PLANNING AND SCHEDULING: FIRST-ORDER LOGIC

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Acknowledgements

- These slides refer to Chapter 8 of the textbook:
  S. Russell and P. Norvig:
  Artificial Intelligence: A Modern Approach
  Prentice Hall, 2003, 2nd Edition (or more recent edition)
- These slides are an adaptation of slides by Min-Yen Kan
- The contributions of these authors are gratefully acknowledged.

Outline

- Why First-Order Logic? (FOL)
- Syntax and semantics of FOL
- Using FOL
- Wumpus world in FOL
- Knowledge engineering in FOL

Pros and Cons of Propositional Logic

☺ Propositional logic is declarative

☺ Propositional logic allows partial/disjunctive/negated information

- (unlike most data structures and databases)

☺ Propositional logic is compositional:

- meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$

☺ Meaning in propositional logic is context-independent

- (unlike natural language, where meaning depends on context)

☹ Propositional logic has very limited expressive power

- (unlike natural language)
- E.g., cannot say "pits cause breezes in adjacent squares" except by writing one sentence for each square

First-Order Logic

- Whereas propositional logic assumes the world contains facts,
- first-order logic (like natural language) assumes the world contains
  - Objects: people, houses, numbers, colors, baseball games, wars, …
  - Relations: red, round, prime, brother of, bigger than, part of, comes between, …
  - Functions: father of, best friend, one more than, plus, …

Syntax of FOL: Basic Elements

• Constants	KingJohn, 2, NUS,...
• Predicates	Brother, >,...
• Functions	Sqrt, LeftLegOf,...
• Variables	x, y, a, b,...
• Connectives	$\neg, \Rightarrow, \wedge, \vee, \Leftrightarrow$
• Equality	=
• Quantifiers	$\forall, \exists$

Atomic Sentences

- Atomic sentence = *predicate* $(term_1,\ldots,term_n)$
  or $term_1 = term_2$

- Term = *function* $(term_1,\ldots,term_n)$
  or *constant*
  or *variable*

- *Brother(KingJohn, RichardTheLionheart)*
- *>(Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))*

Complex Sentences

- Complex sentences are made from atomic sentences using connectives
  - $\neg S$, $S1 \wedge S2$, $S1 \vee S2$, $S1 \Rightarrow S2$, $S1 \Leftrightarrow S2$
- For example:
  - Sibling(KingJohn,Richard) $\Rightarrow$ Sibling(Richard,KingJohn)
  - $>(1,2) \vee \leq(1,2)$
  - $>(1,2) \wedge \neg >(1,2)$

Truth in First-Order Logic

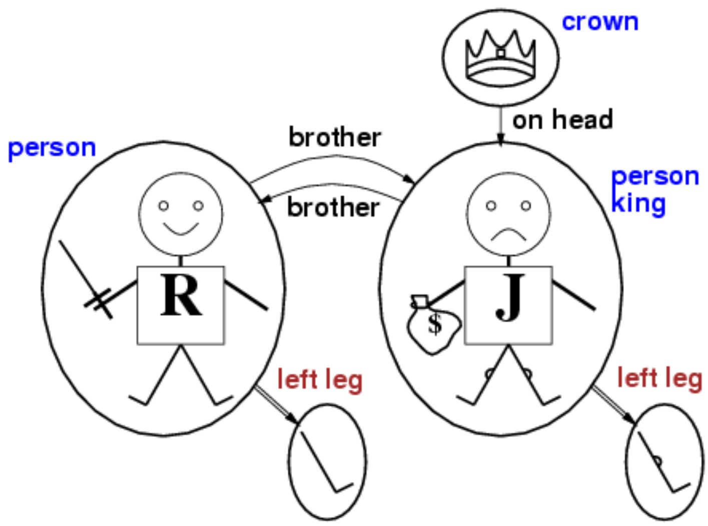
- Sentences are true with respect to a model and an interpretation
- Model contains objects (domain elements) and relations among them
- Interpretation specifies referents for

  constant symbols → objects

  predicate symbols → relations

  function symbols → functional relations
- An atomic sentence *predicate*($term_1,\ldots,term_n$) is true iff the objects referred to by $term_1,\ldots,term_n$ are in the relation referred to by *predicate*

Models for FOL: Example



Universal Quantification

- $\forall$*<variables> <sentence>*
- Everyone at NUS is smart: $\forall x\ At(x,NUS) \Rightarrow Smart(x)$
- $\forall x\ P$ is true in a model *m*
  iff *P* is true with *x* being each possible object in the model
- Roughly speaking, equivalent to the
  conjunction of instantiations of *P*

$$
\begin{aligned}
& At(KingJohn,NUS) \Rightarrow Smart(KingJohn) \\
\wedge\ & At(Richard,NUS) \Rightarrow Smart(Richard) \\
\wedge\ & At(NUS,NUS) \Rightarrow Smart(NUS) \\
\wedge\ & \ldots
\end{aligned}
$$

A Common Mistake to Avoid

- Typically, $\Rightarrow$ is the main connective with $\forall$
- Common mistake: using $\wedge$ as the main connective with $\forall$:

  $\forall x\ At(x,NUS) \wedge Smart(x)$

  means “Everyone is at NUS and everyone is smart”

Existential Quantification

- $\exists$<*variables*> <*sentence*>
- Someone at NUS is smart: $\exists x$ At(x,NUS) $\wedge$ Smart(x)\$
- $\exists x$ *P* is true in a model *m*
  iff *P* is true with *x* being some possible object in the model
- Roughly speaking, equivalent to the disjunction of instantiations of *P*

  At(KingJohn,NUS) $\wedge$ Smart(KingJohn)

  $\vee$ At(Richard,NUS) $\wedge$ Smart(Richard)

  $\vee$ At(NUS,NUS) $\wedge$ Smart(NUS)

  $\vee$ ...

