



# Planning and Scheduling: Representations for Classical Planning



Hochschule  
Bonn-Rhein-Sieg



Prof. Dr.-Ing. Gerhard K. Kraetzschmar

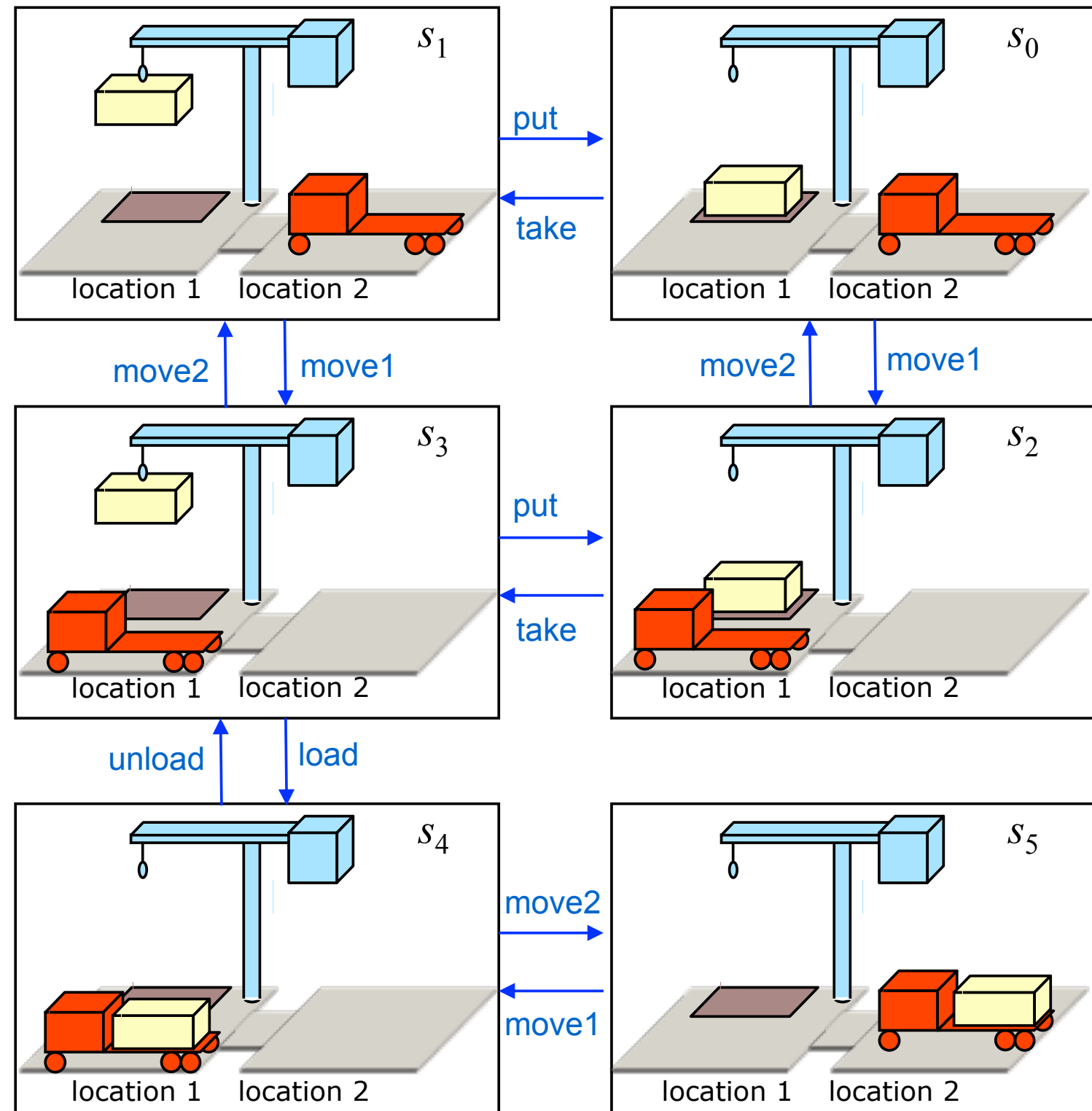
# Acknowledgements

- These slides refer to Chapter 2 of the textbook:  
Malik Ghallab, Dana Nau, and Paolo Traverso:  
Automated Planning: Theory and Practice  
Morgan Kaufmann, 2004
- These slides are an adaptation of slides by Dana Nau
- The contributions of these authors are gratefully acknowledged

# Quick Review of Classical Planning

- Classical planning requires all eight of the restrictive assumptions:

- A0: Finite
- A1: Fully observable
- A2: Deterministic
- A3: Static
- A4: Attainment goals
- A5: Sequential plans
- A6: Implicit time
- A7: Offline planning



# Representations: Motivation

- In most problems, far too many states to try to represent all of them explicitly as  $s_0, s_1, s_2, \dots$
- Represent each state as a set of features, e.g.:
  - A vector of values for a set of variables
  - A set of ground atoms in some first-order language L
- Define a set of operators that can be used to compute state transitions
- Don't give all of the states explicitly
  - Just give the initial state
  - Use the operators to generate the other states as needed

# Classical Representation

- Start with a **function-free** first-order language
  - Finitely many predicate symbols and constant symbols, but no function symbols
  - **Atom**: predicate symbol and args  $on(c_1, c_3), on(c_1, x)$
  - **Ground** expression: contains no variable symbols  $on(c_1, c_3)$
  - **Nonground** expression: at least one variable symbol  $on(c_1, x)$
  - **Substitution**:  $\theta = \{x_1 \leftarrow v_1, x_2 \leftarrow v_2, \dots, x_n \leftarrow v_n\}$ 
    - Each  $x_i$  is a variable symbol; each  $v_i$  is a term
  - **Instance** of an expression  $e$ : result of applying a substitution  $\theta$  to  $e$ 
    - Replace variables of  $e$  simultaneously, not sequentially
- **State**: a set  $s$  of ground atoms
  - The atoms represent the things that are true in one of  $\Sigma$ 's states
  - Only finitely many ground atoms, so only finitely many possible states

# Example of a State

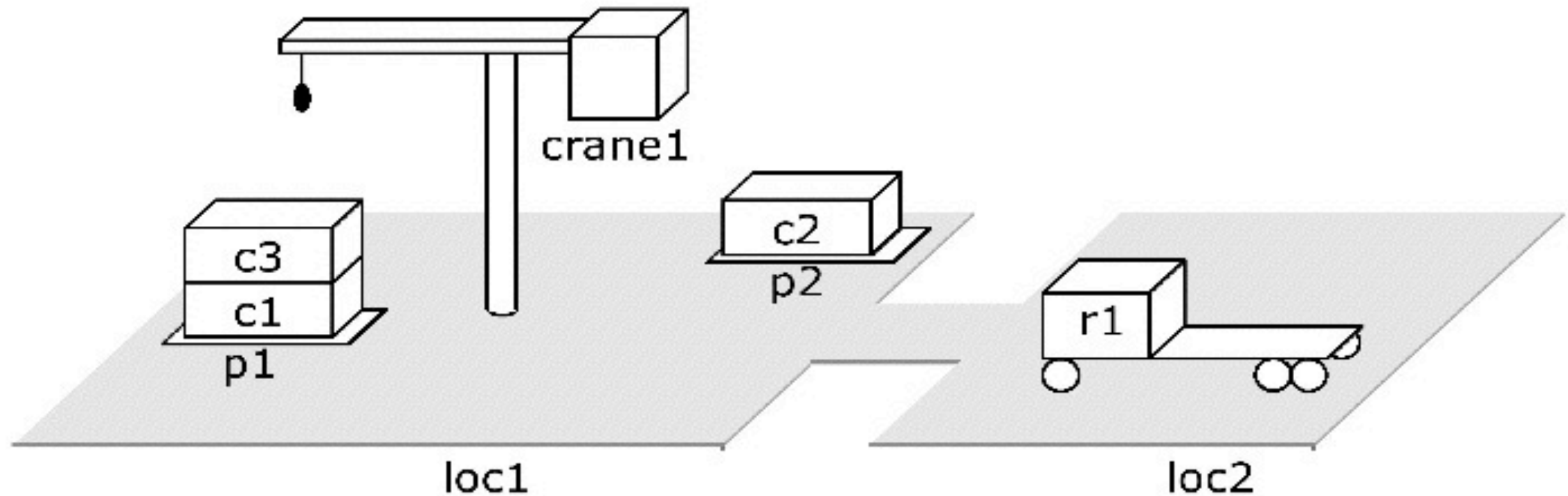


Figure 2.2: The DWR state  $s_1 = \{\text{attached}(p1, \text{loc1}), \text{in}(c1, p1), \text{in}(c3, p1), \text{top}(c3, p1), \text{on}(c3, c1), \text{on}(c1, \text{pallet}), \text{attached}(p2, \text{loc1}), \text{in}(c2, p2), \text{top}(c2, p2), \text{on}(c2, \text{pallet}), \text{belong}(\text{crane1}, \text{loc1}), \text{empty}(\text{crane1}), \text{adjacent}(\text{loc1}, \text{loc2}), \text{adjacent}(\text{loc2}, \text{loc1}), \text{at}(r1, \text{loc2}), \text{occupied}(\text{loc2}), \text{unloaded}(r1)\}$ .

