EXERCISES 5.3

In Problems 1–8, show that $\oint_C f(z)\,dz = 0$, where f is the given function and C is the unit circle $|z| = 1$.

1. $f(z) = z^3 - 1 + 3i$

2. $f(z) = z^2 + \dfrac{1}{z-4}$

3. $f(z) = \dfrac{z}{2z+3}$

4. $f(z) = \dfrac{z-3}{z^2+2z+2}$

5. $f(z) = \dfrac{\sin z}{(z^2-25)(z^2+9)}$

6. $f(z) = \dfrac{e^z}{2z^2+11z+15}$

7. $f(z) = \tan z$

8. $f(z) = \dfrac{z^2-9}{\cosh z}$

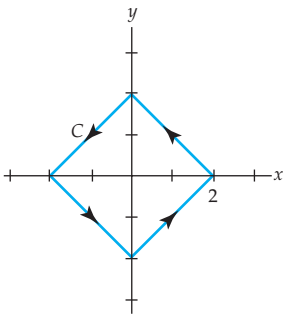

Figure for Problem 9

9. Evaluate $\oint_C \dfrac{1}{z}\,dz$, where C is the contour shown in Figure 9.

10. Evaluate $\oint_C \dfrac{5}{z+1+i}\,dz$, where C is the contour shown in Figure 10.

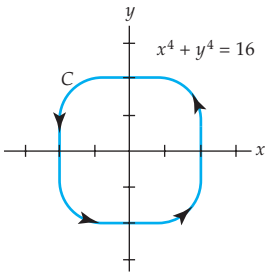

Figure for Problem 10

In Problems 11–22, use any of the results in this section to evaluate the given integral along the indicated closed contour(s).

11. $\oint_C \left(z + \dfrac{1}{z}\right) dz;\ |z| = 2$

12. $\oint_C \left(z + \dfrac{1}{z^2}\right) dz;\ |z| = 2$

13. $\oint_C \dfrac{z}{z^2-\pi^2}\,dz;\ |z| = 3$

14. $\oint_C \dfrac{10}{(z+i)^4}\,dz;\ |z+i| = 1$

15. $\oint_C \dfrac{2z+1}{z^2+z}\,dz;$ **(a)** $|z| = \frac{1}{2}$, **(b)** $|z| = 2$, **(c)** $|z-3i| = 1$

16. $\oint_C \dfrac{2z}{z^2+3}\,dz;$ **(a)** $|z| = 1$, **(b)** $|z-2i| = 1$, **(c)** $|z| = 4$

17. $\oint_C \dfrac{-3z+2}{z^2-8z+12}\,dz;$ **(a)** $|z-5| = 2$, **(b)** $|z| = 9$

18. $\oint_C \left(\dfrac{3}{z+2} - \dfrac{1}{z-2i}\right) dz;$ **(a)** $|z| = 5$, **(b)** $|z-2i| = \frac{1}{2}$

19. $\oint_C \dfrac{z-1}{z(z-i)(z-3i)}\,dz;\ |z-i| = \frac{1}{2}$

20. $\oint_C \dfrac{1}{z^3+2iz^2}\,dz;\ |z| = 1$

21. $\oint_C \operatorname{Ln}(z+10)\,dz;\ |z| = 2$

22. $\oint_C \left[\dfrac{5}{(z-2)^3} + \dfrac{3}{(z-2)^2} - \dfrac{10}{z-2} + 7\csc z\right] dz;\ |z-2| = \frac{1}{2}$