Newton's Second Law ($F = ma$) states that

$$F = (\rho \Delta x) \frac{\partial^2 u}{\partial t^2} \tag{2}$$

where ρ is the linear density of the string (ML^{-1}) and Δx is the length of the segment. The force comes from the tension in the string only - we ignore any external forces such as gravity. The horizontal tension is constant, and hence it is the vertical tension that moves the string vertically (obvious).

Balancing the forces in the horizontal direction gives

$$T(x + \Delta x, t) \cos\theta(x + \Delta x, t) = T(x, t) \cos\theta(x, t) = \tau = const \tag{3}$$

where τ is the constant horizontal tension. Balancing the forces in the vertical direction yields

$$\begin{aligned} F &= T(x + \Delta x, t) \sin\theta(x + \Delta x, t) - T(x, t) \sin\theta(x, t) \\ &= T(x + \Delta x, t) \cos\theta(x + \Delta x, t) \tan\theta(x + \Delta x, t) - T(x, t) \cos\theta(x, t) \tan\theta(x, t) \end{aligned}$$

Substituting (3) and (1) yields

$$\begin{aligned} F &= \tau (\tan\theta(x + \Delta x, t) - \tan\theta(x, t)) \\ &= \tau \left(\frac{\partial u}{\partial x}(x + \Delta x, t) - \frac{\partial u}{\partial x}(x, t) \right). \end{aligned} \tag{4}$$

Substituting F from (2) into Eq. (4) and dividing by Δx gives

$$\rho \frac{\partial^2 u}{\partial t^2}(\xi, t) = \tau \frac{\frac{\partial u}{\partial x}(x + \Delta x, t) - \frac{\partial u}{\partial x}(x, t)}{\Delta x}$$

for $\xi \in [x, x + \Delta x]$. Letting $\Delta x \to 0$ gives the 1-D Wave Equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \qquad c^2 = \frac{\tau}{\rho} > 0. \tag{5}$$

Note that c has units $[c] = \left[\frac{\text{Force}}{\text{Density}}\right]^{1/2} = LT^{-1}$ of speed.

1.1 Boundary conditions

Ref: Guenther & Lee §4.2 (p. 94), Myint-U & Debnath §4.4

In order to guarantee that Eq. (5) has a unique solution, we need initial and boundary conditions on the displacement $u(x, t)$. There are now 2 initial conditions and 2 boundary conditions.

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$$F = (\rho \Delta x) \frac{\partial^2 u}{\partial t^2}$$

where ρ is the linear density of the string (ML^{-1}) and Δx is the length of the segment. The force comes from the tension in the string only - we ignore any external forces such as gravity. The horizontal tension is constant, and hence it is the vertical tension that moves the string vertically (obvious). Balancing the forces in the horizontal direction gives

$$T(x + \Delta x, t) \cos \theta(x + \Delta x, t) = T(x, t) \cos \theta(x, t) = \tau = \text{const}$$

where τ is the constant horizontal tension. Balancing the forces in the vertical direction yields $F = T(x+\Delta x, t) \sin \theta(x+\Delta x, t) - T(x, t) \sin \theta(x, t) = T(x+\Delta x, t) \cos \theta(x+\Delta x, t) \tan \theta(x+\Delta x, t) - T(x, t) \cos \theta(x, t) \tan \theta(x, t)$ Substituting (3) and (1) yields

$$\begin{aligned} F &= \tau(\tan \theta(x + \Delta x, t) - \tan \theta(x, t)) \\ &= \tau \left(\frac{\partial u}{\partial x}(x + \Delta x, t) - \frac{\partial u}{\partial x}(x, t) \right) \end{aligned}$$

Substituting F from (2) into Eq. (4) and dividing by Δx gives

$$\rho \frac{\partial^2 u}{\partial t^2}(\xi, t) = \tau \frac{\frac{\partial u}{\partial x}(x + \Delta x, t) - \frac{\partial u}{\partial x}(x, t)}{\Delta x}$$

for $\xi \in [x, x + \Delta x]$. Letting $\Delta x \to 0$ gives the 1 -D Wave Equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad c^2 = \frac{\tau}{\rho} > 0$$

Note that c has units $[c] = \left[\frac{\text{Forree}}{\text{Density}} \right]^{1/2} = LT^{-1}$ of speed. 1.1 Boundary conditions Ref: Guenther & Lee \$4.2 (p. 94), Myint-U & Debnath \$4.4 In order to guarantee that Eq. (5) has a unique solution, we need initial and boundary conditions on the displacement $u(x, t)$. There are now 2 initial conditions and 2 boundary conditions.

