{3} b(x; 10, 0.6) = 0.05476188 \approx \mathbf{5.48}\%$$

```
> pbinom(3,10,0.6)
[1] 0.05476188
```

This is the probability that the null hypothesis is rejected, when it should not have been rejected. The probability of the Type I error is 5.48%.

b) Find the probability of committing a type II error for the alternatives *p* = 0.3, *p* = 0.4, and *p* = 0.5.

$$\beta = P(type\ II\ error) = P(X > 3\ when\ p = 0.3)$$

$$= \sum_{x=4}^{10} b(x; 10, 0.3) = 0.3503893 \approx \mathbf{35.04}\%$$

```
> 1-pbinom(3,10,0.3)
[1] 0.3503893
```

$$\beta = P(type\ II\ error) = P(X > 3\ when\ p = 0.4)$$

$$= \sum_{x=4}^{10} b(x; 10, 0.4) = 0.6177194 \approx \mathbf{61.77}\%$$

```
> 1-pbinom(3,10,0.4)
[1] 0.6177194
```

$$\beta = P(type\ II\ error) = P(X > 3\ when\ p = 0.5)$$

$$= \sum_{x=4}^{10} b(x; 10,0.5) = 0.828125 \approx \mathbf{82.81}\%$$

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> 1-pbinom(3,10,0.5)
[1] 0.828125
```

The probability of committing a Type II error increases as the values for the probabilities under which β is tested get closer to the hypothetical value of p.

2. **The proportion of adults living in a small town who are college graduates is estimated to be p = 0.6. To test this hypothesis, a random sample of 15 adults is selected. If the number of college graduates in the sample is anywhere from 6 to 12, we shall not reject the null hypothesis that p = 0.6; otherwise, we shall conclude that $p \neq 0.6$.**

   a) Evaluate α assuming that p = 0.6. Use the binomial distribution.

   H_0: p = 0.6
   H_1: p ≠ 0.6

$$\alpha = P(type\ I\ error) = P(6 < X < 12\ when\ p = 0.6)$$

$$\alpha = P(X < 12) - P(X < 6) = 0.972886 - 0.09504741 = 0.8778386 \approx \mathbf{87.78}\%$$

```
> pbinom(12,15,0.6)
[1] 0.972886

> pbinom(6,15,0.6)
[1] 0.09504741
```

Or

```
> pbinom(12,15,0.6)-pbinom(6,15,0.6)
[1] 0.8778386
```

H_0 is being tested at the significance level of 87.78%.

b) Evaluate β for the alternatives *p* = 0.5 and *p* = 0.7.

$$\beta = P(type\ II\ error) = P(6 > X > 12\ when\ p = 0.5)$$

$$= \sum_{x=0}^{6} b(x; 15{,}0.5) + \sum_{x=12}^{15} b(x; 15{,}0.5) = 0.307312 \approx \mathbf{30.73}\%$$

```
> pbinom(6,15,0.5)+(1-pbinom(12,15,0.5))
[1] 0.307312
```

$$\beta = P(type\ II\ error) = P(6 > X > 12\ when\ p = 0.7)$$

$$= \sum_{x=0}^{6} b(x; 15{,}0.7) + \sum_{x=12}^{15} b(x; 15{,}0.7) = 0.1420702 \approx \mathbf{14.21}\%$$

```
> pbinom(6,15,0.7)+(1-pbinom(12,15,0.7))
[1] 0.1420702
```

c) Is this a good test procedure.

The test procedure has a critical region between 6 and 12 and it is a two-tailed test.

No, this is not a good test procedure because the probabilities for both committing a type I or a type II error are high.

Increasing the sample size might be an option to reduce both probabilities.

3. A manufacturer has developed a new fishing line, which the company claims has a mean breaking strength of 15 kilograms with a standard deviation of 0.5 kilogram. To test the hypothesis that μ = 15 kilograms against the alternative that μ < 15 kilograms, a random sample of 50 lines will be tested. The critical region is defined to be xbar < 14.9.
a) Find the probability of committing a type I error when H_0 is true.

$\bar{x} = 15$
$\sigma = 0.5$
n=50
critical region: $\alpha =$ xbar <14.9

H_0: $\mu = 15$
H_1: $\mu < 15$

$$\alpha = P(type\ I\ error) = P(X < 14.9)$$

$$z = \frac{\bar{x} - \mu}{\sigma\sqrt{n}}$$

$$z = \frac{14.9 - 15}{0.5\sqrt{50}} \approx -1.414$$

$$\alpha = P(type\ I\ error) = P(Z < -1.414\ when\ \mu = 15) = 0.07868095 \approx \mathbf{7.89}\%$$

```
> pnorm(-1.414)
[1] 0.07868095
```

b) Evaluate β for the alternatives μ = 14.8 and μ = 14.9 kilograms.

$\mu = 14.8$

$$z = \frac{\bar{x} - \mu}{\sigma\sqrt{n}}$$

$$z = \frac{14.9 - 14.8}{0.5\sqrt{50}} \approx 1.414$$

$\beta = P(type\ II\ error) = P(Z > 1.414\ when\ \mu = 14.8) = 1 - P(Z < 1.414) = 0.07868095 \approx$ $\mathbf{7.87}\%$

```
> 1-pnorm(1.414)
[1] 0.07868095
```

$\mu = 14.9$

$$z = \frac{14.9 - 14.9}{0.5\sqrt{50}} = 0$$

$$\beta = P(type\ II\ error) = P(X > 0\ when\ \mu = 14.9)\ 1 - P(Z < 0) = \mathbf{0.5}$$

```
> 1-pnorm(0)
[1] 0.5
```

4. The average height of females in the freshman class of a certain college has historically been 162.5 centimeters with a standard deviation of 6.9 centimeters. Is there a reason to believe that there has been a change in the average height if a random sample of 50 females in the present freshman class has an average height of 165.2 centimeters? Use a P-value in your conclusion. Assume the standard deviation remains the same.

$\mu = 162.5$
$\bar{x}$=165.2

1. Set null and alternative hypotheses:

To check, if there is reason to believe that the average height of female college students has changed, we can use a one right-tailed test and establish the following hypotheses:

H_0: μ = 162.5
H_1: μ>162.5

2. Choose an appropriate test statistic.

$$z = \frac{\bar{x} - \mu}{\sigma\sqrt{n}}$$

$$z = \frac{165.2 - 162.5}{6.9\sqrt{50}} \approx 2.77$$

3. Compute *p*-value:

Assuming a significance level of 0.05, we can use the z-table (or R) to find its *p*-value:

$$p = p(z > 2.77) = \mathbf{0.0028}$$

```
> 1-pnorm(2.77)
[1] 0.002802815
```

Since the *p*-value of 0.0028 is less than the significance level, the null hypothesis is rejected. Based on the evidence we have, we can conclude, that the average height of female freshman students has increased.