Actuarial Mathematics Homework 7

MORITZ M. KONARSKI

Parmenter Exercises 3–47 to 3–55

3–47. Loan of 6,000. Pay back 800 a year for as long as necessary, then a payment less than 800 at the end. First payment due in 1 year and $i = 0.11$ – find the number of payments and the smaller payment.

$$\begin{aligned}
800 \cdot a_{\overline{n}|} &\leq 6,000 < 800 \cdot a_{\overline{n+1}|} \\
a_{\overline{n}|} &\leq 7.5 < a_{\overline{n+1}|} \\
a_{\overline{n}|} &= 7.5 \\
\frac{1 - v^n}{i} &= 7.5 \\
n &= \frac{\ln(1 - 7.5i)}{\ln(v)} \\
n &= 16.7015
\end{aligned}$$

We know that it takes 16 payments of 800 plus 1 smaller payment to pay off the loan, 17 payments in total. The smaller payment is

$$\begin{aligned}
800 \cdot s_{\overline{16}|} + X &= 6,000(1 + i)^{16} \\
X &= 6,000(1 + i)^{16} - 800 \cdot s_{\overline{16}|} \\
X &= 513.40721
\end{aligned}$$

3–48. Loan of 6,000. Pay back 70 a month for as long as necessary, then a payment less than 70 at the end. First payment due in 1 month and $i = 0.11$ per year – find the number of payments and the smaller payment.

$$\begin{aligned}
i = \sqrt[12]{1.11} - 170 \cdot a_{\overline{n}|} &\leq 6,000 < 70 \cdot a_{\overline{n+1}|} \\
a_{\overline{n}|} &\leq \frac{600}{7} < a_{\overline{n+1}|} \\
a_{\overline{n}|} &= \frac{600}{7} \\
\frac{1 - v^n}{i} &= \frac{600}{7} \\
n &= \frac{\ln(1 - \frac{600}{7}i)}{\ln(v)} \\
n &= 158.79946
\end{aligned}$$

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We know that it takes 158 payments of 70 plus 1 smaller payment to pay off the loan, 159 payments in total. The smaller payment is

$$\begin{aligned}
70 \cdot s_{\overline{158}|} + X &= 6,000(1+i)^{158} \\
X &= 6,000(1+i)^{158} - 70 \cdot s_{\overline{158}|} \\
X &= 55.52574
\end{aligned}$$

3–49. Loan of 6,000. Pay back 800 a year for as long as necessary, then a payment less than 800 at the end. First payment due in 1 year and $i = 0.11$ – find the number of payments and the smaller payment.

$$\begin{aligned}
800 \cdot a_{\overline{n}|} \cdot v \leq 6,000 &< 800 \cdot a_{\overline{n+1}|} \cdot v \\
a_{\overline{n}|} \cdot v \leq 7.5 &< a_{\overline{n+1}|} \cdot v \\
a_{\overline{n}|} &= 7.5(1+i) \\
\frac{1-v^n}{i} &= 7.5(1+i) \\
n &= \frac{\ln(1-7.5i(1+i))}{\ln(v)} \\
n &= 23.70608
\end{aligned}$$

We know that it takes 23 payments of 800 plus 1 smaller payment to pay off the loan, 24 payments in total. The smaller payment is

$$\begin{aligned}
800 \cdot s_{\overline{23}|} + X &= 6,000(1+i)^{24} \\
X &= 6,000(1+i)^{24} - 800 \cdot s_{\overline{23}|} \\
X &= 516.63265
\end{aligned}$$

3–50. Fund of 5,000 is accumulated by n annual payments of 50 by another n payments of 100 plus a final payment of (as small as possible), one year after the final payment. At

$i = 0.08$ find n and the final payment.

$$\begin{aligned}
50 \cdot s_{\overline{n}|}(1+i)^n + 100 \cdot s_{\overline{n}|} &= 5,000 \\
s_{\overline{n}|}(1+i)^n + 2 \cdot s_{\overline{n}|} &= 100 \\
\frac{((1+i)^n)^2 - (1+i)^n + 2(1+i)^n - 2}{i} &= 100 \\
(1+i)^n &= c \\
c^2 - c + 2c - 2 &= 100i \\
0 &= c^2 + c - (2 + 100i) \\
c_{1,2} &= \frac{-1 \pm \sqrt{41}}{2} \\
c_1 < 0 \qquad c_2 &= \frac{-1 + \sqrt{41}}{2} \\
(1+i)^n &= \frac{-1 + \sqrt{41}}{2} \\
n &= \ln\left(\frac{-1 + \sqrt{41}}{2}\right) / \ln(1+i) \\
n &= 12.91342
\end{aligned}$$

We know that it takes 12 payments of 50 and 100 plus 1 smaller payment to accumulate 5,000. The smaller payment is

$$\begin{aligned}
50 \cdot s_{\overline{12}|}(1+i)^{121} + 100 \cdot s_{\overline{12}|} + X &= 5,000 \\
s_{\overline{12}|}(1+i)^{12} + 2 \cdot s_{\overline{12}|} + X &= 100 \\
X &= 100 - s_{\overline{12}|}(1+i)^{12} - 2 \cdot s_{\overline{12}|} \\
X &= 14.25811
\end{aligned}$$

3–51. At what effective monthly rate will payments of 200 at the end of every month for the next 3 years pay of 6500?

$$\begin{aligned}
200 \cdot a_{\overline{36}|} &= 6500 \\
a_{\overline{36}|} &= \frac{6500}{200} \\
\frac{1 - v^{36}}{i} &= \frac{6500}{200} \\
0 &= \frac{6500}{200} i(1+i)^{36} - (1+i)^{36} + 1
\end{aligned}$$

solving with a calculator

$$i = 0.03077$$

3–54. A fund of 25,000 is to be accumulated at the end of 20 years by annual payments of 500 at the end of each year. Find i

$$500 \cdot s_{\overline{20|}} = 25,000$$
$$s_{\overline{20|}} = \frac{25000}{500}$$
$$\frac{(1+i)^{20}-1}{i} = 50$$

solving with a calculator

$$i = 0.04878$$

3–55.

$$100 \cdot s_{\overline{5|}}(1+i)^5 + 200 s_{\overline{5|}} = 2,200$$
$$s_{\overline{5|}}(1+i)^5 + 2 s_{\overline{5|}} = 22$$
$$(1+i)^{10} + (1+i)^5 = 22i$$

solving with a calculator

$$i = 0.66667$$