Л А Б О Р А Т О Р Н А Я Р А Б О Т А I.1

АЛГЕБРАИЧЕСКАЯ ИНТЕРПОЛЯЦИЯ

Основная цель. Научиться строить интерполяции при помощи многочлена Лагранжа. Изучить влияние выбора интерполяционной сетки на сходимость процесса интерполяции.

Теория и основные формулы. На отрезке [0,1] рассмотрим интерполяционную сетку:

$$0 \le x_1 < x_2 < \ldots < x_i < \ldots < x_n \le 1. \qquad \text{(I.1.1)}$$

Пусть $f^h \equiv \{f_i^h\}_{i=1}^n$ - сеточная функция, заданная в узлах сетки (I.1.1). *Интерполяционный многочлен Лагранжа* определяется следующими формулами:

$$\ell_n\left(x, f^h\right) \equiv \sum_{k=1}^{N} \omega_{n,k}(x) \cdot f_k^h; \qquad \text{(I.1.2)}$$

$$\omega_{n,k} \equiv \prod_{\substack{i=1 \\ i \ne k}}^{N} \left(\frac{x - x_i}{x_k - x_i} \right); \; x \in [0,1]. \qquad \text{(I.1.3)}$$

Можно доказать (см. [1-7]), что многочлен, определяемый формулами (I.1.2-3), является единственным решением задачи интерполяции (Определение I.1) в классе многочленов, степени которых не превосходят “n-1” (класс $P_{n-1}[0,1]$). Тот же самый многочлен может быть построен методами, отличными от того, который определяется формулами (I.1.2-3), например интерполяционный многочлен в *форме Ньютона* описан в учебниках [1-7]; здесь мы приведем менее известный метод построения интерполяционного многочлена, который обычно называют *методом Невилле*.

Пусть $m_1, m_2, ..., m_k$ $(1 \le k \le n)$ -натуральные числа, удовлетворяющие условиям:

IA 5 O PA TOPHA $ PA 5 O TA 1.1 Nricsewurcckus unirepootatuar Ochoshas ueas. Hayurrbca crpouts untepnotialuhu npu nomoun MHorowneha JarpaHxka. Hayurntb BIHAHHe BLIGopa HHTepnonguhoHHoù ceTKH Ha cxoaumocts npouecca HHTepnonsuuh. Teopua n ocuosmble phiopuyabl. Ha orpeske [0,1] paccurorphy HHrepnotaluromylo cerry:

$$0 \leq x_1 < x_2 < \ldots < x_1 < \ldots < x_n \leq 1$$

Hyctb $f^4 = \{f_1^4\}_4^2$ - cerounar phiynkilus , andawnar B yarax cerxu (I.1.1). Humepnouratuonnbili nuceoutren Jazpanaca onpezeraetes creayiounsm phiopuyaamı:

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$$\ell_n\left(x, f^n\right) = \sum_{k=1}^n \omega_{n,x}(x) \cdot f_k^*$$

$$\omega_{ex} \equiv \prod_{i=1}^{\infty} \left(\frac{x - x_i}{x_k - x_i}\right); x \in [0,1]$$

