ACTUARIAL MATHEMATICS HOMEWORK 1 AND 2

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Homework 1

1–1.

(a) 20,720.00000
(b) 22,216.24052
(c) 14,202.52055
(d) 16,497.81725

1–2.

(a) 23,590.81417
(b) 22,114.94444
(c) 24,290.01969

1–3. It is not true. It is only true for $n \geq 2$.

$$\begin{aligned} i_1 &= 2 \\ i_2 &= \frac{4}{3} \\ &\vdots \end{aligned}$$

1–4.

(a) $a(0) = \sqrt{1}$; $a(1) = \sqrt{1 + (i^2 + 2i)} = \sqrt{(i+1)^2} = 1 + i$
(b) square roots are continuous and always increasing
(c) $a(1) = \sqrt{i^2 + 2i + 1}$, $a(0.5) = \sqrt{0.25i^2 + 0.5i + 1}$, so $a(0.5) < a(1)$
$a(2) = \sqrt{4i^2 + 8i + 1}$, here $a(1) < a(2)$
(d) for $t = 1$ both equations are equal, $(1 + i)^t$ grows quicker, so for all $t > 1$ this holds

1–5.

(a) for an exponential function $a(0) = 1$, i_n can be found

$$\begin{aligned} i_n &= \frac{a(n) - a(n-1)}{a(n-1)} \\ &= \frac{(1+i)^n - (1+i)^{n-1}}{(1+i)^{n-1}} \\ &= ((1+i)^n - (1+i)^{n-1}) \cdot (1+i)^{-n+1} \\ &= (1+i)^n \cdot (1+i)^{-n+1} - (1+i)^{n-1} \cdot (1+i)^{-n+1} \\ &= (1+i)^1 - (1+i)^0 \\ &= 1 + i - 1 \\ i_n &= i \quad \square \end{aligned}$$

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(b) yes, as the above proof works for all n

1–13.

(a) 681.472
(b) 0.13636
(c) 681.479 – yes it is the same, the lost precision in (b) is the reason for the slight difference

1–14.

(a) 7,092.8357
(b) 6,501,65722
(c) we learned that $d < i$ for one problem. Here they are equal, meaning that if we convert $d = 0.12$ to i it will be larger. d here is equivalent to $i = 0.1363$ which explains why they numbers are different. Higher compound interest rates mean that value increases faster but if one goes back in time they decrease faster too. That's why (b) is lower

1–15.

(a) See FIGURE 1.

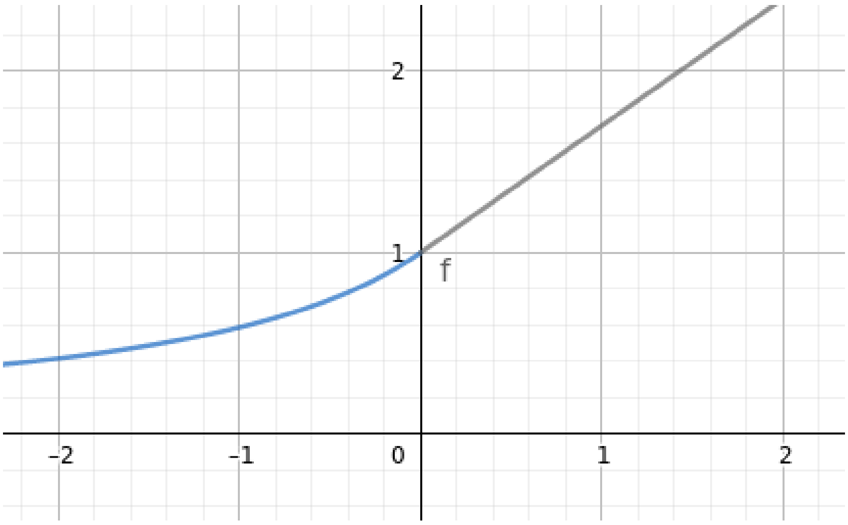

FIGURE 1. Graph of $a(t)$ for simple interest, $i = 0.7$

(b) $1 - it$ is not correct because it does not follow from the formula for accumulation.

1–16.

(a) we proved earlier that i_n is constant for compound interest, and we can express d_n in terms of i_n

$$d = \frac{i}{1+i}$$

1–17.

(a)

$$\begin{aligned} d &= i \cdot v \\ &= i \cdot \frac{1}{1+i} \\ d &= \frac{i}{1+i} \quad \square \end{aligned}$$

(b)

$$\begin{aligned}
d &= 1 - v \\
&= 1 - \frac{1}{1+i} \\
&= \frac{1+i}{1+i} - \frac{1}{1+i} \\
&= \frac{1+i-1}{1+i} \\
d &= \frac{i}{1+i} \quad \square
\end{aligned}$$

(c)

$$\begin{aligned}
i - d &= i \cdot d \\
\frac{d}{1-d} - d &= \frac{d}{1-d} \cdot d \\
\frac{d}{1-d} - \frac{(1-d)d}{1-d} &= \frac{d^2}{1-d} \\
\frac{d - d + d^2}{1-d} &= \frac{d^2}{1-d} \\
\frac{d^2}{1-d} &= \frac{d^2}{1-d} \quad \square
\end{aligned}$$

(d)

$$\begin{aligned}
\frac{1}{d} - \frac{1}{i} &= 1 \\
\frac{1}{d} - \frac{1}{\frac{d}{1-d}} &= 1 \\
\frac{1}{d} - \frac{1-d}{d} &= 1 \\
\frac{1-1+d}{d} &= 1 \\
\frac{d}{d} &= 1 \quad \square
\end{aligned}$$

