Exercise 2–1. What is the effective rate of interest corresponding to an interest rate of 5% compounded quarterly?

Example 2–1. Suppose I have the opportunity to invest \$1 in Bank A which pays 5% interest compounded monthly. What interest rate does Bank B have to pay, compounded daily, to provide an equivalent investment? At any time t in years the amount in the two banks is given by $\left(1 + \frac{0.05}{12}\right)^{12t}$ and $\left(1 + \frac{i}{365}\right)^{365t}$ respectively. Finding the nominal interest rate i which makes these two functions equal is now an easy exercise.

Exercise 2–2. Find the interest rate i. What is the effective rate of interest?

Exercise 2–3. Compute $0.05^{(12)}$.

Exercise 2–4. Show that $\delta = \ln(1 + i)$.

Exercise 2–5. Find the force of interest which is equivalent to 5% compounded daily.

Example 2–2. Suppose the annual interest rate is 5%. What is the present value of a payment of \$2000 payable 10 years from now? The present value is $\$2000(1 + 0.05)^{-10} = \$1227.83$.

Exercise 2–7. Denote by $d^{(m)}$ the rate of discount payable m times per year that is equivalent to a nominal annual rate of interest i. What is the relationship between $d^{(m)}$ and i? Between $d^{(m)}$ and $i^{(m)}$? Hint: Draw the time diagram illustrating the two payments made at time 0 and $1/m$.

Exercise 2–8. Treasury bills (United States debt obligations) pay discount rather than interest. At a recent sale the discount rate for a 3 month bill was 5%. What is the equivalent rate of interest?

Problem 2–3. Calculate the nominal rate of interest convertible once every 4 years that is equivalent to a nominal rate of discount convertible quarterly.

Problem 2–4. Interest rates are not always the same throughout time. In theoretical studies such scenarios are usually modelled by allowing the force of interest to depend on time. Consider the situation in which \$1 is invested at time 0 in an account which pays interest at a constant force of interest δ. What is the amount $A(t)$ in the account at time t? What is the relationship between $A'(t)$ and $A(t)$? More generally, suppose the force of interest at time t is $\delta(t)$. Argue that $A'(t) = \delta(t)A(t)$, and solve this equation to find an explicit formula for $A(t)$ in terms of $\delta(t)$ alone.

Problem 2–5. Suppose that a fund initially containing \$1000 accumulates with a force of interest $\delta(t) = 1/(1 + t)$, for $t > 0$. What is the value of the fund after 5 years?

Problem 2–6. Suppose a fund accumulates at an annual rate of simple interest of i. What force of interest $\delta(t)$ provides an equivalent return?

Problem 2–7. Show that $d = 1 - v$. Is there a similar equation involving $d^{(m)}$?

Problem 2–8. Show that $d = iv$. Is there a similar equation involving $d^{(m)}$ and $i^{(m)}$?

Problem 2–9. Show that if interest is paid at rate i, the amount at time t under simple interest is more than the amount at time t under compound interest provided $t < 1$. Show that the reverse inequality holds if $t > 1$.

Problem 2–10. Compute the derivatives $\frac{d}{di}d$ and $\frac{d}{dv}\delta$.