Queen Mary
University of London

Research Methods in Mathematical Sciences

How to write an (applied maths) paper

Process of writing (chronological order)

A- Building a draft

0. Tentative title
1. Sketch of the abstract (check for one message rule)
2. Structure of the results sections
3. Draw/plot/prepare figures (illustrative and/or data,simulations)

4. Write down the content of each results section/subsection
5. Sketch of the discussion (sketch)
6. Write down the Introduction (+ referencing)
7. Rewrite discussion and abstract

B- Revising your draft

Usually there are no less than 2 rounds and things usually converge after 5-10 rounds

Check for:
- sense of internal coherence
- self-containdness
- Size / style
- acknowledgments
- proof-reading

General tips

When writing your report/paper, have in mind

The purpose of nearly all writing is to communicate.

First goal:
convince the readers that what you communicate is sound.

Second goal:
Make sure the reader understands and appreciates the beauty of the mathematics you have done, and understands its importance in the broader picture.

Care about the reader:
- the audience shapes the style and length
- keep it as much self-contained as possible
- technical parts must be present but must not confuse the message of the paper
(use of appendix sometimes beneficial for readability)
- always remember the 1-2 message key.
- use an adequate software that can handle mathematical expressions: use LaTeX !!!

"Think of yourself as a travel guide, leading the reader through territory charted only by you"
- Ashley Reiter

When we write, we think.
Some researchers start writing "working papers" in parallel with on-going research.
Writing about the idea helps to improve and develop the idea.

Good practice on writing RESULTS SECTION (60-70% of paper´s length)

• Golden rules:

- use only the necessary amount of notation (e.g. do not use notation you are not going to use all over the section)

Example: "On a compact space every real-valued continuous function *f is bounded."*

What does the symbol "f" contribute to the clarity of that statement?

- be consistent

• Order the sections according to readability

- (applied) Theory, simulations, experiments: which come first?
- (pure) Structure your proof: use propositions and lemmas
- make a sketch of the structure of the sections, then a sketch of the structure inside each section (in applied these are subsections, in pure check the logic of definition-proposition/lemmas + proofs, theorems + proofs).

• Concrete stylistic advices:

- Never start a sentence with a symbol
- Use words and symbols in a balanced way
- Avoid to use "any" in mathematics, it can cause confusion! Replace it by "each" or "every"
- Highlight special words: definition, theorem, proof, proposition, lemma…
- Only use the verb "prove" when you include a mathematical proof (otherwise use the verbs show, find, illustrate, etc).
- Important words: "Let", "If/then", "Thus", "Suppose", etc
- Use present tense ("we consider") or "we have considered" rather than past ("we considered"). In abstract, always the present tense!

Good practice on writing INTRODUCTION (10-20% of paper´s length)

- Contextualise & motivate

Build on previous research on the topic
Briefly synthesize related questions that have been addressed
State open questions and / or relevant aspects that are unknown
Focus on what concrete question you address, and suggest its significance

Questions you might ask yourself while writing this section:

- *Does your result strengthen a previous result by giving a more precise characterization of something?*
- *Have you proved a stronger result of an old theorem by weakening the hypotheses or by strengthening the conclusions?*
- *Have you proven the equivalence of two definitions?*
- *Is it a classification theorem of structures which were previously defined but not understood?*
- *Does is connect two previously unrelated aspects of mathematics?*
- *Does it apply a new method to an old problem?*
- *Does it help to explain or understand previous experiments?*
- *Does it provide a new proof for an old theorem?*
- *Is it a special case of a larger question?*

Good practice on writing INTRODUCTION (10-20% of paper´s length)

• Cite
- Use previous references to support
- Reference published papers, reviews, textbooks, not webpages, not wikipedia (work that has been peer-reviewed)
- Warning: your message should not depend on content of other papers you cite without any further explanation: self-containdness is golden…
and equilibrium as well… don´t overload the introduction, speak about the relevant parts only

• Then, depict briefly what you will develop in the results section (your contributions)
- (pure) The statement of the main theorem, and a brief description of the structure of the proof
- (applied) a sketch of the results supporting the main message
Several ways to do so depending on the specific field, but essentially, in all cases: explain the intuition first

• Readability
- avoid here technicalities and minimise notation as much as possible

Important observation:
- This section is where you might lose the people that started to read the paper thanks to an inpiring abstract. Don´t kill them yet!
- If the reader *understands* the problem you want to solve and why it is *important*, he/she is yours!

