Planning and Scheduling: Inference in First-Order Logic

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Acknowledgements

- These slides refer to Chapter 9 of the textbook:
  S. Russell and P. Norvig:
  Artificial Intelligence: A Modern Approach
  Prentice Hall, 2003, 2nd Edition (or more recent edition)
- These slides are an adaptation of slides by Min-Yen Kan
- The contributions of these authors are gratefully acknowledged.

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Outline

- Reducing first-order inference to propositional inference
- Unification
- Generalized Modus Ponens
- Forward chaining
- Backward chaining
- Resolution

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Universal Instantiation (UI)

- Every instantiation of a universally quantified sentence is entailed by it:

$$\frac{\forall v\ \alpha}{\text{Subst}(\{v/g\}, \alpha)}$$

  for any variable *v* and ground term *g*

- E.g., $\forall x\ King(x) \wedge Greedy(x) \Rightarrow Evil(x)$ yields:

  $King(John) \wedge Greedy(John) \Rightarrow Evil(John)$

  $King(Richard) \wedge Greedy(Richard) \Rightarrow Evil(Richard)$

  $King(Father(John)) \wedge Greedy(Father(John)) \Rightarrow Evil(Father(John))$

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Existential Instantiation (EI)

- For any sentence α, variable *v*, and constant symbol *k* that does not appear elsewhere in the knowledge base:

$$\frac{\exists v\ \alpha}{\text{Subst}(\{v/k\}, \alpha)}$$

- E.g., $\exists x\ Crown(x) \wedge OnHead(x, John)$ yields:

$$Crown(C_1) \wedge OnHead(C_1, John)$$

provided C_1 is a new constant symbol, called a Skolem constant

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Reduction to Propositional Inference

- Suppose the KB contains just the following:
  - ∀x King(x) ∧ Greedy(x) ⇒ Evil(x)
  - King(John)
  - Greedy(John)
  - Brother(Richard,John)
- Instantiating the universal sentence in all possible ways, we have:
  - King(John) ∧ Greedy(John) ⇒ Evil(John)
  - King(Richard) ∧ Greedy(Richard) ⇒ Evil(Richard)
  - King(John)
  - Greedy(John)
  - Brother(Richard,John)
- The new KB is propositionalized: proposition symbols are
  - King(John), Greedy(John), Evil(John), King(Richard), etc.

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Reduction

- Every FOL KB can be propositionalized so as to preserve entailment
- A ground sentence is entailed by new KB iff entailed by original KB
- Idea: propositionalize KB and query, apply resolution, return result
- Problem: with function symbols, there are infinitely many ground terms:
  - e.g., Father(Father(Father(John)))

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Reduction

- Theorem: Herbrand (1930). If a sentence α is entailed by an FOL KB, it is entailed by a finite subset of the propositionalized KB
- Idea: For n = 0 to ∞ do
  - create a propositional KB by instantiating with depth-*n* terms
  - see if α is entailed by this KB
- Problem: works if α is entailed, loops if α is not entailed
- Theorem: Turing (1936), Church (1936):
  Entailment for FOL is semidecidable
  (algorithms exist that say yes to every entailed sentence, but
  no algorithm exists that also says no to every nonentailed sentence.)

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Problems with Propositionalization

- Propositionalization seems to generate lots of irrelevant sentences.
- E.g., from:
  - $\forall x\ King(x) \wedge Greedy(x) \Rightarrow Evil(x)$
  - King(John)
  - $\forall y\ Greedy(y)$
  - Brother(Richard,John)
- it seems obvious that Evil(John), but propositionalization produces lots of facts such as Greedy(Richard) that are irrelevant
- With p k-ary predicates and n constants,
  there are $p \cdot n^k$ instantiations.

