Topic Proposal for Report No. 2: Predator-Prey Models

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1 Topic

For my second report of the semester I want to write about predator-prey models. In the 1920s Volterra and Lotka independently derived the same system of differential equations. Volterra sought to describe populations of two interacting species (predator and prey fish), while Lotka tried to describe oscillating chemical reactions [2]. In both cases it is simpler to describe how populations of fish or concentrations of chemicals change than it is to describe then directly and thus differential equations got involved. The Lotka-Volterra model was named in their honor:

$$\begin{cases} \dfrac{dx}{dt} = ax - bxy \\ \dfrac{dy}{dt} = -cy + dxy. \end{cases}$$

Following [2], $x(t)$ is the prey population and $y(t)$ is the predator population. The other variables (all > 0) are defined as:

- a – natural growth rare of prey with no predators,
- b – effect of predators on prey (prey being eaten),
- c – natural death rate of predator with no prey,
- d – effect of prey on predator population (predators multiplying if they have food).

The Lotka-Volterra model will probably be the focus of my report, but other predator-prey models like the Kermack-McKendrick model (accounts for herd immunity and applies to epidemics) and the Jacob-Monod Model (accounts for concentrations of e.g. bacteria and not number of animals) [3] can be covered. A three-species food chain as discussed in [2] is also interesting. If the unexpected nature of predator-prey models should be the focus of my report these equations would be a good thing to focus on.

2 Elements of the Report

- Introduction: brief overview, introduction to topic,
- History: how were the equations developed, by whom

- Finding solutions: with examples
  - Analytic approaches
  - Numerical approaches
- Examples of one or more models computed by me
- Visualizations
  - Graphs of solutions
  - Phase portraits
- Implications and Applications / Conclusion: what is the importance of these equations, where are they applied (specifically)

References

[1] L. Billard. On lotka-volterra predator prey models. *Journal of Applied Probability*, 14(2):375–381, 1977. Retrieved from: `http://www.jstor.org/stable/3213008`.

[2] E. Chauvet, J. E. Paullet, J. P. Previte, and Z. Walls. A lotka-volterra three-species food chain. *Mathematics Magazine*, 75(4):243–255, 2002. Retrieved from: `http://www.jstor.org/stable/3219158`.

[3] F. Hoppensteadt. Predator-prey model. *Scholarpedia*, 1(10):1563, 2006. Retrieved from: `http://www.scholarpedia.org/article/Predator-prey_model`.

[4] G. Israel. On the contribution of volterra and lotka to the development of modern biomathematics. *History and Philosophy of the Life Sciences*, 10(1):37–49, 1988. Retrieved from: `http://www.jstor.org/stable/23328998`.

[5] S. Kingsland. Alfred j. lotka and the origins of theoretical population ecology. *Proceedings of the National Academy of Sciences of the United States of America*, 112(31):9493–9495, 2015. Retrieved from: `https://www.jstor.org/stable/10.2307/26464246`.

[6] P. J. Wangersky. Lotka-volterra population models. *Annual Review of Ecology and Systematics*, 9:189–218, 1978. Retrieved from: `https://www.jstor.org/stable/2096748`.