# Planning and Scheduling: Search-Based Agents and Brute-Force Search

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## Acknowledgements


- These slides refer to Chapter 3 of the textbook:
  S. Russell and P. Norvig:
  Artificial Intelligence: A Modern Approach
  Prentice Hall, 2003, 2nd Edition (or more recent edition)
- These slides are an adaptation of slides by Min-Yen Kan
- The contributions of these authors are gratefully acknowledged.

## Outline

- Problem-solving agents
- Problem types
- Problem formulation
- Example problems
- Basic search algorithms




## Problem-Solving Agents

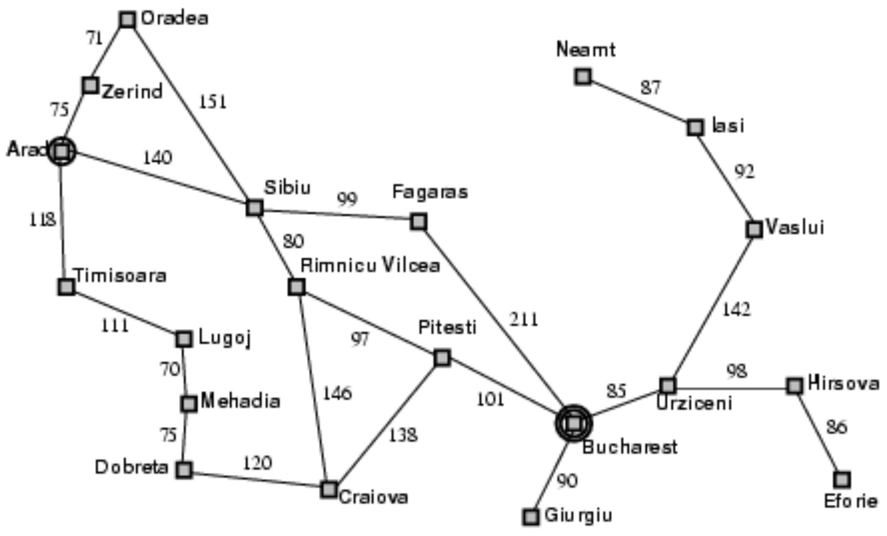
```
function SIMPLE-PROBLEM-SOLVING-AGENT(percept) returns an action
   static: seq, an action sequence, initially empty
           state, some description of the current world state
           goal, a goal, initially null
           problem, a problem formulation

   state ← UPDATE-STATE(state, percept)
   if seq is empty then do
      goal ← FORMULATE-GOAL(state)
      problem ← FORMULATE-PROBLEM(state, goal)
      seq ← SEARCH(problem)
   action ← FIRST(seq)
   seq ← REST(seq)
   return action
```

# Example: Romania

- On holiday in Romania; currently in Arad.
- Flight leaves tomorrow from Bucharest
- Formulate goal:
  - be in Bucharest
- Formulate problem:
  - states: various cities
  - actions: drive between cities
- Find solution:
  - sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest




# Example: Romania



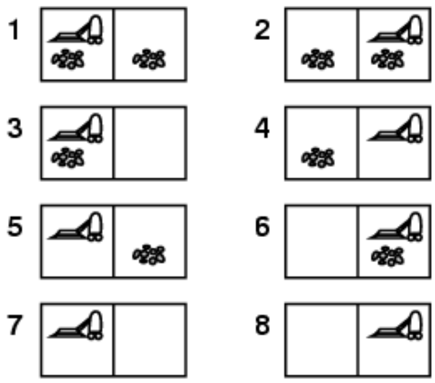
## Problem Types

- Deterministic, fully observable
  - → single-state problem
  - Agent knows exactly which state it will be in; solution is a sequence
- Non-observable
  - → sensorless problem (conformant problem)
  - Agent may have no idea where it is; solution is a sequence
- Nondeterministic and/or partially observable
  - → contingency problem
  - percepts provide new information about current state
  - often interleaved search and execution
- Unknown state space
  - → exploration problem




## Example: Vacuum World

- Single-state, start in #5.
- Solution?

