EXERCISES 5.2

In Problems 1-16, evaluate the given integral along the indicated contour.

1. $\int_C (z+3)\,dz$, where C is $x = 2t$, $y = 4t - 1$, $1 \le t \le 3$

2. $\int_C (2\bar{z} - z)\,dz$, where C is $x = -t$, $y = t^2 + 2$, $0 \le t \le 2$

3. $\int_C z^2\,dz$, where C is $z(t) = 3t + 2it$, $-2 \le t \le 2$

4. $\int_C (3z^2 - 2z)\,dz$, where C is $z(t) = t + it^2$, $0 \le t \le 1$

5. $\int_C \dfrac{z+1}{z}\,dz$, where C is the right half of the circle $|z| = 1$ from $z = -i$ to $z = i$

6. $\int_C |z|^2\,dz$, where C is $x = t^2$, $y = 1/t$, $1 \le t < 2$

7. $\oint_C \mathrm{Re}(z)\,dz$, where C is the circle $|z| = 1$

8. $\oint_C \left(\dfrac{1}{(z+i)^3} - \dfrac{5}{z+i} + 8\right) dz$, where C is the circle $|z+i| = 1$, $0 \le t \le 2\pi$

9. $\int_C (x^2 + iy^3)\,dz$, where C is the straight line from $z = 1$ to $z = i$

10. $\int_C (x^2 - iy^3)\,dz$, where C is the lower half of the circle $|z| = 1$ from $z = -1$ to $z = 1$

11. $\int_C e^z\,dz$, where C is the polygonal path consisting of the line segments from $z = 0$ to $z = 2$ and from $z = 2$ to $z = 1 + \pi i$

12. $\int_C \sin z\,dz$, where C is the polygonal path consisting of the line segments from $z = 0$ to $z = 1$ and from $z = 1$ to $z = 1 + i$

13. $\int_C \mathrm{Im}\,(z - i)\,dz$, where C is the polygonal path consisting of the circular arc along $|z| = 1$ from $z = 1$ to $z = i$ and the line segment from $z = i$ to $z = -1$

14. $\int_C dz$, where C is the left half of the ellipse $\frac{1}{36}x^2 + \frac{1}{4}y^2 = 1$ from $z = 2i$ to $z = -2i$

15. $\oint_C ze^z\,dz$, where C is the square with vertices $z = 0$, $z = 1$, $z = 1+i$, and $z = i$

16. $\int_C f(z)\,dz$, where $f(z) = \begin{cases} 2, & x < 0 \\ 6x, & x > 0 \end{cases}$ and C is the parabola $y = x^2$ from $z = -1 + i$ to $z = 1 + i$

In Problems 17–20, evaluate the given integral along the contour C given in figure

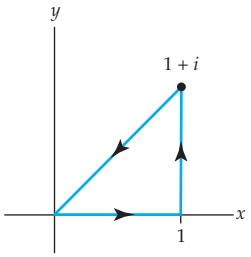

Figure for Problems 17–20

17. $\oint_C x\,dz$

18. $\oint_C (2z-1)\,dz$

19. $\oint_C z^2\,dz$

20. $\oint_C \bar{z}^2\,dz$

In Problems 21–24, evaluate $\int_C (z^2 - z + 2)\,dz$ from i to 1 along the contour C given in the figures.

21.

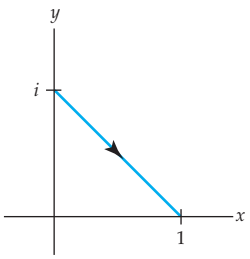

Figure for Problem 21

22.

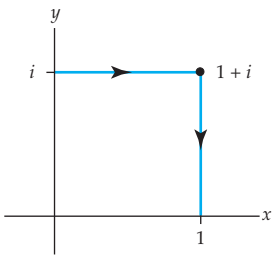

Figure for Problem 22

23.

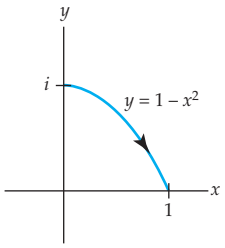

Figure for Problem 23

24.

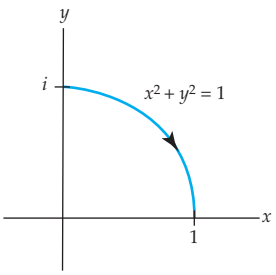

Figure for Problem 24

In Problems 25–28, find an upper bound for the absolute value of the given integral along the indicated contour.

25. $\oint_C \dfrac{e^z}{z^2+1}\,dz$, where C is the circle $|z| = 5$

26. $\displaystyle\int_C \frac{1}{z^2 - 2i}\,dz$, where C is the right half of the circle $|z| = 6$ from $z = -6i$ to $z = 6i$

27. $\displaystyle\int_C (z^2 + 4)\,dz$, where C is the line segment from $z = 0$ to $z = 1 + i$

28. $\displaystyle\int_C \frac{1}{z^3}\,dz$, where C is one-quarter of the circle $|z| = 4$ from $z = 4i$ to $z = 4$