EXERCISES 2.1

In Problems 1–8, evaluate the given complex function f at the indicated points.

1. $f(z) = z^2\bar{z} - 2i$	**(a)** $2i$	**(b)** $1+i$	**(c)** $3-2i$
2. $f(z) = -z^3 + 2z + \bar{z}$	**(a)** i	**(b)** $2-i$	**(c)** $1+2i$
3. $f(z) = \log_e \lvert z\rvert + i\text{Arg}(z)$	**(a)** 1	**(b)** $4i$	**(c)** $1+i$
4. $f(z) = \lvert z\rvert^2 - 2\text{Re}(iz) + z$	**(a)** $3-4i$	**(b)** $2-i$	**(c)** $1+2i$
5. $f(z) = (xy - x^2) + i(3x+y)$	**(a)** $3i$	**(b)** $4+i$	**(c)** $3-5i$
6. $f(z) = e^z$	**(a)** $2-\pi i$	**(b)** $\dfrac{\pi}{3}i$	**(c)** $\log_e 2 - \dfrac{5\pi}{6}i$
7. $f(z) = r + i\cos^2\theta$	**(a)** 3	**(b)** $-2i$	**(c)** $2-i$
8. $f(z) = r\sin\dfrac{\theta}{2} + i\cos 2\theta$	**(a)** -2	**(b)** $1+i$	**(c)** $-5i$

In Problems 9–16, find the real and imaginary parts u and v of the given complex function f as functions of x and y.

9. $f(z) = 6z - 5 + 9i$

10. $f(z) = -3z + 2\bar{z} - i$

11. $f(z) = z^3 - 2z + 6$

12. $f(z) = z^2 + \bar{z}^2$

13. $f(z) = \dfrac{\bar{z}}{z+1}$

14. $f(z) = z + \dfrac{1}{z}$

15. $f(z) = e^{2z+i}$

16. $f(z) = e^{z^2}$

In Problems 17–22, find the real and imaginary parts u and v of the given complex function f as functions of r and θ.

17. $f(z) = \bar{z}$

18. $f(z) = \lvert z\rvert$

19. $f(z) = z^4$

20. $f(z) = z + \dfrac{1}{z}$

21. $f(z) = e^z$

22. $f(z) = x^2 + y^2 - yi$

EXERCISES 2.2

In Problems 1–8, proceed to find the image S' of the set S under the given complex mapping $w = f(z)$.

1. $f(z) = \bar{z}$; S is the horizontal line $y = 3$

2. $f(z) = \bar{z}$; S is the line $y = x$

3. $f(z) = 3z$; S is the half-plane $\text{Im}(z) > 2$

4. $f(z) = 3z$; S is the infinite vertical strip $2 \leq \text{Re}(z) < 3$

5. $f(z) = (1+i)z$; S is the vertical line $x = 2$

6. $f(z) = (1+i)z$; S is the line $y = 2x + 1$

7. $f(z) = iz + 4$; S is the half-plane $\text{Im}(z) \leq 1$

8. $f(z) = iz + 4$; S is the infinite horizontal strip $-1 < \text{Im}(z) < 2$

In Problems 9–14, find the image of the given line under the complex mapping $w = z^2$.

9. $y = 1$

10. $x = -3$

11. $x = 0$

12. $y = 0$

13. $y = x$

14. $y = -x$

EXERCISES 2.3

In Problems 1–6, (**a**) find the image of the closed disk $|z| \leq 1$ under the given linear mapping $w = f(z)$ and (**b**) represent the linear mapping with a sequence of plots

1. $f(z) = z + 3i$

2. $f(z) = z + 2 - i$

3. $f(z) = 3iz$

4. $f(z) = (1 + i)z$

5. $f(z) = 2z - i$

6. $f(z) = (6 - 5i)z + 1 - 3i$

In Problems 7–12, (**a**) find the image of the triangle with vertices 0, 1, and i under the given linear mapping $w = f(z)$ and (**b**) represent the linear mapping with a sequence of plots

7. $f(z) = z + 2i$

8. $f(z) = 3z$

9. $f(z) = e^{i\pi/4}z$

10. $f(z) = \frac{1}{2}i\,z$

11. $f(z) = -3z + i$

12. $f(z) = (1 - i)z - 2$

In Problems 13–16, express the given linear mapping $w = f(z)$ as a composition of a rotation, magnification, and a translation.

13. $f(z) = 3iz + 4$

14. $f(z) = 5\left(\cos\dfrac{\pi}{5} + i\sin\dfrac{\pi}{5}\right)z + 7i$

15. $f(z) = -\dfrac{1}{2}z + 1 - \sqrt{3}i$

16. $f(z) = (3 - 2i)z + 12$