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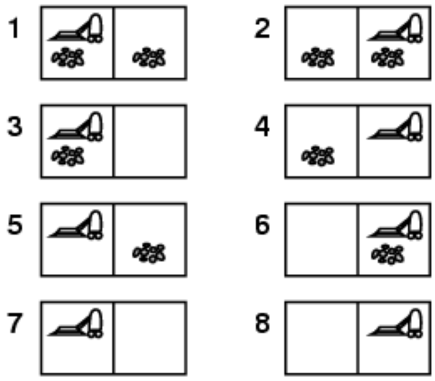
## Example: Vacuum World

- Single-state, start in #5.
- Solution? [Right, Suck]

- Sensorless,
  start in {1,2,3,4,5,6,7,8}
  e.g., right goes to {2,4,6,8}
- Solution?





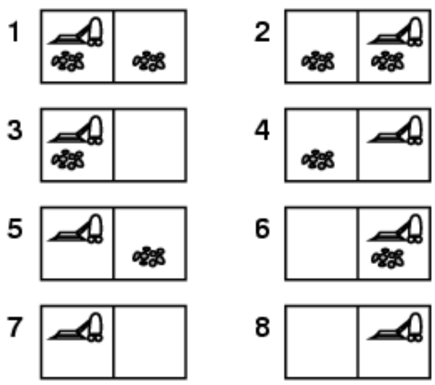


## Example: Vacuum World

- Sensorless, start in {1,2,3,4,5,6,7,8}
  e.g., right goes to {2,4,6,8}
- Solution? [Right,Suck,Left,Suck]

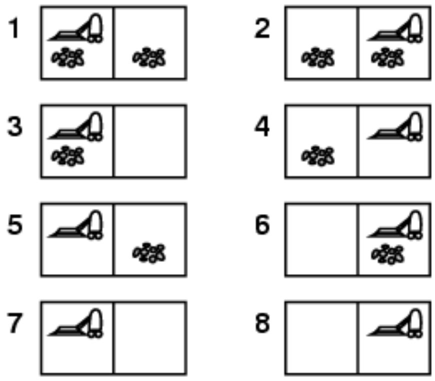
- Contingency
  - Nondeterministic:
    Suck may dirty a clean carpet
  - Partially observable:
    location, dirt at current location.
  - Percept:
    [L, Clean], i.e., start in #5 or #7

  - Solution?



## Example: Vacuum World

- Sensorless, start in {1,2,3,4,5,6,7,8}
  e.g., right goes to {2,4,6,8}
- Solution? [Right,Suck,Left,Suck]

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  - Nondeterministic:
    Suck may dirty a clean carpet
  - Partially observable:
    location, dirt at current location.
  - Percept:
    [L, Clean], i.e., start in #5 or #7
  - Solution? [Right, if dirt then Suck]



## Single-State Problem Formulation

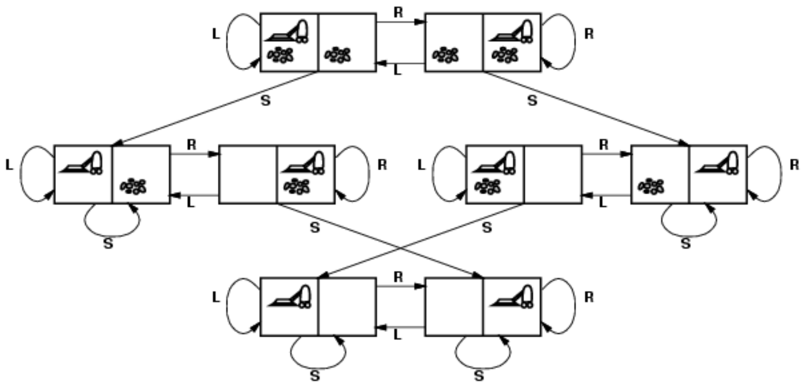
- A problem is defined by four items:
  - initial state e.g., "at Arad"
  - actions or successor function S(x) = set of action–state pairs
    - e.g., S(Arad) = {<Arad → Zerind, Zerind>, … }
  - goal test, can be
    - explicit, e.g., x = "at Bucharest"
    - implicit, e.g., Checkmate(x)
  - path cost (additive)
    - e.g., sum of distances, number of actions executed, etc.
    - c(x,a,y) is the step cost, assumed to be ≥ 0
- A solution is a sequence of actions leading from the initial state to a goal state

