

Research Methods in Mathematical Sciences

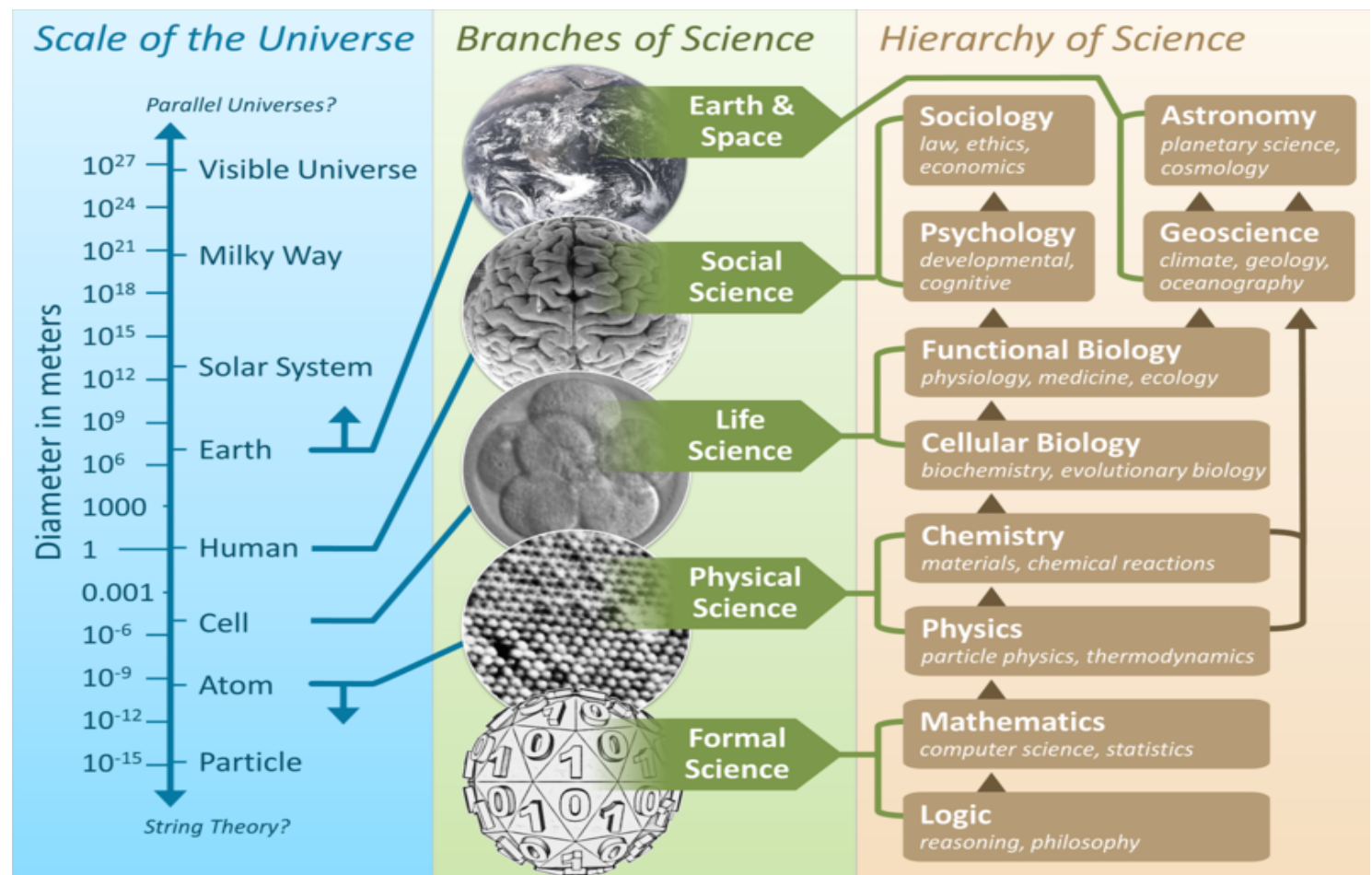
mathematics, science and the scientific method

What is science?

What is science?

