American University-Central Asia

SYLLABUS
NUMERICAL METHODS FOR EQUATIONS OF MATHEMATICAL PHYSICS
(MAT 410, ID 3968)

Fall 2020 (September 1 – December 12)

1. Instructor: Sklyar Sergey Nikolaevich - Professor, Doctor of Science in Physics and Mathematics, Office: 415, Phone: +998(312)91-50-00(Ext: 426), E-mail: sklyar_s@auca.kg

2. Class meetings and Volume of academic load: 2 lessons per week (one lesson = 75 minutes; lecture and laboratory practice; total 6 credits, 15 working weeks).

3. Consultations: according to the preliminary arrangement with instructor (room 415).

4. Brief course description: Course will focus on the study of both classical and modern numerical methods for solving the equations of mathematical physics. We will consider boundary-value and initial-boundary-value problems for the advection-diffusion, Poisson and non-stationary heat equations.

5. Prerequisites: Equations of mathematical physics (MAT 360), Numerical methods (MAT 407).

6. Textbooks:

- Richard L. Burden, J. Douglas Faires. Numerical Analysis. Brooks/Cole Cengage Learning, Ninth Edition, 2011.
- Alfio Quarteroni, Riccardo Sacco, Fausto Saleri. Numerical Mathematics. 2000 Springer-Verlag New York, Inc.
- Additional materials are presented on Google Classroom.

7. Objectives: There are several distinct goals of the course. One definite goal is to prepare the students to be competent practitioners, capable of solving a large range of problems, evaluating numerical results and understanding how and why results might be bad. Another goal is to prepare the applied mathematics students to take additional courses (such as courses of mathematical modeling in geophysics or fluid dynamics) and to write theses in applied mathematics.

8. Requirements and knowledge evaluation:

Grading

Grades will be based on a total of 100 points, coming from**:**

- Performance of original laboratory projects – 80 points (max),
- Final Exam- theory and typical problems – 20 points (max).

The final grade of the student will calculated in conformity with a following scale:

$$\mathbf{0 \le F \le 40 < D \le 45 < C\text{-} \le 50 < C \le 60 < C\text{+} \le 65}$$

$$\mathbf{65 < B\text{-} \le 70 < B \le 80 < B\text{+} \le 85 < A\text{-} \le 90 < A \le 100.}$$

Attendance and class activity are expected. Attendance affects grades: missing 10 or more classes for any reasons will result in a grade of “F” in the course. Policy and Schedule of projects delivery and defense is defined by the instructor additionally. Instructor reserves the right to change or modify this syllabus as needed; any changes will be announced during class.

9. Course content and tentative academic calendar:

I. Lectures

- ☛ ***Introduction and theoretical foundations.*** Spaces of mesh functions, norms of mesh functions, equivalent norms. Finite-difference approximation of the first and second derivatives and their properties. Finite-difference approximation of a differential problem. Stability and convergence of the finite-difference problem, Lax theorem about convergence. Operators of monotone type (M-operator), stability of the finite-difference problem with mesh operator of monotone type.
- ☛ ***Boundary-value problems for the advection-diffusion equation.*** Finite-difference method. A family of difference schemes for the equation of advection-diffusion transport. The error of the approximation, a sufficient condition for stability and estimation of convergence. Schemes with central and upwind differences, schemes of Samarskii A.A. and Ilyin A.M., the numerical viscosity. Projective variant of the integral-interpolation method for the advection-diffusion problems. Approximation of general type boundary conditions.
- ☛ ***Initial-boundary-value problems for the heat equation.*** Finite-difference method. A family of finite-difference schemes for the heat problems. Explicit, Crank-Nicholson, implicit schemes. Approximation, stability and convergence of schemes. Von Neuman stability analysis.
- ☛ ***Two-dimensional Dirichlet problem for the Poisson equation.*** Finite-difference method. Approximation, stability and convergence for the appropriate elliptic difference schemes. Using iterative methods to solve two-dimensional elliptic difference equations: Jacobi relaxation and Gauss-Seidel techniques.

II. Laboratory practice

Performance of original laboratory projects (description on an electronic resource Google Classroom)

☛ ***Project 1:***
"To compare the effectiveness of various difference schemes for solving the boundary-value advection-diffusion transport problems. Programming, numerical analysis and graphic illustration of the results"
Dates: 10.09-08.10.2020

☛ ***Project 2:***
"To compare the effectiveness of various difference schemes for solving the Initial-boundary-value heat problems. Programming, numerical analysis and graphic illustration of the results"
Dates: 08.10-12.11.2020

☛ ***Project 3:***
"Using iterative methods to solve two-dimensional elliptic difference equations. Construction a test problems, programming, numerical analysis and graphic illustration of the results"
Dates: 12.11-10.12.2020