EXERCISES 9

1. Large sheets of metal have faults in random positions but on average have 1 fault per 10m^2.

   What is the probability that a sheet 5m×8m will have at most one fault?

2. If 250 litres of water are known to be polluted with 10^6 bacteria what is the probability that a sample of 1cc of the water contains no bacteria?

3. Suppose vehicles arrive at a signalised road intersection at an average rate of 360 per hour and the cycle of the traffic light is set at 40 seconds. In what percentage of cycles will the number of vehicles arriving be (a) exactly 5, (b) less than 5? If, after the lights change to green, there is time to clear only 5 vehicles before the signal changes to red again, what is the probability that waiting vehicles are not cleared in one cycle?

4. Previous results indicate that 1 in 1000 transistors are defective on average.

   (a) Find the probability that there are 4 defective transistors in a batch of 2000.
   (b) What is the largest number, N, of transistors that can be put in a box so that the probability of no defectives is at least 1/2?

5. A manufacturer sells a certain article in batches of 5000. By agreement with a customer the following method of inspection is adopted: A sample of 100 items is drawn at random from each batch and inspected. If the sample contains 4 or fewer defective items, then the batch is accepted by the customer. If more than 4 defectives are found, every item in the batch is inspected. If inspection costs are 75p per hundred articles, and the manufacturer normally produces 2% of defective articles, find the average inspection costs per batch.

6. A book containing 150 pages has 100 misprints. Find the probability that a particular page contains (a) no misprints, (b) 5 misprints, (c) at least 2 misprints, (d) more than 1 misprint.

7. For a particular machine, the probability that it will break down within a week is 0.009. The manufacturer has installed 800 machines over a wide area. Calculate the probability that (a) 5, (b) 9, (c) less than 5, (d) more than 4 machines breakdown in a week.

8. At a given university, the probability that a member of staff is absent on any one day is 0.001. If there are 800 members of staff, calculate the probabilities that the number absent on any one day is (a) 6, (b) 4, (c) 2, (d) 0, (e) less than 3, (f) more than 1.

9. The number of failures occurring in a machine of a certain type in a year has a Poisson distribution with mean 0.4. In a factory there are ten of these machines. What is

   (a) the expected total number of failures in the factory in a year?
   (b) the probability that there are fewer than two failures in the factory in a year?

10. A factory uses tools of a particular type. From time to time failures in these tools occur and they need to be replaced. The number of such failures in a day has a Poisson distribution with mean 1.25. At the beginning of a particular day there are five replacement tools in stock. A new delivery of replacements will arrive after four days. If all five spares are used before the new delivery arrives then further replacements cannot be made until the delivery arrives. Find

   (a) the probability that three replacements are required over the next four days.
   (b) the expected number of replacements actually made over the next four days.

Answers

1. 0.0916

2. 0.0183

3. (a) 0 .15629   (b) 0.6288

4. (a) 0.09022   (b) 693 or less

5. 268.5p

6. (a) 0.51342m   (b) 0.00056,   (c) 0.14430,   (d) 0.14430

7. (a) 0.12038,   (b) 0.10698,   (c) 0.15552,   (d) 0.84448

8. (a) 0.00016,   (b) 0.00767,   (c) 0.14379,   (d) 0.44933,   (e) 0.95258,   (f) 0.19121

9. (a) 4   (b) 0.0916

10. (a) 0.1404 (b) 4.123