Signals & Systems

Sara Taghizadeh

Overview

- Signal
- Signal processing
- Signal Classification
  - Continuous time signals
  - Discrete time signals
  - Digital signals
  - Analog signals
- Systems
  - Impulse response
  - System properties
  - Linear time invariant (LTI systems)

Overview

- Sampling
- Frequency domain (Fourier Analysis)
- Discrete time fourier transform (DTFT)
  - DTFT important properties
- Aliasing
  - Aliasing in time domain
  - Aliasing in frequency domain
- Nyquist theorem
- Frequency Response

Overview

- Filter design
  - Filter type
  - FIR vs IIR
  - Filter order
  - Filter characteristics
- Summary

Signal

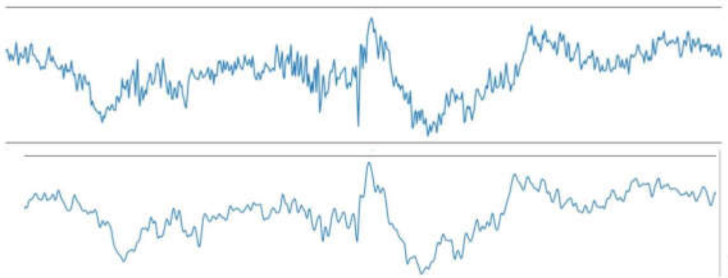
- Signal is something that conveys information
- Is used for communicating information between humans or between humans and machines
- Examples of signals:
  - Audio signals
    - sound, human voice, sound of a car, music instruments
  - Biological signals
    - EEG, EMG, ECG, etc.
  - Images, and Videos

Signal processing

- Is a subfield of mathematics and electrical engineering **[1]**
- Deals with analyzing and modifying signals **[1]**
- Application of signal processing
  - Audio and speech processing
  - Video and image processing
  - Biological signal processing
  - Biomedical imaging

Signal processing applications

- Biological signal processing



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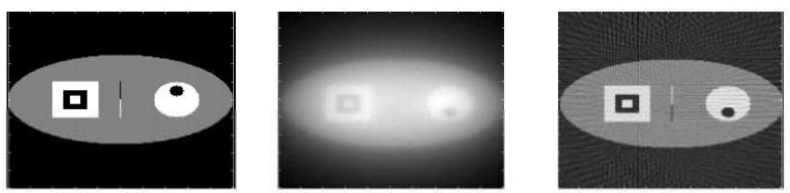
Signal processing applications

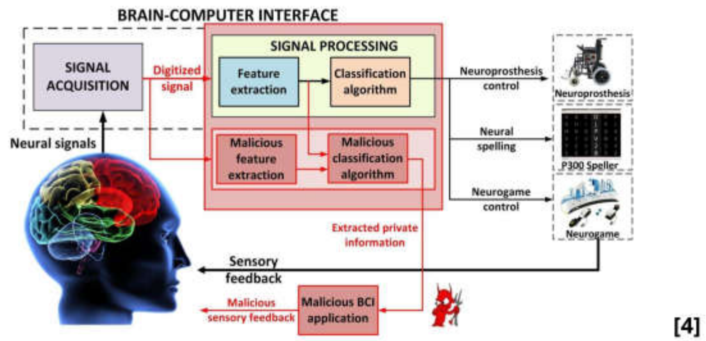
- Biomedical Image processing



Original image Direct back-projection Filtered back-projection

[3]

Signal processing applications



Brain Computer Interface

https://www.cnn.com/2017/04/12/health/brain-computer-interface-partner/index.html