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Unification

- We can get the inference immediately if we can find a substitution θ such that *King(x)* and *Greedy(x)* match *King(John)* and *Greedy(y)*

  θ = {x/John,y/John} works

- Unify(α,β) = θ if αθ = βθ

p	q	θ
Knows(John,x)	Knows(John,Jane)	
Knows(John,x)	Knows(y,OJ)	
Knows(John,x)	Knows(y,Mother(y))	
Knows(John,x)	Knows(x,OJ)	

- Standardizing apart eliminates overlap of variables,
  e.g., Knows(z_{17},OJ)

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- Unify(α,β) = θ if αθ = βθ

p	q	θ
Knows(John,x)	Knows(John,Jane)	{x/Jane}}
Knows(John,x)	Knows(y,OJ)	
Knows(John,x)	Knows(y,Mother(y))	
Knows(John,x)	Knows(x,OJ)	

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p	q	θ
Knows(John,x)	Knows(John,Jane)	{x/Jane}}
Knows(John,x)	Knows(y,OJ)	{x/OJ,y/John}}
Knows(John,x)	Knows(y,Mother(y))	
Knows(John,x)	Knows(x,OJ)	

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- Unify(α,β) = θ if αθ = βθ

p	q	θ
Knows(John,x)	Knows(John,Jane)	{x/Jane}}
Knows(John,x)	Knows(y,OJ)	{x/OJ,y/John}}
Knows(John,x)	Knows(y,Mother(y))	{y/John,x/Mother(John)}}
Knows(John,x)	Knows(x,OJ)	

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Unification

- We can get the inference immediately if we can find a substitution θ such that *King(x)* and *Greedy(x)* match *King(John)* and *Greedy(y)*

  θ = {x/John,y/John} works

- Unify(α,β) = θ if αθ = βθ

p	q	θ
Knows(John,x)	Knows(John,Jane)	{x/Jane}}
Knows(John,x)	Knows(y,OJ)	{x/OJ,y/John}}
Knows(John,x)	Knows(y,Mother(y))	{y/John,x/Mother(John)}}
Knows(John,x)	Knows(x,OJ)	{fail}

- Standardizing apart eliminates overlap of variables, e.g., Knows(z_{17},OJ)

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Unification

- To unify *Knows(John,x)* and *Knows(y,z)*,

  θ = {y/John, x/z }
  or θ = {y/John, x/John, z/John}

- The first unifier is more general than the second.
- There is a single most general unifier (mgu) that is unique up to renaming of variables.

  mgu = { y/John, x/z }

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The Unification Algorithm: UNIFY

```
function UNIFY(x, y, θ) returns a substitution to make x and y identical
  inputs: x, a variable, constant, list, or compound
          y, a variable, constant, list, or compound
          θ, the substitution built up so far

  if θ = failure then return failure
  else if x = y then return θ
  else if VARIABLE?(x) then return UNIFY-VAR(x, y, θ)
  else if VARIABLE?(y) then return UNIFY-VAR(y, x, θ)
  else if COMPOUND?(x) and COMPOUND?(y) then
      return UNIFY(ARGS[x], ARGS[y], UNIFY(OP[x], OP[y], θ))
  else if LIST?(x) and LIST?(y) then
      return UNIFY(REST[x], REST[y], UNIFY(FIRST[x], FIRST[y], θ))
  else return failure
```


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The Unification Algorithm: UNIFY-VAR

function UNIFY-VAR(var, x, θ) **returns** a substitution
  inputs: var, a variable
    x, any expression
    θ, the substitution built up so far

  if $\{var/val\} \in \theta$ **then return** UNIFY(val, x, θ)
  else if $\{x/val\} \in \theta$ **then return** UNIFY(var, val, θ)
  else if OCCUR-CHECK?(var, x) **then return** failure
  else return add $\{var/x\}$ to θ

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Generalized Modus Ponens (GMP)

$$\frac{p_1', p_2', \ldots, p_n', (p_1 \wedge p_2 \wedge \ldots \wedge p_n \Rightarrow q)}{q\theta}$$

where $p_i'\theta = p_i\,\theta$ for all *i*

p_1' is *King*(*John*)
p_2' is *Greedy*(*y*)
θ is {x/John,y/John}
q θ is *Evil*(*John*)

p_1 is *King*(*x*)
p_2 is *Greedy*(*x*)
q is *Evil*(*x*)

- GMP used with KB of definite clauses (exactly one positive literal)
- All variables assumed universally quantified

