An Adaptive Noise Cancelation Model for Removal of Noise from Modeled ECG Signals

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***Abstract*—In this paper an adaptive noise cancelation (ANC) model is presented to remove baseline wander (BW) noise from mathematically modeled ECG signals. The ANC model is designed to have a trade-off between the correlation properties of noise and reference signals. Matlab is used to simulate ECG signals artificially, to represent different sinus rhythms and leads of ECG waveform. Furthermore contamination of an important artifact (baseline wander) is simulated for normal ECG lead II, and then identified using LMS algorithm and its preconditioned versions: NLMS and TDLMS algorithms, to get denoised ECG signals. Experimental results are presented for a comparison of these adaptive algorithm, which shows preference of TDLMS algorithm over the rest.**

***Index Terms*—Mathematical model, almost periodic, adaptive algorithm, adaptive noise canceler.**

I. Introduction

Adaptive filters can be identified as self-designing systems that rely on a recursive algorithm to become able to perform adequately in an environment where knowledge of the relevant statistics is not available. They have ability to detect time varying potentials and to track the dynamic variations of biomedical signals. An important biomedical signal processing application is denoising of the electrocardiogram (ECG) signals. ECG signals are non-stationary bioelectrical signals, used to detect the beats of heart rate variability (HRV). These signals can be split into different segments and intervals according to the phases of cardiac conduction. ECG signal include valuable clinical information, but frequently the valuable clinical information is corrupted by various kinds of noise, including baseline wander (BW), which is caused by variable contact between the electrodes and the skin.

The least mean square (LMS) algorithm is a widely used adaptive filtering algorithm and have found its place in biomedical signal processing community as well [1]–[3]. LMS algorithm is a stochastic gradient algorithm which offers the easiest and reliable adaptive method to denoise and identify a signal. Its performance depends on the power spectral density (i.e. eigenvalue spread) of the input autocorrelation matrix [4]. When contamination of noise in the desired signal increases, its spectral power increases and this results in poor performance of LMS algorithm. Several approaches have been put forward to improve the performance of LMS algorithm, two such approaches are Normalized LMS (NLMS) algorithm and transform domain LMS algorithm [5]. Application of these techniques in biomedical signal processing is very limited.

This study proposes a mathematical development of ECG waveform by employing the almost periodic nature of the waveform. The generating function is obtained by taking the superposition of linear combination of basis functions, while Fourier coefficients, and values of model parameters (amplitude, duration, time) for ECG lead-II are employed from the online model [6]. Afterwards an adaptive noise canceler (ANC) is presented for identification and removal of BW artifacts from modeled ECG. The reference signal of this noise canceler is obtained by using a novel technique for setting a trade-off between the correlation properties of input noise and reference signal. It is done by passing noise signal through an unknown filter, to have a reference signal with different correlation properties. After that performance of NLMS and discrete cosine transform based TDLMS algorithms is analyzed in identification and removal of estimated BW noise from contaminated ECG signals. Experimental results are obtained for noise cancelation of normal ECG waveform and it is observed that in case of BW noise of frequency 0.25 Hz, DCT-TDLMS algorithm has better convergence properties as compared with NLMS and LMS algorithms.

II. Adaptive Noise Canceler for Removal of Baseline Wander (BW) From Modeled ECG Signals

The block diagram of proposed adaptive noise canceler (ANC) for denoising ECG signals is shown in Figure1.

The signal $x(n)$, consists of the desired clean signal $ecg(n)$ and contaminated noise $g(n) = BW(n)$, such that

$$x(n) = ecg(n) + BW(n) \tag{1}$$

The noise source produces a noise signal which is recorded simultaneously with noise $BW(n)$ and the reference signal $u(n)$, obtained by passing noise signal $g(n) = BW(n)$ through an unknown filter with frequency response:

$$H(z) = \frac{\sqrt{1 - \alpha^2}}{1 - \alpha z^{-1}}$$

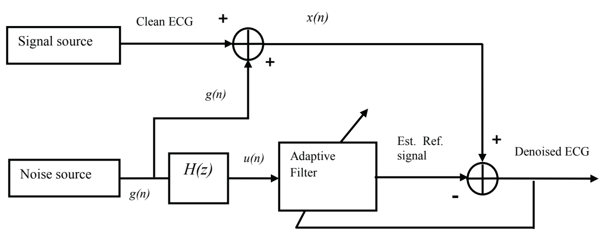

Fig. 1. Proposed Adaptive Noise Cancelation model for Denoising Modeled ECG signal.

