In Problems 1–6, expand the given function in a Laurent series valid for the given annular domain.

1. $f(z) = \dfrac{\cos z}{z}$, $0 < |z|$

2. $f(z) = \dfrac{z - \sin z}{z^5}$, $0 < |z|$

3. $f(z) = e^{-1/z^2}$, $0 < |z|$

4. $f(z) = \dfrac{1 - e^z}{z^2}$, $0 < |z|$

5. $f(z) = \dfrac{e^z}{z - 1}$, $0 < |z - 1|$

6. $f(z) = z \cos \dfrac{1}{z}$, $0 < |z|$

In Problems 7–12, expand $f(z) = \dfrac{1}{z(z - 3)}$ in a Laurent series valid for the indicated annular domain.

7. $0 < |z| < 3$

8. $|z| > 3$

9. $0 < |z - 3| < 3$

10. $|z - 3| > 3$

11. $1 < |z - 4| < 4$

12. $1 < |z + 1| < 4$

In Problems 13–16, expand $f(z) = \dfrac{1}{(z - 1)(z - 2)}$ in a Laurent series valid for the given annular domain.

13. $1 < |z| < 2$

14. $|z| > 2$

15. $0 < |z - 1| < 1$

16. $0 < |z - 2| < 1$

In Problems 17–20, expand $f(z) = \dfrac{z}{(z + 1)(z - 2)}$ in a Laurent series valid for the given annular domain.

17. $0 < |z + 1| < 3$

18. $|z + 1| > 2$

19. $1 < |z| < 2$

20. $0 < |z - 2| < 3$

In Problems 21 and 22, expand $f(z) = \dfrac{1}{z(1 - z)^2}$ in a Laurent series valid for the given annular domain.

21. $0 < |z| < 1$

22. $|z| > 1$