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Soundness of GMP

- Need to show that

$$p_1', \ldots, p_n', (p_1 \wedge \ldots \wedge p_n \Rightarrow q) \models q\theta$$

  provided that $p_i'\theta = p_i\theta$ for all i

- Lemma: For any sentence p, we have $p \models p\theta$ by UI
  - $(p_1 \wedge \ldots \wedge p_n \Rightarrow q) \models (p_1 \wedge \ldots \wedge p_n \Rightarrow q)\theta = (p_1\theta \wedge \ldots \wedge p_n\theta \Rightarrow q\theta)$
  - $p_1', \ldots, p_n' \models p_1' \wedge \ldots \wedge p_n' \models p_1'\theta \wedge \ldots \wedge p_n'\theta$
  - From 1 and 2, $q\theta$ follows by ordinary Modus Ponens

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Example Knowledge Base

- The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.
- Prove that Col. West is a criminal

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Example Knowledge Base

… it is a crime for an American to sell weapons to hostile nations:

American(x) ∧ Weapon(y) ∧ Sells(x,y,z) ∧ Hostile(z) ⇒ Criminal(x)

Nono … has some missiles, i.e., ∃x Owns(Nono,x) ∧ Missile(x):

Owns(Nono,M_1) and Missile(M_1)

… all of its missiles were sold to it by Colonel West

Missile(x) ∧ Owns(Nono,x) ⇒ Sells(West,x,Nono)

Missiles are weapons:

Missile(x) ⇒ Weapon(x)

An enemy of America counts as "hostile“:

Enemy(x,America) ⇒ Hostile(x)

West, who is American …

American(West)

The country Nono, an enemy of America …

Enemy(Nono,America)

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Forward Chaining Algorithm

function FOL-FC-ASK(KB, α) **returns** a substitution or *false*

repeat until *new* is empty
- *new* ← { }
- **for each** sentence r **in** KB **do**
  - $(p_1 \wedge \ldots \wedge p_n \Rightarrow q) \leftarrow$ STANDARDIZE-APART(r)
  - **for each** θ such that $(p_1 \wedge \ldots \wedge p_n)\theta = (p'_1 \wedge \ldots \wedge p'_n)\theta$ for some $p'_1, \ldots, p'_n$ in KB
    - $q' \leftarrow$ SUBST(θ, q)
    - **if** q' is not a renaming of a sentence already in KB or *new* **then do**
      - add q' to *new*
      - $\phi \leftarrow$ UNIFY(q', α)
      - **if** ϕ is not *fail* **then return** ϕ
- add *new* to KB

return *false*

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Forward Chaining Proof

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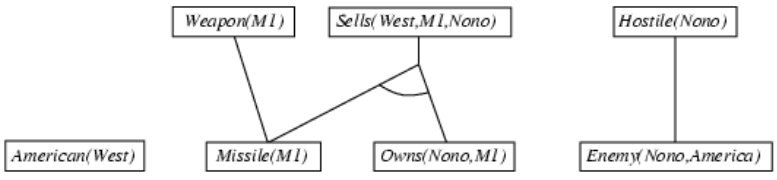
Forward Chaining Proof



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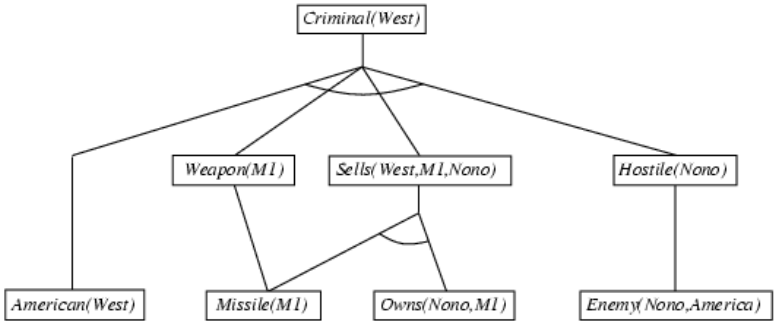
Forward Chaining Proof



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Properties of Forward Chaining

- Sound and complete for first-order definite clauses
- Datalog = first-order definite clauses + no functions
- FC terminates for Datalog in finite number of iterations
- May not terminate in general if α is not entailed
- This is unavoidable: entailment with definite clauses is semidecidable

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Efficiency of Forward Chaining

Incremental forward chaining:
no need to match a rule on iteration *k*
if a premise wasn't added on iteration *k-1*

⇒ match each rule whose premise contains a newly added positive literal

Matching itself can be expensive:

