EXERCISES 6.4

In Problems 1–6, use an appropriate Laurent series to find the indicated residue.

1. $f(z) = \dfrac{2}{(z-1)(z+4)}$; $\operatorname{Res}(f(z), 1)$

2. $f(z) = \dfrac{1}{z^3(1-z)^3}$; $\operatorname{Res}(f(z), 0)$

3. $f(z) = \dfrac{4z-6}{z(2-z)}$; $\operatorname{Res}(f(z), 0)$

4. $f(z) = (z+3)^2 \sin\left(\dfrac{2}{z+3}\right)$; $\operatorname{Res}(f(z), -3)$

5. $f(z) = e^{-2/z^2}$; $\operatorname{Res}(f(z), 0)$

6. $f(z) = \dfrac{e^{-z}}{(z-2)^2}$; $\operatorname{Res}(f(z), 2)$

In Problems 7–16, use (1), (2), or (4) to find the residue at each pole of the given function.

7. $f(z) = \dfrac{z}{z^2+16}$

8. $f(z) = \dfrac{4z+8}{2z-1}$

9. $f(z) = \dfrac{1}{z^4+z^3-2z^2}$

10. $f(z) = \dfrac{1}{(z^2-2z+2)^2}$

11. $f(z) = \dfrac{5z^2-4z+3}{(z+1)(z+2)(z+3)}$

12. $f(z) = \dfrac{2z-1}{(z-1)^4(z+3)}$

13. $f(z) = \dfrac{\cos z}{z^2(z-\pi)^3}$

14. $f(z) = \dfrac{e^z}{e^z-1}$

15. $f(z) = \sec z$

16. $f(z) = \dfrac{1}{z \sin z}$

In Problems 17–20, use Cauchy's residue theorem, where appropriate, to evaluate the given integral along the indicated contours.

17. $\displaystyle\oint_C \frac{1}{(z-1)(z+2)^2}\,dz$ (**a**) $|z| = \frac{1}{2}$ (**b**) $|z| = \frac{3}{2}$ (**c**) $|z| = 3$

18. $\displaystyle\oint_C \frac{z+1}{z^2(z-2i)}\,dz$ (**a**) $|z| = 1$ (**b**) $|z-2i| = 1$ (**c**) $|z-2i| = 4$

19. $\displaystyle\oint_C z^3 e^{-1/z^2}\,dz$ (**a**) $|z| = 5$ (**b**) $|z+i| = 2$ (**c**) $|z-3| = 1$

20. $\displaystyle\oint_C \frac{1}{z \sin z}\,dz$ (**a**) $|z-2i| = 1$ (**b**) $|z-2i| = 3$ (**c**) $|z| = 5$