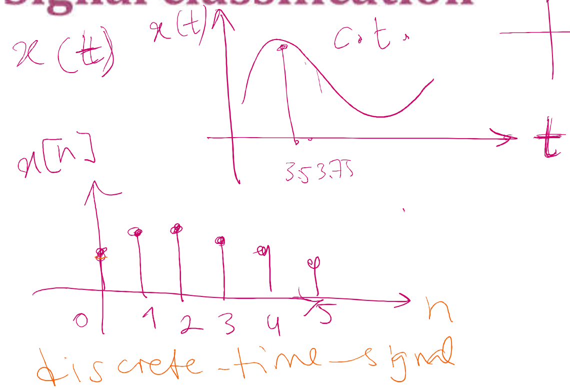
Signal

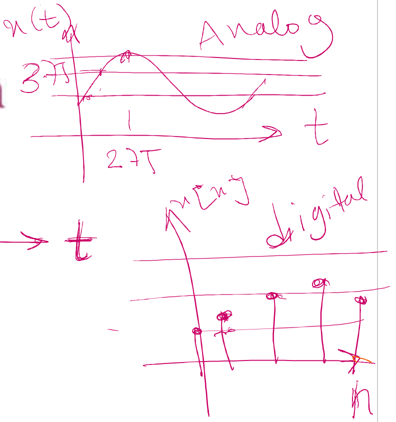
- A signal can be represented as a function of an ***independent variable***
- The independent variable can be time, space, etc
- Here we assume the independent variable is ***time***
- The signal amplitude is the ***dependent variable***
- Can classify signals into different groups

Signal classification

- ***Continuous signal***: the independent variable (time) takes continuous values
- Continuous signal is denoted by x(t)
- ***Discrete signal***: the independent variable (time) takes discrete values
- Discrete signal is denoted by x[n]
- ***Analog signal***: the dependent variable takes continuous values
- ***Digital signal:*** both dependent and independent variables are discrete
- In computers we deal with digital signals

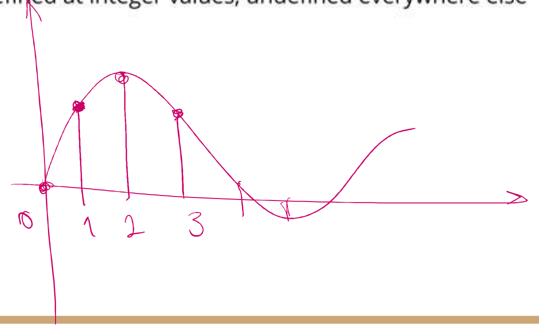
Signal classification





Discrete time signals

- Represented as a sequence of numbers
- Defined at integer values, undefined everywhere else



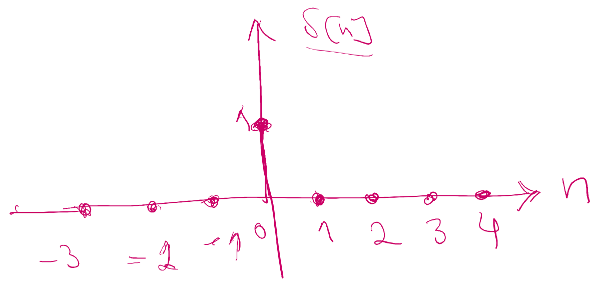
Signal classification

- ***One dimensional (1D)*** signal is represented as a function of one variable (usually time)
  - x(t), x[n]
- ***Two dimensional (2D)*** signal is represented as a function of two variables
  - f(x,t)
  - Images

$x[n]$

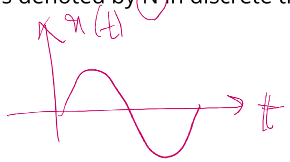
Important Signals

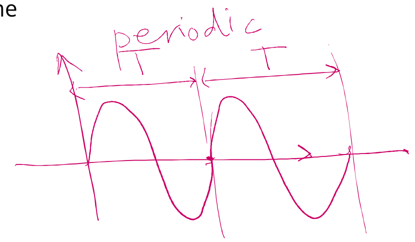
- One specific discrete time signal of special importance in signal processing is the ***unit impulse (delta)*** function



Periodic signals

- A ***periodic signal*** is a signal that repeats
- The period is the number for which the signal repeats
- A signal that is not periodic is referred to as an ***aperiodic signal***
- Is denoted by T in continuous time
- Is denoted by N in discrete time





