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- 1. Definition What is the 3x + 1 Problem
  - G: number theoretic function
  - T: map of N+1 to N+1 (positive integers)
  - L: definition of the function and all its attributes
    - stopping time
    - Ch: explanation of stopping time
    - total stopping time counts n as the 0th number
    - total stopping time
    - trajectory of a number
- 2. Conjecture
  - What is the Collatz conjecture?
    - Ch: because it is an integer function, T has three possible options
    - G: the conjecture is equivalent to saying that for every int there is a finite stopping time
    - Cr: height of all sequences is finite
    - Ch: if stopping time and height are finite for all N the conjecture holds
    - Cr: if the trajectory does not contain 1, it is infinite
    - L: research areas
      - \* number theory
      - \* dynamical systems
      - \* ergodic theory
      - \* theory of computation
      - \* stochastic processes

- \* computer science
- What is cool about it?
  - L: the problem itself is not that important, more of a challenge
  - L: the general type of function is pretty popular rn and important
  - L: cool because it is simple to state but hard to prove
  - L: hard because it is pseudorandom and proofs need structure
  - L: subsidiary conjectures that also seem hard to prove
- 3. History of the Conjecture Where does it come from?
  - Ch: Paul Erdos quote
  - Ch: different names that the conjecture has
  - L: attributed to Collatz, hence the name, 1930s
  - L: at least around since the 50s
  - L: 1970s math literature began
  - L: 1971 saw the first appearance of this exact problem
  - L: another Erdos quote
  - Cr: Everett proved that almost all odd ints have finite stopping time
  - T: computational verification for all  $N \le 10^2$
- 4. Closer look at some attributes of the function
  - L: many unproved conjectures regarding the original conjecture exist
  - L: general decreasing slope is equal to  $-0.5 \log(3/4)$
  - L: most trajectories just follow a downward logarithm, some are split and interesting
  - L: iterates can be arbitrarily larger than the starting values
  - Ch: sum of even integers equals sum of odd integers plus number of odd integers maybe check this thing computationally?
  - Plotting a graph
    - Ch: graph of the elements of a couple sequences, inspired by this and lagarias
    - L: one can plot  $\log(T^k)$  vs k
    - L: cool plot of n = 649, make inspired diagram
    - L: illustrate pseudo-randomness using graph
    - compare the approximation in Cr with the actual stopping time for some values
  - Cycles
    - Ch: for all integers we get three more cycles at  $\{0\}$ ,  $\{-5\}$ ,  $\{-17\}$ , conjectured to be all cycles
    - Ch: non-trivial cycle of at least 272,500,658 length
    - L: minimal unknown cycles must be of length 10 billion
    - G: proof that any non-trivial cycle must have thousands of terms
  - Stochastic Approximations
    - L: creation of probabilistic models used to describe discrete processes
    - L: stems from the fact that the number of even and odd iterates is basically equal – this is the assumption that gives rise to the slope and general stopping time
    - L: because it seems so random, people are describing it probabilistically, this describes groups of trajectories at best
    - L: T(x) seems to be pseudorandom
    - Cr: for sufficiently large into the collatz function yiels basically random

#### variables

- Height of the graph
  - Cr: height can also be called the cardinality of the trajectory
  - Cr: approximation of the height of any given x cool to visualize
- Stopping time
  - Ch: average stopping time of odd numbers should be around 9.477955
  - L: generally is takes 6.95212 log n steps to reach 1
  - L: total stopping time is equal to the number of even numbers in the sequence
  - L: largest total stopping time is at most 41.677647 log n − quantitatively predicts that divergent trajectories do not exist
  - G: most into have small stopping times, can be as large as you want though

## 2 General features

- 15 pages
- all of the latex fancies
- quotations?
- enumerate and itemize?
- images of graphs in .eps or .svg
- indents
- footnotes
- acknowledgements

# 3 Controversy

- what is the general sentiment
- what is actually the case
- not immediately obvious revelation
- less than 250 words
- reasonable amount of creativity
- quotes for context
- paraphrasing

# 4 Structure

#### 4.1 Title

#### 4.2 Abstract

- present tense, passive voice
- catch interest of people in related fields
- no symbols if possible

## 4.3 Table of Contents

### 4.4 Introduction

- contextualize and get the reader interested
- why this topic
- how did I do it
- what was found
- what does it mean

## 4.5 Body

- 4.5.1 Section 1
- 4.5.2 Section 2

#### 4.6 Conclusion

• relate to other peoples findings

# 4.7 Acknowledgements

### 4.8 References

## 5 Talk

- tell what you are going to say
- tell it
- tell what you said
- $\leq 7$  lines per slide
- 2 slides per minute
- tell a story but tell the audience where you will end up
- explain the problem
- what is the controversy
- use examples

- only hint at how things are being proven
- give context for your results
- convince the reader or listener
- appreciate the beauty of the mathematics
- $\bullet\,$  abstract in passive voice, present tense