Week 4 Assignment

MATH 511 Christina Morgenstern

About 10% of users do not close Windows properly. Suppose that Windows is installed in a public library that is used by random people in a random order:

1. **On the average, how many users of this computer *do not* close Windows properly before someone *does* close it properly?**

Since there are two outcomes of a trial resulting in either failure (of not closing Windows properly) or success (of closing Windows properly), the given example deals with a Bernoulli process.

The trials will be repeated until a fixed number of successes occurs (somebody closing Windows properly first time).

The number *X* of trials required to produce k successes is a negative binomial random variable and its probability distribution is called the negative binomial distribution:

$$b^*(x; k, p) = \binom{x-1}{k-1} p^k q^{x-k}, \qquad x = k, k+1, k+2, \ldots.$$

Since in this case *k=1*, the number of trials required for a single success, the distribution is a special case of the negative binomial distribution, the **geometric distribution**.

If repeated independent trials (of closing Windows) result in a success with probability *p* and a failure with probability *q=1-p*, then the probability distribution of the random variable *X*, the number of the trial on which the first success occurs is:

$$g(x, p) = pq^{x-1}, \qquad x = 1, 2, 3 \ldots.$$

Using the formula to calculate the mean of the random variable *X,* following a geometric distribution, I can obtain the average number of users, that do not close Windows properly, before somebody does close it properly.

p= 0.9 (success of closing windows properly)

$$\mu = \frac{1}{p}$$

$$\mu = \frac{1}{0.9} \approx \mathbf{1.111}$$

On average, 1.1 users do not close Windows properly, before somebody does close it properly.

2. **Suppose that in one day, 25 users make use of Windows. How many users are expected to have closed Windows properly? How many of them are expected to *not* close Windows properly?**

From the information given, the variable *X* follows a **binomial distribution**.

n=25 (number of trials)
p=0.9 (success case - closing Windows properly)
q=0.1 (failure case - closing Windows not properly)

X = binomial random variable, representing the number of successes in 25 trials. Using the formula for the mean, I can calculate the average number of users that can close Windows properly.

$$\mu = np$$
$$\mu = 25(0.9)$$
$$\mu = \mathbf{22.5}$$

$$\sigma^2 = npq$$
$$\sigma^2 = 25(0.9)(0.1)$$
$$\sigma^2 = 2.25$$
$$\sigma = \mathbf{1.5}$$

About 22.5$\pm$1.5 users will be able to close Windows properly on a day with 25 users.

n=25 (number of trials)
p=0.1 (success case - closing windows not properly)
q=0.9 (failure case - closing windows properly)

X = binomial random variable, representing the number of successes in 25 trials. Using the formula for the mean, I can calculate the average number of users that cannot close Windows properly.

$$\mu = np$$
$$\mu = 25(0.1)$$
$$\mu = \mathbf{2.5}$$

$$\sigma^2 = npq$$
$$\sigma^2 = 25(0.9)(0.1)$$
$$\sigma^2 = 2.25$$
$$\sigma = \mathbf{1.5}$$

About 2.5$\pm$1.5 users won´t be able to close Windows properly on a day with 25 users.

The number of computer shutdowns during any month has a Poisson distribution, averaging 0.25 shutdowns per month.

3. **What is the probability of at least 3 computer shutdowns during the next year?**

0.25 shutdowns per month -> (0.25) (12) = average shutdowns per year (Property of Proportionality)
$\lambda t = 3$

Let X be the Poisson random variable of the number of computer shutdowns in the next year. At least 3 computer shutdowns mean, that there are equal than or more than 3 shutdowns in a year.

$$P(X \geq 3) = 1 - P(\leq 2) = 1 - \sum_{x=0}^{2} p(x; 3)$$

Use of R to compute the probability:

```
> 1-ppois(2,3)
[1] 0.5768099
```

This calculation war verified using table A2 from the textbook:

$$P(X \geq 3) = 1 - P(\leq 2) = 1 - \sum_{x=0}^{2} p(x; 3) = 1 - 0.4232 = 0.5768$$

The probability, that at least 3 computer shutdowns are going to happen the next year is about 0.5778.

4. **During the next year, what is the probability of at least 3 months (out of 12) with exactly 1 computer shutdown each?**

$\lambda t = 1$ (Arrival rate: exactly 1 Computer shutdown per month)

Let X be the Poisson random variable of the number of computer shutdowns in the next year. At least 3 months with exactly one computer shutdown means:

$$P(3 \leq X \leq 12) = \sum_{x=3}^{12} p(x; 1)$$

Use of R to compute the probability:

```
> sum=0
> for(i in 3:12)
+ sum=sum+dpois(i, 1)
> sum
[1] 0.0803014
```

The probability, that there are at least 3 months with exactly 1 computer shutdown in the next year is 0.0803.