where $|\alpha| < 1$, α is a correlation parameter and controls the spectral properties of input signals. $\alpha = 0$ corresponds to the case when condition number is close to 1, condition number increase with an increase in the value of α. The ECG signals and contamination of noise is obtained by using a mathematical model which is based upon the superposition of linear combination of basis functions, and is summarized below:

A. Mathematical Simulation of ECG Signals

An ECG waveform is comprised of peaks and troughs with P, Q, R, S, T, and U events, associated with the artrial and ventricular depolarization/repolarisation. These activities produce potential variation between the ECG electrodes positioned on different limbs and chest wall, and result in a time varying waveform. ECG model parameters are used for mechanical detection and classification of various sinus rhythms. Modeling the ECG signal is concerned with making the best simulation of the signal followed by obtaining the parameters of the model. More accurate model parameters will make the simulator more reliable. The proposed Mathematical model is designed by employing the almost periodic nature of ECG waveform and is based on three model parameters: amplitude (measured in mV), duration (in sec.) and incident time (in sec.) of events.

Let us denote the number of heartbeat per minute by N_{hb}, and take the set of significant waveforms of ECG lead as $J = P, Q, R, S, T, U$. Then an event $j \in J$ in the ECG waveform, at time t, can be identified by an almost periodic function $E_j(t)$ of time period $\tau_{hb}(t)$ such that $E_j(t) \approx E_j(t+\tau_{hb}(t))$, and the Fourier series representation of $E_j(t)$ would be [7]

$$E_j(t) = A_j + \sum_{n=1}^{N_{hb}} U_j(n) cos\{2\pi n \int_{t_j}^{t} f_{hb}(t)dt\} \tag{2}$$

where $f_{hb}(t) = 1/\tau_{hb}(t)$ is the fundamental frequency of heartbeat. A_j and $U_j(n)$ are Fourier coefficients, while parameter t_j is the incident time of event $j \in J$. Generally almost periodic behavior of waveform is governed by time dependent period $\tau_{hb}(t)$, but for ease of computations, we assume that it is function of heartbeat only, i.e., $\tau_{hb}(t) = \frac{60}{N_{hb}}$. Simplifying and setting $\omega_{hb} = 2\pi f_{hb}(t) = \frac{2\pi}{\tau_{hb}(t)}$, (2) yields

$$E_j(t) = A_j + \sum_{n=1}^{N_{hb}} U_j(n) cos\{\omega_{hb}(t - t_j)n\} \tag{3}$$

If a_j , and d_j denote respectively the amplitude and duration of event $j \in J$, then each event is supposed to occur in a cycle of period $\tau_j = \frac{\tau_{hb}(t)}{d_j}$, with frequency $f_j = \frac{1}{\tau_j}$. Given the values of model parameters a_j , d_j and t_j, for each $j \in J$, an appropriate choice of functions can be made to find the determine Fourier coefficients A_j & $U_j(n)$ independently, as is done in appendix-A. The superposition of events E_j can be a good approach to develop a comprehensive set of all the waveforms in the ECG lead. For a known value of N_{hb}, the ECG waveform is approximated as:

$$\begin{aligned} ECG(t) &= \sum_{j\in J} E_j(t) \\ &= A_o + \sum_{j\in J}\sum_{n=1}^{N_{hb}} U_j(n) cos\{\omega_{hb}(t - t_j)n\} \end{aligned} \tag{4}$$

where $A_o = \sum_{j\in J} A_j$ adjusts the deviation of base line of ECG waveform from zero voltage. Since occurrence of an event in independent of N_{hb}, (4) can be rewritten as:

$$ECG(t) = A_o + \sum_{n=1}^{N_{hb}}\sum_{j\in J} U_j(n) cos\{\omega_{hb}(t - t_j)n\} \tag{5}$$

