Actuarial Mathematics Homework 8

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Parmenter Exercises 3–60 to 3–66

3–60. Money is borrowed, extracted in 5 annual payments X. Repayment starts 1 year after the last extraction. It is repaid in 20 annual payments, first 100, then 200, etc. If $i = 0.132$, find X.

$$Xs_{\overline{5}|} = 100a_{\overline{20}|} + 100\left[\frac{a_{\overline{20}|} - 20v^{20}}{i}\right]$$

$$X = \frac{100a_{\overline{20}|} + 100\left[\frac{a_{\overline{20}|} - 20v^{20}}{i}\right]}{s_{\overline{5}|}}$$

$$X = 719.85057$$

3–61. I could not solve this one.

3–63. Present value of a perpetuity. After 1 year 100 is paid, then 200, etc. When 1500 is paid, the amount remains constant forever.

$$A = Pa_{\overline{15}|} + Q\left[\frac{a_{\overline{15}|} - 15v^{15}}{i}\right] + (a_{\overline{\infty}|} - a_{\overline{15}|}) \cdot 1,500//A = 100a_{\overline{15}|} + 100\left[\frac{a_{\overline{15}|} - 15v^{15}}{i}\right] + (a_{\overline{\infty}|} - a_{\overline{15}|}) \cdot$$

$$A = 12,652.20497$$

3–64. Annuity for 20 years, first payment immediately, $i = 0.11$, payments increase by 10% each year, starting with 1,000. We can find the geometric sum.

$$\ddot{a}_{\overline{20}|} = \frac{S_{20}}{(1+i)^{19}}$$

$$\ddot{a}_{\overline{20}|} = \frac{1,000(1.1)^{20}\left(\frac{1-r^20}{1-r}\right)}{(1+i)^{19}}$$

$$\ddot{a}_{\overline{20}|} = \frac{1,000(1.1)^{20}\left(\frac{1-\frac{1.1}{1.11}^20}{1-\frac{1.1}{1.11}}\right)}{(1+i)^{19}}$$

$$\ddot{a}_{\overline{20}|} = 20,215.10754$$

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3–66. Annuity for 20 years, first payment immediately, $i = 0.09$, payments are 1, 4, 9, 16, ..., starting immediately.

$$\ddot{a}_{\overline{20|}} = \frac{\sum_{j=1}^{20} j^2 (1+i)^{20-j}}{(1+i)^{19}}$$

$$\ddot{a}_{\overline{20|}} = 887.15787$$