

NUMERICAL METHODS

- 1. Mathematical modeling and computational mathematics.
- 2. Vector's space and mesh functions, norm and its properties, equivalent norms in finite-dimensional space.
- 3. Interpolation by using the Lagrange polynomial. Error estimation. Convergence of interpolation process.
- 4. Optimal nodes selection for the polynomial interpolation. Chebyshev polynomials.
- 5. Spline interpolation. Definition of a spline. Linear spline construction. Error estimation.
- 6. Spline interpolation. Definition of a spline. Cubic spline construction. Error estimation.
- 7. Quadrature formula of composite type. Formula of rectangles, errors estimation.
- 8. Quadrature formula of composite type. Trapezoidal rule, error estimation.
- 9. Quadrature formula of composite type. Simpson's formula, error estimation.
- 10. Round-off errors and computer arithmetic. Relative error estimation. Finite-digit arithmetic and relative errors for the arithmetic operations.
- 11. Direct methods for solving systems of linear algebraic equations. Tridiagonal matrices, Double Sweep Method, analysis of stability and correctness.
- 12. Iterative methods for solving nonlinear equations. Simple iteration method and its geometric interpretation. Theorem on convergence for simple iteration method.
- 13. Iterative methods for solving nonlinear equations. Newton's method and its geometric interpretation. Modified versions of the Newton's method, secant method, its geometrical interpretation.
- 14. Runge-Kutta methods for solving nonlinear differential equations. The general scheme of explicit N-stage method. Runge-Kutta methods of the first and second orders.



1. Approximate the following integral

$$\int_{0}^{2} (2x-1)dx,$$

using the Trapezoidal rule (rectangle rule with right point, left point, central point. Simpson's formula), on uniform grid with number of nodes n=3 (n=5). Compare the result to the exact value of the integral.

- 2. Construct the Lagrange polynomial that has values 2, 1, 3 at points 0, 1, and 2.
- 3. Construct a linear spline (cubic spline) that has values 2, 1, 3, at points 0,1, and 2.
- 4. Find the norms $\|\vec{u}\|_1$, $\|\vec{u}\|_2$, $\|\vec{u}\|_{\infty}$ of the vector $\vec{u} = (1,2,3)$.
- 5. Let us consider the equation f(x)=0, where $f(x)=x^3$, and Newton's method (the method of simple iteration). Build a formula for the iterative process and locate the area of convergence of the method to the root of the equation.