## Selecting a State Space

- Real world is absurdly complex
  - → state space must be **abstracted** for problem solving
- (Abstract) state = set of real states
- (Abstract) action = complex combination of real actions
  - e.g., "Arad → Zerind" represents a complex set of possible routes, detours, rest stops, etc.
- For guaranteed realizability, **any** real state "in Arad" must get to **some** real state "in Zerind"
- (Abstract) solution =
  - set of real paths that are solutions in the real world
- Each abstract action should be "easier" than the original problem

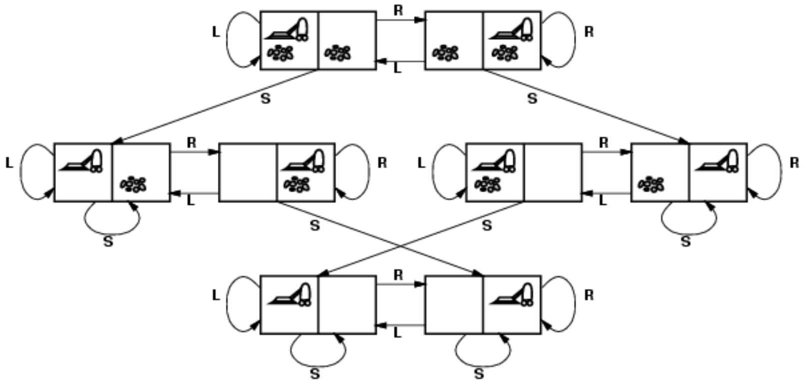



## Vacuum World State Space Graph




- states?
- actions?
- goal test?
- path cost?

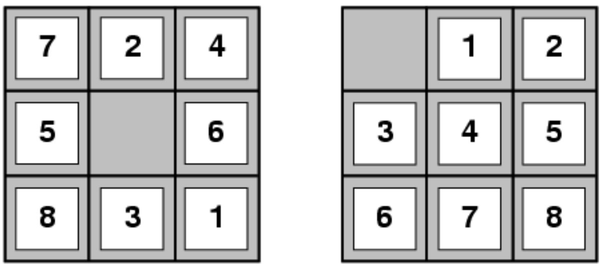
## Vacuum World State Space Graph




- states? integer dirt and robot location
- actions? Left, Right, Suck
- goal test? no dirt at all locations
- path cost? 1 per action



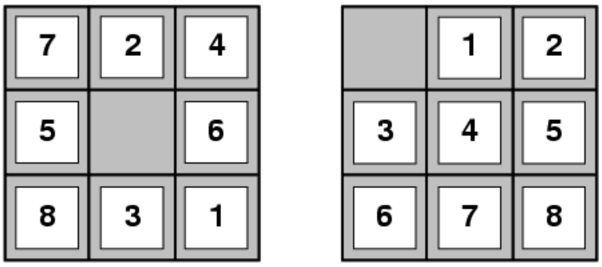

## Example: The 8-Puzzle




Start State Goal State

- states?
- actions?
- goal test?
- path cost?

## Example: The 8-Puzzle

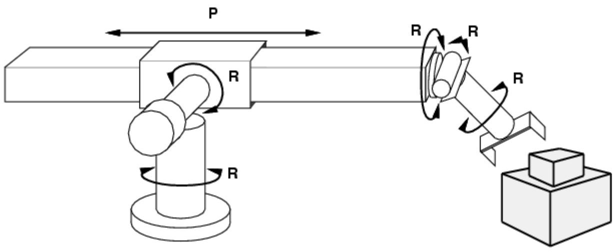



Start State Goal State

- states? locations of tiles
- actions? move blank left, right, up, down
- goal test? = goal state (given)
- path cost? 1 per move
- [Note: optimal solution of n-Puzzle family is **NP-hard**]




## Example: Robotic Assembly




- states? real-valued coordinates of robot joint angles, parts of the object to be assembled
- actions? continuous motions of robot joints
- goal test? complete assembly
- path cost? time to execute

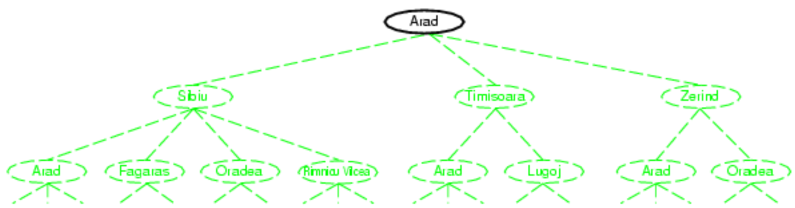
# Tree Search Algorithms

- Basic idea:
  - offline, simulated exploration of state space
    by generating successors of already-explored states
    (a.k.a. **expanding** states)