(e)

$$\begin{aligned}
d(1 + \frac{i}{2}) &= i(1 - \frac{d}{2}) \\
d + \frac{d \cdot i}{2} &= i - \frac{i \cdot d}{2} \\
d - i &= -\frac{2 \cdot d \cdot i}{2} \\
d - i &= -d \cdot i \\
i - d &= d \cdot i \qquad \text{see (c)} \quad \square
\end{aligned}$$

(f)

$$i\sqrt{1-d} = d\sqrt{1+i}$$
$$i^2(1-d) = d^2(1+i)$$
$$1+i = \frac{i}{d}, \quad 1-d = \frac{d}{i}$$
$$i^2(\frac{d}{i}) = d^2(\frac{i}{d})$$
$$di = di \quad \square$$

1–18.

(a)

$$\frac{d^3}{(1-d)^2} = \frac{d^3}{(d/i)^2} = di^2$$

(b)

$$\frac{(i-d)^2}{1-v} = \frac{(i^2/(1+i))^2}{d} = \frac{(i^2/(i/d))^2}{d} = \frac{(di)^2}{d} = di^2$$

(c) **this one is different**

$$(i-d)d = \frac{i^2}{1+i}d = \frac{i^2}{i/d}d = d^2i$$

(d)

$$i^3 - i^3d = i^3(1-d) = i^3(\frac{d}{i}) = di^2$$

(e)

$$di^2$$

Homework 2

1–20.

(a) $i = 12.550881\%$
(b) $i = 12.557092\%$ best for the investor
(c) $i = 12.44\%$ best for the trust fund

1–21. After 21 months at $i^{(4)} = 0.16$ \$3000 become \$3,947.795.

1–22.

(a) $i = 12.36\%$
(b) $d^{(4)} = 11.4857\%$
(c) $i^{(12)} = 11.7106\%$

1–23.

(a) $i = 7\%$ for 6 months
(b) $i = 14\%$
(c) $i = 14.49\%$ per year
(d) $d^{(4)} = 6.802\%$

1–24.

$$\begin{aligned}
1 + \frac{i^{(n)}}{n} &= \frac{1 + \frac{i^{(6)}}{6}}{1 + \frac{i^{(8)}}{8}} \\
1 + \frac{i^{(n)}}{n} &= \sqrt[n]{1+i} \\
\sqrt[n]{1+i} &= \frac{\sqrt[6]{1+i}}{\sqrt[8]{1+i}} \\
(1+i)^{1/n} &= (1+i)^{1/6} \cdot (1+i)^{-1/8} \\
(1+i)^{1/n} &= (1+i)^{8/48-6/48} = (1+i)^{1/24} \\
e^{\ln(1+i)^{1/n}} &= e^{\ln(1+i)^{1/24}} \\
e^{(1/n)\cdot\ln(1+i)} &= e^{(1/24)\cdot\ln(1+i)} \\
(1/n)\cdot\ln(1+i) &= (1/24)\cdot\ln(1+i) \\
1/n &= 1/24 \\
n &= 24
\end{aligned}$$

1–25.

$$
\begin{aligned}
d^{(7)}: \quad 1-d &= \left[1-\frac{d^{(m)}}{m}\right]^{m} \\
&= \left[1-\frac{d^{(7)}}{7}\right]^{7} \\
d^{(7)} &= 7\left(1-\sqrt[7]{1-d}\right) \\
1-d &= (1+i)^{-1} \\
i^{(5)}: \quad (1+i)^{-1} &= \left(\left[1+\frac{i^{(m)}}{m}\right]^{m}\right)^{-1} \\
&= \left[1+\frac{i^{(5)}}{5}\right]^{-5} \\
d^{(7)} &= 7\left(1-\sqrt[7]{\left[1+\frac{i^{(5)}}{5}\right]^{-5}}\right) \\
d^{(7)} &= 7\left(1-\left[1+\frac{i^{(5)}}{5}\right]^{-5/7}\right) \\
d^{(7)} &= 7-7\left[1+\frac{i^{(5)}}{5}\right]^{-5/7}
\end{aligned}
$$

1–26.

$$
\begin{aligned}
v\left(1+\frac{i^{(3)}}{3}\right) &= \left(1+\frac{i^{(30)}}{30}\right)\left(1-\frac{d^{(5)}}{d}\right)\sqrt{1-d} \\
(1-d)\left(\sqrt[3]{1+i}\right) &= \left(\sqrt[30]{1+i}\right)\left(\sqrt[5]{1-d}\right)\sqrt{1-d} \\
(1-d)\cdot\sqrt[3]{1+i} &= \sqrt[30]{1+i}\cdot(1-d)^{1/5}\cdot(1-d)^{1/2} \\
(1-d)\cdot(1+i)^{1/3} &= (1+i)^{1/30}\cdot(1-d)^{7/10} \\
(1+i)^{-1/30}\cdot(1+i)^{1/3} &= (1-d)^{7/10}\cdot(1-d)^{-1} \\
(1+i)^{3/10} &= (1-d)^{-3/10} \\
(1-d)^{-1} &= (1+i) \\
\left((1-d)^{-1}\right)^{3/10} &= (1-d)^{-3/10} \\
(1-d)^{-3/10} &= (1-d)^{-3/10} \quad \square
\end{aligned}
$$