# Operators

- Operator: a triple  $o = (\text{name}(o), \text{precond}(o), \text{effects}(o))$ 
  - $\text{name}(o)$  is a syntactic expression of the form  $n(x_1, \dots, x_k)$ 
    - $n$ : operator symbol - must be unique for each operator
    - $x_1, \dots, x_k$  : variable symbols (parameters)
      - must include every variable symbol in  $o$
  - $\text{precond}(o)$ : preconditions
    - Literals that must be true in order to use the operator
  - $\text{effects}(o)$ : effects
    - Literals the operator will make true

$\text{take}(k, l, c, d, p)$

;; crane  $k$  at location  $l$  takes  $c$  off of  $d$  in pile  $p$

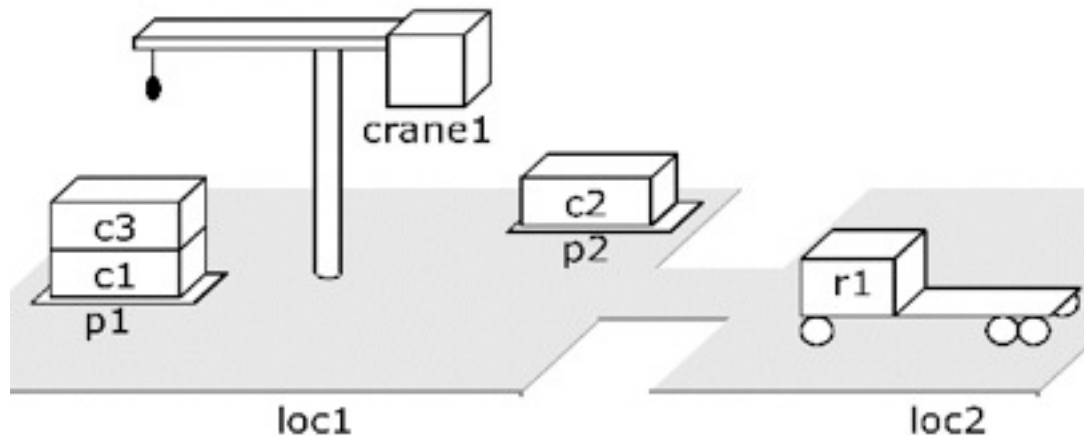
precond:  $\text{belong}(k, l), \text{attached}(p, l), \text{empty}(k), \text{top}(c, p), \text{on}(c, d)$

effects:  $\text{holding}(k, c), \neg \text{empty}(k), \neg \text{in}(c, p), \neg \text{top}(c, p), \neg \text{on}(c, d), \text{top}(d, p)$



# Actions

- Action: ground instance (via substitution) of an operator



$\text{take}(k, l, c, d, p)$

;; crane  $k$  at location  $l$  takes  $c$  off of  $d$  in pile  $p$

precond:  $\text{belong}(k, l), \text{attached}(p, l), \text{empty}(k), \text{top}(c, p), \text{on}(c, d)$

effects:  $\text{holding}(k, c), \neg \text{empty}(k), \neg \text{in}(c, p), \neg \text{top}(c, p), \neg \text{on}(c, d), \text{top}(d, p)$

$\text{take}(\text{crane1}, \text{loc1}, \text{c3}, \text{c1}, \text{p1})$

;; crane  $\text{crane1}$  at location  $\text{loc1}$  takes  $\text{c3}$  off  $\text{c1}$  in pile  $\text{p1}$

precond:  $\text{belong}(\text{crane1}, \text{loc1}), \text{attached}(\text{p1}, \text{loc1}),$   
 $\text{empty}(\text{crane1}), \text{top}(\text{c3}, \text{p1}), \text{on}(\text{c3}, \text{c1})$

effects:  $\text{holding}(\text{crane1}, \text{c3}), \neg \text{empty}(\text{crane1}), \neg \text{in}(\text{c3}, \text{p1}),$   
 $\neg \text{top}(\text{c3}, \text{p1}), \neg \text{on}(\text{c3}, \text{c1}), \text{top}(\text{c1}, \text{p1})$



# Notation

- Let  $S$  be a set of literals. Then
  - $S^+ = \{\text{atoms that appear positively in } S\}$
  - $S^- = \{\text{atoms that appear negatively in } S\}$
- More specifically, let  $a$  be an operator or action. Then
  - $precond^+(a) = \{\text{atoms that appear positively in } a\}$
  - $precond^-(a) = \{\text{atoms that appear negatively in } a\}$
  - $effects^+(a) = \{\text{atoms that appear positively in } a\}$
  - $effects^-(a) = \{\text{atoms that appear negatively in } a\}$

$take(k, l, c, d, p)$

;; crane  $k$  at location  $l$  takes  $c$  off of  $d$  in pile  $p$

precond:  $belong(k, l), attached(p, l), empty(k), top(c, p), on(c, d)$

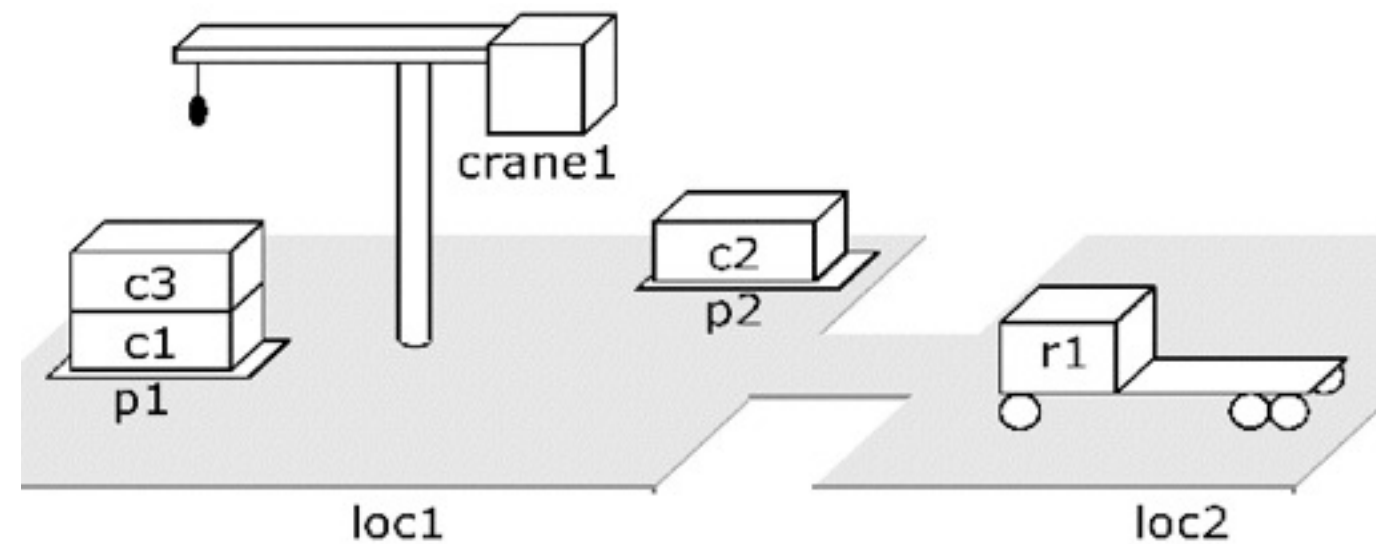
effects:  $holding(k, c), \neg empty(k), \neg in(c, p), \neg top(c, p), \neg on(c, d), top(d, p)$