Frequency

- Represents the number of cycles in one second
- Denoted by f and has a unit of Hz (hertz)
- We can represents signals frequency as a function of time (t)
- We can also represent signals as function of frequency (f)

$$(Hz)\ f = 1/T\ (s)$$

frequency

period

Signal manipulation

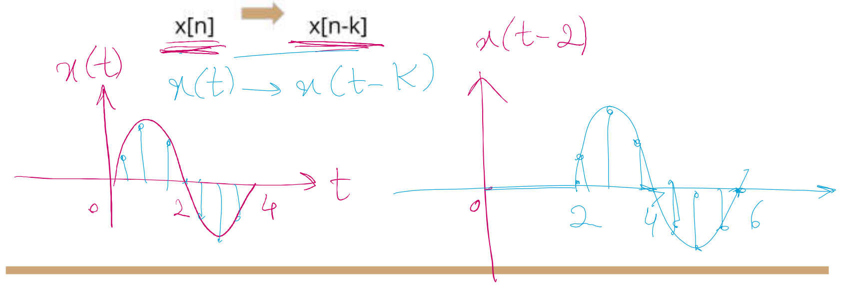
Time shift

Shift the signal by k ➡ Replace n with n-k

[0, 1, 2, 3, ...
-1, -2, ...]

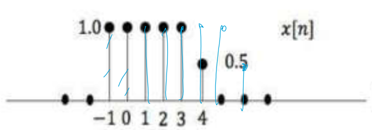
x[n] ➡ x[n-k]

x(t) → x(t-K)



Question

The following is a discrete time signal. Sketch x[n-2] !



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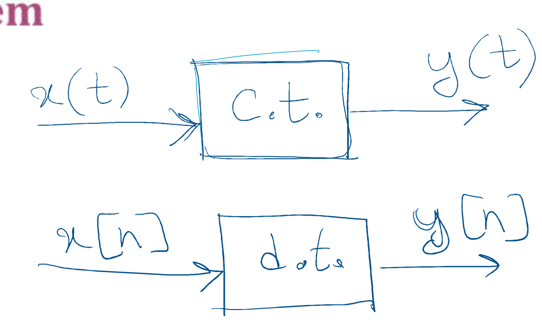
System

- System takes an input and maps it to an output
- Some examples of systems are:
  - Amplifiers, Filters, etc
- Continuous time systems have a continuous time input and output
- Discrete time systems have a discrete time input and output

System

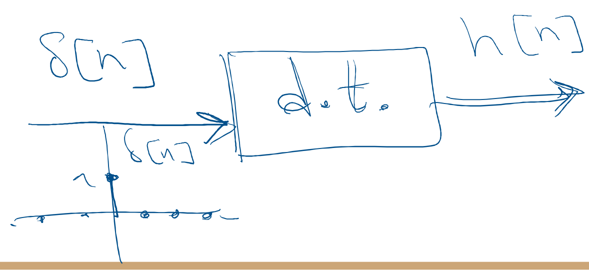


Discrete time Systems

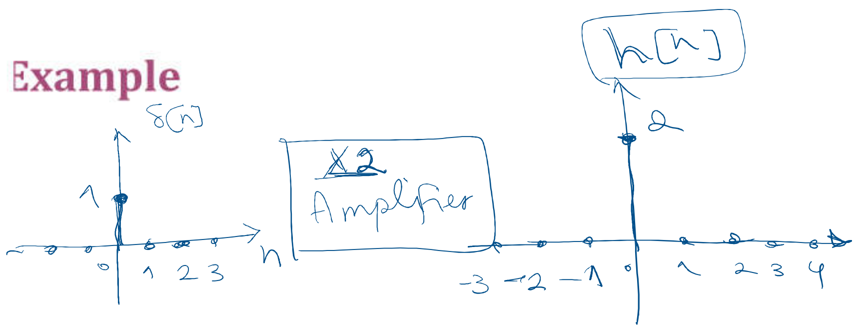
- Maps a ***discrete time input*** signal to a ***discrete time output*** signal

Impulse Response

- If the input to the system is the unit impulse signal, the output of the system is called the ***impulse response***
- The impulse response in represented by h[n]

