Exercises for the course

Machine Learning 2

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Exercise Sheet 6

Exercise 1: Markov Model Forward Problem (20 P)

A Markov Model can be seen as a joint distribution over states at each time step $q_1, \dots, q_T$ where $q_t \in \{S_1, \dots, S_N\}$, and where the probability distribution has the factored structure:

$$P(q_1, \dots, q_T) = P(q_1) \cdot \prod_{t=2}^{T} P(q_t | q_{t-1})$$

Factors are the probability of the initial state and conditional distributions at every time step.

(a) *Show* that the following relation holds:

$$P(q_{t+1} = S_j) = \sum_{i=1}^{N} P(q_t = S_i) P(q_{t+1} = S_j | q_t = S_i)$$

for $t \in \{1, \dots, T-1\}$ and $j \in \{1, \dots, N\}$.

Exercise 2: Hidden Markov Model Forward Problem (20 P)

A Hidden Markov Model (HMM) can be seen as a joint distribution over hidden states $q_1, \dots, q_T$ at each time step and corresponding observation $O_1, \dots, O_T$. Like for the Markov Model, we have $q_t \in \{S_1, \dots, S_N\}$. The probability distribution of the HMM has the factored structure:

$$P(q_1, \dots, q_T, O_1, \dots, O_T) = P(q_1) \cdot \prod_{t=2}^{T} P(q_t | q_{t-1}) \cdot \prod_{t=1}^{T} P(O_t | q_t)$$

Factors are the probability of the initial state and conditional distributions at every time step.

(a) *Show* that the following relation holds:

$$P(O_1, \dots, O_t, O_{t+1}, q_{t+1} = S_j) = \sum_{i=1}^{N} P(O_1, \dots, O_t, q_t = S_i) P(q_{t+1} = S_j | q_t = S_i) P(O_{t+1} | q_{t+1} = S_j)$$

for $t \in \{1, \dots, T-1\}$ and $j \in \{1, \dots, N\}$.

Exercise 3: Programming (60 P)

Download the programming files on ISIS and follow the instructions.

Exercise Sheet 6

1 Markov Model Forward Problem

a) $$P(q_{t+1} = S_j) = P(q_1) \cdot P(q_2 | q_1) \cdot \ldots P(q_t | q_{t-1}) \cdot P(q_{t+1} = S_j | q$$

$$= \sum_{i=1}^{N} \underbrace{P(q_1) P(q_2|q_1) \cdot \ldots \cdot P(q_t = S_i | q_{t-1})}_{P(q_t = S_i)} \cdot P(q_{t+1} = S_j | q_t = S_i)$$

2 Hidden Markov Model Forward Problem

$$P(O_1, \ldots, O_t, O_{t+1}, q_{t+1} = S_j) = P(q_1) P(q_2 | q_1) \cdot \ldots$$
$$\cdot P(O_1 | q_1) P(O_2 | q_2) \cdot \ldots \cdot P(O_{t+1} | q_{t+1} = S_j)$$
$$= \sum_{i=1}^{N} P(q_1) \cdot P(q_2 | q_1) \cdot \ldots \cdot P(q_t = S_i | q_{t-1})$$
$$\cdot P(O_1 | q_1) \cdot P(O_2 | q_2) \cdot \ldots \cdot P(O_t | q_t = S_i) \cdot P(q_{t+1} = S_j | q_t = S_i)$$
$$\cdot P(O_{t+1} | q_{t+1} = S_j)$$