$effects^+(take(k, l, c, d, p)) = \{holding(k, c), top(d, p)\}$

$effects^-(take(k, l, c, d, p)) = \{empty(k), in(c, p), top(c, p), on(c, d)\}$

# Applicability

- An action  $a$  is applicable to a state  $s$  if  $s$  satisfies  $precond(a)$ 
  - i.e., if  $precond^+(a) \subseteq s \wedge precond^-(a) \cap s = \emptyset$
- Here are an action and a state that it is applicable to:



`take(crane1,loc1,c3,c1,p1)`

`:: crane crane1 at location loc1 takes c3 off c1 in pile p1`

`precond: belong(crane1,loc1), attached(p1,loc1),  
          empty(crane1), top(c3,p1), on(c3,c1)`

`effects: holding(crane1,c3), ¬empty(crane1), ¬in(c3,p1),  
         ¬top(c3,p1), ¬on(c3,c1), top(c1,p1)`

# Result of Performing an Action

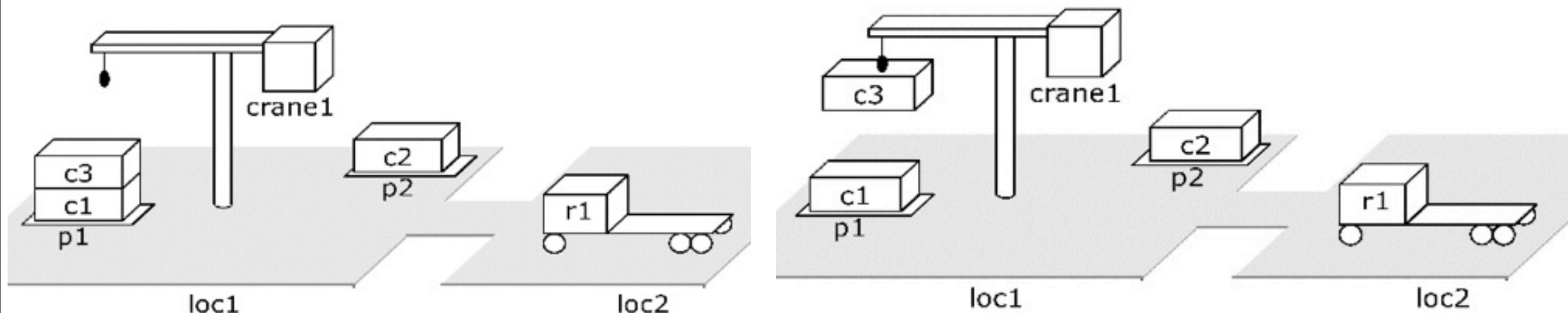
- If  $a$  is applicable to  $s$ , the result of performing it is
  - $\gamma(s, a) = (s \setminus effects^-(a)) \cup effects^+(a)$
  - Delete the negative effects, and add the positive ones

`take(crane1, loc1, c3, c1, p1)`

`:: crane crane1 at location loc1 takes c3 off c1 in pile p1`

`precond: belong(crane1, loc1), attached(p1, loc1),  
empty(crane1), top(c3, p1), on(c3, c1)`

`effects: holding(crane1, c3),  $\neg$ empty(crane1),  $\neg$ in(c3, p1),  
 $\neg$ top(c3, p1),  $\neg$ on(c3, c1), top(c1, p1)`



# Planning Domains: Language plus Operators

- Corresponds to a set of state-transition systems

- Example:  
operators for the  
DWR domain

`move( $r, l, m$ )`

;; robot  $r$  moves from location  $l$  to location  $m$

precond: `adjacent( $l, m$ ), at( $r, l$ ),  $\neg$  occupied( $m$ )`

effects: `at( $r, m$ ), occupied( $m$ ),  $\neg$  occupied( $l$ ),  $\neg$  at( $r, l$ )`

`load( $k, l, c, r$ )`

;; crane  $k$  at location  $l$  loads container  $c$  onto robot  $r$

precond: `belong( $k, l$ ), holding( $k, c$ ), at( $r, l$ ), unloaded( $r$ )`

effects: `empty( $k$ ),  $\neg$  holding( $k, c$ ), loaded( $r, c$ ),  $\neg$  unloaded( $r$ )`

`unload( $k, l, c, r$ )`

;; crane  $k$  at location  $l$  takes container  $c$  from robot  $r$

precond: `belong( $k, l$ ), at( $r, l$ ), loaded( $r, c$ ), empty( $k$ )`

effects:  `$\neg$  empty( $k$ ), holding( $k, c$ ), unloaded( $r$ ),  $\neg$  loaded( $r, c$ )`

`put( $k, l, c, d, p$ )`

;; crane  $k$  at location  $l$  puts  $c$  onto  $d$  in pile  $p$

precond: `belong( $k, l$ ), attached( $p, l$ ), holding( $k, c$ ), top( $d, p$ )`

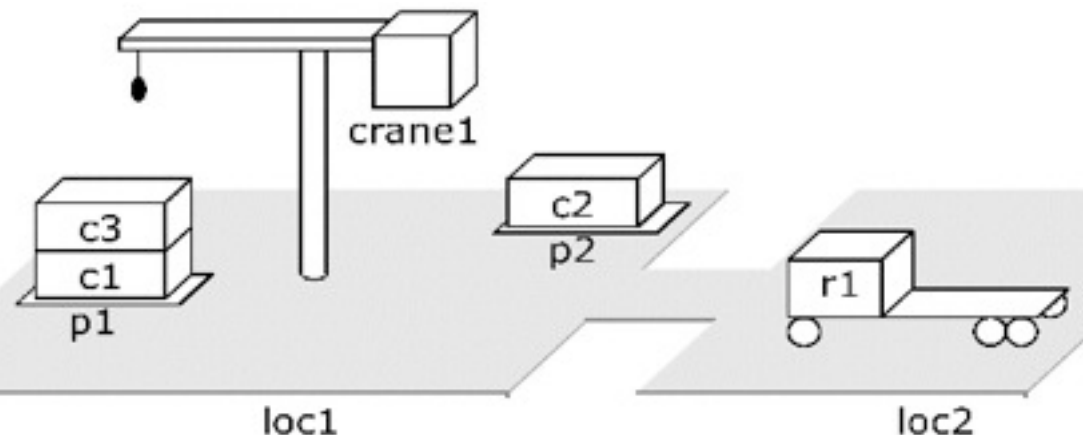
effects:  `$\neg$  holding( $k, c$ ), empty( $k$ ), in( $c, p$ ), top( $c, p$ ), on( $c, d$ ),  $\neg$  top( $d, p$ )`

`take( $k, l, c, d, p$ )`

;; crane  $k$  at location  $l$  takes  $c$  off of  $d$  in pile  $p$

precond: `belong( $k, l$ ), attached( $p, l$ ), empty( $k$ ), top( $c, p$ ), on( $c, d$ )`

effects: `holding( $k, c$ ),  $\neg$  empty( $k$ ),  $\neg$  in( $c, p$ ),  $\neg$  top( $c, p$ ),  $\neg$  on( $c, d$ ), top( $d, p$ )`





# Planning Problems

- Given a planning domain (language  $L$ , operators  $O$ )
  - Statement of a planning problem: a triple  $P = (O, s_0, g)$ 
    - is the collection of operators  $O$
    - is a state (the initial state)  $s_0$
    - is a set of literals (the goal formula)  $g$
  - The actual planning problem:  $P = (\Sigma, s_0, S_g)$ 
    - $s_0, S_g$  are as above
    - $\Sigma = (S, A, \gamma)$  is a state-transition system
    - $S = \{\text{all sets of ground atoms in } L\}$
    - $A = \{\text{all ground instances of operators in } O\}$
    - $\gamma = \text{the state-transition function determined by the operators}$
- We often say “planning problem”  
when we mean the statement of the problem