Most simple ECG machines are capable of recording at least three leads: lead I, lead II and lead III. The most commonly used lead is Lead II, which is the best lead for interpreting the heart's rhythm. Lead II is the view from the patient's right arm to his left leg. All further discussion is made for ECG lead II waveform, whose model parameters (Amplitude(mV), Duration(sec.), Time(sec.)) are employed from an online model [6], and are given in table-I. Starting with event P, the signals are sampled with 1mSec resolution for 5 seconds time, with $n = 0.001 : 0.001 : 5$.

TABLE I
PARAMETERS FOR ECG LEAD-II.

Parameters	P	Q	R	S	T	U
Amplitude	0.25	0.15	1.6	0.25	0.35	0.035
Duration	0.09	0.066	0.11	0.066	0.142	0.0476
Time	0.1	0.25	0.3	0.39	0.545	0.733

Figure2(b) shows the waveform of normal ECG-lead II, while the waveform shown in Figure2(a) is contaminated by BW noise. The noise signal $BW(n)$ is simulated by the mathematical function $Asin(2\pi f_{bw} n)$, where f_{bw} is the high frequency component of respiratory sinus arrhythmia (RSA), and A is amplitude of wandering from baseline.

III. LMS BASED ADAPTIVE NOISE CANCELERS

Consider an FIR filter of length N with a tap-weight vector $\mathbf{w}_n$, at instant n. The LMS algorithm minimizes the

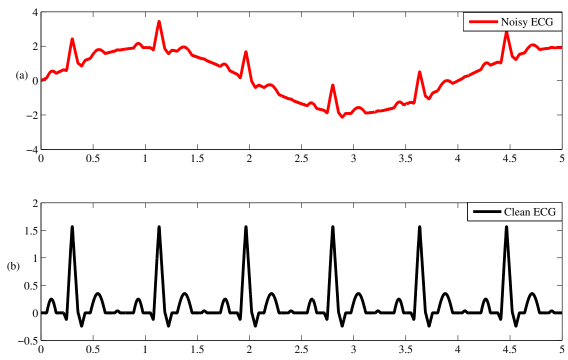

Fig. 2. Simulated ECG signal of normal sinus rhythm: (a). Contaminated by BW of $f_{BW} = 0.25$, (b) Clean for $\alpha = 0.8$.

instantaneous objective function (MSE),

$$J(n) = e^2(n)$$

where $e(n) = s(n) - \mathbf{w}_n^T \mathbf{a}_n$. The vectors $a_i \in \mathcal{R}^N$ are formed by input signals $u(i)$ in such a way that

$$a_i = \begin{bmatrix} u(i) & u(i-1) & \cdots & u(i-N+1) \end{bmatrix}^T; \; 1 \le i \le n$$

Using input signals we can define the $n \times N$ data matrix A_n as:

$$A_n = \begin{pmatrix} u(1) & 0 & \cdots & 0 \\ u(2) & u(1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ u(N) & u(N-1) & \cdots & u(1) \\ u(N+1) & u(N) & \cdots & u(2) \\ \vdots & \vdots & \ddots & \vdots \\ u(n-1) & u(n-2) & \cdots & u(n-N) \\ u(n) & u(n-1) & \cdots & u(n-N+1) \end{pmatrix}$$

Define $X = E\left[\mathbf{a}_n \mathbf{a}_n^T\right]$ as the autocorrelation matrix of input vector $\mathbf{a}_n$, and $p = E\left[\mathbf{a}_n s_n\right]$ as the crosscorrelation vector.

