The $3n + 1$ Problem

Moritz Konarski, 12.02.2020

The $3n + 1$ problem (also known as the Collatz conjecture) is defined as

$$s(n) = \begin{cases} n/2 & \text{if } n \text{ is even} \\ 3n + 1 & \text{if } n \text{ is odd.} \end{cases}$$

A *Collatz sequence* is the sequence of numbers produced by the repeated application of this function starting at n. For example the Collatz sequence for $n = 20$ is $\{20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, \dots\}$. Starting at any positive integer, it is conjectured that all Collatz sequences end in the cycle $4, 2, 1, 4, 2, 1, \dots$. This behavior has been computationally verified for a very large amount of numbers, but a proof has not been found.

Outline

I want to inform about the Collatz conjecture and explore the current state of research regarding it or why it seems so difficult to prove. I need to do more research to really focus the report on a specific thing.

- Introduction
  - What is the Collatz conjecture?
  - Previous research on it
- Body Part
  - Why is it important?
    - New ideas might be needed to prove this conjecture
    - What implications does it have for other areas of mathematics?
  - How could this conjecture be proven?
- Conclusion
  - Could it be proven soon?
  - What would a proof mean?

References

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