Preliminary MIDTERM EXAM, Spring - 2019

COMPLEX VARIABLES

1. Evaluate the expression and write your answer in the form $a+bi$

a) $\dfrac{(2-3i)^2}{2+3i}$ b) $(6+i)(7-2i)$

2. Find z^5 if $z=\dfrac{1+i}{1-i}$

3. Find all solutions $z^3=\sqrt{3}-3i$

4. Find the real and imaginary part of the following functions and evaluate them at point $z=1+i$

a) $\dfrac{1}{(z+1)^2}$ b) $z^2+3iz-3$

5. Find the limits.

a) $\lim\limits_{z\to 0}\dfrac{\operatorname{Re} z^2}{|z|^2}$ b) $\lim\limits_{z\to i}\dfrac{z^2+iz+2}{z-i}$

6. Use the Cauchy-Riemann equations to show that the given function is analytic and find $f'(i)$ if it possible

a) $f(z)=3z^2-iz^3$ c) $f(z)=\dfrac{5x}{2+i}-3iy$

b) $f(z)=4z^3+i\bar{z}$ b) $f(z)=e^{3z}+(2-i)z^2$

7. Verify if function *u(x,y)* is harmonic and find harmonic conjugate if it possible.

a) $u(x,y)=y-\dfrac{y}{x^2+y^2}$

b) $u(x,y)=x^2+y^2-5$