ACTUARIAL MATHEMATICS HOMEWORK 6

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Parmenter Exercises 3

3–3.

$$
\begin{aligned}
i &= 0.13 \\
X \cdot a_{\overline{12}|} &= 6,500 \\
X &= \frac{6,500}{a_{\overline{12}|}} \\
&= \frac{6,500}{\frac{1-(1/1.13)^{12}}{0.13}} \\
X &= 1,098.40955
\end{aligned}
$$

3–4.

$$
\begin{aligned}
i &= 0.13 \\
i^{(12)} &= 0.12284 \\
X \cdot a_{\overline{144}|} &= 6,500 \\
X &= \frac{6,500}{a_{\overline{144}|}} \\
&= \frac{6,500}{\frac{1-(1/1.12284)^{144}}{0.12284}} \\
X &= 798.46005
\end{aligned}
$$

3–5. 450 at the beginning of each year from 1977 to 1997, what is the value at the end of 1996?

$$
\begin{aligned}
i &= 0.08 \\
d &= \frac{0.08}{1.08} \\
X &= \ddot{s}_{\overline{19}|} \cdot 450 \\
&= \frac{((1.08^{19}-1))(1.08)}{d} \cdot 450 \\
X &= 20,142.88393
\end{aligned}
$$

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3–6. 1,000 p.a. for 8 years, $i = 0.08$

(a) value one year before first payment

$$X = a_{\overline{8}|} \cdot 1,000$$
$$X = 5,746.63894$$

(b) value one year after last payment

$$X = \ddot{s}_{\overline{8}|} \cdot 1,000$$
$$X = 11,487.55784$$

(c) value at the time of the fifth payment

$$X = s_{\overline{5}|} \cdot 1,000$$
$$X = 5,866.60096$$

(d) number of years until present value is double its value

$$2 \cdot a_{\overline{8}|} = a_{\overline{n}|}$$
$$2 \cdot \frac{1 - v^8}{i} = \frac{1 - v^n}{i}$$
$$2 - 2v^8 = 1 - v^n$$
$$v^n = 2v^8 - 1$$
$$n = \frac{\ln(2v^8 - 1)}{\ln(v)}$$
$$n = 32.73122$$

(e) number of years until present value is triple its value

$$3 \cdot a_{\overline{8}|} = a_{\overline{n}|}$$
$$3 \cdot \frac{1 - v^8}{i} = \frac{1 - v^n}{i}$$
$$3 - 3v^8 = 1 - v^n$$
$$v^n = 3v^8 - 2$$
$$3v^8 - 2 = -0.37919$$

This problem cannot be solved because you cannot take a logarithm of a negative number. The largest possible increase for these parameters is 2.17518. This is because for an increase of factor k we find

$$k - kv^8 = 1 - v^n$$
$$v^n = k(1 - v^8) - 1$$
$$k(1 - v^8) - 1 > 0$$
$$k < \frac{1}{1 - v^8}$$
$$k < 2.17518$$

3–7. Prove the identities

(a)

$$\begin{aligned}
a_{\overline{m+n}|} &= a_{\overline{m}|} + v^m \cdot a_{\overline{n}|} \\
\frac{1-v^{m+n}}{i} &= \frac{1-v^m}{i} + \frac{v^m(1-v^n)}{i} \\
1-v^{m+n} &= 1-v^m+v^m-v^{m+n} \\
1 &= 1 \quad \square
\end{aligned}$$

(b)

$$\begin{aligned}
a_{\overline{m-n}|} &= a_{\overline{m}|} - v^m \cdot s_{\overline{n}|} \\
\frac{1-v^{m-n}}{i} &= \frac{1-v^m}{i} - v^m a_{\overline{n}|}(1+i)^n \\
\frac{1-v^{m-n}}{i} &= \frac{1-v^m}{i} - v^m \frac{1-v^n}{i}(1+i)^n \\
1-v^{m-n} &= 1-v^m-v^m(1-v^n)(1+i)^n \\
1-v^{m-n} &= 1-v^m-v^{m-n}+v^m \\
1-v^{m-n} &= 1-v^{m-n} \\
1 &= 1 \quad \square
\end{aligned}$$

