EXERCISES 13

1. A store is preparing the launch of a new product and is considering various price points at which to sell it. A market survey of the selling price at 25 competitorshas a mean selling price of $90.50. It is known that the population standard deviation for this product price is $7.50 and that the population is normally distributed. Construct a 90% confidence interval for the population mean price of this product.

2. A sample of 900 Canadian households shows spends an average of $7868 a year on eating out. Assume that the spending habits of all Canadian households is normally distributed with a standard deviation of $2070. Construct a 99% confidence interval for the average amount that Canadian households spend on eating out each year.

3. A Regina city planner is interested in the average commuting time for U of R off-campus students. She finds that a sample of 53 students has an average commuting time of 23.1 minutes. Assuming that the commuting time for all U of R off-campus students has $\sigma=8.3$ minutes, use her findings to construct a 95% confidence interval for U of R commuting times.

4. A student association wants to estimate average student debt. It is known that population standard deviation for student debt is $11,800. How large a sample should be selected so that a 99% confidence interval can estimate the true average student debt with a maximum error of $800?

5. A financial auditor wants to estimate the mean payout for automobile accidents at an insurance company by sampling 26 random cases. The sample mean is found to be $17,415. The distribution of accident payouts can be assumed to be normal.

   a) Assuming that the population standard deviation for payouts is $2,600, construct a 95% confidence interval for the true mean accident payout.
   b) Repeat (a) if we can no longer assume knowledge of σ, but must instead use a sample standard deviation of s=$2600.

6. A healthy invididual has an MCV (average volume of a red blood cell) of 90 femtoliters (fl). (Note: a femtoliter is 10^{-15} litres). A particular chemotherapy drug is believed to cause a change in MCV levels. A sample of 35 patients taking this particular drug has a mean volume of 87 fl with a standard deviation of 8.3 fl.

   a) Construct a 99% confidence interval for the true mean MCV level in patients undergoing this particular chemotherapy.

   b) At the $\alpha=0.01$ level of significance, test the claim that this drug changes MCV levels.

   c) Compute the P-value for this test. Would your decision in (b) change at the 0.1 level of significance? Would it change at the 0.001 level of significance?

7. The total amount of time that an individual will spend in REM (Rapid Eye Movement) sleep during a full week is, on average, 11.2 hours. Some health researchers believe that the problem of "light pollution" (being exposed to light almost continuously, even at night, from streetlights, etc) has detrimental health effects, including a reduction in REM sleep. To test this hypothesis, a random sample of 40 volunteers spend a week sleeping with an additional light source in their room. For this sample, the average amount of time spent in REM sleep was 10.9 hours with a standard deviation of 1.2 hours. Calculate the P-value for this test. How would you respond to the researchers' claim at the

   a) 0.1 level of significance? b) 0.05 level of significance? c) 0.01 level of signifiance?

8. A student wishes to estimate the average level of student debt of recent U of R graduates. How large a sample should (s)he select if (s)he wishes to be accurate within $250, 9 times out of 10? From previous studies, the student will assume that σ=$3551.

9. A published research paper in astrophysics states that 27 of 300 ESP systems (i.e. solar systems containing at least one planet) have low levels of metallicity (i.e. a low proportion of elements other than hydrogen and helium). Construct a 90% confidence interval of the true proportion of ESP systems with low metallicity.

10. A newly introduced blood test is claimed to accurately determine the presence (or lack) of the dreaded Mathandstatitis disease in 87% of all patients. To test this claim, a sample of 200 patients is tested. In 164 cases, the test accurately determined the presence (or lack) of Mathandstatitis. Test the claim at the α=0.02 level and calculate the P-value.

11. A medical researcher is interested in the proportion of patients that develop serious side effects to a newly introduced drug.

    a) How many patients should be sampled, if the results are to be correct within 2%, with confidence level 99%?

    b) Based on an existing drug with similar properties, the researcher believes that the percentage of patients with serious side effects is going to be less than 1%. How does this effect the required sample size for the proposed study?

12. A fertilizer distributor sells fertilizers in 20 kg bags. A customer feel that the distributor sells underweight bags. To support his hunch, he purchases 8 bags of fertilizer. They have an average weight of 19.65 kg with standard deviation 0.7 kg. At the .05 level of significance, does the customer have a case against the distributor?

Answers:

1. $\$88.03 < \mu < \$92.97$
2. $\$7690 < \mu < \$8046$
3. $20.9 \text{ min} < \mu < 25.3 \text{ min}$
4. n= 1442.57
5. a) $\$16415.60 < \mu < \$18,414.40$ b) $\$16364.60 < \mu < \$18465.40$
6. a) $83.39 \text{ fl} < 90.61 \text{ fl}$ b) FTR H0.
   c) Reject H0 at the 0.05 level of significance, since $0.0324<0.05$. We would not, however, change our answer at the 0.001 level of significance (i.e. we would not reject H0).
7. (a) We would reject H0 at the 0.01 level of significance.
   (b)/(c) We would not reject H0 at 0.05 or 0.1 level of significance.
8. The student needs to sample a minimum of 546 recent U of R graduates.
9. $0.063 < p < 0.117$
10. FTR H0
11. a) Minimum of 4148 patients must be sampled.
    b) Only 165 patients must be sampled
12. We do not reject H0. At $\alpha=0.05$, there is insufficient evidence for the customer's claim.