Actuarial Mathematics Homework 13

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Parmenter Exercises 5

5–1. Find a price to yield 8% convertible semiannually for a 10–year, 1,000 face value bond that has 9% convertible semiannually.

$$P = (Fr)a_{\overline{n}|i} + Cv^n$$

$$P = (1,000 \cdot 0.045)a_{\overline{20}|0.04} + 1,000v^{20}$$

$$P = 45a_{\overline{20}|0.04} + 1,000v^{20}$$

$$P = 1,067.95163$$

5–2. 1,000 par value bond, $r = 0.055$, semiannually at Jan. 1 and Jul. 1. Redeemed on Jul. 1, 2001. Bond is bought Jan. 1 1999. Find a price so it yields 12% semiannually.

$$P = (1,000 \cdot 0.055)a_{\overline{5}|i} + 1000 \cdot v^5$$

$$P = 55 \cdot a_{\overline{5}|0.06} + 1000 \cdot v^5$$

$$P = 978.93818$$

5–5.

$$P_1 = 879.58 = 50a_{\overline{n}|} + 1000v^n$$

$$v^n = \frac{879.58 - \frac{50}{i}}{1000 - \frac{50}{i}}$$

$$n = \frac{\log\left(\frac{879.58 - \frac{50}{i}}{1000 - \frac{50}{i}}\right)}{\log v} n = 22$$

$$P_2 = 70a_{\overline{22}|} + 1000v^{22} P_2 = 1120.41582$$

5–6.

$$P = 9a_{\overline{20}|} + 110v^{20}$$

$$A_1 = 8a_{\overline{20}|} + 100v^{20}$$

$$A_2 = 10a_{\overline{20}|} + 100v^{20}$$

$$P = A_1 + 0.1A_2$$

$$P = 8a_{\overline{20}|} + 100v^{20} + 0.1(10a_{\overline{20}|} + 100v^{20})$$

$$P = 9a_{\overline{20}|} + 110v^{20}$$

Date: November 30, 2020.

5–8. Using the iterative method, we can find i using

$$i = \frac{Fr(1 - v^n)}{P - Cv^n}$$
$$i = 0.03293$$

per half year

5–9. Using the iterative method for i, and $r = i + 0.01$, we use the iterative method to find i

$$i = \frac{Fr(1 - v^n)}{P - Cv^n}$$
$$i = 0.07266$$

5–10. We start with a 6% annual coupon, for 5 years, with a yield of 4%. A new bond has a 5% coupon rate, how long does it have to run to have the same yield rate.

$$P = a_{\overline{5}|}Fr + Fv^5$$
$$F = 1$$
$$P = a_{\overline{5}|} \cdot 0.06 + v^5$$
$$P = 0.05a_{\overline{n}|} + v^n$$
$$P = 0.05\frac{1 - v^n}{i} + v^n$$
$$a_{\overline{5}|}r + v^5 = \frac{0.05}{i} - \frac{0.05v^n}{i} + v^n$$
$$v^n = \frac{0.06a_{\overline{5}|} + v^5 - \frac{0.05}{i}}{1 - \frac{0.05}{i}}$$
$$n = 11.22578$$

5–11.

$$P = Fra_{\overline{n}|} + Cv^n$$
$$n = \frac{\ln\left(\frac{P - \frac{Fr}{i}}{C - \frac{Fr}{i}}\right)}{\ln v}$$
$$n = 9$$
$$2n = 18$$
$$P = 100 \cdot 0.05 \cdot a_{\overline{18}|0.075} + 100 \cdot v^{18}$$
$$P = 76.82317$$

5–13.

(a) just after the 7th coupon has been paid

$$B_7 = 45a_{\overline{13}|} + 1000v^{13}$$
$$B_7 = 1049.92824$$

(b) 4 months after the 7th coupon has been paid

$$X = B_7(1 + \frac{2}{3}i) = 1077.92633$$

(c) just before the 8th coupon is paid

$$X = B_7(1 + i) = 1091.92537$$

5–14. $P = 978.93818$

(a) June 30, 1999, 11:59 pm

$$P(1 + i) = 1037.67447$$

(b) Jul 1, 2000 12:01 am

$$P(1 + i) - Fr = 982.67447$$

(c) March 1, 2001 - value on Jan 1, 2001 plus a bit

$$X = B_5 \cdot (1 + \frac{2}{6}i) = 1015.18868$$

(d) March 1, 2001 - value on Jan 1, 2001 plus a bit

$$X = B_5 \cdot (1 + \frac{23}{24}i) = 1052.51180$$

5–16. A screenshot of my program.

Dur	Coupon	Int	PA	BV
0	0.00000	0.00000	0.00000	1028.56024
1	35.00000	25.71401	9.28599	1019.27424
2	35.00000	25.48186	9.51814	1009.75610
3	35.00000	25.24390	9.75610	1000.00000

FIGURE 1. 5–16

5–17. A screenshot of my program.

Dur	Coupon	Int	PA	BV
0	0.00000	0.00000	0.00000	986.12454
1	35.00000	39.44498	-4.44498	990.56953
2	35.00000	39.62278	-4.62278	995.19231
3	35.00000	39.80769	-4.80769	1000.00000

FIGURE 2. 5–17

5–18. A screenshot of my program.