## ■ Plan:

- any sequence of actions  $\sigma = \langle a_1, a_2, \dots, a_n \rangle$  such that each  $a_i$  is a ground instance of an operator in  $O$

- The plan is a solution for  $P = (O, s_0, g)$  if it is executable and achieves  $g$

- i.e., if there are states  $s_0, s_1, \dots, s_n$  such that

$$\begin{aligned}\gamma(s_0, a_1) &= s_1 \\ \gamma(s_1, a_2) &= s_2 \\ &\vdots \\ \gamma(s_{n-1}, a_n) &= s_n \\ s_n &\vdash g\end{aligned}$$



# Example: DWR – Dock Worker Robot

- Let  $P_1 = (O, s_1, g_1)$  where

- $O$  is the set of operators given earlier

- $s_1$  is as shown:

- $g_1 = \{loaded(r_1, c_3), at(r_1, loc_2)\}$

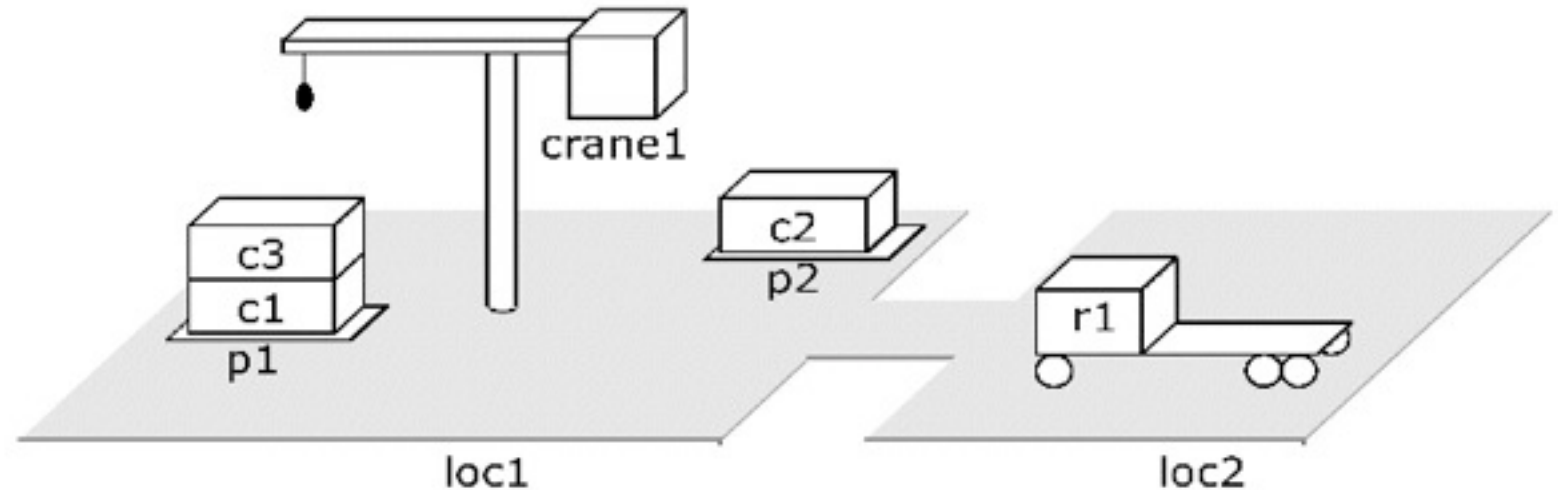
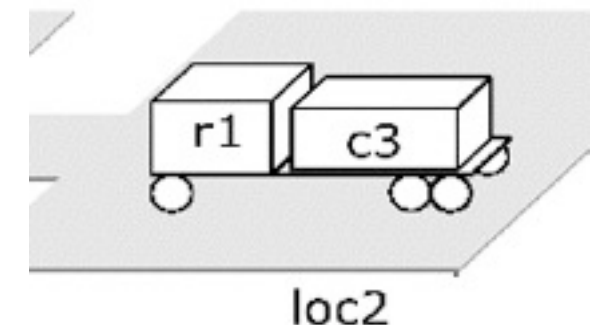
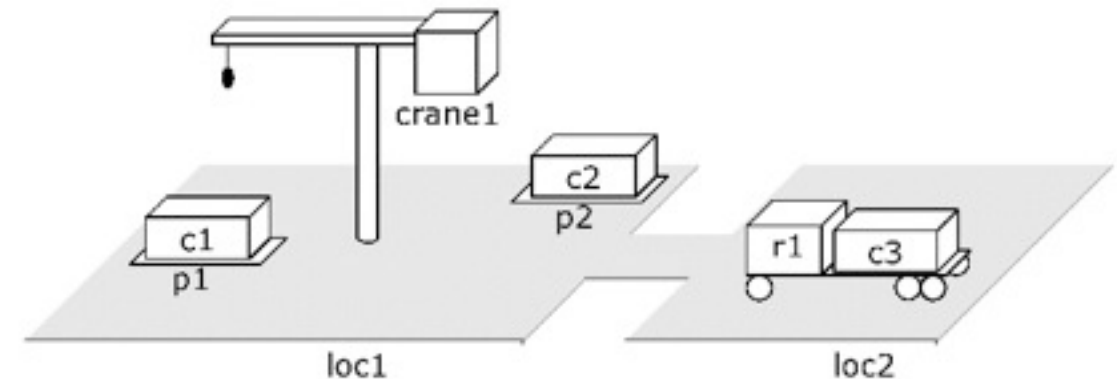
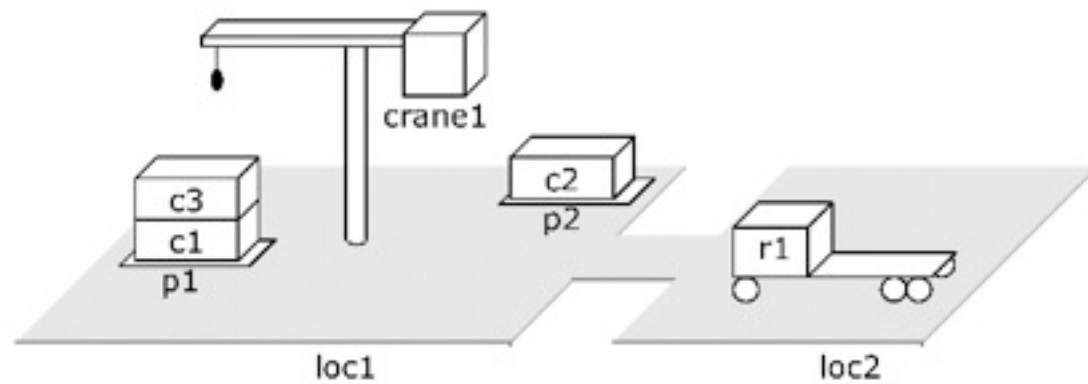


Figure 2.2: The DWR state  $s_1 = \{attached(p1, loc1), in(c1, p1), in(c3, p1), top(c3, p1), on(c3, c1), on(c1, pallet), attached(p2, loc1), in(c2, p2), top(c2, p2), on(c2, pallet), belong(crane1, loc1), empty(crane1), adjacent(loc1, loc2), adjacent(loc2, loc1), at(r1, loc2), occupied(loc2), unloaded(r1)\}$ .