(c)

$$\begin{aligned}
s_{\overline{m+n}|} &= s_{\overline{m}|} + (1+i)^m s_{\overline{n}|} \\
a_{\overline{m+n}|}(1+i)^{m+n} &= a_{\overline{m}|}(1+i)^m + a_{\overline{n}|}(1+i)^{m+n} \\
(a_{\overline{m}|} + v^m \cdot a_{\overline{m}|})(1+i)^{m+n} &= \\
a_{\overline{m}|}(1+i)^{m+n} + (1+i)^{-m} \cdot a_{\overline{m}|}(1+i)^{m+n} &= \\
a_{\overline{m}|}(1+i)^{m+n} + a_{\overline{n}|}(1+i)^n &= a_{\overline{m}|}(1+i)^m + a_{\overline{n}|}(1+i)^{m+n} \\
(1-v^m)\cdot v^{-m-n} + (1-v^n)\cdot v^{-n} &= (1-v^m)\cdot v^{-m} + (1-v^n)\cdot v^{-m-n} \\
v^{-m-n} - v^{-m} + v^{-n} - 1 &= v^{-m} - 1 + v^{-m-n} - v^{-m} \\
-1 &= -1 \quad \square
\end{aligned}$$

(d)

$$\begin{aligned}
s_{\overline{m-n}|} &= s_{\overline{m}|} - (1+i)^m a_{\overline{n}|} \\
a_{\overline{m-n}|} v^{-m+n} &= a_{\overline{m}|} v^{-m} - a_{\overline{n}|} v^{-m} \\
(1-v^{m-n})v^{-m+n} &= (1-v^m)v^{-}m - v^m(1-v^n) \\
v^{-m+n} - 1 &= v^{-m} - 1 - v^{-m} + v^{-m+n} \\
-1 &= -1 \quad \square
\end{aligned}$$

3–13. Account at 25 years is 85,000. For the first 10 years, $1,000$ are deposited. For the next 15 years $1,000 + X$ is deposited yearly. Find X if $i = 0.07$

$$\begin{aligned}85,000 &= 1,000 \cdot s_{\overline{10}|} \cdot (1+0.07)^{15} + s_{\overline{15}|} \cdot (1,000 + X)\\ X &= \frac{85,000 - 1,000 \cdot s_{\overline{10}|} \cdot (1.07)^{15} - 1,000 \cdot s_{\overline{15}|}}{s_{\overline{15}|}}\\ X &= 865.57138\end{aligned}$$

3–18. At the beginning of the first 10 years 500 are deposited. At the end of the next 15 years 300 are deposited. If $i = 0.08$, find the value of the annuity 3 years before the first payment.

$$\begin{aligned}X &= v^3 \cdot a_{\overline{10}|} \cdot 500 + v^{13} \cdot a_{\overline{15}|} \cdot 300\\ X &= 3,607.53024\end{aligned}$$

3–19. Mortgage of 60,000 paid monthly. Interest convertible semiannually is $i^{(2)} = 0.12$

$$i^{(12)} = 6(\sqrt[6]{1 + i^{(2)}} - 1)$$

(a) payments if it's paid for 25 years

$$\begin{aligned}X &= \frac{60,000}{a_{\overline{25 \cdot 12}|}}\\ X &= 6,831.93091\end{aligned}$$

(b) payments if it's paid for 20 years

$$\begin{aligned}X &= \frac{60,000}{a_{\overline{20 \cdot 12}|}}\\ X &= 6,831.93091\end{aligned}$$

(c) payments if it's paid for 10 years

$$\begin{aligned}X &= \frac{60,000}{a_{\overline{10 \cdot 12}|}}\\ X &= 6,831.94730\end{aligned}$$

3–20. Mortgage of 60,000 paid monthly. Interest convertible semiannually is $i^{(2)} = 0.16$

$$i^{(12)} = 6(\sqrt[6]{1 + i^{(2)}} - 1)$$

(a) payments if it's paid for 25 years

$$\begin{aligned}X &= \frac{60,000}{a_{\overline{25 \cdot 12}|}}\\ X &= 8,960.49930\end{aligned}$$

(b) payments if it's paid for 20 years

$$\begin{aligned}X &= \frac{60,000}{a_{\overline{20 \cdot 12}|}}\\ X &= 8,960.49930\end{aligned}$$

(c) payments if it's paid for 10 years

$$X = \frac{60,000}{a_{\overline{10\cdot 12}|}}$$

$$X = 8,960.49980$$

3–21. A man deposits 2,500 for 25 years at the beginning of each year, $i = 0.07$. Then he with draws the money in 20 annual withdrawals at $i = 0.11$. What is the size of the withdrawals?

$$2,500 \cdot \ddot{a}_{\overline{25}|} = X \cdot a_{\overline{20}|}$$

$$X = \frac{2,500 \cdot \ddot{a}_{\overline{25}|}}{a_{\overline{20}|}}$$

$$X = 3,914.61140$$

3–22. I couldn't solve this problem.

3–24. I couldn't solve this problem.