Practice Problems

1. Ten percent of computer parts produced by a certain supplier are defective. What is the probability that a sample of 10 parts contains more than 3 defective ones?

2. On the average, two tornadoes hit major U.S. metropolitan areas every year. What is the probability that more than five tornadoes occur in major U.S. metropolitan areas next year?

   (As you probably saw in the news, a real tornado occurred near downtown Dallas on Sep 8, 2010.)

3. A lab network consisting of 20 computers was attacked by a computer virus. This virus enters each computer with probability 0.4, independently of other computers.

   a) Find the probability that the virus enters at least 10 computers.

   b) A computer manager checks the lab computers, one after another, to see if they were infected by the virus. What is the probability that she has to test at least 6 computers to find the first infected one?

4. On the average, 1 computer in 800 crashes during a severe thunderstorm. A certain company had 4,000 working computers when the area was hit by a severe thunderstorm.

   a) Compute the expected value and variance of the number of crashed computers.

   b) Compute the probability that less than 10 computers crashed.

   c) Compute the probability that exactly 10 computers crashed.

   (You may use a suitable approximation.)

5. A baker put 500 raisins into dough, mixed well, and made 100 cookies. You take a random cookie. What is the probability of finding at least 4 raisins in it?

   (Hint: use Poisson approximation to the Binomial distribution of the number of raisins in one cookie.)

6. Identical computer components are shipped in boxes of 5. About 15% of components have defects. Boxes are tested in a random order.

   a) What is the probability that a randomly selected box has only non-defective components?

   b) What is the probability that at least 8 of randomly selected 10 boxes have only non-defective components?

7. The Stanley Cup winner is determined in the final series between two teams. The first team to win 4 games wins the Cup. Suppose that Dallas Stars advance to the final series, and they have a probability of 0.55 to win each game, and the game results are independent of each other. Find the probability that

   a) Dallas Stars wins the Stanley Cup

   b) seven games are required to determine the Cup winner

   (Hint: Without loss of generality, you can assume that the series continues until 7 games are played, even if the Cup winner is determined earlier. This "change of Stanley Cup rules" will not change the answer to the problem!)

8. Suppose that the number of inquiries arriving at a certain interactive system follows a Poisson distribution with arrival rate of 12 inquiries per minute.
Find the probability of 10 inquiries arriving

a) in a 1-minute interval;

b) in a 3-minute interval.

c. What is the expectation and the variance of the number of arrivals during each of these intervals?

Answers:

1. $P(X > 3)$ =0.0128
2. $P(X > 5) = = 0.017$
3. a. $P(X \geq 10) = 0.2447$
   b. 0.0778
4. a. $E(X) = 5$ and $Var(X) = = 4.994$
   b. $P(X < 10) = 0.968$
   c. $P(X = 10) = 0.018$
5. $P(X \geq 4) =$ 0.735
6. **a)** $P(X = 5) =$ 0.44 .
   b) 0.0249
7. **a)** $P($ Dallas wins $) =$ 0.6083
   b) $P($ 7 games required $) =$ 0.3032
8. **a)** 0.1048
   b. 0.0000002337
   c. for a 1-min interval, $E(X) = Var(X) = 12$, and for a 3-min interval, $E(X) = Var(X) = 36$.