

EXERCISES 1

1–14 ■ Evaluate the expression and write your answer in the form $a + bi$.

1. $(5 - 6i) + (3 + 2i)$
2. $(4 - \frac{1}{2}i) - (9 + \frac{5}{2}i)$
3. $(2 + 5i)(4 - i)$
4. $(1 - 2i)(8 - 3i)$
5. $\overline{12 + 7i}$
6. $\overline{2i(\frac{1}{2} - i)}$
7. $\frac{1 + 4i}{3 + 2i}$
8. $\frac{3 + 2i}{1 - 4i}$
9. $\frac{1}{1 + i}$
10. $\frac{3}{4 - 3i}$
11. i^3
12. i^{100}
13. $\sqrt{-25}$
14. $\sqrt{-3}\sqrt{-12}$

15–17 ■ Find the complex conjugate and the modulus of the number.

15. $12 - 5i$
16. $-1 + 2\sqrt{2}i$
17. $-4i$

18. Prove the following properties of complex numbers.

- (a) $\overline{z + w} = \overline{z} + \overline{w}$
 - (b) $\overline{zw} = \overline{z}\overline{w}$
 - (c) $\overline{z^n} = \overline{z}^n$, where n is a positive integer
- [Hint: Write $z = a + bi$, $w = c + di$.]

19–24 ■ Find all solutions of the equation.

19. $4x^2 + 9 = 0$
20. $x^4 = 1$
21. $x^2 + 2x + 5 = 0$
22. $2x^2 - 2x + 1 = 0$
23. $z^2 + z + 2 = 0$
24. $z^2 + \frac{1}{2}z + \frac{1}{4} = 0$

25–28 ■ Write the number in polar form with argument between 0 and 2π .

25. $-3 + 3i$
26. $1 - \sqrt{3}i$
27. $3 + 4i$
28. $8i$

29–32 ■ Find polar forms for zw , z/w , and $1/z$ by first putting z and w into polar form.

29. $z = \sqrt{3} + i$, $w = 1 + \sqrt{3}i$
30. $z = 4\sqrt{3} - 4i$, $w = 8i$
31. $z = 2\sqrt{3} - 2i$, $w = -1 + i$
32. $z = 4(\sqrt{3} + i)$, $w = -3 - 3i$

33–36 ■ Find the indicated power using De Moivre's Theorem.

33. $(1 + i)^{20}$
34. $(1 - \sqrt{3}i)^5$
35. $(2\sqrt{3} + 2i)^5$
36. $(1 - i)^8$

37–40 ■ Find the indicated roots. Sketch the roots in the complex plane.

37. The eighth roots of 1
38. The fifth roots of 32
39. The cube roots of i
40. The cube roots of $1 + i$

41–46 ■ Write the number in the form $a + bi$.

41. $e^{i\pi/2}$
42. $e^{2\pi i}$
43. $e^{i\pi/3}$
44. $e^{-i\pi}$
45. $e^{2+i\pi}$
46. $e^{\pi+i}$

47. Use De Moivre's Theorem with $n = 3$ to express $\cos 3\theta$ and $\sin 3\theta$ in terms of $\cos \theta$ and $\sin \theta$.

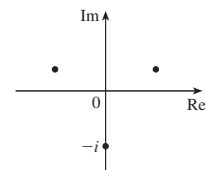
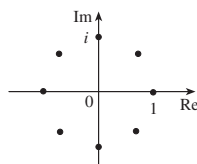
ANSWERS

1. $8 - 4i$
3. $13 + 18i$
5. $12 - 7i$
7. $\frac{11}{13} + \frac{10}{13}i$
9. $\frac{1}{2} - \frac{1}{2}i$
11. $-i$
13. $5i$
15. $12 + 5i$
17. $4i$
19. $\pm \frac{3}{2}i$
21. $-1 \pm 2i$
23. $-\frac{1}{2} \pm (\sqrt{7}/2)i$
25. $3\sqrt{2}[\cos(3\pi/4) + i\sin(3\pi/4)]$
27. $5\{\cos[\tan^{-1}(\frac{4}{3})] + i\sin[\tan^{-1}(\frac{4}{3})]\}$
29. $4[\cos(\pi/2) + i\sin(\pi/2)]$, $\cos(-\pi/6) + i\sin(-\pi/6)$, $\frac{1}{2}[\cos(-\pi/6) + i\sin(-\pi/6)]$

$$31. 4\sqrt{2}[\cos(7\pi/12) + i\sin(7\pi/12)], (2\sqrt{2})[\cos(13\pi/12) + i\sin(13\pi/12)], \frac{1}{4}[\cos(\pi/6) + i\sin(\pi/6)]$$

$$33. -1024 \quad 35. -512\sqrt{3} + 512i$$

$$37. \pm 1, \pm i, (1/\sqrt{2})(\pm 1 \pm i) \quad 39. \pm(\sqrt{3}/2) + \frac{1}{2}i, -i$$



$$41. i \quad 43. \frac{1}{2} + (\sqrt{3}/2)i \quad 45. -e^2$$

$$47. \cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta, \sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$$