Exercises

1. A truth serum has the property that 90% of the guilty suspects are properly judged while, of course, 10% of guilty suspects are improperly found innocent. On the other hand, innocent suspects are misjudged 1% of the time. If the suspect was selected from a group of suspects of which only 5% have ever committed a crime, and the serum indicates that he is guilty, what is the probability that he is innocent?

2. The probabilities that a service station will pump gas into 0, 1, 2, 3, 4, or 5 or more cars during a certain 30-minute period are 0.03, 0.18, 0.24, 0.28, 0.10, and 0.17, respectively. Find the probability that in this 30-minute period

(a) more than 2 cars receive gas;

(b) at most 4 cars receive gas;

(c) 4 or more cars receive gas.

3. From a group of 4 men and 5 women, how many committees of size 3 are possible

(a) with no restrictions?

(b) with 1 man and 2 women?

(c) with 2 men and 1 woman if a certain man must be on the committee?

4. In a certain city district the need for money to buy drugs is stated as the: reason for 75% of all thefts. Find the probability that among the next 5 theft cases reported in this district,

(a) exactly 2 resulted from the need for money to buy drugs;

(b) at most 3 resulted from the need for money to buy drugs.

5. A manufacturing company uses an acceptance scheme on production items before they are shipped. The plan is a two-stage one. Boxes of 25 are readied for shipment and a sample of 3 is tested for defectives. If any defectives are found, the entire box is sent back for 100% screening. If no defectives are found, the box is shipped.

(a) What is the probability that a box containing 3 defectives will be shipped?

(b) What is the probability that a box containing only 1 defective will be sent back for screening?

(c) Suppose that the manufacturing company decides to change its acceptance scheme. Under the new scheme an inspector takes one at random, inspects it, and then replaces it in the box; a second inspector does likewise. Finally, a third inspector goes through the same procedure. The box is not shipped if any of the three find a defective. Answer (a)and (b) under this new plan.

6. Suppose that, on average, 1 person in 1000 makes a numerical error in preparing his or her income tax return. If 10,000 forms are selected at random and examined

(a) Find the probability that 6, 7, or 8 of the forms contain an error.

(b) Find the mean and variance of the random variable X representing the number of persons among 10,000 who make an error in preparing their income tax returns.

7.The probability that a patient recovers from a delicate heart operation is 0.9. Of the next 100 patients having this operation, what is the probability that

(a) between 84 and 95 inclusive survive?

(b) fewer than 86 survive?

8. An electrical firm manufactures light bulbs that have a length of life that, is approximately normally distributed with a standard deviation of 40 hours. If a sample of 30 bulbs has an average life of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm.

9. A random sample of 12 graduates of a certain secretarial school typed an average of 79.3 words per minute with a standard deviation of 7.8 words per minute. Assuming a normal distribution for the number of words typed per minute, find a 95% confidence interval for the average number of words typed by all graduates of this school.

10. The special paper which is producible at the factory must have the thickness equals 0.05 mm. If the average thickness of the paper differs from the standard, then the equipment, on which paper is produced, is subject to adjustment. A random sample of 100 sheets of paper was taken and it was founded that the sample mean $\bar{x} = 0.051$ mm and the sample standard deviation $s = 0.002$ mm. At the significance level $\alpha = 0.05$, evaluate - should adjust the equipment or not? Find the p-value.

Answers

1. 0.1743

2. (a) 0.55. (b) 0.83. (c) 0.27.
3. (a) 84 (b) 40 (c)15
4. (a) 0.0879. (b) 0.3672
5. (a) 0.6696. (b) 0.12 (c) 0.6815 and 0.1153

6. (a) 0.2657. (b) 10 and 10

7. (a) 0.9514. (b) 0.0668

8. $765 < \mu < 795$.

9. $74.34 < \mu < 84.26$.

10. RH_0