

Preliminary FINAL EXAM, Spring - 2019

COMPLEX VARIABLES

1. Evaluate the expression and write your answer in the form $a + bi$

$$\frac{5 + 2i}{4 - 7i}$$

2. Find z^3 if $z = 3 - 3\sqrt{3}i$

3. Find all solutions

a) $z^5 = 1 + \sqrt{3}i$

b) $\cos(z) = 5$

4. Use the Cauchy-Riemann equations to decide whether the following functions are analytic, if it is the case, compute its derivative

a) $f(z) = z^2 - 2iz - 1$

b) $f(z) = \frac{z}{z}$

5. Show that $u(x, y)$ is harmonic in some domain and find a harmonic conjugate $v(x, y)$ when $u(x, y) = 3y - x(1 + y)$

6. Find the principal value of $(\sqrt{3} - i)^{3i}$

7. Obtain the integral $\int_C (z - 2\bar{z})dz$ along the straight-line path from $z = 1 + 4i$ to $z = 2 + 7i$.

8. Evaluate $\int_{3+i}^{4+2i} (3z^2 + 2\cos z - 4e^{2z})dz$

9. Determine $\oint_C \frac{e^{z+1}}{(z-1)^2(z+2i)^3} dz$ where C is the path:

a) $|z + 1| = 1$ b) $|z - 1 - i| = 2$ c) $|z - 1 - 3i| = 2$ d) $|z - 1 + i| = 2$