Database indexing allows O(1) retrieval of known facts

- e.g., query *Missile(x)* retrieves *Missile(M_1)*

Forward chaining is widely used in deductive databases

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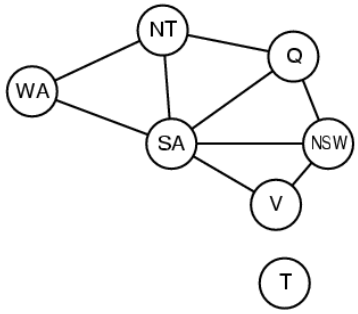
Hard Matching Example



Diff(wa,nt) ∧ Diff(wa,sa) ∧ Diff(nt,q)
∧ Diff(nt,sa) ∧ Diff(q,nsw) ∧ Diff(q,sa)
∧ Diff(nsw,v) ∧ Diff(nsw,sa) ∧ Diff(v,sa)
⇒ Colorable()

Diff(Red,Blue)
Diff(Red,Green)
Diff(Green,Red)
Diff(Green,Blue)
Diff(Blue,Red)
Diff(Blue,Green)

- *Colorable*() is inferred iff the CSP has a solution
- CSPs include 3SAT as a special case, hence matching is NP-hard

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Backward Chaining Algorithm

function FOL-BC-ASK(KB, goals, θ) **returns** a set of substitutions
  inputs: KB, a knowledge base
    $goals$, a list of conjuncts forming a query
    θ, the current substitution, initially the empty substitution $\{\}$
  local variables: ans, a set of substitutions, initially empty

  if $goals$ is empty **then return** $\{\theta\}$
  $q' \leftarrow$ SUBST(θ, FIRST($goals$))
  for each r **in** KB where STANDARDIZE-APART(r) = $(p_1 \wedge \ldots \wedge p_n \Rightarrow q)$
    and $\theta' \leftarrow$ UNIFY(q, q') succeeds
    $ans \leftarrow$ FOL-BC-ASK(KB, $[p_1, \ldots, p_n | \text{REST}(goals)]$, COMPOSE($\theta, \theta'$)) $\cup$ ans
  return ans

SUBST(COMPOSE(θ_1, θ_2), p) = SUBST(θ_2, SUBST(θ_1, p))

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Backward Chaining Example

Criminal(West)

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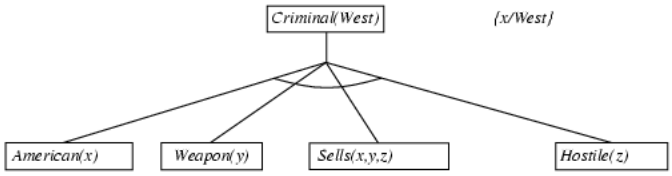
Backward Chaining Example



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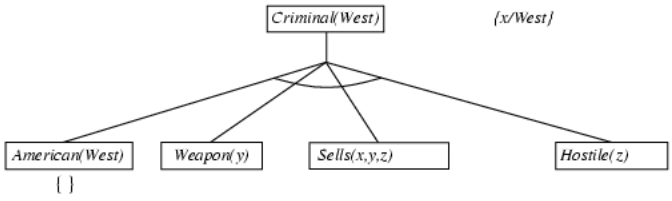
Backward Chaining Example



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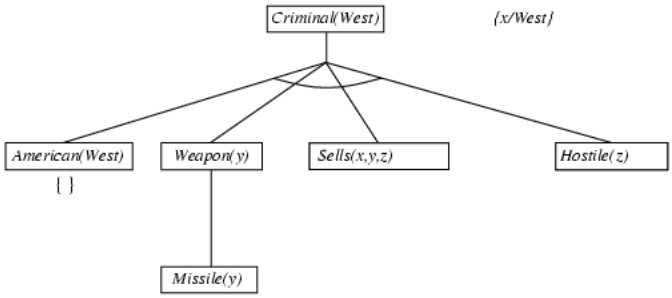
Backward Chaining Example