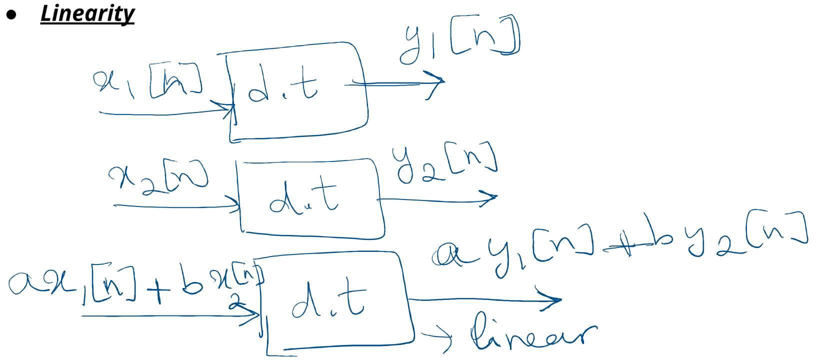


Example



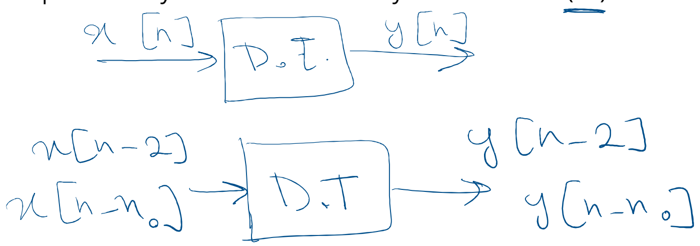
Some important properties of discrete-time systems

- ***Linearity***



Some important properties of discrete-time systems

- ***Time Invariance***: implies that if we shift the input by a value(n0) the output of the system will be shifted by the same value(n0)



Time Invariant

LTI systems

- Time invariance and linearity property together make up an important class of systems
- These systems are referred to as ***Linear Time Invariant (LTI) systems***
- LTI systems have great importance in signal and system analysis

LTI systems

Convolution:

- The output of an LTI system can be represented by the convolution of the input with the impulse response of the system
- Convolution is denoted by " * "
- Convolution is an operation like multiplication and summation

$$\delta[n] \rightarrow \boxed{S} \rightarrow h[n] \qquad y[n] = x[n] * h[n]$$

$$x[n] \rightarrow \boxed{S} \rightarrow y[n] = ?$$

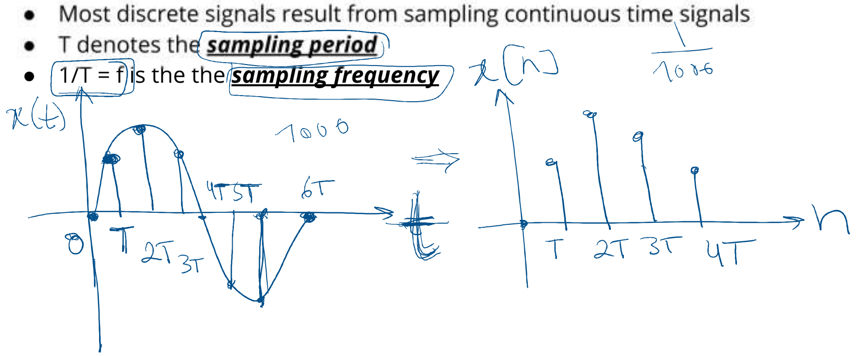
Some important operations on discrete time systems

Convolution:

- If the impulse response of an LTI system is known, the output corresponding to any input can be calculated

Sampling

- Most discrete signals result from sampling continuous time signals
- T denotes the ***sampling period***
- 1/T = f is the the ***sampling frequency***



Sampling

-

Fourier Analysis

- So far we talked about signals as a function of time (time domain)
- Another way to think of signals is a function of their frequency components
- Fourier analysis converts a signal from time domain to frequency domain

time domain → freq domain
Fourier Analysis

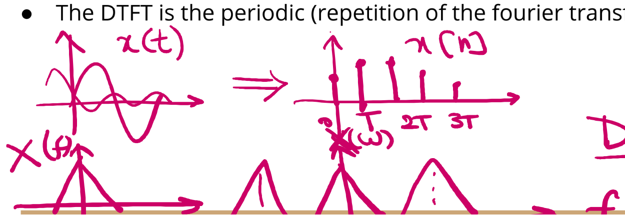
Fourier Analysis

[ttps://www.coursera.org/lecture/cryo-em/1-d-sine-waves-and-their-sums-78uHF](https://www.coursera.org/lecture/cryo-em/1-d-sine-waves-and-their-sums-78uHF)

