宿題 1

$B_\tau(y)$ を再帰的に表現する。

$$B_\tau(y) = \sum_{y^{(\tau+1)},\dots,y^{(\mathrm{m})}=1}^{c} \exp\left(\sum_{k=\tau+2}^{m} \boldsymbol{\zeta}^{\mathrm{T}} \boldsymbol{\varphi}_i^{(k)} \left(y^{(k)},\, y^{(k-1)} \right) + \boldsymbol{\zeta}^{\mathrm{T}} \boldsymbol{\varphi}_i^{(\tau+1)} \left(y^{(\tau+1)},\, y \right) \right) \tag{1}$$

$$= \sum_{y^(\tau+1)=1}^{c} \cdot \sum_{y^{(\tau+2)},\dots,y^{(\mathrm{m})}=1}^{c} \exp\left\{ \sum_{k=\tau+2}^{m} \boldsymbol{\zeta}^{\mathrm{T}} \boldsymbol{\varphi}_i^{(k)} \left(y^{(k)},\, y^{(k-1)} \right) \right\} \exp\left(\boldsymbol{\zeta}^{\mathrm{T}} \boldsymbol{\varphi}_i^{(\tau+1)} \left(y^{(\tau+1)},\, y \right) \right) \tag{2}$$

ここで，

$$\sum_{y^{(\tau+2)},\dots,y^{(\mathrm{m})}=1}^{c} \exp\left\{ \sum_{k=\tau+2}^{m} \boldsymbol{\zeta}^{\mathrm{T}} \boldsymbol{\varphi}_i^{(k)} \left(y^{(k)},\, y^{(k-1)} \right) \right\}$$

$$= \sum_{y^{(\tau+2)},\dots,y^{(\mathrm{m})}=1}^{c} \exp\left\{ \sum_{k=\tau+3}^{m} \boldsymbol{\zeta}^{\mathrm{T}} \boldsymbol{\varphi}_i^{(k)} \left(y^{(k)},\, y^{(k-1)} \right) + \boldsymbol{\zeta}^{\mathrm{T}} \boldsymbol{\varphi}_i^{(\tau+2)} \left(y^{(\tau+2)},\, y^{(\tau+1)} \right) \right\} \tag{3}$$

$$= B_{\tau+1}\left(y^{(\tau+1)} \right) \tag{4}$$

となるから，結局 $B_\tau(y)$ は，

$$B_\tau(y) = \sum_{y^(\tau+1)=1}^{c} B_{\tau+1}\left(y^{(\tau+1)} \right) \exp\left(\boldsymbol{\zeta}^{\mathrm{T}} \boldsymbol{\varphi}_i^{(\tau+1)} \left(y^{(\tau+1)},\, y \right) \right) \tag{5}$$

という形で再帰表現される。