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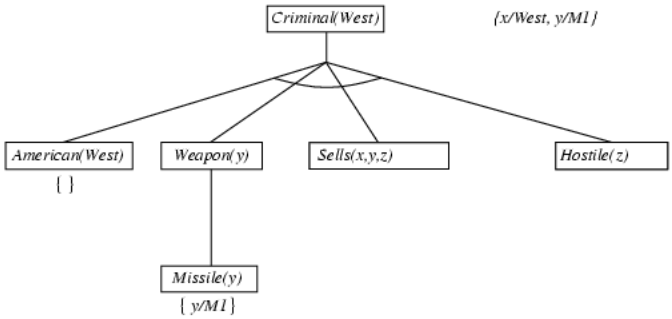
Backward Chaining Example



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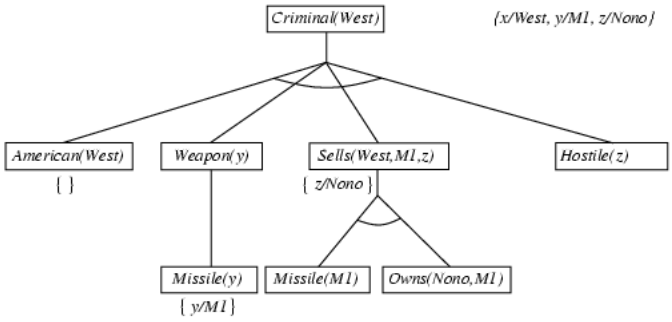
Backward Chaining Example



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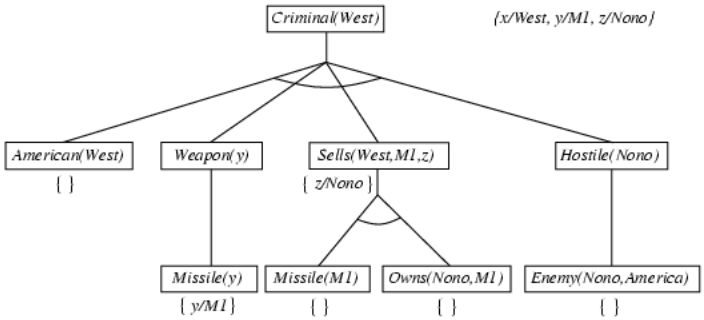
Backward Chaining Example



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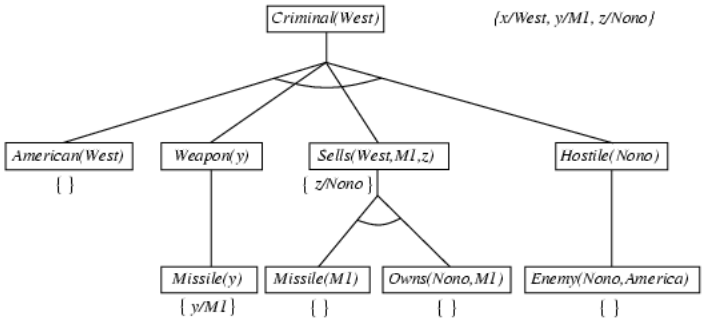
Backward Chaining Example



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Properties of Backward Chaining

- Depth-first recursive proof search: space is linear in size of proof
- Incomplete due to infinite loops
  - ⇒ fix by checking current goal against every goal on stack
- Inefficient due to repeated subgoals (both success and failure)
  - ⇒ fix using caching of previous results (extra space)
- Widely used for logic programming

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Logic Programming: Prolog

- Algorithm = Logic + Control
- Basis: backward chaining with Horn clauses + bells & whistles

  Widely used in Europe, Japan (basis of 5th Generation project)

  Compilation techniques ⇒ 60 million LIPS
- Program = set of clauses = `head :- literal`$_1$`, … literal`$_n$.

```
criminal(X) :- american(X), weapon(Y), sells(X,Y,Z), hostile(Z).
```

- Depth-first, left-to-right backward chaining
- Built-in predicates for arithmetic etc., e.g., `X is Y*Z+3`
- Built-in predicates that have side effects (e.g., input and output
- predicates, assert/retract predicates)
- Closed-world assumption ("negation as failure")
  - e.g., given `alive(X) :- not dead(X).`
  - `alive(joe)` succeeds if `dead(joe)` fails

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Prolog

- Appending two lists to produce a third:

```
append([],Y,Y).

append([X|L],Y,[X|Z]) :- append(L,Y,Z).
```

