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Parallel Numerics

Exercise 6: Domain Decomposition

1 Domain Decomposition

i) Name the two main types of domain decomposition approaches.

ii) Which direct solver for the decomposed system do you know (assume that our problem is linear)?

iii) Give the Schur-complement and the right-hand-side for the following system:

$$\begin{pmatrix} A & 0 & C \\ 0 & B & D \\ C^T & D^T & E \end{pmatrix} \begin{pmatrix} u_A \\ u_B \\ u_E \end{pmatrix} = \begin{pmatrix} b_A \\ b_B \\ b_E \end{pmatrix}$$

iv) List the steps of the algorithm solving the decomposed system based on the Schur-complement and LU-decomposition.

2 Robin-Robin Coupling for Non-Overlapping DD

In the lecture, we have learned that solving the Poisson equation

$$\begin{aligned} -\Delta u &= f && \text{in} \quad \Omega, \\ u &= 0 && \text{on} \quad \partial\Omega \end{aligned}$$

on Ω is equivalent to solving the decomposed system

$$\begin{aligned} -\Delta u_1 &= f && \text{in} \quad \Omega_1, \\ u_1 &= 0 && \text{on} \quad \partial\Omega_1 \backslash \Gamma, \\ u_1 &= u_2 && \text{on} \quad \Gamma, \end{aligned}$$

$$\begin{aligned} -\Delta u_2 &= f && \text{in} \quad \Omega_2, \\ u_2 &= 0 && \text{on} \quad \partial\Omega_2 \backslash \Gamma, \\ \frac{\partial u_1}{\partial n_1} &= -\frac{\partial u_2}{\partial n_2} && \text{on} \quad \Gamma. \end{aligned}$$

with two non-overlapping subdomains Ω_1 and Ω_2. As an example, we have studied the 1D equation

$$u_{xx} = 0 \;\text{ in }\;]0;8[, \quad u(0) = 0,\, u(8) = 8, \tag{1}$$

started with the initial approximations

$$u_1(x) = 0 \;\text{ in }\; \Omega_1 =]0;4[, \quad u_2(x) = 2(x-8) \;\text{ in }\; \Omega_2 =]4;8[, \tag{2}$$

and tried to solve the problem iteratively with a block-Gauss-Seidel iteration, which did not converge.

i) Show that the solution of the modified decomposed system

$$\begin{aligned}
-\Delta u_1 &= f && \text{in } \Omega_1,\\
u_1 &= 0 && \text{on } \partial\Omega_1\backslash\Gamma,\\
\frac{\partial u_1}{\partial n_1} + \theta u_1 &= -\frac{\partial u_2}{\partial n_2} + \theta u_2 && \text{on } \Gamma,\\[1ex]
-\Delta u_2 &= f && \text{in } \Omega_2,\\
u_2 &= 0 && \text{on } \partial\Omega_2\backslash\Gamma,\\
\frac{\partial u_1}{\partial n_1} - \theta u_1 &= -\frac{\partial u_2}{\partial n_2} - \theta u_2 && \text{on } \Gamma.
\end{aligned}$$

with $\theta > 0$ has the same solution as the decomposed system given above.

ii) Perform (by hand) one block-Gauss-Seidel iteration for the modified decomposed system for problem (1) with initial approximations from (2).

iii) For which values of θ do you get convergence?