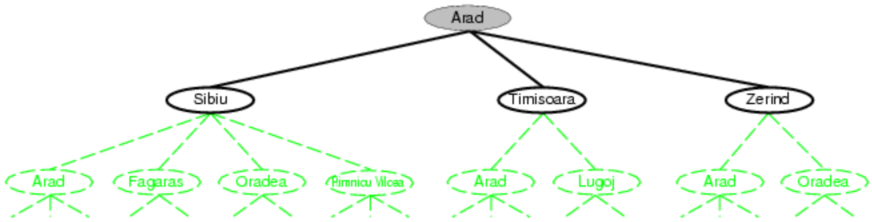
```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
   initialize the search tree using the initial state of problem
   loop do
      if there are no candidates for expansion then return failure
      choose a leaf node for expansion according to strategy
      if the node contains a goal state then return the corresponding solution
      else expand the node and add the resulting nodes to the search tree
```




# Tree Search Example



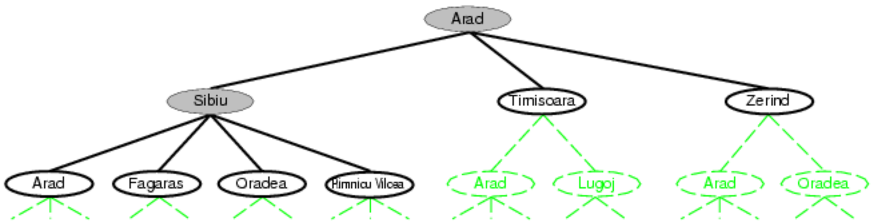
## Tree Search Example







## Tree Search Example

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## Implementation: General Tree Search

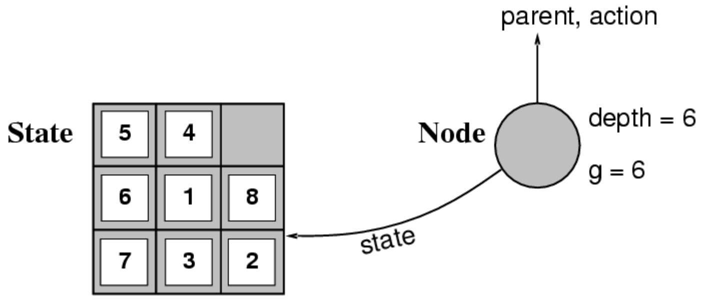
```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
  loop do
     if fringe is empty then return failure
     node ← REMOVE-FRONT(fringe)
     if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
     fringe ← INSERTALL(EXPAND(node, problem), fringe)

function EXPAND( node, problem) returns a set of nodes
  successors ← the empty set
  for each action, result in SUCCESSOR-FN[problem](STATE[node]) do
     s ← a new NODE
     PARENT-NODE[s] ← node;  ACTION[s] ← action;  STATE[s] ← result
     PATH-COST[s] ← PATH-COST[node] + STEP-COST(node, action, s)
     DEPTH[s] ← DEPTH[node] + 1
     add s to successors
  return successors
```



## Implementation: States vs. Nodes

- A state is a (representation of) a physical configuration
- A node is a data structure constituting part of a search tree
  It includes state, parent node, action, path cost g(x), depth




- The **Expand** function creates new nodes, filling in the various fields and using the **SuccessorFn** of the problem to create the corresponding states.

## Search Strategies

- A search strategy is defined by picking the **order of node expansion**
- Different search strategies result from different ways of handling `fringe`
- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?
- Time and space complexity are measured in terms of
  - $b$: maximum branching factor of the search tree
  - $d$: depth of the least-cost solution
  - $m$: maximum depth of the state space (may be $\infty$)

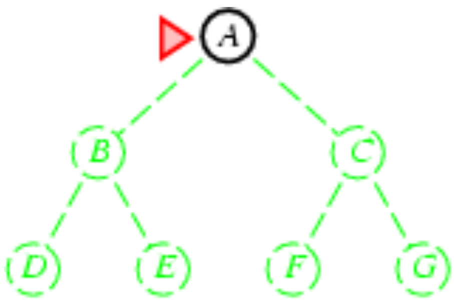



## Uninformed Search Strategies

- **Uninformed** search strategies use only the information available in the problem definition
  - Breadth-first search
  - Uniform-cost search
  - Depth-first search
  - Depth-limited search
  - Iterative deepening search

