EXERCISES 11

1. In 2010, the mean score for the three sections of the SAT was $\mu = 1509$, with a standard deviation $\sigma = 339$. (source: *College Entrance Examination Board, College-Bound Seniors: Total Group Profile (National) Report, 1966-67 through 2009-10*)

a) One student takes the SAT. What is her expected score on the three parts?
b) One hundred students take the SAT. What is the expected value of their average score (i.e. expected value of the sample mean, $\bar{x}$)? What is the standard error for the sampling distribution?
c) One hundred students take the SAT. What is the probability that their average score is less than 1450?
d) One hundred students take the SAT. What is the probability that their average score is between 1450 and 1600?
e) One hundred students take the SAT. What is the probability that their average score is above 1600?

2. An experiment has the following probability distribution:

X	1	2	3	4
$P(X)$	0.4	0.1	0.3	0.2

a) What are the expected value and standard deviation for a single randomly-chosen value of X?
b) You randomly select a sample of size $n = 50$. What is the expected value for the sample mean, $\bar{x}$? What is the standard error for the sampling distribution?
c) You randomly select a sample of size $n = 50$. What is the probability that the sample mean is less than 2.2?
d) You randomly select a sample of size $n = 50$. What is the probability that the sample mean is between 2.2 and 2.5?

3. You pay \$1 to play a game in which you roll one standard six-sided die. You lose your dollar if the die is 1, 2, 3 or 4. You get your dollar back if the die is a 5, and if the die is a 6 you get your dollar back plus \$2 more (total of \$3).
a) Calculate expected value and standard deviation for a single toss. (Be sure to include the dollar you pay to play the game.)
b) If you play the game 100 times, what are the expected value and standard error for the sampling distribution?
c) If you play the game 100 times, what is the probability that your average outcome will be positive? (That is, you walk away with more money than what you had before the game.)
d) If you play the game 100 times, your average winnings have a 90% probability of being below what value?

4. You flip a fair coin 100 times. Define random variable Y = proportion of heads.
a) What is the expected proportion of heads for this experiment?
b) What is the standard error for the sampling distribution?
c) What is the probability that the proportion of heads is between 0.45 and 0.60?
d) What is the probability that the proportion of heads is greater than 0.60?

5. Lab results indicate that a certain drug is effective 75% of the time (success) and ineffective 25% of the time (failure). As a trial, the drug is administered to a sample of 1000 patients.
a) What is the expected proportion of successes for this experiment?
b) What is the standard error for the sampling distribution?
c) What is the probability that between 710 and 770 of the patients are helped by the drug? (Remember to think in terms of proportion: i.e. 710 is what proportion of the 1000 tested? 770 is what proportion of the 1000 tested?)
d) What is the probability that at least 770 of the patients are helped by the drug?

1.a) 1509; b) 1509, 33.9; c) 0.0409; d) 0.9554; e) 0.0037

2.a) 2.3, $\sqrt{1.41}$; b) 2.3, $\sqrt{\frac{1.41}{50}}$; c) 0.2743; d) 0.6087

3.a) $-\frac{1}{3}$, $\frac{\sqrt{11}}{3}$; b) $-\frac{1}{3}$, $\frac{\sqrt{11}}{30}$; c) 0.0013; d) 1.28 $\left(\frac{\sqrt{11}}{30}\right)-\frac{1}{3}$

4.a) 0.5; b) 0.05; c) 0.8185; d) 0.0228

5.a) 0.75; b) $\sqrt{\frac{3}{16000}} = \sqrt{0.0001875}$; c) 0.9261; d) 0.0721