Encyclopedia Britannica says

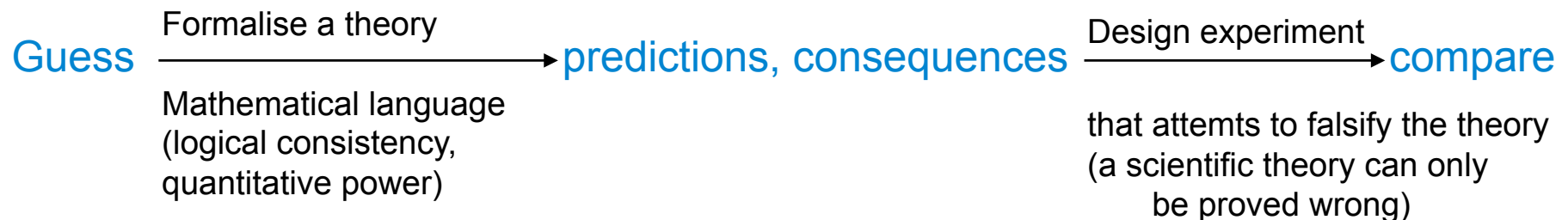
Any system of knowledge that is concerned with the physical world and its phenomena and that entails unbiased observations and systematic experimentation. In general, a science involves a pursuit of knowledge covering general truths or the operations of fundamental laws.



Scientific method

Scientific approach followed to describe and understand a phenomenon

1. Observation
2. Educated guess (theory)
3. The theory should make observable predictions.
4. If predictions do not match observation, theory is wrong.



For a theory to be considered scientific

1. Needs to follow ***Scientific method***.

2. ***Popper's Falsifiability*** (the theory should be able to make predictions that can be proved false by experiments or observations).

+ Occam's razor on simplicity (ex: Ptolomeus vs Copernicus)



Karl Popper

A theory in the empirical sciences can never be proven, but it can be falsified !

Example:

Newton laws were 'correct' but untrue (some small discrepancies with experiments suggested that these were wrong or at least incomplete)

Truth has to do with mathematics and with philosophy

"In science we are never sure to be right, we are only sure when we're wrong" R. Feynman

What is a pseudoscience

Claim, belief, or practice which is [presented as scientific](#) but does not adhere to a valid scientific method, lacks supporting evidence or plausability, cannot be reliably tested (falsifiability), enters in [contradiction with verified physical theories](#).

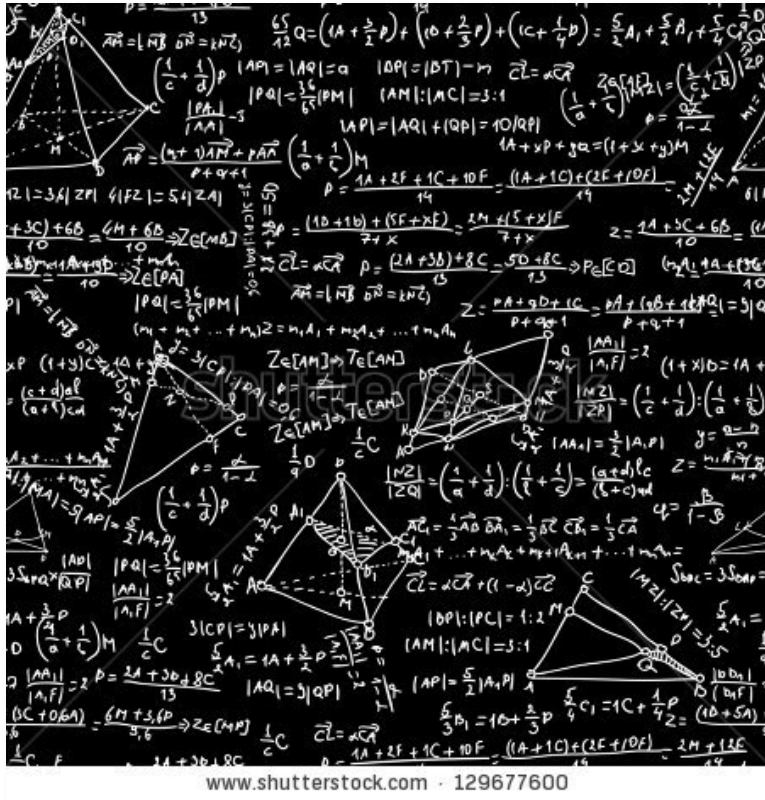
Often characterised by the use of [vague](#), contradictory, exaggerated claims, an over-reliance on confirmation rather than rigorous attempts at refutation,...

