Probabilistic Reasoning Assignment IV

Alexander Moriarty *BRS University of Applied Sciences*
email: alexander@dal.ca github: @moriarty

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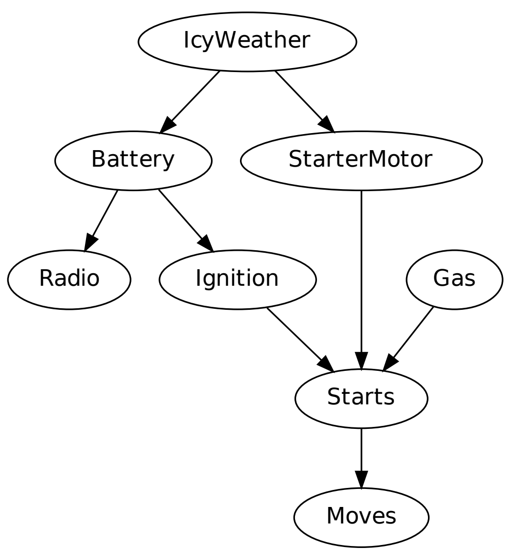
1 Car Diagnosis

Consider the network for car diagnosis shown in Figure 14.18.

1.a

Extend the network with the Boolean variables *IcyWeather* and *StarterMotor*.

Figure 1: *Car Diagnosis Network Extended*



1.b

Give reasonable conditional probability tables for all the nodes.

(a) *Icy Weather (IW)*

P(IcyWeather)
0.25

(b) *Battery (B)*

IW	P(Battery)
t	0.90
f	0.99

(c) *Starter Motor (SM)*

IW	P(StarterMotor)
t	0.90
f	0.99

(d) *Radio (R)*

B	P(Radio)
t	0.99
f	0.01

(e) *Ignition (I)*

B	P(Ignition)
t	0.99
f	0.01

(f) *Gas (G)*

P(Gas)
0.95

(g) *Starts (S)*

I	SM	G	P(Starts)
t	t	t	0.99
t	t	f	0.00
t	f	t	0.00
t	f	f	0.00
f	t	t	0.00
f	t	f	0.00
f	f	t	0.00
f	f	f	0.00

(h) *Moves (M)*

S	P(Moves)
t	0.99
f	0.01

Table 1: *Conditional probability tables for car diagnosis network*

1.c

Many independent values are contained in the joint probability distribution for eight Boolean nodes, assuming that no conditional independence relations are known to hold among them?

There are 2^8 possible values for the 8 boolean variables, but we know they will all sum up to 1 so we get one value for free.

$$2^8 - 1 \quad = \quad 255 \tag{1}$$

1.d

How many independent probability values do your network tables contain?

From Table: 1		Values
a	2^0	1
b	2^1	2
c	2^1	2
d	2^1	2
e	2^1	2
f	2^0	1
g	2^3	8
h	2^1	2
Total	$2 \cdot 2^0 + 5 \cdot 2^1 + 2^3$	20

Table 2: *Number of independent probability values*

1.e

The conditional distribution for *Starts* could be described as a *noisy-AND* distribution. Define this family in general and relate it to the *noisy-OR* distribution.

Because all of the variables must be true for the motor to start we can think of it as:

$$Starts \quad \approx \quad Ignition \wedge StarterMotor \wedge Gas \tag{1}$$

Using De Morgan's we know this could equivalently use OR:

$$Starts \quad \approx \quad \neg(\neg Ignition \vee \neg StarterMotor \vee \neg Gas) \tag{2}$$

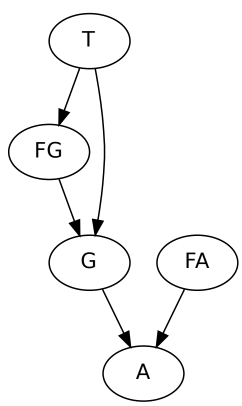
It's noisy because there is still the possibility that all variables are true/false and the motor doesn't start.

2 Nuclear Power Station

In your local nuclear power station, there is an alarm that senses when a temperature gauge exceeds a given threshold. The gauge measures the temperature of the core. Consider the Boolean variables $A(alarm\ sounds)$, $FA(alarm\ is\ faulty)$, and $FG(gauge\ is\ faulty)$ and the multi-valued nodes $G(gauge\ reading)$ and $T(actual\ core\ temperature)$

Figure 2: .

(a) *Bayesian Network of Power Domain*



(b) *Question 2.c*

	$T = Normal$		$T = High$	
	FG	$\neg FG$	FG	$\neg FG$
$G = Normal$	$1 - y$	$1 - x$	y	x
$G = High$	y	x	$1 - y$	$1 - x$

(c) *Question 2.d*

	$T = Normal$		$T = High$	
	FG	$\neg FG$	FG	$\neg FG$
A	0	0	0	1
$\neg A$	1	1	1	0

2.a

Draw a Bayesian network for this domain, given that the gauge is more likely to fail when the core tempurature gets too high.
Please see Figure 2a

2.b

Is your network a polytree?
No, there are two paths from T to G.

2.c

Suppose there are just two possible actual and measured temperatures, normal and high; the probability that the gauge gives the correct temperature is x when it is working, but y when it is faulty.

Give the conditional probability table associated with G
Please see Figure 2b

2.d

Suppose the alarm works correctly unless it is faulty, in which case it never sounds. Give the conditional probability table associated with A.
Please see Figure 2c

2.e

Suppose the alarm and gauge are working and the alarm sounds. Calculate an expression for the probability that the temperature of the core is too high, in terms of the various conditional probabilities in the network.
We know FA and A do not influence T:

$$\begin{aligned}
P(T \mid A, \neg FA, \neg FG) &= P(T \mid G, \neg FG) && (1)\\
P(T \mid G, \neg FG) &= P(G \mid T, \neg FG) P(T \mid \neg FG) && (2)\\
P(T \mid G, \neg FG) &= P(G \mid T, \neg FG) P(\neg FG \mid T) P(T) && (3)\\
P(T \mid G, \neg FG) &= \frac{P(G \mid T, \neg FG) P(\neg FG \mid T) P(T)}{P(G \mid T, \neg FG) P(\neg FG \mid T) P(T) + P(G \mid \neg T, \neg FG) P(\neg FG \mid \neg T) P(\neg T)} && (4)
\end{aligned}$$