# Example



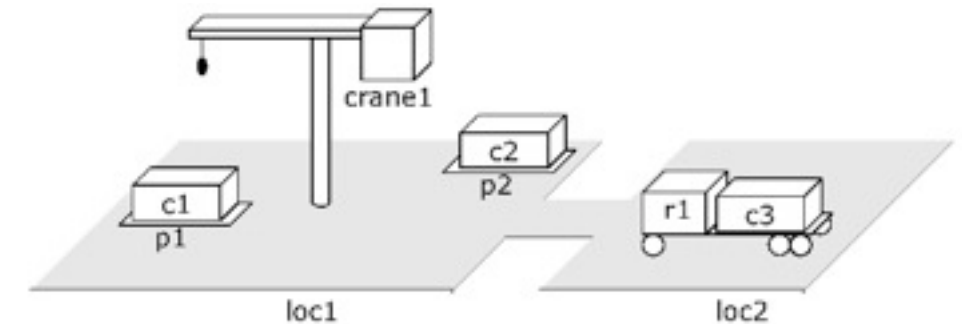
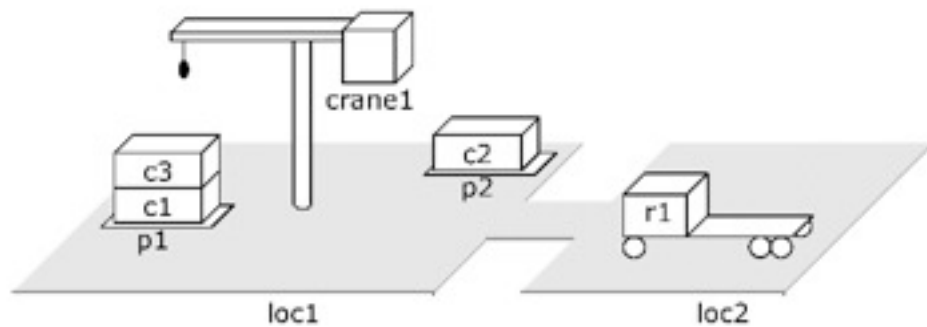
■ Here are three solutions for P:

```
< take(crane1, loc1, c1, p1)  
  move(r1, loc2, loc1),  
  load(crane1, loc1, c3, r1),  
  move(r1, loc1, loc2) >
```

```
< move(r1, loc2, loc1),  
  take(crane1, loc1, c1, p1)  
  load(crane1, loc1, c3, r1),  
  move(r1, loc1, loc2) >
```

```
< take(crane1, loc1, c1, p1)  
  move(r1, loc2, loc1),  
  move(r1, loc1, loc2),  
  move(r1, loc2, loc1),  
  load(crane1, loc1, c3, r1),  
  move(r1, loc1, loc2) >
```

# Example



- This one is redundant: can remove actions and still have a solution

```
< take(crane1, loc1, c1, p1)  
  move(r1, loc2, loc1),  
  move(r1, loc1, loc2),  
  move(r1, loc2, loc1),  
  load(crane1, loc1, c3, r1),  
  move(r1, loc1, loc2) >
```

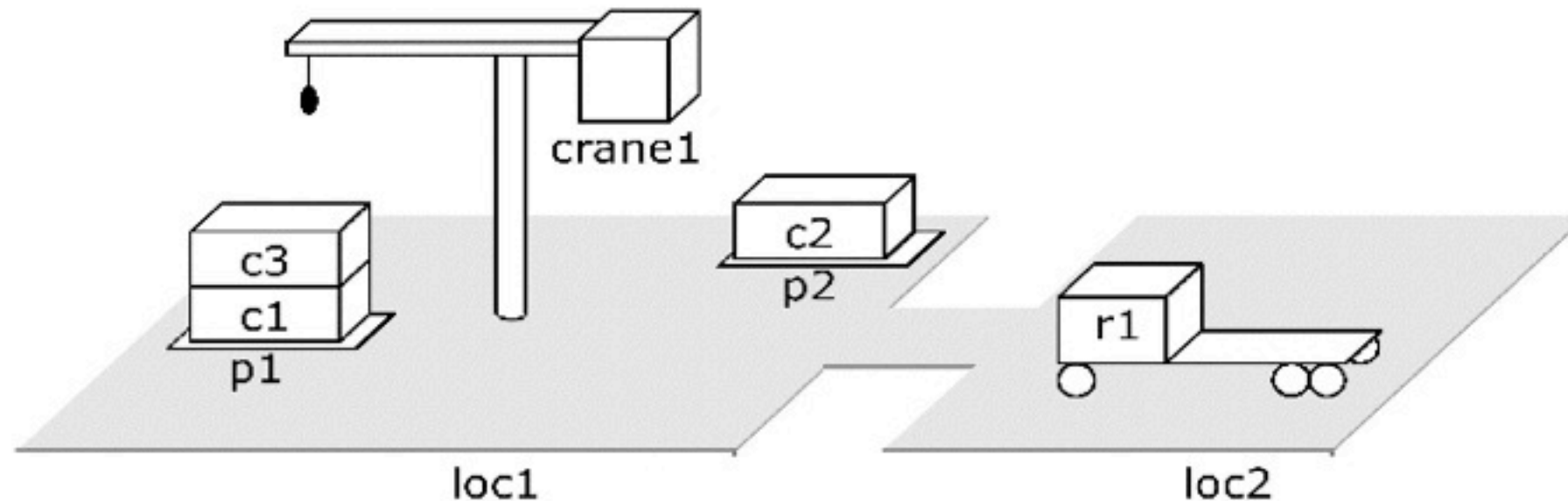
- These two are irredundant and shortest

```
< take(crane1, loc1, c1, p1)  
  move(r1, loc2, loc1),  
  load(crane1, loc1, c3, r1),  
  move(r1, loc1, loc2) >
```

```
< move(r1, loc2, loc1),  
  take(crane1, loc1, c1, p1)  
  load(crane1, loc1, c3, r1),  
  move(r1, loc1, loc2) >
```

# Set-Theoretic Representation

- Like classical representation, but restricted to propositional logic



- States:
  - Instead of a collection of ground atoms ...
  - $\{on(c_1, pallet), on(c_1, r1), on(c1, c2), \dots, at(r_1, l_1), at(r_1, l_2), \dots\}$
  - ... use a collection of propositions (Boolean variables):
  - $\{on-c1-pallet, on-c1-r1, on-c1-c2, \dots, at-r1-l1, at-r1-l2, \dots\}$

# Set-Theoretic Representation

- Instead of an operator like this one,

$\text{take}(k, l, c, d, p)$

$::$  crane  $k$  at location  $l$  takes  $c$  off of  $d$  in pile  $p$

precond:  $\text{belong}(k, l), \text{attached}(p, l), \text{empty}(k), \text{top}(c, p), \text{on}(c, d)$

effects:  $\text{holding}(k, c), \neg \text{empty}(k), \neg \text{in}(c, p), \neg \text{top}(c, p), \neg \text{on}(c, d), \text{top}(d, p)$

- ... there are lots of actions like this one

$\text{take-crane1-loc1-c3-c1-p1}$

precond:  $\text{belong-crane1-loc1}, \text{attached-p1-loc1},$   
 $\text{empty-crane1}, \text{top-c3-p1}, \text{on-c3-c1}$

delete:  $\text{empty-crane1}, \text{in-c3-p1}, \text{top-c3-p1}, \text{on-c3-p1}$

add:  $\text{holding-crane1-c3}, \text{top-c1-p1}$

- Exponential blow-up

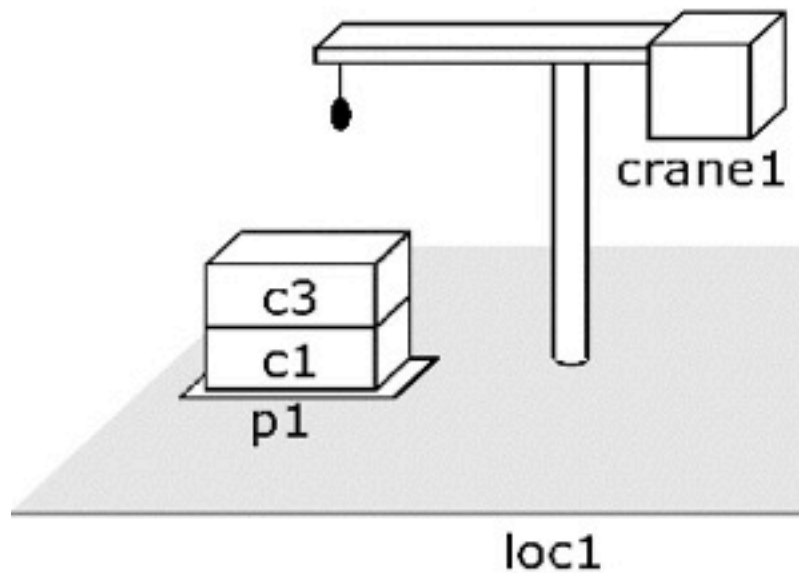
- If a classical operator contains  $n$  atoms and each atom has arity  $k$ ,

- then it corresponds to  $c^{nk}$  actions where  $c = \|\{\text{constantsymbols}\}\|$



