EXERCISES 8

In Problems 1-2, verify that the following probability experiments represent hypergeometric probability experiments. Then determine the values of N, n, k and list the possible values of the random variable X.

1. In Michigan's Winfall Lottery, a player must choose 6 num-bers between 1 and 49, inclusive. Six balls numbered between 1 and 49 are then randomly selected from an urn. The random variable X represents the number of matching numbers.

2. A manufacturer received an order of 250 computer chips. Unfortunately, 12 of the chips are defective. To test the shipment, the quality-control engineer randomly selects 20 chips from the box of 250 and tests them. The random vari-able X represents the number of defective chips in the sample of 20.

In Problems 3-4, a hypergeometric probability experiment is conducted with the given parameters. Compute the probability of obtaining x successes.

3. $N = 150, n = 20, k = 30, x = 5$

4. $N = 230, n = 15, k = 200, x = 12$

In Problems 5-6, compute the mean and standard deviation of the hypergeometric random variable X.

5. $N = 150, n = 20, k = 30$

6. $N = 230, n = 15, k = 200$

7. In Michigan's Classic Lotto 47 Lottery, a player must choose 6 numbers between 1 and 47, in-clusive. Six balls numbered from 1 and 47 are then randomly selected from an urn. The random variable X represents the number of matching numbers.

(a) What is the probability of matching 3 numbers?
(b) What is the probability of matching 4 numbers?
(c) What is the probability of matching 5 numbers?
(d) What is the probability of matching 6 numbers?
(e) A winning ticket is one in which the player matches 3, 4, 5, or 6 numbers. What is the probability of purchasing a winning ticket? Would it be unusual to purchase a win-ning ticket?
(f) What is the mean and standard deviation of the random variable X? For a randomly selected ticket, how many numbers do you expect to match?

8. A manufacturer received an order of 250 computer chips. Unfortunately, 12 of the chips are defec-tive. To test the shipment, the quality-control engineer ran-domly selects 20 chips from the box of 250 and tests them. The random variable X represents the number of defective chips in the sample.

(a) What is the probability of obtaining 4 defective chips?
(b) What is the probability of obtaining 3 defective chips?
(c) What is the probability that the quality-control engineer will not find any defective chips?
(d) What is the probability of obtaining 14 defective chips?
(e) How many defective chips would you expect to select?

9. A hung jury is one that is unable to come to a unanimous decision regarding the guilt of the defendant. Suppose that there is a pool of 30 potential jurors, but 2 of the 30 potential jurors would never be willing to convict, regardless of the evidence presented. What is the proba-bility that the trial will result in a hung jury, regardless of the evidence, if the jury consists of 12 randomly selected jurors?

10. Suppose that a concrete manufactur-er has made 200 concrete cylinders that are supposed to withstand 4,000 pounds per square inch of pressure. As the quality-control manager, you decide to randomly test 4 of the cylinders to be sure they are manufactured to specifica-tion. You will only accept the shipment if all 4 cylinders pass the inspection. What is the probability that the shipment is accepted:

(a) If 10% of the 200 cylinders are defective?
(b) If 20% of the 200 cylinders are defective?
(c) If 40% of the 200 cylinders are defective?
(d) If 60% of the 200 cylinders are defective?
(e) If 80% of the 200 cylinders are defective?
(f) Draw a horizontal axis and label it Percent Defective.

Draw a vertical axis and label it Probability Accept Ship-ment. Plot probability accept shipment against the percent defective and connect the points in a smooth curve. This curve is referred to as an **operating characteristic curve**.

Answers

1. $N = 49, n = 6, k = 6, X = 0, 1, 2, \ldots, 6$

2. $N = 250, n = 20, k = 12, X = 0, 1, 2, \ldots, 12$

3. 0.1856

4. 0.1939

5. $\mu_X = 4, \sigma_X = 1.67$

6. $\mu_X = 13.0, \sigma_X = 1.26$

7. **(a)** 0.01986
(b) 0.001146
(c) 0.0000229
(d) 0.000000093
(e) 0.02103
(f) 0.766; 0.772; 0.766

8. **(a)** 0.0087
(b) 0.0507
(c) 0.3590
(d) 0
(e) 0.96

9. 0.6483

10. **(a)** 0.6539
(b) 0.4065
(c) 0.1270
(d) 0.0245
(e) 0.0014
(f)