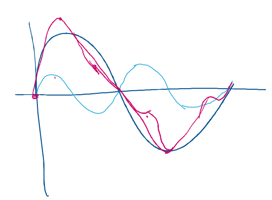
$t \Longleftrightarrow f$

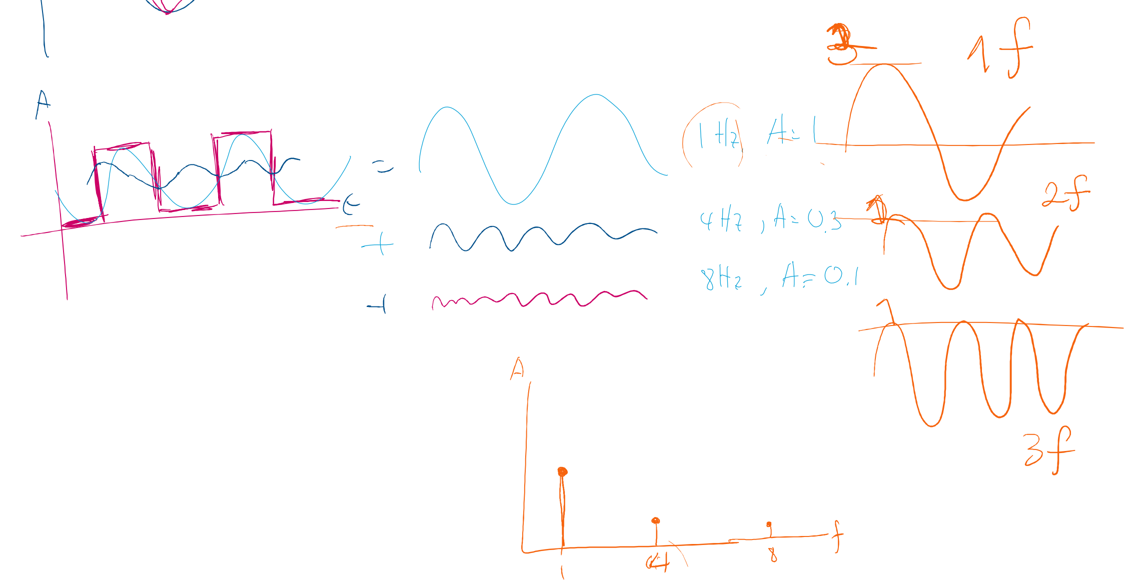
Discrete Time Fourier Transform (DTFT)

- Recall: last week we introduced the fourier transform
- Fourier transform (FT) is the frequency representation of a continuous time signal
- The frequency representation of a ***discrete*** and ***aperiodic*** signal is called the ***discrete time fourier transform (DTFT)***
- The DTFT is the periodic (repetition of the fourier transform)