# State-Variable Representation

- A state variable is like a field in a record structure



$\text{load}(c, r, l)$

;; robot  $r$  loads container  $c$  at location  $l$

precond:  $\text{rloc}(r) = l, \text{cpos}(c) = l, \text{rload}(r) = \text{nil}$

effects:  $\text{rload}(r) \leftarrow c, \text{cpow}(c) \leftarrow r$

$\text{unload}(c, r, l)$

;; robot  $r$  unloads container  $c$  at location  $l$

precond:  $\text{rloc}(r) = l, \text{rload}(r) = c$

effects:  $\text{rload}(r) \leftarrow \text{nil}, \text{cpow}(c) \leftarrow l$

- $\{\text{top}(p_1) = c_3, \text{cpow}(c_3) = c_1, \text{cpow}(c_1) = \text{pallet}, \dots\}$

- Classical and state-variable representations take similar amounts of space
  - Each can be translated into the other in low-order polynomial time

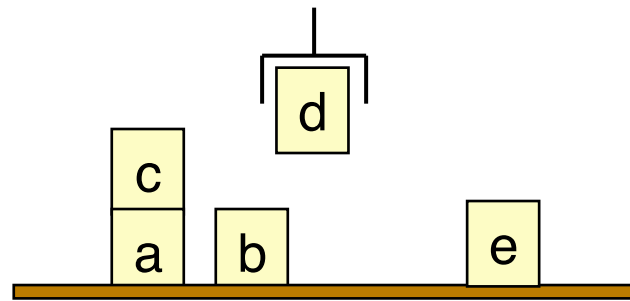


# Example: The Blocks World

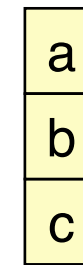
- Infinitely wide table, finite number of children's blocks
- Ignore where a block is located on the table
- A block can sit on the table or on another block
- Want to move blocks from one configuration to another

■ e.g.,

■ initial state



goal



- Can be expressed as a special case of DWR
  - But the usual formulation is simpler
- I'll give classical, set-theoretic, and state-variable formulations
  - For the case where there are five blocks

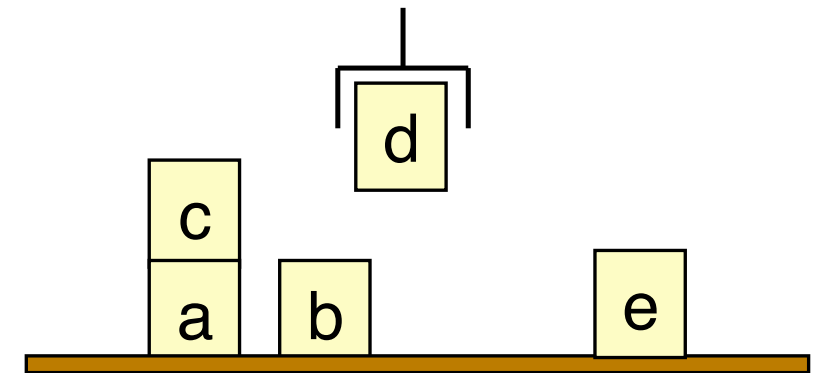
# Classical Representation: Symbols

- Constant symbols:

- The blocks: a, b, c, d, e

- Predicates:

- `ontable(x)` - block x is on the table
- `on(x,y)` - block x is on block y
- `clear(x)` - block x has nothing on it
- `holding(x)` - the robot hand is holding block x
- `handempty` - the robot hand isn't holding anything



# Classical Operators

## **unstack(x,y)**

Precond:  $\text{on}(x,y), \text{clear}(x), \text{handempty}$

Effects:  $\sim\text{on}(x,y), \sim\text{clear}(x), \sim\text{handempty},$   
 $\text{holding}(x), \text{clear}(y)$

## **stack(x,y)**

Precond:  $\text{holding}(x), \text{clear}(y)$

Effects:  $\sim\text{holding}(x), \sim\text{clear}(y),$   
 $\text{on}(x,y), \text{clear}(x), \text{handempty}$

## **pickup(x)**

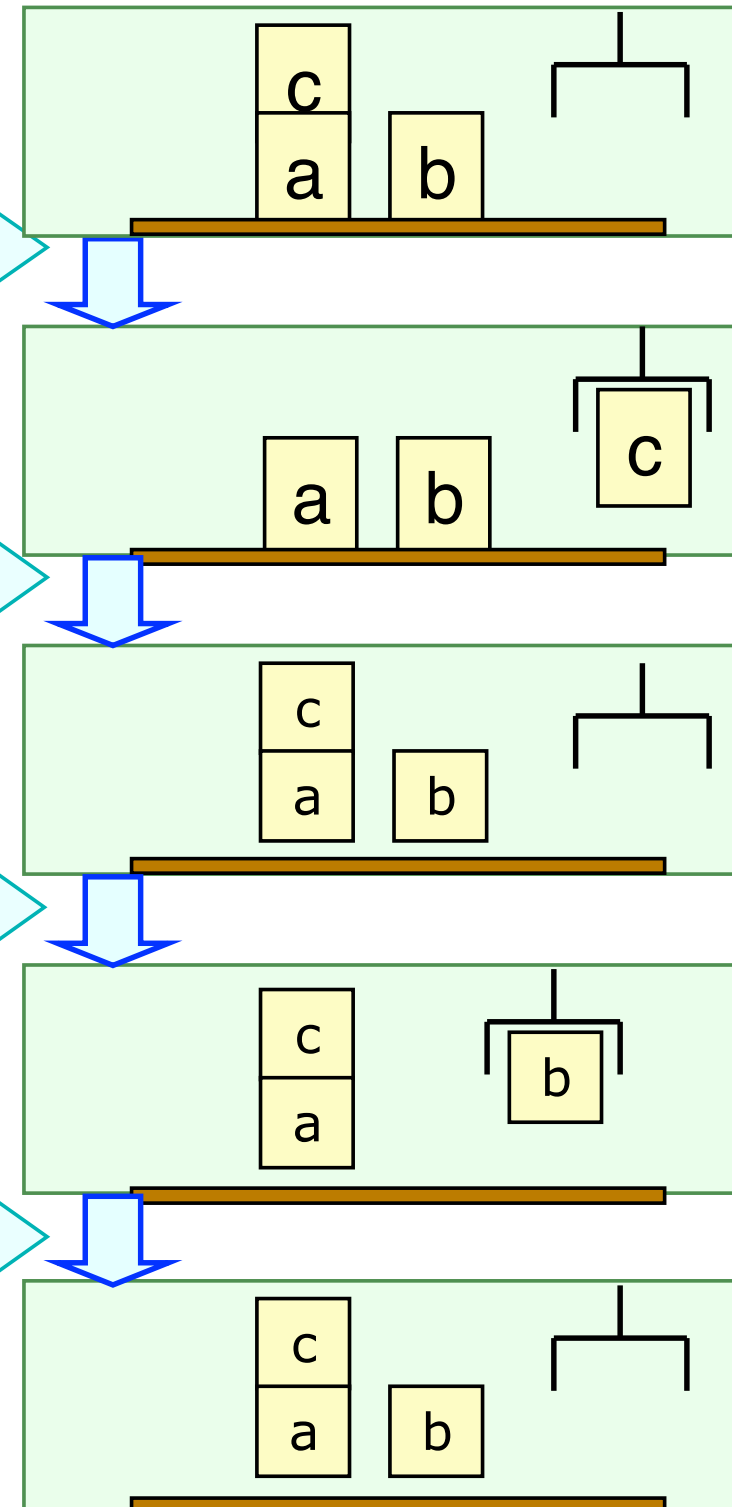
Precond:  $\text{ontable}(x), \text{clear}(x), \text{handempty}$

Effects:  $\sim\text{ontable}(x), \sim\text{clear}(x),$   
 $\sim\text{handempty}, \text{holding}(x)$