Another Common Mistake to Avoid

- Typically, $\wedge$ is the main connective with $\exists$
- Common mistake: using $\Rightarrow$ as the main connective with $\exists$:

  $\exists x$ At(x,NUS) $\Rightarrow$ Smart(x)

  is true if there is anyone who is not at NUS!

Properties of Quantifiers

- $\forall x \; \forall y$ is the same as $\forall y \; \forall x$
- $\exists x \; \exists y$ is the same as $\exists y \; \exists x$
- $\exists x \; \forall y$ is not the same as $\forall y \; \exists x$
- $\exists x \; \forall y \; Loves(x,y)$
  - "There is a person who loves everyone in the world"
- $\forall y \; \exists x \; Loves(x,y)$
  - "Everyone in the world is loved by at least one person"
- Quantifier duality: each can be expressed using the other
- $\forall x \; Likes(x,IceCream) \qquad \neg\exists x \; \neg Likes(x,IceCream)$
- $\exists x \; Likes(x,Broccoli) \qquad \neg\forall x \; \neg Likes(x,Broccoli)$

Equality

- $term_1 = term_2$ is true under a given interpretation iff $term_1$ and $term_2$ refer to the same object
- E.g., definition of Sibling in terms of Parent:
- $\forall x,y \; Sibling(x,y) \Leftrightarrow [\neg(x = y) \land \exists m,f \; \neg (m = f)$
  $\land \; Parent(m,x) \land Parent(f,x) \land Parent(m,y) \land Parent(f,y)]$

Using FOL: The Kinship Domain

- Brothers are siblings
  - $\forall x,y\ Brother(x,y) \Leftrightarrow Sibling(x,y)$
- One's mother is one's female parent
  - $\forall m,c\ Mother(c) = m \Leftrightarrow (Female(m) \wedge Parent(m,c))$
- "Sibling" is symmetric
  - $\forall x,y\ Sibling(x,y) \Leftrightarrow Sibling(y,x)$

Using FOL: The Set Domain

- $\forall s\ Set(s) \Leftrightarrow (s = \{\}) \vee (\exists x,s_2\ Set(s_2) \wedge s = \{x|s_2\})$
- $\neg\exists x,s\ \{x|s\} = \{\}$
- $\forall x,s\ x \in s \Leftrightarrow s = \{x|s\}$
- $\forall x,s\ x \in s \Leftrightarrow [\ \exists y,s_2\}\ (s = \{y|s_2\} \wedge (x = y \vee x \in s_2))]$
- $\forall s_1,s_2\ s_1 \subseteq s_2 \Leftrightarrow (\forall x\ x \in s_1 \Rightarrow x \in s_2)$
- $\forall s_1,s_2\ (s_1 = s_2) \Leftrightarrow (s_1 \subseteq s_2 \wedge s_2 \subseteq s_1)$
- $\forall x,s_1,s_2\ x \in (s_1 \cap s_2) \Leftrightarrow (x \in s_1 \wedge x \in s_2)$
- $\forall x,s_1,s_2\ x \in (s_1 \cup s_2) \Leftrightarrow (x \in s_1 \vee x \in s_2)$

Interacting with FOL KBs

- Suppose a wumpus-world agent is using an FOL KB and perceives a smell and a breeze (but no glitter) at *t=5*:

  `Tell`(KB,Percept([Smell,Breeze,None],5))

  `Ask`(KB,∃a BestAction(a,5))

- I.e., does the KB entail some best action at *t=5*?
- Answer: *Yes*, {*a/Shoot*} ← substitution (binding list)
- Given a sentence *S* and a substitution σ,
- *Sσ* denotes the result of plugging σ into *S*; e.g.,

  S = Smarter(x,y)

  σ = {x/Hillary,y/Bill}

  Sσ = Smarter(Hillary,Bill)

- `Ask`(KB,S) returns some/all σ such that KB ⊨ Sσ

Knowledge Base for the Wumpus World

- Perception
  - ∀t,s,b Percept([s,b,Glitter],t) ⇒ Glitter(t)
- Reflex
  - ∀t Glitter(t) ⇒ BestAction(Grab,t)

Deducing Hidden Properties

- $\forall x,y,a,b$ *Adjacent*([x,y],[a,b]) $\Leftrightarrow$ [a,b] $\in$ {[x+1,y], [x-1,y],[x,y+1],[x,y-1]}