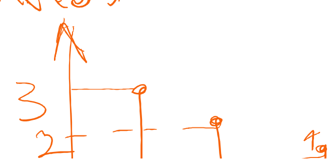
DTFT

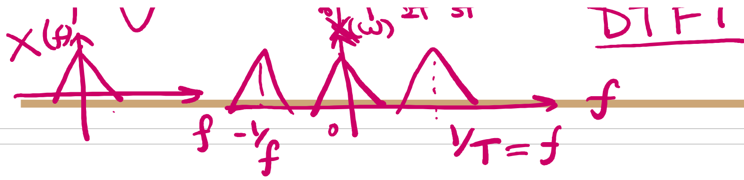


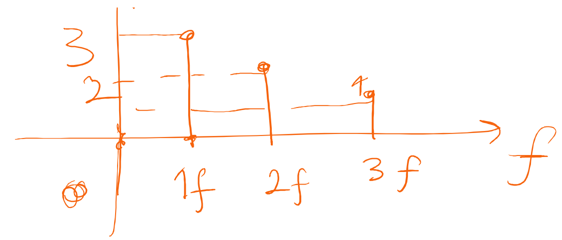


$X(f)$

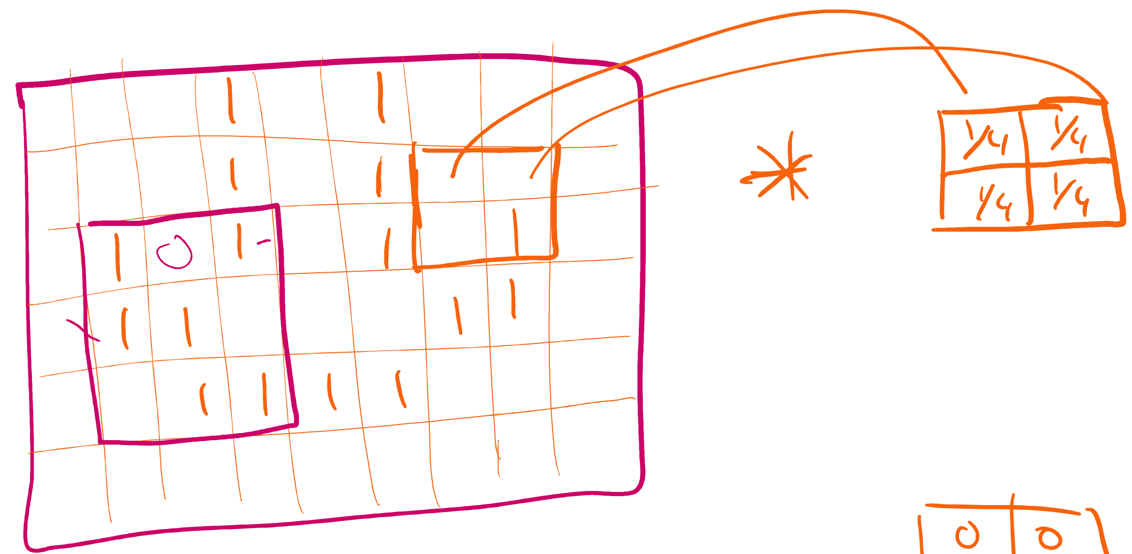
$t \Longleftrightarrow f$







Example



$$= 0\left(\tfrac{1}{4}\right) + 0\left(\tfrac{1}{4}\right) + 0\left(\tfrac{1}{4}\right) + 1\left(\tfrac{1}{4}\right)$$

$$= 0.25$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{bmatrix} = \boxed{0.25}$$

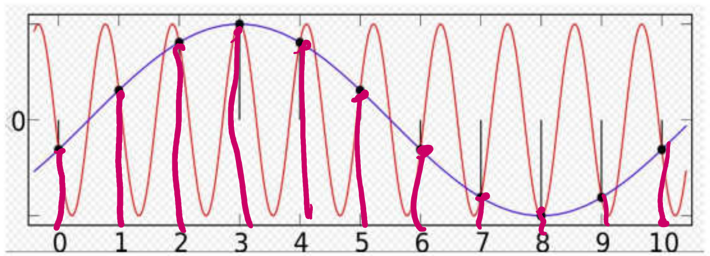
$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Aliasing

- Assume we want to recover the original continuous time signal from the sampled discrete time signal
- If the recovered continuous signal is different from the original one ***<u>Aliasing</u>*** has occurred
- If multiple signals result from the reconstruction they are called aliases
- If the frequency components overlap this is called ***<u>Aliasing</u>***

Aliasing

- Aliasing in time domain



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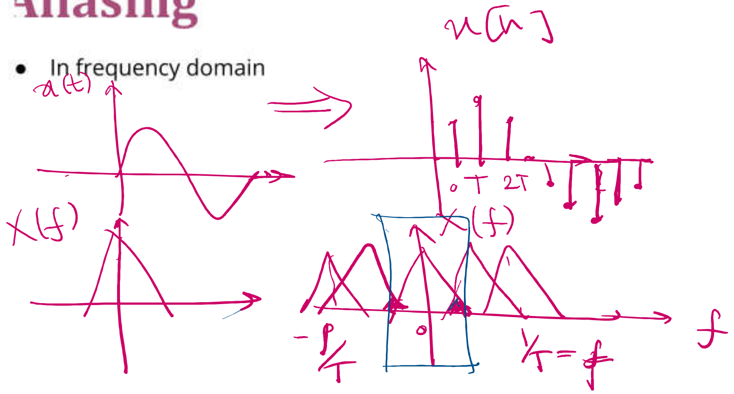
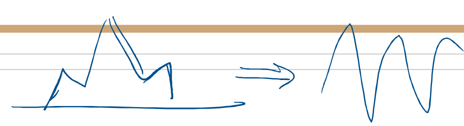
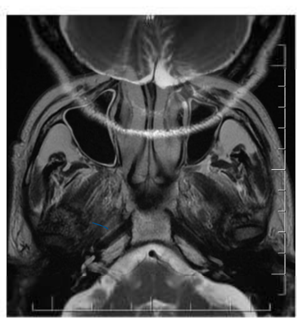
Aliasing

