EXERCISES 6.3

In Problems 1–4, show that $z = 0$ is a removable singularity of the given function. Supply a definition of $f(0)$ so that f is analytic at $z = 0$.

1. $f(z) = \dfrac{e^{2z} - 1}{z}$

2. $f(z) = \dfrac{z^3 - 4z^2}{1 - e^{z^2/2}}$

3. $f(z) = \dfrac{\sin 4z - 4z}{z^2}$

4. $f(z) = \dfrac{1 - \frac{1}{2}z^{10} - \cos z^5}{\sin z^2}$

In Problems 5–10, determine the zeros and their order for the given function.

5. $f(z) = (z + 2 - i)^2$

6. $f(z) = z^4 - 16$

7. $f(z) = z^4 + z^2$

8. $f(z) = \sin^2 z$

9. $f(z) = e^{2z} - e^z$

10. $f(z) = ze^z - z$

In Problems 11–14, the indicated number is a zero of the given function. Use a Maclaurin or Taylor series to determine the order of the zero.

11. $f(z) = z(1 - \cos^2 z);\ z = 0$

12. $f(z) = z - \sin z;\ z = 0$

13. $f(z) = 1 - e^{z-1};\ z = 1$

14. $f(z) = 1 - \pi i + z + e^z;\ z = \pi i$

In Problems 15–26, determine the order of the poles for the given function.

15. $f(z) = \dfrac{3z - 1}{z^2 + 2z + 5}$

16. $f(z) = 5 - \dfrac{6}{z^2}$

17. $f(z) = \dfrac{1 + 4i}{(z + 2)(z + i)^4}$

18. $f(z) = \dfrac{z - 1}{(z + 1)(z^3 + 1)}$

19. $f(z) = \tan z$

20. $f(z) = \dfrac{\cot \pi z}{z^2}$

21. $f(z) = \dfrac{1 - \cosh z}{z^4}$

22. $f(z) = \dfrac{e^z}{z^2}$

23. $f(z) = \dfrac{1}{1 + e^z}$

24. $f(z) = \dfrac{e^z - 1}{z^2}$

25. $f(z) = \dfrac{\sin z}{z^2 - z}$

26. $f(z) = \dfrac{\cos z - \cos 2z}{z^6}$