EXERCISES 4.3

Complex Trigonometric Functions

In Problems 1–8, express the value of the given trigonometric function in the form $a + ib$.

1. $\sin(4i)$

2. $\cos(-3i)$

3. $\cos(2 - 4i)$

4. $\sin\left(\frac{\pi}{4} + i\right)$

5. $\tan(2i)$

6. $\cot(\pi + 2i)$

7. $\sec\left(\frac{\pi}{2} - i\right)$

8. $\csc(1 + i)$

In Problems 9–12, find all complex values z satisfying the given equation.

9. $\sin z = i$

10. $\cos z = 4$

11. $\sin z = \cos z$

12. $\cos z = i \sin z$

In Problems 13–16, verify the given trigonometric identity.

13. $\sin(-z) = -\sin z$

14. $\cos(z_1 + z_2) = \cos z_1 \cos z_2 - \sin z_1 \sin z_2$

15. $\overline{\cos z} = \cos \bar{z}$

16. $\sin\left(z - \frac{\pi}{2}\right) = -\cos z$

In Problems 17–20, find the derivative of the given function.

17. $\sin\left(z^2\right)$

18. $\cos\left(ie^z\right)$

19. $z \tan \frac{1}{z}$

20. $\sec\left(z^2 + (1 - i)z + i\right)$

Complex Hyperbolic Functions

In Problems 21–24, express the value of the given hyperbolic function in the form $a + ib$.

21. $\cosh(\pi i)$

22. $\sinh\left(\frac{\pi}{2} i\right)$

23. $\cosh\left(1 + \frac{\pi}{6} i\right)$

24. $\tanh(2 + 3i)$

In Problems 25–28, find all complex values z satisfying the given equation.

25. $\cosh z = i$

26. $\sinh z = -1$

27. $\sinh z = \cosh z$

28. $\sinh z = e^z$

In Problems 29–32, verify the given hyperbolic identity.

29. $\cosh^2 z - \sinh^2 z = 1$

30. $\sinh(z_1 + z_2) = \sinh z_1 \cosh z_2 + \cosh z_1 \sinh z_2$

31. $|\sinh z|^2 = \sinh^2 x + \sin^2 y$

32. $\operatorname{Im}(\cosh z) = \sinh x \sin y$

In Problems 33–36, find the derivative of the given function.

33. $\sin z \sinh z$

34. $\tanh z$

35. $\tanh(iz - 2)$

36. $\cosh\left(iz + e^{iz}\right)$