Dur	Coupon	Int	PA	BV
0	0.00000	0.00000	0.00000	1067.95163
1	45.00000	42.71807	2.28193	1065.66970
2	45.00000	42.62679	2.37321	1063.29648
3	45.00000	42.53186	2.46814	1060.82834
4	45.00000	42.43313	2.56687	1058.26148
5	45.00000	42.33046	2.66954	1055.59194
6	45.00000	42.22368	2.77632	1052.81561
7	45.00000	42.11262	2.88738	1049.92824
8	45.00000	41.99713	3.00287	1046.92537
9	45.00000	41.87701	3.12299	1043.80238
10	45.00000	41.75210	3.24790	1040.55448
11	45.00000	41.62218	3.37782	1037.17666
12	45.00000	41.48707	3.51293	1033.66372
13	45.00000	41.34655	3.65345	1030.01027
14	45.00000	41.20041	3.79959	1026.21068
15	45.00000	41.04843	3.95157	1022.25911
16	45.00000	40.89036	4.10964	1018.14947
17	45.00000	40.72598	4.27402	1013.87545
18	45.00000	40.55502	4.44498	1009.43047
19	45.00000	40.37722	4.62278	1004.80769
20	45.00000	40.19231	4.80769	1000.00000

FIGURE 3. 5–18

5–19. Code for the program is available upon request. The above questions are solved using this program.

5–20.

$$B_{n-1} = Fra_{\overline{1}|} + Cv^1$$
$$F = C$$
$$B_{n-1} = F(ra_{\overline{1}|} + v^1)$$
$$B_{n-1} = 5000(0.03a_{\overline{1}|} + v^1)$$
$$B_{n-1} = 5024.39024$$

5–21.

$$B_{n-1} = Fra_{\overline{n-1}|} + Cv^{n-1}$$
$$\frac{B_{n-1}}{F} = \frac{r}{i} - \frac{rv^{n-1}}{i} + v^{n-1}$$
$$v^{n-1} = \frac{B_{n-1}}{F} - \frac{r}{i}$$
$$n = \frac{\ln\left(\frac{B_{n-1}}{F} - \frac{r}{i}\right)}{\ln v} + 1$$
$$n = 2$$
$$P = 45a_{\overline{2}|} + 1000v^2$$
$$P = 972.49911$$
$$D = F - P = 27.5$$

5–22.

$$\sum_{j=1}^{20} B_j \cdot i = 910.03968$$

5–23. Principal adjustment is PA

$$PA_{18} = 36$$
$$PA_{18} = Fr - B_{t-1}i$$
$$36 = Fr - i((Fr)a_{\overline{40-17}|} - Cv^{40-17})$$
$$r = \frac{36 + iCv^{23}}{F(1 - a_{\overline{23}|})}$$
$$r = 0.07375$$
$$PA_{29} = Fr - B_{t-1}$$
$$PA_{29} = 737.5 - i(737.5a_{\overline{12}|} - Cv^{12})$$
$$PA_{29} = 68.33875$$

5–24.

(a) 12% per year $\rightarrow i = \sqrt{1.12} - 1 = 0.0583$

$$P = 1000ra_{\overline{20}|} + Cv^{20}$$
$$P = 903.46609$$

(b) 1% per month $\rightarrow i = 1.01^6 - 1 = 0.06152$

$$P = 1000ra_{\overline{20}|} + Cv^{20}$$
$$P = 869.48008$$

5–25.

$$\bar{d} = \frac{\sum_{j=1}^{20} jv^j 50 + 50v^{20}1000}{\sum_{j=1}^{20} v^j 50 + 50v^{20}1000}$$

$$\bar{d} = 22.10101$$

5–26. 1000 face par value, 10 year bond, $r = 0.03$ for 5, then $r = 0.035$ for the last 5. Find the price if the following yield is anticipated

(a) earn 7% as half year

$$P = P_1 + v^{10}P_2$$

$$P = 1102.44089$$

(b) earn 14% per year, $i = \sqrt{1.14} - 1$

$$P = P_1 + v^{10}P_2$$

$$P = 1131.10854$$

5–27. 10 year bond, 1000 par value, coupon starts at 200, decreases by 20 each time, until it reaches 20.

(a) find P for $i = 0.12$

$$P = (Da)_{\overline{10|}} + 1000v^{10}P = 1046.93607$$

(b) find the yield rate if the bond is purchased at face value, solved through iteration

$$i = \frac{(10 - a_{\overline{20|}})20}{P - 1000v^{10}} i = 0.12963$$

5–28. 1000 bond, 15 year bond, 60 per coupon, callable for the last 10 dates. Find the price to guarantee a yield rate of:

(a) 7%, the yield rate is higher than the coupon rate, we should consider the last value

$$P = 1000 \cdot a_{\overline{30|}} + 1000v^{30}$$

$$P = 875.90959$$

(b) 5%, the yield rate is smaller than the coupon rate, we consider the first value

$$P = 1000 \cdot a_{\overline{20|}} + 1000v^{20}$$

$$P = 1124.62210$$

(c) 6%, in this case the redemption date does not matter. Because the coupon rate and the yield rate are the same, for a price of 1000, regardless of redemption date, the bond will yield 6%.