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- The $3x + 1$ Problem

1.0.2

- Name
- Date
- University, department, etc

1.1 Overview

1.1.1

- What is the $3x + 1$ Problem?
- What is the Collatz conjecture?
- What is cool about it?
- History of the conjecture
- closer look at attributes of it
  - plotting graphs?
  - cycles
  - stochastic approximations
  - height of the graph
  - stopping time

1.2 What is the $3x + 1$ Problem?

1.2.1

- based on the Collatz function $C(x)$
- often written in literature as the $3x + 1$ function $T(x)$

1.2.2

- number theoretic function – deals only with integers
- map of positive natural numbers to the same
- stopping time
- total stopping time
- trajectory of a number

1.2.3 Stopping time

1.2.4 Total Stopping time

1.2.5 Trajectory

- show the graph of a cool one?

1.3 What is the collatz conjecture?

1.3.1 Possible behavior

- integer function, so three possible paths

1.3.2 The Conjecture

- for each natural number the collatz sequence contains one
- alternative ways to phrase that observation
- many unproven subsidiary conjectures

1.3.3 research areas

- Lagarias describes applications in these fields

1.4 What is cool about it?

1.4.1

- Paul Erdos quote

1.4.2

- the problem itself is not that important does not have immediate applications
- general type of functions is pretty popular right now
- is seen as difficult because it is so random where maths generally needs order to prove things
- it is simple to state and hard to prove which is cool
- ...

1.5 History of the Conjecture

1.5.1 background

- many different names based on people who studied it
- started with Collatz etc
- how research developed
- today we have over 10ˆ20 numbers verified

1.6 Attributes of the function

1.6.1 Cycles

- non-trivial cycle at {1}
- if we expand to all integers we get three more cycles
- Ch had a cycle of at least 272 mil in length, Lagarias say 10 billion
- Garner proved at least 10s of thousands

1.6.2 Stochastic approximations

- show the pseudo-randomness with the help of a graph
- interesting because stochastic models are used to approach deterministic systems
- we assume that the number of add iterated and even iterates is about the same
- because it seems random people are using probability distributions to describe groups of these functions

1.6.3 Height of the Graph

- height can be called the cardinality of the trajectory?
- how many approximation of the height of a function
- graph actual height of the function vs the approximation?

1.6.4 Stopping time

- most ints have large stopping times, even though they can be very large
- average stopping time for odd integers should be around 9.477955
- general total stopping time estimation
- total stopping time is equal to the number of even iterates in the sequence
- upper bound for the total stopping time is 41. . . . log n – suggests that all sequences are finite
- graph the stopping times for some functions vs their approximations?

1.6.5

- logarithmically the slope of the function is equal x
- most trajectories follow that shape
- some are split and more interesting
- iterates can be arbitrarily larger than the starting values
- sum of even ints equals the sum of odd ints plus the number of odd ints?