- Query: `append(A,B,[1,2]) ?`
- Answers:

```
A=[]     B=[1,2]
A=[1]    B=[2]
A=[1,2]  B=[]
```


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Resolution: Brief Summary

- Full first-order version:

$$\frac{\ell_1 \vee \cdots \vee \ell_k, \qquad m_1 \vee \cdots \vee m_n}{(\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n)\theta}$$

  where $\texttt{Unify}(\ell_i, \neg m_j) = \theta$.
- The two clauses are assumed to be standardized apart so that they share no variables.
- For example,

$$\frac{\neg Rich(x) \vee Unhappy(x), \quad Rich(Ken)}{Unhappy(Ken)}$$

  with θ = {x/Ken}
- Apply resolution steps to CNF(KB ∧ ¬α); complete for FOL

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Conversion to CNF

- Everyone who loves all animals is loved by someone:

  $\forall x\, [\forall y\, Animal(y) \Rightarrow Loves(x,y)] \Rightarrow [\exists y\, Loves(y,x)]$

1. Eliminate biconditionals and implications

   $\forall x\, [\neg \forall y\, \neg Animal(y) \vee Loves(x,y)] \vee [\exists y\, Loves(y,x)]$

2. Move ¬ inwards: $\neg \forall x\, p \equiv \exists x\, \neg p$, $\neg \exists x\, p \equiv \forall x\, \neg p$

   $\forall x\, [\exists y\, \neg(\neg Animal(y) \vee Loves(x,y))] \vee [\exists y\, Loves(y,x)]$

   $\forall x\, [\exists y\, \neg\neg Animal(y) \wedge \neg Loves(x,y)] \vee [\exists y\, Loves(y,x)]$

   $\forall x\, [\exists y\, Animal(y) \wedge \neg Loves(x,y)] \vee [\exists y\, Loves(y,x)]$

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Conversion to CNF

3. Standardize variables: each quantifier should use a different one

   $\forall x\,[\exists y\, Animal(y) \wedge \neg Loves(x,y)] \vee [\exists z\, Loves(z,x)]$

4. Skolemize: a more general form of existential instantiation.

   Each existential variable is replaced by a Skolem function of the enclosing universally quantified variables:
   $\forall x\,[Animal(F(x)) \wedge \neg Loves(x,F(x))] \vee Loves(G(x),x)$

5. Drop universal quantifiers:

   $[Animal(F(x)) \wedge \neg Loves(x,F(x))] \vee Loves(G(x),x)$

6. Distribute $\vee$ over $\wedge$:

   $[Animal(F(x)) \vee Loves(G(x),x)] \wedge [\neg Loves(x,F(x)) \vee Loves(G(x),x)]$

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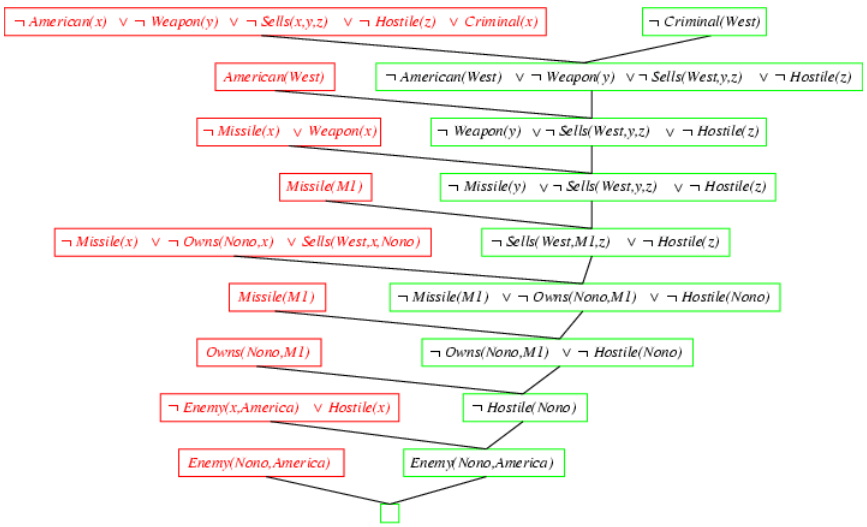
Resolution Proof: Definite Clauses



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