## Breadth-First Search

- Principle:
  - Expand shallowest unexpanded node
- Implementation:
  - **`fringe`** is a FIFO queue, i.e., new successors go at end







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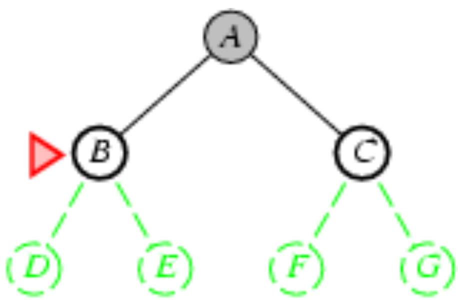

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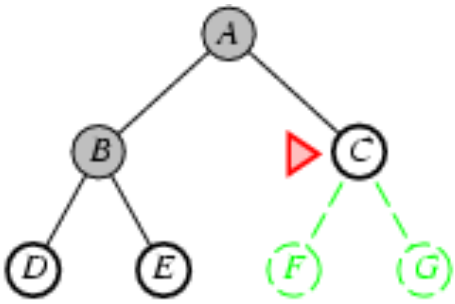





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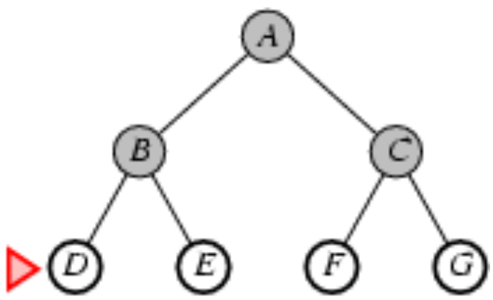

## Properties of Breadth-First Search

- Complete? Yes (if b is finite)
- Time? $1+b+b^2+b^3+\ldots +b^d + b(b^{d+1}-b) = O(b^{d+1})$
- Space? $O(b^{d+1})$ (keeps every node in memory)
- Optimal? Yes (if cost = 1 per step)
- **Space** is the bigger problem (more than time)




## Uniform-Cost Search

- Principle:
  - Expand least-cost unexpanded node
- Implementation:
  - **`fringe`** = queue ordered by path cost
- Equivalent to breadth-first if step costs all equal
- Complete? Yes, if step cost ≥ ε
- Time? # of nodes with g ≤ cost of optimal solution, $O(b^{ceiling(C^*/\varepsilon)})$ where C* is the cost of the optimal solution
- Space? # of nodes with g ≤ cost of optimal solution, $O(b^{ceiling(C^*/\varepsilon)})$
- Optimal? Yes – nodes expanded in increasing order of g(n)

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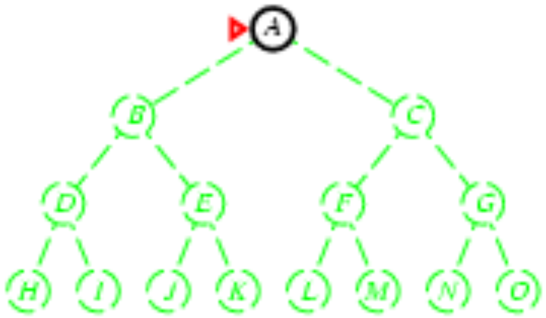
## Depth-First Search

- Principle:
  - Expand deepest unexpanded node
- Implementation:
  - **`fringe`** = LIFO queue, i.e. put successors at front







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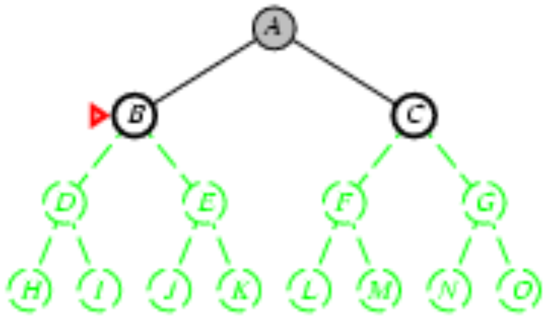

