EXERCISES 6.1

In Problems 1–12, use known results to expand the given function in a Maclaurin series. Give the radius of convergence R of each series.

1. $f(z) = \dfrac{z}{1+z}$

2. $f(z) = \dfrac{1}{4-2z}$

3. $f(z) = \dfrac{1}{(1+2z)^2}$

4. $f(z) = \dfrac{z}{(1-z)^3}$

5. $f(z) = e^{-2z}$

6. $f(z) = ze^{-z^2}$

7. $f(z) = \sinh z$

8. $f(z) = \cosh z$

9. $f(z) = \cos\dfrac{z}{2}$

10. $f(z) = \sin 3z$

11. $f(z) = \sin z^2$

12. $f(z) = \cos^2 z$ [*Hint*: Use a trigonometric identity.]

In Problems 13 and 14, use the Maclaurin series for e^z to expand the given function in a Taylor series centered at the indicated point z_0. [*Hint*: $z = z - z_0 + z_0$.]

13. $f(z) = e^z,\ z_0 = 3i$

14. $f(z) = (z-1)e^{-3z},\ z_0 = 1$

In Problems 15-22, expand the given function in a Taylor series centered at the indicated point z_0. Give the radius of convergence R of each series.

15. $f(z) = \dfrac{1}{z},\ z_0 = 1$

16. $f(z) = \dfrac{1}{z},\ z_0 = 1+i,$

17. $f(z) = \dfrac{1}{3-z},\ z_0 = 2i$

18. $f(z) = \dfrac{1}{1+z},\ z_0 = -i,$

19. $f(z) = \dfrac{z-1}{3-z},\ z_0 = 1$

20. $f(z) = \dfrac{1+z}{1-z},\ z_0 = i$

21. $f(z) = \cos z,\ z_0 = \pi/4$

22. $f(z) = \sin z,\ z_0 = \pi/2$