The mean square error $J(n)$ is minimized by continuously updating the weight vector $\mathbf{w}_n$ as each new input signal is received, according to the equation:

$$\mathbf{w}_{n+1} = \mathbf{w}_n + 2\mu e(n) \mathbf{a}_n \tag{6}$$

where μ is a positive constant that controls the rate of convergence. To ensure the stability of the adaptive process, value of μ must satisfy the condition:

$$0 < \mu < \frac{1}{\lambda_{\max}} \tag{7}$$

$\lambda_{\max}$ is the largest eigenvalue of the autocorrelation matrix $X = E\left[\mathbf{a}_n \mathbf{a}_n^T\right]$ and is given by the maximum of the power spectrum of input signal $\mathbf{a}_n$.

For stationary input and an appropriate choice of μ, the minimum value of $e(n)$ generates a Cauchy sequence $\{\mathbf{w}_n\}_{n=1}^{\infty}$ from (6) in $\mathcal{R}^N$. But since $\mathcal{R}^N$ is a Banach space [8], there exists an optimum weight vector $\mathbf{w}_o \in \mathcal{R}^N$, such that $\mathbf{w}_n \to \mathbf{w}_o$ as $n \to \infty$. Value of $\mathbf{w}_o$, as given by Wiener- Hopf equation [4], is:

$$\mathbf{w}_o = \mathrm{X}^{-1} p \tag{8}$$

Let us define the misalignment vector $\mathbf{m}_n$ as :

$$\mathbf{m}_n = \mathbf{w}_n - \mathbf{w}_o$$

Then $\mathbf{m}_n = \|\mathbf{m}_n\|_2 = \|\mathbf{w}_n - \mathbf{w}_o\|_2 \to 0$ as $n \to \infty$.

It is not difficult to show that under independence assumption,

$$E[\mathbf{m}_{n+1}] = (I - 2\mu X) E[\mathbf{m}_n] \tag{9}$$

This relation is used in literature [4], [9] to show that convergence behavior of the LMS algorithm is directly linked to the eigenvalue spread of X. For highly correlated input, X has high eigenvalue spread, and convergence of the algorithm can be extremely slow. To improve the convergence speed , we need to reduce eigenvalue spread of X by using some decorrelation techniques. We may overcome this problem by employing preconditioning theory from numerical linear algebra . Here we briefly describe some algorithms which have been derived from conventional LMS algorithm by using techniques similar to that theory.

A. Normalized LMS Algorithm

The normalized LMS (NLMS) algorithm, which was developed as a constrained optimization problem [9], can be considered as a preconditioned LMS algorithm. The preconditioner $(\psi I + \mathbf{a}_n \mathbf{a}_n^T)^{-1}$ is a regularized inverse of $\mathbf{a}_n \mathbf{a}_n^T$, where $\psi \approx 0$. The update equation is given by:

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu e(n) (\psi I + \mathbf{a}_n \mathbf{a}_n^T)^{-1} \mathbf{a}_n$$

Using matrix inversion lemma, we have

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \frac{\mu}{\psi + \mathbf{a}_n^T \mathbf{a}_n} e(n) \mathbf{a}_n \tag{10}$$

where ψ is selected to be small enough when compared with $\mathbf{a}_n^T \mathbf{a}_n$. NLMS has fast convergence as compared with the conventional LMS algorithm, but has a drawback of increased misadjustment.

B. TD-LMS Algorithm

Transform domain LMS algorithms is a class of robust preconditioned algorithms having good tracking capabilities in non stationary environments. Application of an orthogonal transform , followed by a power normalization step, has the ability to reduce the eigenvalue spread of input correlation matrix, which results in an increase of convergence speed of the algorithm [4]. Here we give a brief description of the TD-LMS algorithm.