- In frequency domain





Aliasing



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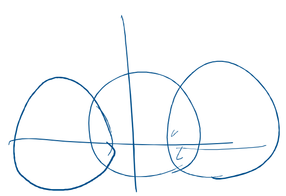
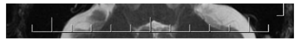


0	0	0
1	0	0
1	1	0

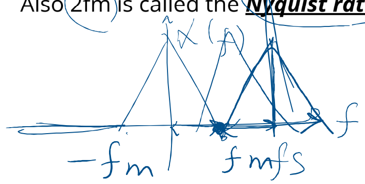


[6]

Nyquist Theorem

- To prevent aliasing we utilize the ***Nyquist theorem***
- If highest frequency component in signal is at f_m
- The sampling rate fs has to be at least 2fm

- Also 2fm is called the ***Nyquist rate***



Frequency Response

- The frequency representation of the impulse response (h[n]) is called the ***Frequency response*** (H(w))

$$h[n] \longrightarrow H(\omega)$$

$$\boxed{h(n)} \qquad \boxed{H(\omega)}$$

Important note

$\omega = 2\pi f$
rad hz

Time Domain		Frequency Domain
$*$ Convolution	⟷	Multiplication $\cdot$
$\cdot$ Multiplication	⟷	Convolution $*$

$$x[n] * y[n] \Longleftrightarrow X(\omega) \cdot Y(\omega)$$

$$x[n] \cdot y[n] \Longleftrightarrow X(\omega) * Y(\omega)$$

Filters

- One special type of ***LTI systems***
- Have many applications
- Can be used to remove, reduce, amplify the frequency content of signals
- Have to consider different parameters when designing filters
- Filters perform their operation on signals in ***time domain using convolution(*)***, and using ***multiplication in the frequency domain***

Example

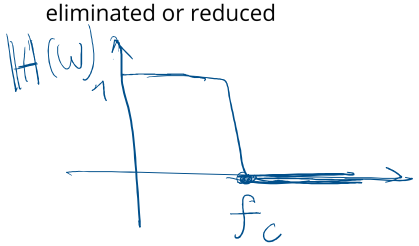
- How do filters perform:

$$x[n] \rightarrow \boxed{h[n]} \rightarrow \quad y[n] = x[n] * h[n]$$

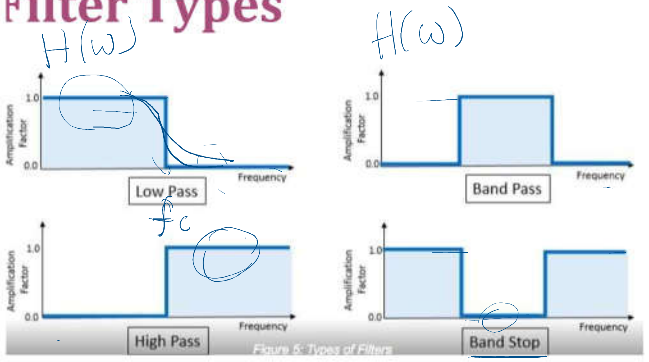
$$X(\omega) \rightarrow \boxed{H(\omega)} \rightarrow \quad Y(\omega) = X(\omega) \cdot H(\omega)$$

Filters

- ***Cut off frequency*** :
  - Also called the corner frequency (hz)
  - Is the frequency at which some frequency content begins to be eliminated or reduced

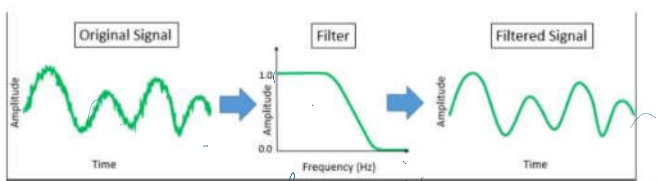


Filter Types



[7]