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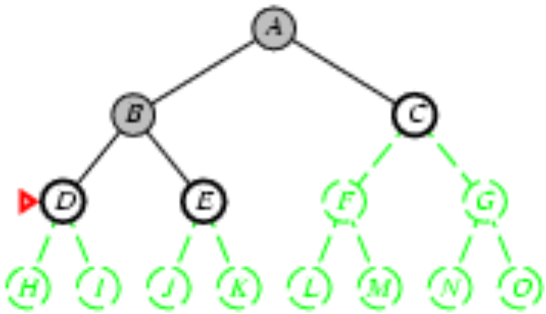





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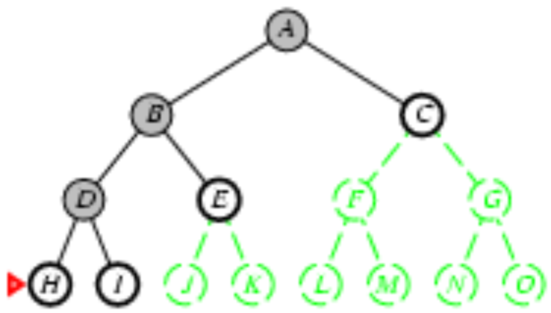

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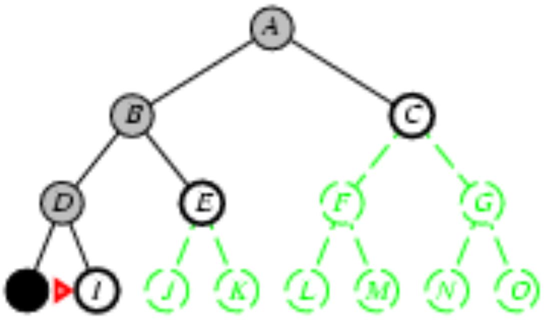





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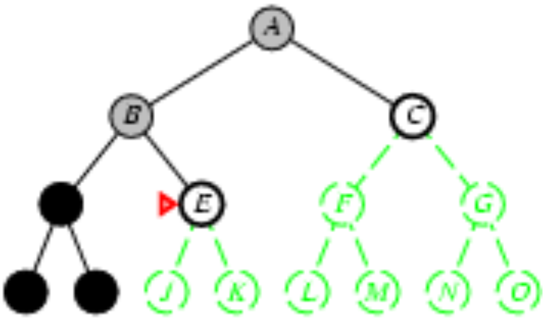

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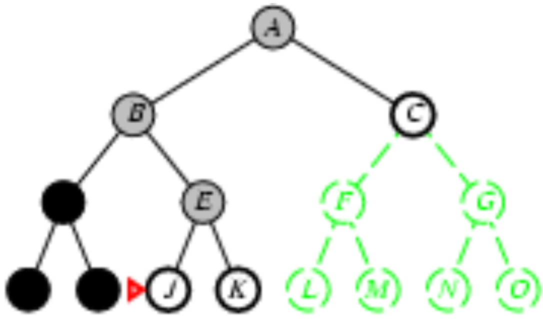





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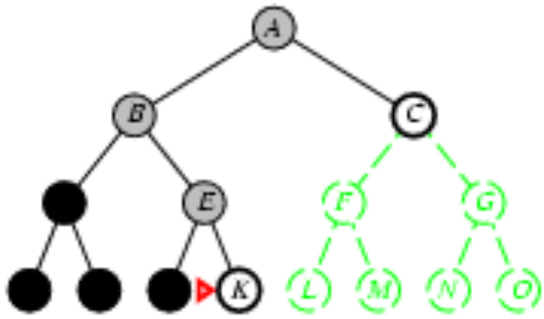

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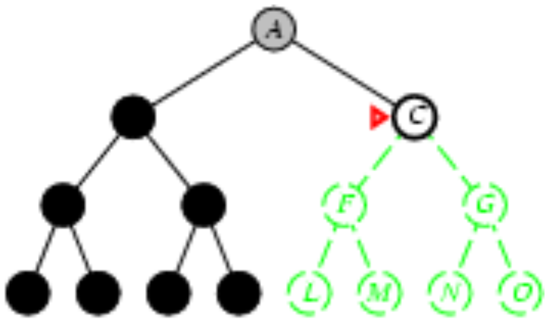