The input vector $\mathbf{a}_n$, and weight vector $\mathbf{w}_n$ are transformed to $\hat{\mathbf{a}}_n = \mathrm{T}\mathbf{a}_n$ and $\hat{\mathbf{w}}_n = \mathrm{T}\mathbf{w}_n$ respectively, through an orthogonal transform T. With error estimate$e(n) = s(n) - \hat{\mathbf{w}}_n^T \hat{\mathbf{a}}_n$, and power normalization

$$\sigma_n^2(i) = \beta \sigma_{n-1}^2(i) + (1-\beta)\hat{\mathbf{a}}_n^2(i) \quad ; \quad i = 0, 1, \cdots, N-1,$$

where $0 < \beta < 1$, the weight vector update equation is:

$$\hat{\mathbf{w}}_{n+1} = \hat{\mathbf{w}}_n + 2\mu D^{-1} e(n)\,\hat{\mathbf{a}}_n \tag{11}$$

with $D = diag\left[\sigma_n^2(0), \sigma_n^2(1), \cdots, \sigma_n^2(N-1)\right]$.

An analytical approach [10] has shown significant decrease in the eigenvalue spread of the input correlation matrix of a first order Markov signal after application of discrete fourier (DFT) and discrete cosine (DCT) transforms, followed by power normalization. For current noise cancelation problem, we use DCT based TDLMS algorithm, because it has better control on the eigenvalue spread of reference signals. Moreover it has better compatibility with our model of ECG waveform.

IV. Experimental Results

In order to compare the performance of iterative algorithms for identification of BW noise in modeled ECG signal and then generation of denoised ECG curve, we make use of Matlab to apply LMS, NLMS and DCT-TDLMS algorithm on our modeled ECG signals contaminated by BW noise of frequency 0.25. Learning curves of MSE are noted for all the three algorithms (Figure3), for an ECG recording of 5 second duration with 1mSec resolution of sampling and time $n = 0.001 : 0.001 : 5$. Furthermore denoised ECG waveform obtained by application of these algorithms is compared with clean ECG waveform in all cases (see Figure 4). Although all the three algorithms are found efficient in cancelation of noise components and identification of clean ECG signals from contaminated ones, but from a closer look at learning curved of ECG waveform in Figure 5(a) & Figure5(b), preference of DCT-TDLMS is found over the rest. For all computations, correlation parameter is kept fixed at $\alpha = 0.8$, and filter length is $N = 5$. Checking performance of all algorithms for different values of BW-amplitudes, it is found that because of its invariance under the correlation properties of reference signals, DCT-TDLMS algorithm can perform well even for amplitude of BW noise equal to amplitude of event R wave, while performance of rest two becomes poor for amplitude higher than that of three-forth of event R.

V. Conclusion

The mathematical model presented in this paper is able to simulate complete ECG waveform of different leads with model parameters. Furthermore, it is flexible enough to be modeled with added noises, such as artifact of baseline wander (BW). This model is then used to compare the performance of LMS, NLMS and DCT-TDLMS algorithms for cancelation of BW noise from modeled ECG signals. An adaptive noise canceler is designed for the purpose by offering a trade-off between the correlation properties of noise and reference signals. Experimental results for MSE learning curves and denoised ECG waveforms are presented to show the comparative performance of three algorithm. This comparison results in preference of TDLMS algorithm under given circumstances.

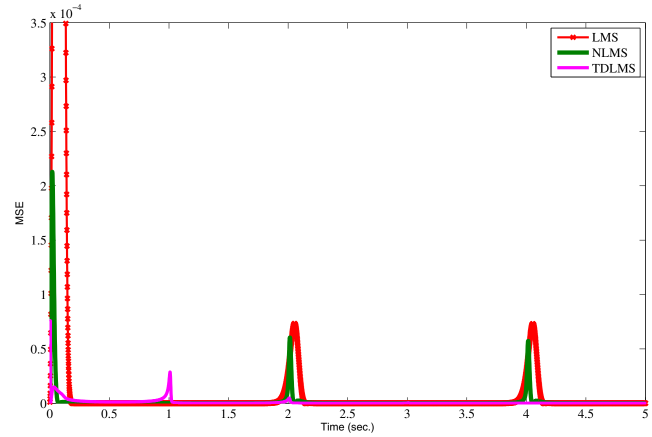

Fig. 3. Learning curves of MSE of TDLMS, NLMS and LMS for $\alpha = 0.8$.

