EXERCISES 2.6

Limits

In Problems 1–16 compute the given complex limit.

1. $\lim_{z\to 2i}\left(z^2-\bar{z}\right)$

2. $\lim_{z\to 1+i}\dfrac{z-\bar{z}}{z+\bar{z}}$

3. $\lim_{z\to 1-i}\left(|z|^2-i\bar{z}\right)$

4. $\lim_{z\to 3i}\dfrac{\operatorname{Im}\left(z^2\right)}{z+\operatorname{Re}(z)}$

5. $\lim_{z\to \pi i} e^z$

6. $\lim_{z\to i} ze^z$

7. $\lim_{z\to 2+i}\left(e^z+z\right)$

8. $\lim_{z\to i}\left(\log_e\left|x^2+y^2\right|+i\arctan\dfrac{y}{x}\right)$

9. $\lim_{z\to 2-i}\left(z^2-z\right)$

10. $\lim_{z\to i}\left(z^5-z^2+z\right)$

11. $\lim_{z\to e^{i\pi/4}}\left(z+\dfrac{1}{z}\right)$

12. $\lim_{z\to 1+i}\dfrac{z^2+1}{z^2-1}$

13. $\lim_{z\to -i}\dfrac{z^4-1}{z+i}$

14. $\lim_{z\to 2+i}\dfrac{z^2-(2+i)^2}{z-(2+i)}$

15. $\lim_{z\to z_0}\dfrac{(az+b)-(az_0+b)}{z-z_0}$

16. $\lim_{z\to -3+i\sqrt{2}}\dfrac{z+3-i\sqrt{2}}{z^2+6z+11}$

17. $\lim_{z\to\infty}\dfrac{z^2+iz-2}{(1+2i)z^2}$

18. $\lim_{z\to\infty}\dfrac{iz+1}{2z-i}$

19. $\lim_{z\to i}\dfrac{z^2-1}{z^2+1}$

20. $\lim_{z\to -i/2}\dfrac{(1-i)z+i}{2z+i}$

21. $\lim_{z\to\infty}\dfrac{z^2-(2+3i)z+1}{iz-3}$

22. $\lim_{z\to i}\dfrac{z^2+1}{z^2+z+1-i}$

Continuity

In Problems 23–30, show that the function f is continuous at the given point.

23. $f(z) = z^2 - iz + 3 - 2i;\ z_0 = 2 - i$

24. $f(z) = z^3 - \dfrac{1}{z};\ z_0 = 3i$

25. $f(z) = \dfrac{z^3}{z^3 + 3z^2 + z};\ z_0 = i$

26. $f(z) = \dfrac{z - 3i}{z^2 + 2z - 1};\ z_0 = 1 + i$

27. $f(z) = \begin{cases} \dfrac{z^3 - 1}{z - 1}, & |z| \neq 1 \\ 3, & |z| = 1 \end{cases};\ z_0 = 1$

28. $f(z) = \begin{cases} \dfrac{z^3 - 1}{z^2 + z + 1}, & |z| \neq 1 \\ \dfrac{-1 + i\sqrt{3}}{2}, & |z| = 1 \end{cases};\ z_0 = \dfrac{1 + i\sqrt{3}}{2}$

29. $f(z) = \bar{z} - 3\,\mathrm{Re}(z) + i;\ z_0 = 3 - 2i$

30. $f(z) = \dfrac{\mathrm{Re}(z)}{z + iz} - 2z^2;\ z_0 = e^{i\pi/4}$

In Problems 31–36, show that the function f is discontinuous at the given point.

31. $f(z) = \dfrac{z^2 + 1}{z + i};\ z_0 = -i$

32. $f(z) = \dfrac{1}{|z| - 1};\ z_0 = i$

33. $f(z) = \mathrm{Arg}(z);\ z = -1$

34. $f(z) = \mathrm{Arg}(iz);\ z_0 = i$

35. $f(z) = \begin{cases} \dfrac{z^3 - 1}{z - 1}, & |z| \neq 1 \\ 3, & |z| = 1 \end{cases};\ z_0 = i$

36. $f(z) = \begin{cases} \dfrac{z}{|z|}, & z \neq 0 \\ 1, & z = 0 \end{cases};\ z_0 = 0$