Good practice on writing DISCUSSION/CONCLUSION (1%-10% paper´s length)

• Summary of the main findings (avoid again technicalities and notation)

• Frame your research in the larger picture
- discuss the implications of your work for state of the art
- relate your findings to other works (**this should be avoided in the introduction**!!)

• Suggest further work
- open problems, future work
- applications of your work

Good practice on writing ABSTRACT (usually less than 200 words)

People most likely will read your title, and if you´re lucky, your abstract: the rest depends on you

• Clear, concise

• Briefly contextualise

• Avoid unnecessary mathematical notation

• The abstract needs to be attractive and easy to read to
- catch the interest of those that work in your field
- catch the interest of those that work in close fields

Good practice on writing TITLE

The title is the first contact that readers will have with your paper.
It must communicate something of the substances to the experts in your field as well as to the novices who will be interested.

Advice: read plenty of papers, look at the arxiv!

Working group session

SOME EXAMPLES OF TITLES AND ABSTRACTS
(From *Mathematical Writing*, Franco Vivaldi)

ABSTRACT. Let F be a rational map of degree $n \geqslant 2$ of the Riemann sphere $\overline{\mathbb{C}}$. We develop a theory of equilibrium states for the class of Hölder continuous functions f for which the pressure is larger than $\sup f$. We show that there exists a unique conformal measure (reference measure) and a unique equilibrium state, which is equivalent to the conformal measure with a positive continuous density.

- Some symbols are introduced but not used anymore in the abstract.

- Formulae (sup f) shoudn´t be mixed with text in the abstract.

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> ABSTRACT. We consider rational maps of the Riemann sphere, of degree greater than 1. We develop a theory of equilibrium states for the class of Hölder continuous functions f for which the pressure is larger than the supremum of f. We show that …

The symbol f in the second sentence is not strictly necessary, and can easily be disposed of.

> We develop a theory of equilibrium states for the class of Hölder continuous functions for which the pressure is larger than the supremum of the function.

SOME EXAMPLES OF TITLES AND ABSTRACTS
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ABSTRACT. The logistic map is a well-studied map of the unit interval into itself. However, if we treat x as a discrete variable, as is done in any computer, then every orbit is eventually periodic. Thus the aperiodic behaviour that the continuous map displays for some value of the parameter r cannot be obtained from computer simulation. We investigated the differences and the similarities between the dynamics of a continuous map and its discrete approximation. We found that the limit cycles of a discrete map follow the unstable periodic orbits of the corresponding continuous map.

- There is a brief motivation / contextualisation – GOOD
- the symbol x is introduced without explanation – BAD
- the symbols x and r are introduced but not used anymore - BAD
- the main message (if we discretize the domain of the logistic map, then we obtain new phenomena worth studying) is a bit lost.
- We “found” – VAGUE
- We “found” – USED PAST TENSE

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ABSTRACT. We consider the logistic map, a well-studied map of the unit interval which depends on a parameter. If the domain of this map is discretised, as happens in any computer simulation, then, necessarily, all orbits become eventually periodic. Thus the aperiodic orbits observed for certain parameter values no longer exist. We investigate differences and similarities between the original map and its discrete approximations. We provide evidence that the limit cycles of the discrete map converge to the unstable orbits of the original map, in a sense to be made precise.

SOME EXAMPLES OF TITLES AND ABSTRACTS
(From *Mathematical Writing*, Franco Vivaldi)

ABSTRACT. Let $f : X \to X, X = [0,1)$, be an IET (interval exchange transformation) ergodic with respect to the Lebesgue measure on X. Let $f_t : X_t \to X_t$ be the IET obtained by inducing f to $X_t = [0,t)$, $0 < t < 1$. We show that

$$X_{wm} = \{0 < t < 1 : f_t \text{ is weakly mixing}\}$$

is a residual subset of X of full Lebesgue measure. The result is proved by establishing a Diophantine sufficient condition on t for f_t to be weakly mixing.

- Too many symbols!

- Avoid technicalities (formal definitions for instance) in the abstract

- Interval exchange transformation (IET) instead of IET (interval exchange transformation)

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ABSTRACT. Let f be an interval exchange transformation (IET) of the unit interval, ergodic with respect to the Lebesgue measure, and let f_t be the IET obtained by inducing f on the sub-interval $[0,t)$, with $0 < t < 1$. We show that the set of values of t for which f_t is weakly mixing is a residual subset of full Lebesgue measure. The result is proved by establishing a Diophantine condition on t, which is sufficient for weak mixing.

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