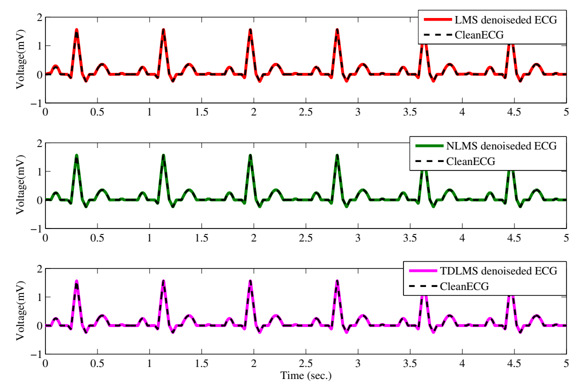

Fig. 4. Denoised ECG curves using (a) LMS , (b) NLMS, (c) TDLMS algorithms for $\alpha = 0.8$.

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Appendix

In section-II-A, let $\tau_{hb}(t) = 2L$ then $\tau_j = \frac{2L}{d_j}$, or equivalently $d_j = \frac{2L}{\tau_j}$ for each $j \in J$. Thus Fourier coefficients of each $j \in J$ can be computed over an interval $[\frac{-L}{\tau_j}, \frac{L}{\tau_j}]$. It is clear from ECG waveform of Figure 6 [11], that the events P,T and U are semi elliptic, while events Q, R and S are conical.

Keeping an eye on the geometrical picture of these events, we can choose similar algebraic expressions for the corresponding functions $E_j(t)$.

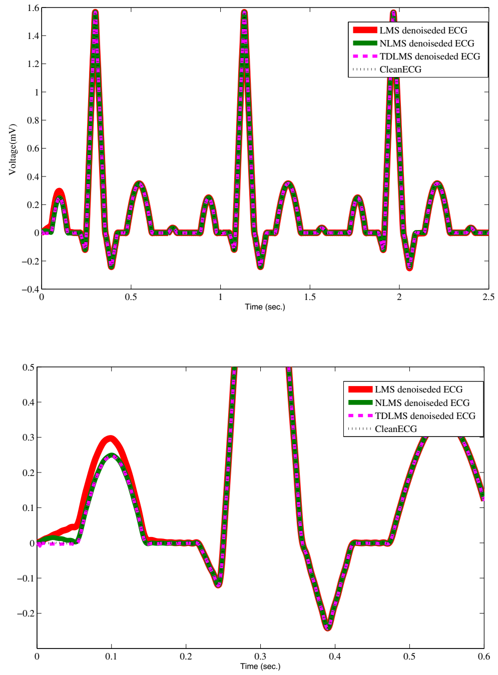

Fig. 5. Waveform of clean ECG signal vs denoised ECG signal using LMS, NLMS and TDLMS for $\alpha = 0.8$: (a). Learning curve for 1.7 seconds , (b). Closer view of learning curve for 0.6 seconds.

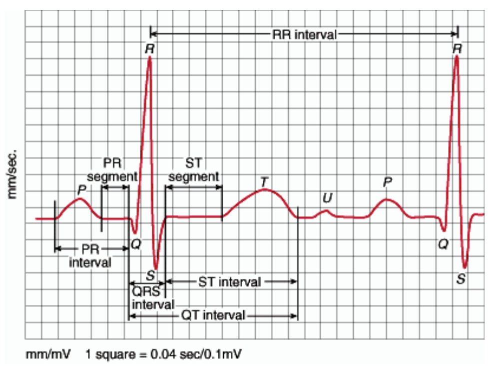

Fig. 6. Normal ECG waveform of lead II

An appropriate representation of $E_j(t)$, for event $j \in \{P, T, U\}$ is:

$$E_j(t) = a_j cos(\pi \frac{\tau_j}{2L} t) \quad ; \quad \frac{-L}{\tau_j} \leq t \leq \frac{L}{\tau_j} \tag{12}$$