Properties of squares:

- $\forall s,t$ *At*(Agent,s,t) $\wedge$ Breeze(t) $\Rightarrow$ Breezy(s)

Squares are breezy near a pit:

- Diagnostic rule---infer cause from effect
  $\forall s$ Breezy(s) $\Rightarrow$ [$\exists r$ Adjacent(r,s) $\wedge$ Pit(r)]
- Causal rule---infer effect from cause
  $\forall r$ Pit(r) $\Rightarrow$ [$\forall s$ Adjacent(r,s) $\Rightarrow$ Breezy(s)]

Knowledge Engineering in FOL

1. Identify the task
2. Assemble the relevant knowledge
3. Decide on a vocabulary of predicates, functions, and constants
4. Encode general knowledge about the domain
5. Encode a description of the specific problem instance
6. Pose queries to the inference procedure and get answers
7. Debug the knowledge base

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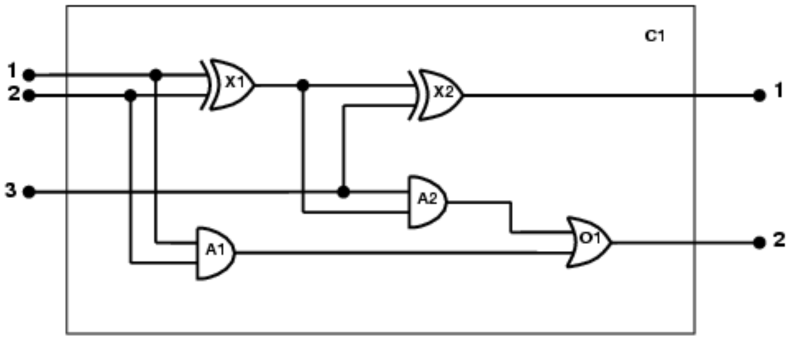
The Electronic Circuits Domain

One-bit full adder



The Electronic Circuits Domain

1. Identify the task
   - Does the circuit actually add properly? (circuit verification)
2. Assemble the relevant knowledge
   - Composed of wires and gates; Types of gates (AND, OR, XOR, NOT)
   - Irrelevant: size, shape, color, cost of gates
3. Decide on a vocabulary
   - Alternatives:

     Type(X_1) = XOR

     Type(X_1, XOR)

     XOR(X_1)

The electronic circuits domain

4. Encode general knowledge of the domain
   - $\forall t_1,t_2$ Connected(t_1, t_2) $\Rightarrow$ Signal(t_1) = Signal(t_2)
   - $\forall t$ Signal(t) = 1 $\vee$ Signal(t) = 0
   - $1 \neq 0$
   - $\forall t_1,t_2$ Connected(t_1, t_2) $\Rightarrow$ Connected(t_2, t_1)
   - $\forall g$ Type(g) = OR $\Rightarrow$ Signal(Out(1,g)) = 1 $\Leftrightarrow$ $\exists n$ Signal(In(n,g)) = 1
   - $\forall g$ Type(g) = AND $\Rightarrow$ Signal(Out(1,g)) = 0 $\Leftrightarrow$ $\exists n$ Signal(In(n,g)) = 0
   - $\forall g$ Type(g) = XOR $\Rightarrow$ Signal(Out(1,g)) = 1 $\Leftrightarrow$ Signal(In(1,g)) $\neq$ Signal(In(2,g))
   - $\forall g$ Type(g) = NOT $\Rightarrow$ Signal(Out(1,g)) $\neq$ Signal(In(1,g))

The Electronic Circuits Domain

5. Encode the specific problem instance

Type(X_1) = XOR   Type(X_2) = XOR
Type(A_1) = AND   Type(A_2) = AND
Type(O_1) = OR

Connected(Out(1,X_1),In(1,X_2))   Connected(In(1,C_1),In(1,X_1))
Connected(Out(1,X_1),In(2,A_2))   Connected(In(1,C_1),In(1,A_1))
Connected(Out(1,A_2),In(1,O_1))   Connected(In(2,C_1),In(2,X_1))
Connected(Out(1,A_1),In(2,O_1))   Connected(In(2,C_1),In(2,A_1))
Connected(Out(1,X_2),Out(1,C_1))   Connected(In(3,C_1),In(2,X_2))
Connected(Out(1,O_1),Out(2,C_1))   Connected(In(3,C_1),In(1,A_2))

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The electronic circuits domain

6. Pose queries to the inference procedure
   - What are the possible sets of values of all the terminals for the adder circuit?

     $\exists i_1,i_2,i_3,o_1,o_2$ Signal(In(1,C_1)) = i_1 ∧ Signal(In(2,C_1)) = i_2 ∧ Signal(In(3,C_1)) = i_3 ∧ Signal(Out(1,C_1)) = o_1 ∧ Signal(Out(2,C_1)) = o_2
7. Debug the knowledge base
   - May have omitted assertions like $1 \neq 0$

Summary

- First-order logic:
  - objects and relations are semantic primitives
  - syntax: constants, functions, predicates, equality, quantifiers
- Increased expressive power: sufficient to define the wumpus world