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SS 2016

Parallel Numerics

Exercise 5: Sparse Approximate Inverse

1 Sparse Approximate Inverses

i) What is preconditioning and why is it necessary/useful?

ii) Remember the Richardson iteration and apply preconditioning to it. What happens when you use $P = A$ or $P = I$ as preconditioning matrix?

iii) What are the main advantages of the SPAI (Sparse Approximate Inverse) algorithm?

iv) How is it possible to parallelize the computation of a static SPAI algorithm performing no pattern updates? What about the load balancing? (Give only thoughts, no pseudocode)

v) Implement a SPAI for a tridiagonal matrix resulting from the heat equation, looking like:

$$A = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & -2 \end{pmatrix}$$

Use the same pattern for M^{-1}, i.e. $M^{-1} = \mathcal{J}(A)$.

vi) Implement a preconditioned CG with SPAI for the beforementioned matrix. Compare runtimes \ iterations with and without preconditioning.