2.7 Nonhomogeneous ODEs

Kreyzsig

Abstract

This is a test abstract that is just supposed to fill some space here. There are two common methods for finding particular solutions : Undetermined Coefficients and Variation of Parameters. Both have their advantages and disadvantages as you will see in the next couple of sections [1].

1. Introduction

We now advance from homogeneous to nonhomogeneous linear ODEs.
Consider the second-order nonhomogeneous linear ODE

$$y'' + p(x)y' + q(x)y = r(x) \tag{1}$$

where $r(x) \neq 0$. We shall see that a "general solution" of [1] is the sum of a general solution of the corresponding homogeneous ODE

$$y'' + p(x)y' + q(x)y = 0 \tag{2}$$

and a "particular solution" of [1]. These two new terms "general solution of [1]" and "particular solution of [1]" are defined as follows.

1.1. Definition

General Solution, Particular Solution
A **general solution** of the nonhomogeneous ODE [1] on an open interval I is a solution of the form

$$y(x) = y_h(x) + y_p(x); \tag{3}$$

here, $y_h = c_1y_1 + c_2y_2$ is a general solution of the homogeneous ODE [2] on I and y_p is any solution of [1] on I containing no arbitrary constants.

A **particular solution** of [1] on I is a solution obtained from [3] by assigning specific values to the arbitrary constants c_1 and c_2 in y_h.

Our task is now twofold, first to justify these definitions and then to develop a method for finding a solution y_p of [1].

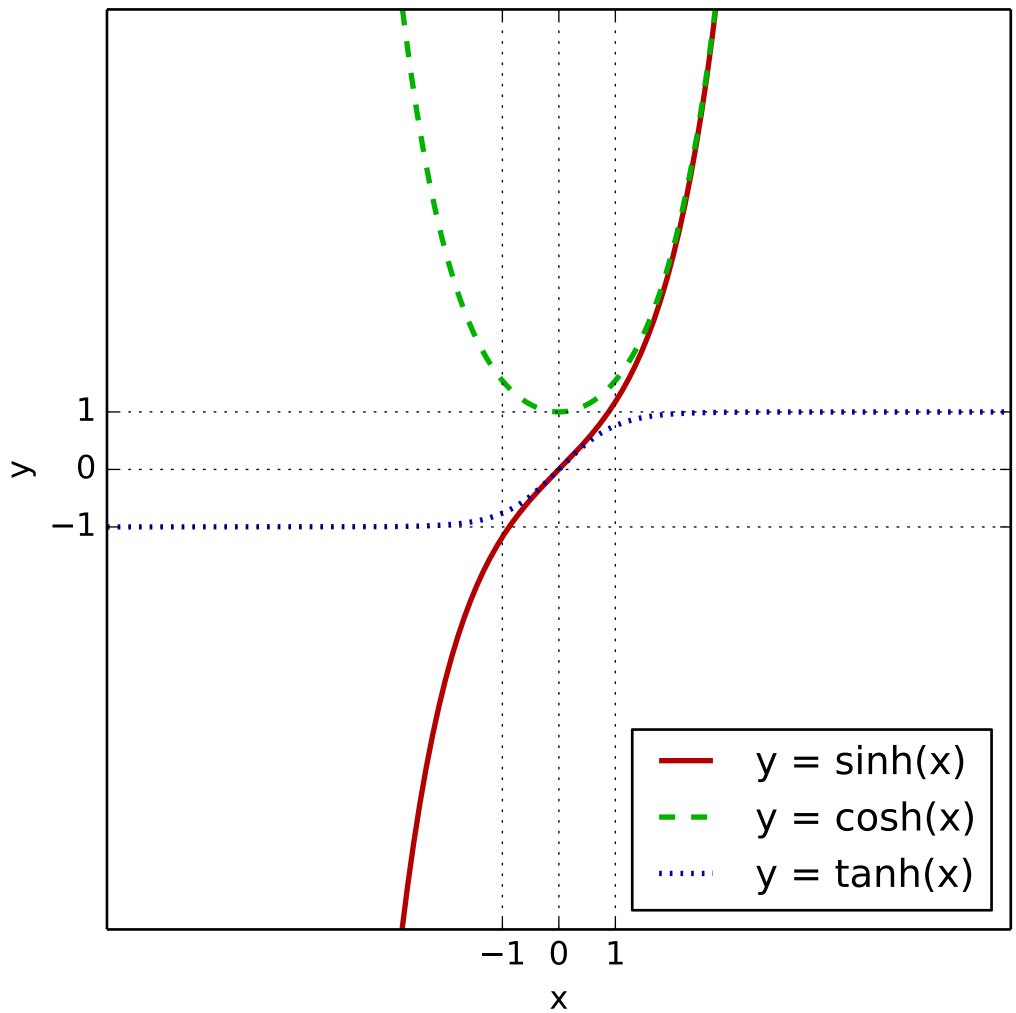
Accordingly, we first show that a general solution as just defined satisfies [1] and that the solutions of [1] and [2] are related in a very simple way.

1.2. Theorem 1

Relations of Solutions of [1] to Those of [2]

1. *The sum of a solution $\boldsymbol{y}$ of [1] on some open interval $\boldsymbol{I}$ and a solution $\tilde{\boldsymbol{y}}$ of [2] on I is a solution of [1] on I. In particular, [3] is a solution of [1] on I.*
2. *The difference of two solutions of [1] on $\boldsymbol{I}$ is a solution of [2] on $\boldsymbol{I}$*

1.3. Hyperbolic Functions



2. References

1. Kreyzsig's *Advanced Engineering Mathematics Textbook*, p. 79-
2. Paul's Online Math Notes, http://tutorial.math.lamar.edu/Classes/DE/NonhomogeneousDE.aspx
3. Wikipedia, Hyperbolic Functions, https://upload.wikimedia.org/wikipedia/commons/7/76/Sinh_cosh_tanh.svg