Lots of them have been scientifically disproved. Many of them however cannot be disproved as they are not falsifiable.

Can you name some examples?

Is mathematics a science?

Is mathematics a science? Controversial topic



- * Some think NO as it doesn't follow standard scientific method
it is a **philosophy**, not a science
- * Some think it follows a restricted version (logic & reasoning + reproducibility):
it is a **formal science**
- * Some actually think it follows scientific method (platonic world): it is a **natural science**

Eventually all reduces to the following philosophical question:

Is mathematics created or discovered?

[Mathematical realism?](#)

[Current maths is only one of many?](#)

Pure vs Applied Mathematics and the scientific method

Some views of mathematics by a pure mathematician

- Mathematics is an abstract language (and as such mathematicians discover -or create- new parts of this language).
- It's a formal (not natural) science whose results and predictions are reproducible.
- A pure mathematician doesn't care necessarily about the scientific method, although different mathematicians have different opinions on the subject.
- Some mathematicians consider maths as a natural science that addresses the description of the **real** platonic world.
- Pure mathematicians focus on advancing our mathematical knowledge, mainly **proving** new **theorems**. The central cornerstone of pure mathematics is mathematical proof.
- They **discover (or create)** new mathematical topics, find links between mathematical fields, conjecture unknown mathematical statements and prove such conjectures.
- Pure mathematicians focus mostly on general statements rather on particular statements. They like generality.

Pure vs Applied Mathematics and the scientific method

Some views of mathematics by a **pure** applied mathematician

- Mathematics is the language of the physical world (i.e. it's a tool)
- An applied mathematician **uses** mathematics to describe reality and therefore bothers with the scientific method (different philosophical approach than pure mathematicians).
- The applied mathematics uses mathematics to **model** real world phenomena and **solve concrete problems**
(for instance, traduce a physical phenomenon into a mathematical framework where mathematical techniques can provide answers to physical questions)
- An applied mathematician with interests in...
 - * physics is very similar to a theoretical physicist
 - * chemistry is very similar to a theoretical chemist
 - * biology is very similar to a theoretical biologist

Pure vs Applied Mathematics and the scientific method

To summarise:

- * A mathematical statement can be proved true or false
- * A scientific theory (with mathematical consistency) can only be proved wrong
- * Pure mathematicians develop new mathematics *per se*
- * Applied mathematicians use mathematics (and develop new mathematics) to respond to scientific questions

In practice, there is high overlap between pure and applied mathematics as:

- applied mathematicians also prove theorems from time to time
- pure mathematicians develop mathematical fields that also have real world applicability:

Abstract mathematical fields (pure maths) have found incredibly fruitful applications:

- * Differential geometry (Einstein's theory of general relativity)
- * Number theory (cryptography, RSA algorithm to encrypt emails)
- * Logic and axiomatic foundations of maths (computer science)

Real world phenomena / physics oriented areas have generated new mathematical fields:

- * Differential equations

Pure vs Applied Mathematics and the scientific method

In research, try to be both a pure and an applied mathematician

- the pure maths approach would yield questions such as

- * Can I prove this or that statement about the problem to be true?
- * Up to what level of generality? Can I generalise this statement?
- * What are the minimal properties that need to hold for that statement to hold true?

- the applied maths approach would yield questions such as

- * what can I learn about the problem by using this or that technique?
- * can I apply this or that technique in a different problem?

- A good mathematician should have training in both “philosophical” approaches.
- Depending on the specialisation, you will bend towards pure or applied mathematics.
- Some mathematicians do both!

WORKING GROUP SESSION

Which of these are: (i) science, (ii) not a science yet, (iii) pseudoscience, (iv) none of the above. **Justify**

- | | |
|--------------------|------------------|
| - Physics | - Astronomy |
| - Economics | - Astrology |
| - Biology | - Homeopathy |
| - Psychology | - Psychoanalysis |
| - Sociology | - Chakras |
| - Philosophy | - Reiki |
| - Deidic Religions | - Chemistry |
| - String theory | - Mathematics |

WORKING GROUP SESSION

>> Written assignment

What is mathematics for you?

(30min, maximum 1 page)

Hand in to Lucas or Reto before you leave!