## **putdown(x)**

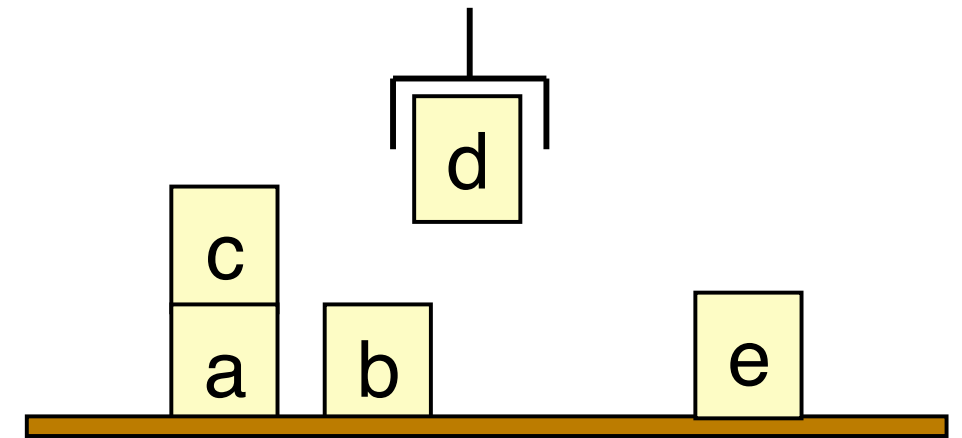
Precond:  $\text{holding}(x)$

Effects:  $\sim\text{holding}(x), \text{ontable}(x),$   
 $\text{clear}(x), \text{handempty}$



# Set-Theoretic Representation: Symbols

- For five blocks, there are 36 propositions
- Here are 5 of them:
  - `ontable-a`      - block a is on the table
  - `on-c-a`        - block c is on block a
  - `clear-c`        - block c has nothing on it
  - `holding-d`      - the robot hand is holding block d
  - `handempty`     - the robot hand isn't holding anything



# Set-Theoretic Actions

- Fifty different actions
- Four of them:

## **unstack-c-a**

Pre: on-c,a, clear-c, handempty  
Del: on-c,a, clear-c, handempty  
Add: holding-c, clear-a

## **stack-c-a**

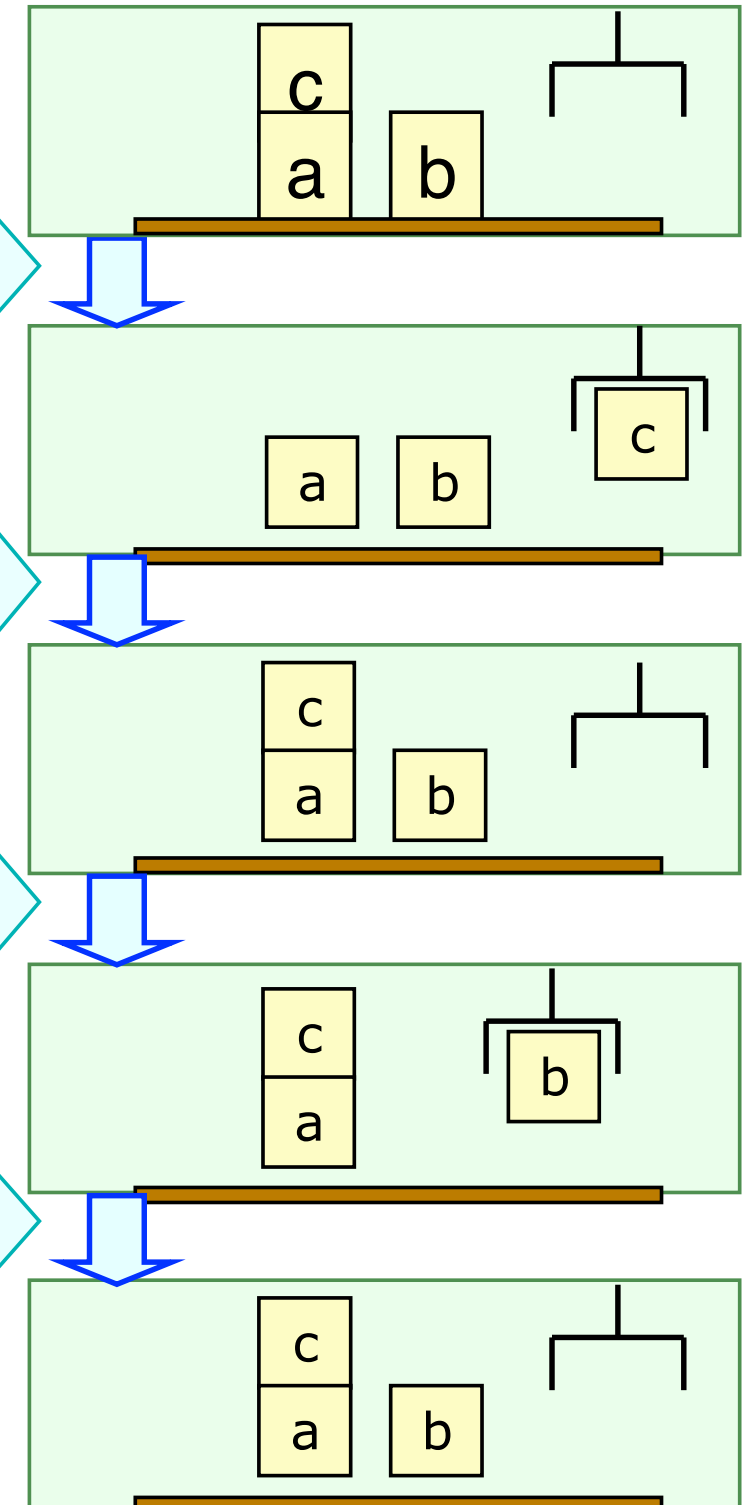
Pre: holding-c, clear-a  
Del: holding-c,  $\sim$ clear-a  
Add: on-c-a, clear-c, handempty

## **pickup-c**

Pre: on-table-c, clear-c, handempty  
Del: on-table-c, clear-c, handempty  
Add: holding-c

## **putdown-c**

Pre: holding-c  
Del: holding-c  
Add: on-table-c, clear-c, handempty



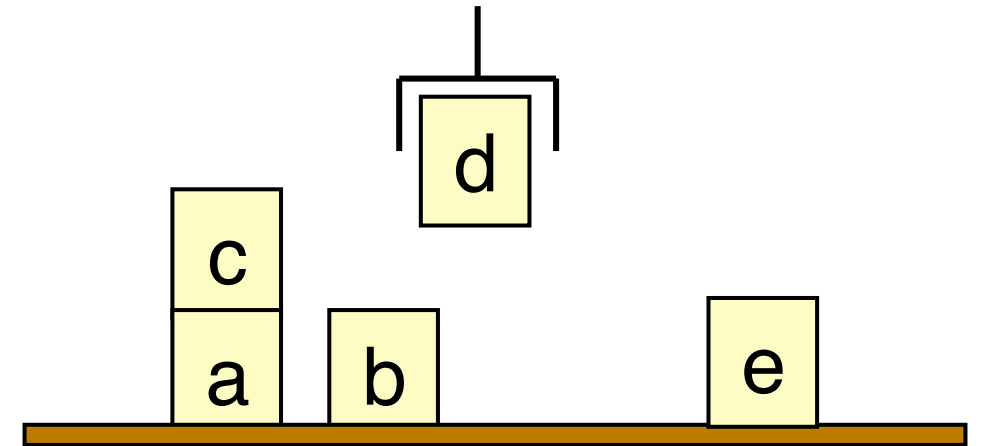
# State-Variable Representation: Symbols

- Constant symbols:

- a, b, c, d, e of type block
- 0, 1, table, nil of type other

- State variables:

- $\text{pos}(x) = y$  if block x is on block y
- $\text{pos}(x) = \text{table}$  if block x is on the table
- $\text{pos}(x) = \text{nil}$  if block x is being held
- $\text{clear}(x) = 1$  if block x has nothing on it
- $\text{clear}(x) = 0$  if block x is being held or has another block on it
- $\text{holding} = x$  if the robot hand is holding block x
- $\text{holding} = \text{nil}$  if the robot hand is holding nothing





# State-Variable Operators

## **unstack(x : block, y : block)**

Precond:  $\text{pos}(x)=y, \text{clear}(y)=0, \text{clear}(x)=1, \text{holding}=\text{nil}$

Effects:  $\text{pos}(x)=\text{nil}, \text{clear}(x)=0, \text{holding}=x, \text{clear}(y)=1$