Moxko Aoka3arb (cM. [1-7]), "To Mnoroutinen, onpezenzesibuiù phiopuynaun (1.1.2-3), samates eauncracumbly pemennem 3añaun nurrepnotiauun (OnpeñeneHue I.1) B KIacce struorotutenos, creneun Kotopblx He npegocxourt "n-1" (kracc P_-L[0,1]). Tot we caubsiu MAOro $*$ atel Moxer 6 brib noctpo Meto_laMH, OTIM4HLIMH OT TOTO, KOTOPLIÜ onpe,TeTSETCS QopMy. nanpumep merepnotisumounbili menter apwere Hbomona omucan **B** yue6Hukax $[1-7]$; 37ecb Mbl npusezen MeHee H3BeCTHbIù MeTOA nocrpoeHus HHTeprotsunohhoro mHorouneua, kotopbili o6blyHo HazbIBaroT Memodom Hesuzze. IIycts $\mathrm{m}_1, \mathrm{m}_2, \ldots, \mathrm{m}_\mathrm{k} (1 \leq \mathrm{k} \leq \mathrm{n})$ - - HarypaTsHble unc-aa, y Aosiersopaioume yctoBHSM:

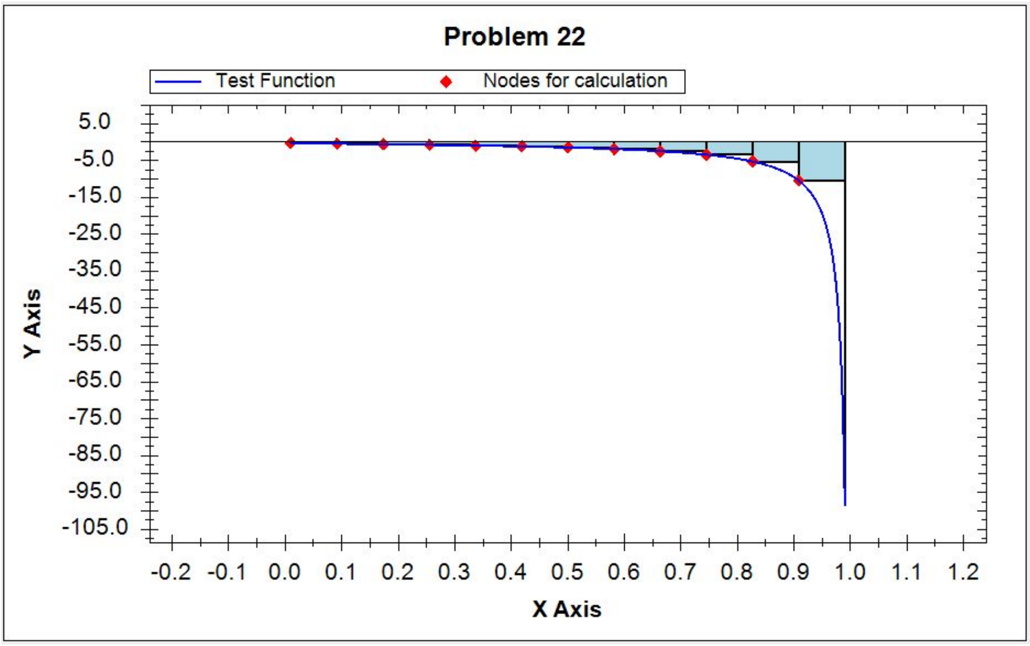
The below table shows the calculated value of the integral in problem 22 for different numbers of nodes and the different methods used in this project.

Problem 22

Number of Nodes	Approximate Value of Integral						
	Rectangle	Rectangle V1 (Theta)		Rectangle V2 (Theta)		Trapezia	Simpson's Rule
		0.25	0.75	0.25	0.75		
3	-0.81	-1.28	-3.95	-12.98	-37.30	-25.14	-9.73
5	-1.42	-1.87	-4.41	-7.50	-19.66	-13.58	-6.27
9	-2.02	-2.44	-4.47	-5.06	-11.14	-8.10	-4.79
17	-2.58	-2.96	-4.68	-4.10	-7.14	-5.62	-4.24
33	-3.06	-3.38	-4.54	-3.82	-5.34	-4.58	-4.07
65	-3.44	-3.67	-4.35	-3.82	-4.58	-4.20	-4.04
129	-3.70	-3.84	-4.21	-3.89	-4.27	-4.08	-4.03

One interesting occurence is this case is that for rectangles of V2, Trapezia, and Simpson's Rule, the values start large and negative and then become smaller as the all approach a value of about -4. The normal rectangles and rectangles of V1 with theta 0.25 start small and negative and approach -4 from the other direction. Rectangles of V1 for theta 0.75 start small, increase in magnitude and then decrease again as they approach -4.

