EXERCISES 2.4

In Problems 1–14, find the image of the given set under the mapping $w = z^2$. Represent the mapping by drawing the set and its image.

1. the ray $\arg(z) = \dfrac{\pi}{3}$
2. the ray $\arg(z) = -\dfrac{3\pi}{4}$
3. the line $x = 3$
4. the line $y = -5$
5. the line $y = -\dfrac{1}{4}$
6. the line $x = \dfrac{3}{2}$
7. the positive imaginary axis
8. the line $y = x$
9. the circular arc $|z| = \dfrac{1}{2}$, $0 \le \arg(z) \le \pi$
10. the circular arc $|z| = \dfrac{4}{3}$, $-\dfrac{\pi}{2} \le \arg(z) \le \dfrac{\pi}{6}$
11. the triangle with vertices 0, 1, and $1 + i$
12. the triangle with vertices 0, $1 + 2i$, and $-1 + 2i$
13. the square with vertices 0, 1, $1 + i$, and i
14. the polygon with vertices 0, 1, $1 + i$, and $-1 + i$
15. Find the image of the ray $\arg(z) = \pi/6$ under each of the following mappings.

    (a) $f(z) = z^3$ **(b)** $f(z) = z^4$ **(c)** $f(z) = z^5$
16. Find the image of the first quadrant of the complex plane under each of the following mappings.

    (a) $f(z) = z^2$ **(b)** $f(z) = z^3$ **(c)** $f(z) = z^4$

In Problems 17–22, find the value of the given principal nth root function at the given value of z.

17. $z^{1/2}$, $z = -i$
18. $z^{1/2}$, $z = 2 + i$
19. $z^{1/3}$, $z = -1$
20. $z^{1/3}$, $z = -3 + 3i$
21. $z^{1/4}$, $z = -1 + \sqrt{3}i$
22. $z^{1/5}$, $z = -4\sqrt{3} + 4i$

In Problems 23–30, find the image of the given set under the principal square root mapping $w = z^{1/2}$. Represent the mapping by drawing the set and its image.

23. the ray $\arg(z) = \dfrac{\pi}{4}$
24. the ray $\arg(z) = -\dfrac{2\pi}{3}$
25. the positive imaginary axis
26. the negative real axis
27. the arc $|z| = 9$, $-\dfrac{\pi}{2} \le \arg(z) \le \pi$
28. the arc $|z| = \dfrac{4}{7}$, $-\dfrac{\pi}{2} \le \arg(z) \le \dfrac{\pi}{4}$
29. the parabola $x = \dfrac{9}{4} - \dfrac{y^2}{9}$
30. the parabola $x = \dfrac{y^2}{10} - \dfrac{5}{2}$