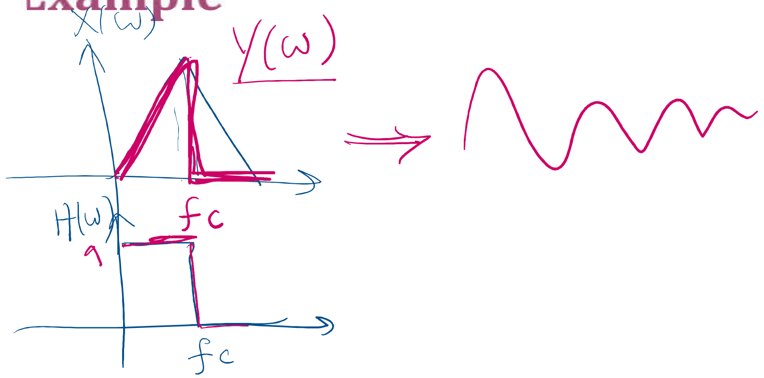
Example



[7]

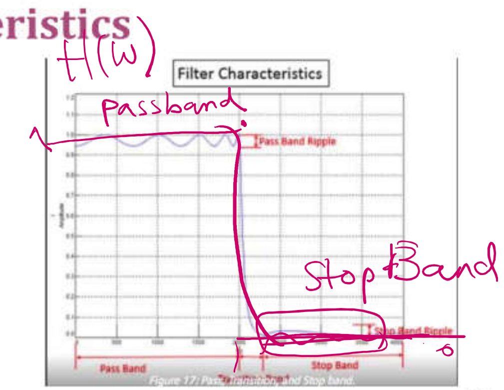
$x[n] * h[n]$

Example



Filter Characteristics

- Pass Band
- Transition Band
- Stop Band

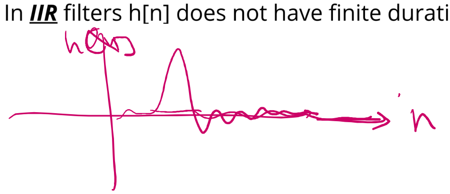


[7]

FIR vs IIR

h[n]

- ***FIR*** stands for Finite impulse response h[n]
- ***IIR*** stands for Infinite impulse response
- Remember the impulse response (the system output to the delta function) h[n]
- In ***FIR*** filters h[n] has a finite duration
- In ***IIR*** filters h[n] does not have finite duration(it is infinite)



FIR vs IIR

FIR Filter Equation: $y(n) = \sum_{k=0}^{N} a(k)x(n-k)$

IIR Filter Equation: $y(n) = \sum_{k=0}^{N} a(k)x(n-k) + \sum_{j=0}^{P} b(j)\underbrace{y(n-j)}_{\text{Output used recursively}}$

[7]

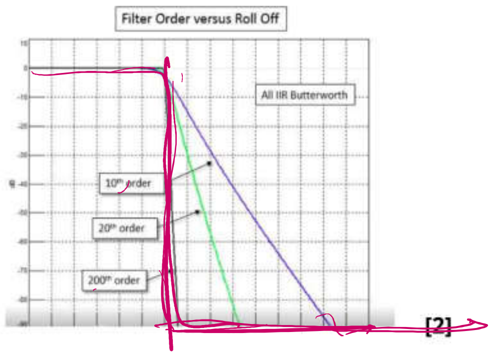
FIR vs IIR

- In ***FIR*** output corresponding to each input is calculated only using the input
- In ***IIR*** the output is calculated using the input and past output
- This is why IIR filters are computationally faster than FIR filters

Filter Order

- Refers delay in the filter
- Higher the order of the filter shaper the transition

Filter Order N



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References

1] https://en.wikipedia.org/wiki/Signal_processing

2]https://github.com/neurotechuoft/Workshops/blob/master/workshop_2018
2019/notebooks/exercises/wk2b_intro_to_signal_processing.ipynb

3] https://q.utoronto.ca/courses/65501/files/868141?module_item_id=150010

4] http://brl.ee.washington.edu/neural-engineering/bci-security/

5] Digital Signal Processing, Allan V. Oppenheim

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