The behavior of the normal rectangles can be explained by showing the following graph. Because of the shape of this graph, the normal rectangle's sum is always smaller than the value of the integral, but as the number of rectangles increases, this error becomes smaller and the value approaches -4. For rectangles of V1 with theta 0.25 the behavior is very similar, theta of 0.25 means a shift of 0.25 of the interval towards the next interval and this shift is too small to make a difference is these measurements. Below see normal rectangles.



The behavior of the rectangles of variant one with theta 0.75 can be explained by the fact that the slope is >1 for the last part of the function, which also makes up most of the value of it. Theta of 0.75 means that the node is shifted by 0.75 of the interval length towards the next interval. From a

The below table shows the calculated value of the integral in problem 22 for different numbers of nodes and the different methods used in this project.

~~Nimber~~	Ampoin				
	Recinge	Rectange V(MEder)			
	Recturde V_2(mest				
3	-0.81	0.25	0.75	0.73	
5	-142	-128	-395	-1298	-3
9	202	-187	+41	-7.50	-1
17	258	244	447	-506	-
33	325	296	468	410	
65	344	363	454	382	
128	370	384	435	382	

One interesting occurence is this case is that for rectangles of V2, Traperia, and Simpson's Rule, the values start large and negative and then become smaller as the all approach a value of about - 4. The normal rectangles and rectangles of $V1$ with theta 0.25 start small and negative and approach - 4 from the other direction. Rectangles of V1 for theta 0.75 start small, increase in magnitude and then decrease again as they approach -4 The behavior of the normal rectangles can be explained by showing the following graph. Because of the shape of this graph, the normal rectangle's sum is always smaller than the value of the integral, but as the number of rectangles increases, this error becomes smaller and the value approaches - 4. For rectangles of V1 with theta 0.25 the behavior is very similar, theta of 0.25 means a

shift of 0.25 of the interval towards the next interral and this shift is too small to make a diffe these measurements. Below sec normal rectangles.
Problem 22 5.0
5.0 -15.0 -25.0
= Tealfunction -35.0
$\frac{-1}{2} = 45.0 =$ $\geq \frac{1}{5} =$ 255.0 -65.0 -75.0 -85.0 -95.0 -105.0

The behavior of the rectangles of variant one with theta 0.75 can be explained by the fact that the slope is > 1 for the last part of the function, which also makes up most of the value of it. Theta of 0.75 means that the node is shifted by 0.75 of the interval length towards the next interval. From a

6. *Biased*
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Anybody's Son Will Do

Gwynne Dyer

Ordinary people would be loathe to do the sorts of things that soldiers may be called upon to do—but societies seem to need soldiers. As Dyer explains in this chapter excerpted from his book *War,* the means of socializing men out of the civilian role and into the soldier/killer role has become institutionalized as a result of centuries of experience.

. . . All soldiers belong to the same profession, no matter what country they serve, and it makes them different from everybody else. They have to be different, for their job is ultimately about killing and dying, and those things are not a natural vocation for any human being. Yet all soldiers are born civilians. The method for turning young men into soldiers—people who kill other people and expose themselves to death—is basic training. It's essentially the same all over the world, and it always has been, because young men everywhere are pretty much alike.

Human beings are fairly malleable, especially when they are young, and in every young man there are attitudes for any army to work with: the inherited values and postures, more or less dimly recalled, of the tribal warriors who were once the model for every young boy to emulate. Civilization did not involve a sudden clean break in the way people behave, but merely the progressive distortion and redirection of all the ways in which people in the old tribal societies used to behave, and modern definitions of maleness still contain a great deal of the old warrior ethic. The anarchic machismo of the primitive warrior is not what modern armies really need in their soldiers, but it does provide them with promising raw material for the transformation they must work in their recruits.