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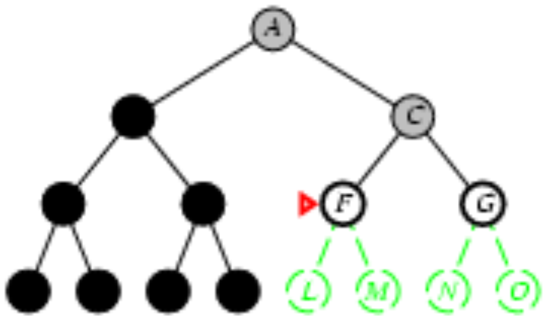

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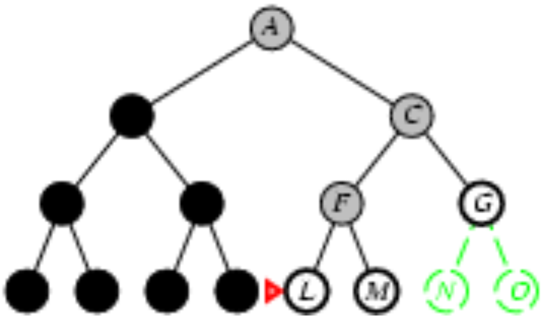






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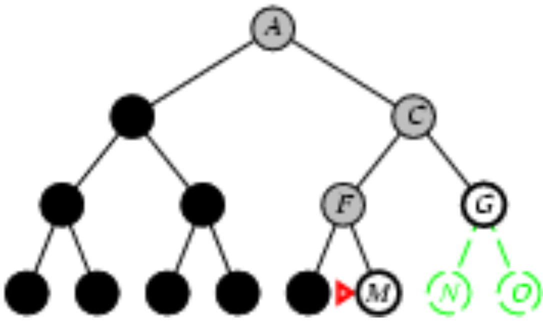


# Properties of Depth-First Search

- Complete? No: fails in infinite-depth spaces, spaces with loops
  - Modify to avoid repeated states along path
  - → complete in finite spaces
- Time? $O(b^m)$: terrible if m is much larger than d
  - but if solutions are dense, may be much faster than breadth-first
- Space? O(bm), i.e., linear space!
- Optimal? No




# Depth-Limited Search

- Principle:
  - depth-first search with depth limit l, i.e., nodes at depth l have no successors
- Recursive implementation:

```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
   RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
   cutoff-occurred? ← false
   if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
   else if DEPTH[node] = limit then return cutoff
   else for each successor in EXPAND(node, problem) do
       result ← RECURSIVE-DLS(successor, problem, limit)
       if result = cutoff then cutoff-occurred? ← true
       else if result ≠ failure then return result
   if cutoff-occurred? then return cutoff else return failure
```

## Iterative Deepening Search

- Principle:
  - Perform depth-limited search with increasing limit l
- Implementation:

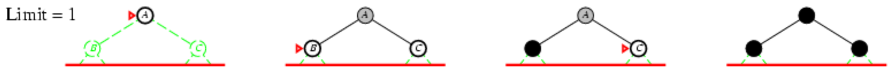
```
function ITERATIVE-DEEPENING-SEARCH(problem) returns a solution, or failure
    inputs: problem, a problem

    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH(problem, depth)
        if result ≠ cutoff then return result
```