## **stack(x : block, y : block)**

Precond:  $\text{holding}=x, \text{clear}(x)=0, \text{clear}(y)=1$

Effects:  $\text{holding}=\text{nil}, \text{clear}(y)=0, \text{pos}(x)=y, \text{clear}(x)=1$

## **pickup(x : block)**

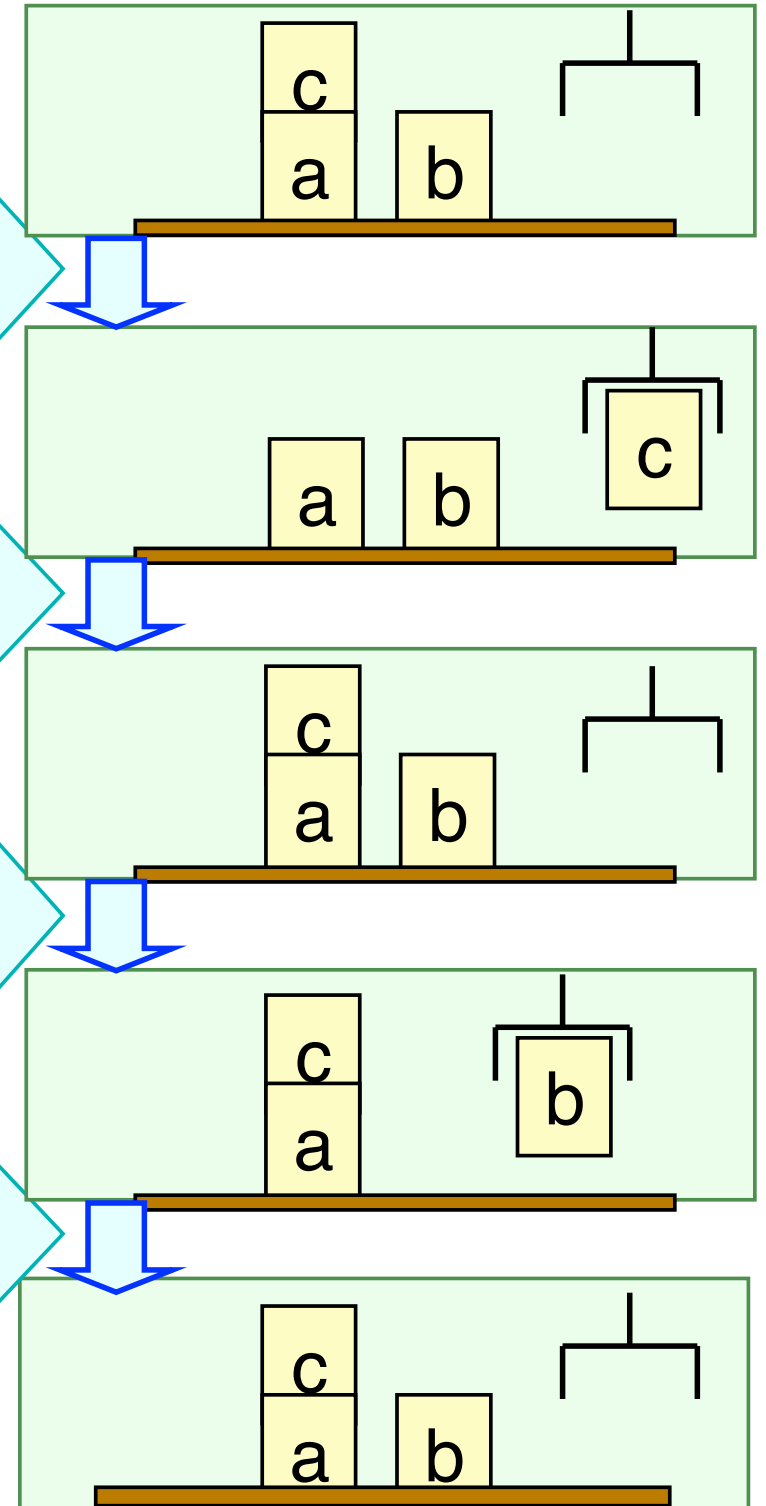
Precond:  $\text{pos}(x)=\text{table}, \text{clear}(x)=1, \text{holding}=\text{nil}$

Effects:  $\text{pos}(x)=\text{nil}, \text{clear}(x)=0, \text{holding}=x$

## **putdown(x : block)**

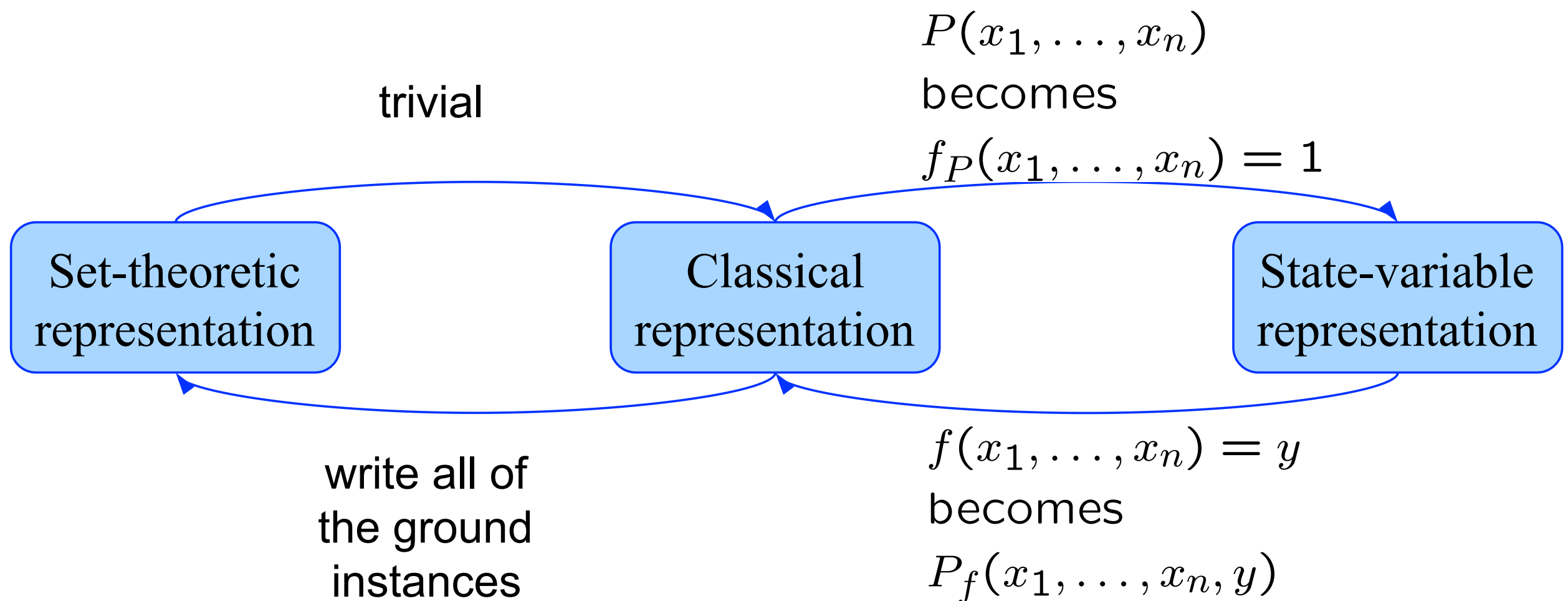
Precond:  $\text{holding}=x$

Effects:  $\text{holding}=\text{nil}, \text{pos}(x)=\text{table}, \text{clear}(x)=1$



# Expressive Power

- Any problem that can be represented in one representation can also be represented in the other two
- Can convert in linear time and space, except for the following:
  - Converting to set-theoretic from either of the others can incur exponential blowup



- Classical representation
  - The most popular for classical planning, partly for historical reasons
- Set-theoretic representation
  - Can take much more space than classical representation
  - Useful in algorithms that manipulate ground atoms directly
    - e.g., planning graphs (Chapter 6), satisfiability (Chapters 7)
  - Useful for certain kinds of theoretical studies
- State-variable representation
  - Equivalent to classical representation
  - Less natural for logicians, more natural for engineers
  - Useful in non-classical planning problems as a way to handle numbers, functions, time