Just how this transformation is wrought varies from time to time and from country to country. In totally militarized societies—ancient Sparta, the samurai class of medieval Japan, the areas controlled by organizations like the Eritrean People's Liberation Front today[1]—it begins at puberty or before, when the young boy is immersed in a disciplined society in which only the military values are allowed to penetrate. In more sophisticated modern societies, the process is briefer and more concentrated, and the way it works is much more visible. It is, essentially, a conversion process in an almost religious sense—and as in all conversion phenomena, the emotions are far more important than the specific ideas. . . .

> When I was going to school, we used to have to recite the Pledge of Allegiance every day. They don't do that now. You know, we've got kids that come in here now, when they first get here, they don't know the Pledge of Allegiance to the flag. And that's something—that's like a cardinal sin. . . . My daughter will know that stuff by the time she's three; she's two now and she's working on it. . . . You know, you've got to have

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[1]Eritrea, an Italian colony from 1885 to 1941, was annexed by Ethiopia in 1962. After a 30-year civil war, Eritrea gained its independence in 1992.—Ed.

.20 Anybody's Son Will Do cimmenyererer Ordinary people would be loathe to do the sorts of things that soldiers may be called upon to do - but societies seem to need soldiers. As Dyer explains in this chapter excerpted from his book War, the means of socializing men out of the civilian role and into the soldier/killer role has become institutionalized as a result of centuries of experience. All soldiers belong to the same profession, Just how this transformation is wrought no matter what country they serve, and it varies from time to time and from country to makes them different from everybody else. country. In totally militarized societies - anThey have to be different, for their job is ulti- - cient Sparta, the samurai class of medieval mately about killing and dying, and those Japan, the areas controlled by organizations things are not a natural vocation for any like the Eritrean People's Liberation Front to: human being. Yet all soldiers are born civil- day'- it begins at puberty or before, when ians. The method for turning young men into the young boy is immersed in a disciplined sosoldiers- people who kill other people and ciety in which only the military values are expose themselves to death - is basic training. allowed to penetrate. In more sophisticated Ifsessentially the same all over the world, and modern societies, the process is briefer and italways has been, because young men every- - more concentrated, and the way it works is where are pretty much alike. much more visible. It is, essentially, a converHuman beings are fairly malleable, espe- - sion process in an almost religious sense- cally when they are young, and in every and as in all conversion phenomena, the emoyoung man there are attitudes for any army to tions are far more important than the specific work with: the inherited values and postures, ideas. more or less dimly recalled, of the tribal war- When I was going to school, we used to have to nors who were once the model for every recite the Pledge of Allegiance every day. They zoung boy to emulate. Civilization did not in- don't do that now. You know, we've got kids "solve a sudden clean break in the way people that come in here now, when they first get here, behave, but merely the progressive distortion they don't know the Pledge of Allegiance to the and redirection of all the ways in which people flag. And that's something - that's like a cardiin the old tribal societies used to behave, and nal sin..... My daughter will know that stuff by modern definitions of maleness still contain a the time she's three; she's two now and she's Feat deal of the old warrior ethic. The anar- working on it.... You know, you've got to have tic machismo of the primitive warrior is not what modern armies really need in their sol- Giers, but it does provide them with promising There material for the transformation they must work in their recruits.

$$\overrightarrow{OM} = \begin{pmatrix} 0 \\ 105 \end{pmatrix} + s \begin{pmatrix} 0 \\ 210,23 \end{pmatrix} = \begin{pmatrix} 0 \\ 105 \end{pmatrix} + s \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 105 + s \end{pmatrix}$$

$$|\overrightarrow{OM}| = \sqrt{0^2 + (105 + s)^2} = \sqrt{11025 + 210s + s^2}$$

$$|\overrightarrow{OM}| = 105 + s$$

$$\overrightarrow{OM} = \begin{pmatrix} 0 \\ 105 \end{pmatrix} + s \begin{pmatrix} 0 \\ 210,23 \end{pmatrix} = \begin{pmatrix} 0 \\ 105 \end{pmatrix} + s \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 105 + s \end{pmatrix}$$

$$|\overrightarrow{OM}| = \sqrt{0^2 + (105 + 5)^2} = \sqrt{11025} + 210s + s^2$$

$$|\overrightarrow{OM}| = 105$$