Then,

$$A_j = \frac{1}{2L} \int_{\frac{-L}{\tau_j}}^{\frac{L}{\tau_j}} a_j cos(\pi \frac{\tau_j}{2L} t) dt = \frac{2a_j}{\pi \tau_j}$$

and,

$$U_j(n) = \frac{1}{L} \int_{\frac{-L}{\tau_j}}^{\frac{L}{\tau_j}} a_j cos(\pi \frac{\tau_j}{2L} t) cos(\frac{n\pi}{L} t) dt$$
$$= \frac{4a_j \tau_j}{\pi(\tau_j^2 - 4n^2)} cos(\frac{n\pi}{\tau_j})$$

Next for $j \in \{R\}$

$$E_j(t) = \begin{cases} \frac{a_j \tau_j}{L} t + a_j & , \quad \frac{-L}{\tau_j} \leq t < 0 \\ -\frac{a_j \tau_j}{L} t + a_j & , \quad 0 \leq t \leq \frac{L}{\tau_j} \end{cases} \tag{13}$$

Then,

$$A_j = \frac{1}{2L} \left\{ \int_{\frac{-L}{\tau_j}}^{0} (\frac{a_j \tau_j}{L} t + a_j) dt + \int_{0}^{\frac{L}{\tau_j}} (-\frac{a_j \tau_j}{L} t + a_j) dt \right\} = \frac{a_j}{2\tau_j}$$

and,

$$U_j(n) = \frac{1}{L} \left\{ \int_{\frac{-L}{\tau_j}}^{0} (\frac{a_j \tau_j}{L} t + a_j) cos(\frac{n\pi}{L} t) dt + \int_{0}^{\frac{L}{\tau_j}} (-\frac{a_j \tau_j}{L} t + a_j) cos(\frac{n\pi}{L} t) dt \right\}$$
$$= \frac{2a_j \tau_j}{(n\pi)^2} \left\{ 1 - cos(\frac{n\pi}{\tau_j}) \right\}$$

The waveforms of events Q and S are similar to that of reciprocal of waveform of event R. Therefore, it is enough to find the Fourier coefficients of $E_j(t)$ for $j \in \{R\}$ only, because functions corresponding to $j \in \{Q, S\}$ are just negative of $E_R(t)$.

References

[1] S. Seyedtabaii, *Analysis of heart rate variation filtering using LMS based adaptive systems*. WSEAS Transactions on Signal Processing 4(4), 2008.

[2] M. Z. U. Rahman, R. A. Shaik, et al., *Adaptive noise removal in the ECG using the Block LMS algorithm*, IEEE proceeding of 2nd International Conference ICAST, 2009.

[3] S.Z. Islam, R. Jidin, and M.A. Mohd. Ali, *Performance Study of Adaptive Filtering Algorithms for Noise Cancellation in ECG Signals*, IEEE Proceedings of ICICS, 2009.

[4] B. Farhang-Boroujeny, *Adaptive filters: theory and applications*, John Wiley & Sons, Inc. 1998.

[5] S. Narayan, A. Peterson, and M. Narasimha, *Transform domain LMS algorithm*, IEEE Transactions on Acoustics, Speech and Signal Processing, 1983.

[6] R. Karthik, *ECG simulation using Matlab*, 2006.

[7] C. Corduneanu, N. Gheorghiu, et al. *Almost periodic functions*, Chelsea Publishing Company New York, 1989.

[8] E. Kreyszig, *Introductory functional analysis with applications*, Vol. 1, wiley, 1989.

[9] S. Haykin, *Adaptive Filter Theory*, 2^{nd} edition, Prentice Hall, 1991.

[10] F. Beaufays, *Transform-domain adaptive filters: An analytical approach*, IEEE Transactions on Signal Processing 43, 1995.

[11] Michael J. Shea, M. Electrocardiography (ECG). (2013); Available from: www.merckmanuals.com/professional/cardiovascular_disorders /cardiovascular_tests_and_procedures/electrocardiography_ecg.html.