## Iterative Deepening Search l =0

Limit = 0

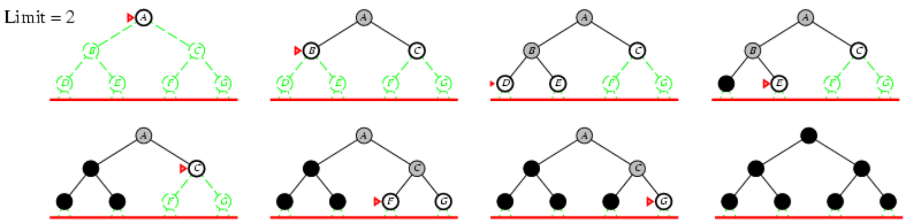
# Iterative Deepening Search l =1



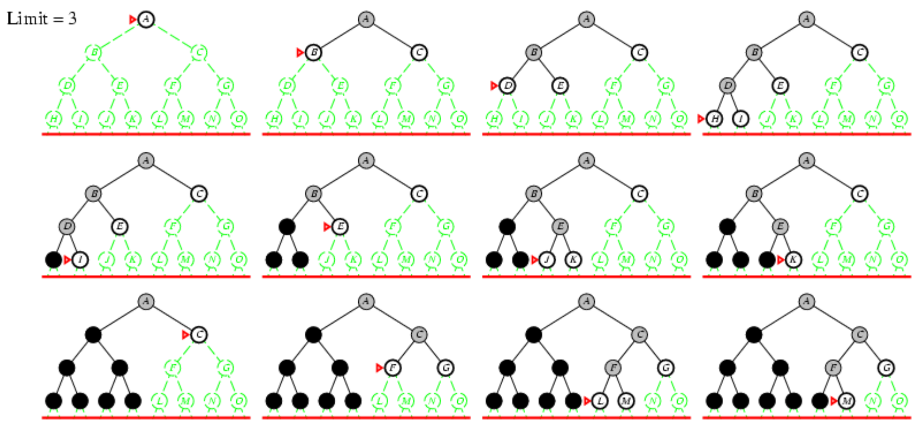




# Iterative Deepening Search l =2



## Iterative Deepening Search l =3






## Iterative Deepening Search

- Number of nodes generated in a depth-limited search to depth d with branching factor b:

$$N_{DLS} = b^0 + b^1 + b^2 + \ldots + b^{d-2} + b^{d-1} + b^d$$

- Number of nodes generated in an iterative deepening search to depth d with branching factor b:

$$N_{IDS} = (d+1)b^0 + d\, b^1 + (d-1)b^2 + \ldots + 3b^{d-2} + 2b^{d-1} + 1b^d$$

- Example: for b = 10, d = 5,

$$N_{DLS} = 1 + 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 = 111{,}111$$

$$N_{IDS} = 6 + 50 + 400 + 3{,}000 + 20{,}000 + 100{,}000 = 123{,}456$$

- Overhead = (123,456 - 111,111)/111,111 = 11%

## Properties of Iterative Deepening Search

- Complete? Yes
- Time? $(d+1)b^0 + d\, b^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
- Space? $O(bd)$
- Optimal? Yes, if step cost = 1

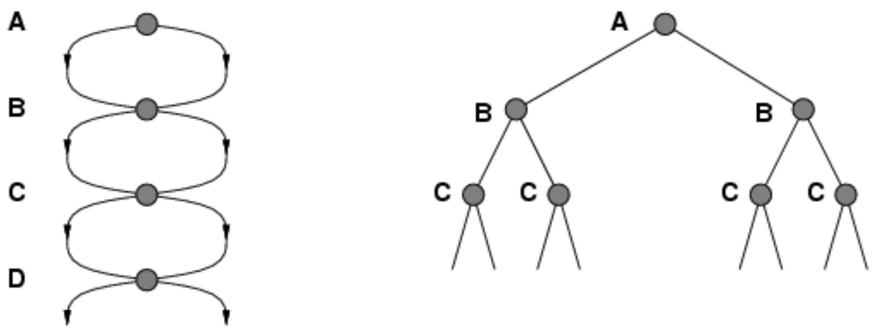



## Summary of Algorithms

| Criterion | Breadth-First | Uniform-Cost | Depth-First | Depth-Limited | Iterative Deepening |
|---|---|---|---|---|---|
| Complete? | Yes | Yes | No | No | Yes |
| Time | $O(b^{d+1})$ | $O(b^{\lceil C^*/\epsilon \rceil})$ | $O(b^m)$ | $O(b^l)$ | $O(b^d)$ |
| Space | $O(b^{d+1})$ | $O(b^{\lceil C^*/\epsilon \rceil})$ | $O(bm)$ | $O(bl)$ | $O(bd)$ |
| Optimal? | Yes | Yes | No | No | Yes |

## Repeated States

- Failure to detect repeated states can turn a linear problem into an exponential one!







## Graph Search

**function** GRAPH-SEARCH( *problem*, *fringe*) **returns** a solution, or failure
  *closed* ← an empty set
  *fringe* ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)
  **loop do**
    **if** *fringe* is empty **then return** failure
    *node* ← REMOVE-FRONT(*fringe*)
    **if** GOAL-TEST[*problem*](STATE[*node*]) **then return** SOLUTION(*node*)
    **if** STATE[*node*] is not in *closed* **then**
      add STATE[*node*] to *closed*
      *fringe* ← INSERTALL(EXPAND(*node*, *problem*), *fringe*)

## Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored
- Variety of uninformed search strategies
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms