Contents

1 Structure

1. Definition – What is the $3x + 1$ Problem
   - G: number theoretic function
   - T: map of N+1 to N+1 (positive integers)
   - L: definition of the function and all its attributes
     - stopping time
     - Ch: explanation of stopping time
     - total stopping time counts n as the 0th number
     - total stopping time
     - trajectory of a number
2. Conjecture
   - What is the Collatz conjecture?
     - Ch: because it is an integer function, T has three possible options
     - G: the conjecture is equivalent to saying that for every int there is a finite stopping time
     - Cr: height of all sequences is finite
     - Ch: if stopping time and height are finite for all N the conjecture holds
     - Cr: if the trajectory does not contain 1, it is infinite
     - L: research areas
       * number theory
       * dynamical systems
       * ergodic theory
       * theory of computation
       * stochastic processes

        * computer science
    - What is cool about it?
        - L: the problem itself is not that important, more of a challenge
        - L: the general type of function is pretty popular rn and important
        - L: cool because it is simple to state but hard to prove
        - L: hard because it is pseudorandom and proofs need structure
        - L: subsidiary conjectures that also seem hard to prove
3. History of the Conjecture – Where does it come from?
    - Ch: Paul Erdos quote
    - Ch: different names that the conjecture has
    - L: attributed to Collatz, hence the name, 1930s
    - L: at least around since the 50s
    - L: 1970s math literature began
    - L: 1971 saw the first appearance of this exact problem
    - L: another Erdos quote
    - Cr: Everett proved that almost all odd ints have finite stopping time
    - T: computational verification for all N <= 10^20
4. Closer look at some attributes of the function
    - L: many unproved conjectures regarding the original conjecture exist
    - L: general decreasing slope is equal to -0.5 log(3/4)
    - L: most trajectories just follow a downward logarithm, some are split and interesting
    - L: iterates can be arbitrarily larger than the starting values
    - Ch: sum of even integers equals sum of odd integers plus number of odd integers – maybe check this thing computationally?
    - Plotting a graph
        - Ch: graph of the elements of a couple sequences, inspired by this and lagarias
        - L: one can plot log(T^k) vs k
        - L: cool plot of n = 649, make inspired diagram
        - L: illustrate pseudo-randomness using graph
        - compare the approximation in Cr with the actual stopping time for some values
    - Cycles
        - Ch: for all integers we get three more cycles at {0}, {-5}, {-17}, conjectured to be all cycles
        - Ch: non-trivial cycle of at least 272,500,658 length
        - L: minimal unknown cycles must be of length 10 billion
        - G: proof that any non-trivial cycle must have thousands of terms
    - Stochastic Approximations
        - L: creation of probabilistic models used to describe discrete processes
        - L: stems from the fact that the number of even and odd iterates is basically equal – this is the assumption that gives rise to the slope and general stopping time
        - L: because it seems so random, people are describing it probabilistically, this describes groups of trajectories at best
        - L: T(x) seems to be pseudorandom
        - Cr: for sufficiently large ints the collatz function yiels basically random

variables
- Height of the graph
  - Cr: height can also be called the cardinality of the trajectory
  - Cr: approximation of the height of any given x – cool to visualize
- Stopping time
  - Ch: average stopping time of odd numbers should be around 9.477955
  - L: generally is takes 6.95212 log n steps to reach 1
  - L: total stopping time is equal to the number of even numbers in the sequence
  - L: largest total stopping time is at most 41.677647 log n – quantitatively predicts that divergent trajectories do not exist
  - G: most ints have small stopping times, can be as large as you want though

2 General features

- 15 pages
- all of the latex fancies
- quotations?
- enumerate and itemize?
- images of graphs in .eps or .svg
- indents
- footnotes
- acknowledgements

3 Controversy

- what is the general sentiment
- what is actually the case
- not immediately obvious revelation
- less than 250 words
- reasonable amount of creativity
- quotes for context
- paraphrasing

4 Structure

4.1 Title

4.2 Abstract

- present tense, passive voice
- catch interest of people in related fields
- no symbols if possible

4.3 Table of Contents

4.4 Introduction

- contextualize and get the reader interested
- why this topic
- how did I do it
- what was found
- what does it mean

4.5 Body

4.5.1 Section 1

4.5.2 Section 2

4.6 Conclusion

- relate to other peoples findings

4.7 Acknowledgements

4.8 References

---

5 Talk

- tell what you are going to say
- tell it
- tell what you said
- ≤ 7 lines per slide
- 2 slides per minute
- tell a story but tell the audience where you will end up
- explain the problem
- what is the controversy
- use examples

- only hint at how things are being proven
- give context for your results
- convince the reader or listener
- appreciate the beauty of the mathematics
- abstract in passive voice, present tense