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A mathematical model for analysis of ECG waves in a normal subject

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Abstract

The modelling of ECG wave is one of the essential tools in explaining the activity of the heart ECG voltage vector. In the present paper a mathematical model of the ECG wave has been proposed, assuming the human body to be equivalent to a cylindrical composite dielectric and conducting medium and the heart to be a harmonic bio-signal generator located in this medium. This model has been utilised to explain the harmonic amplitude orientation pattern at a number of electrode locations on the frontal plane of a normal human body. It has been observed that there exists a close proximity of the nature of the mathematically predicted data with that of the experimental data.
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1. Introduction

The ECG heart electric field vector signal is generated due to the beating of the heart by the excitation signal produced at the S.A. node. This electric field is then propagated through the composite aqueous dielectric and conducting medium of the human body and is available on the surface of the human body in the form of time varying voltage signal called ECG signal. The voltage signal between the Wilson Central Terminal and an electrode location on the human body is called the monopolar ECG [1,2] and that between any two-electrode locations is called the bipolar ECG [1,2]. The physiological activity of heart is

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analysed from these monopolar and bipolar ECG records and the nature of normality or abnormality of the heart function is estimated from this analysis. The analysis of the ECG signals is generally made on the basis of some ECG models [1,2] and the accuracy of the results depends on the mathematical reasoning of these models. In the widely used classical 12-lead ECG model [1,2] the human body is assumed to be a uniform conducting spherical medium and the Wilson's Central Terminal (WCT) is selected to be the reference point of measurement. A large number of many other models have been proposed by various workers in order to have more accurate analysis of the ECG signals and hence more accurate clinical estimation of the physiological activity of the heart. The investigations on the analysis of ECG waves on the basis of these models are still being continued by various workers [1,2]. Biel et al. [3], Lacroix et al. [4] and Kors et al. [5] have proposed different techniques of analysis of ECG waves. Nygaard et al. [6] and many other workers have proposed different compression techniques of the large amount of ECG digital data in a computer based ECG monitor for accurate analysis of ECG waves in both time and frequency planes. The detection of the ECG wave components [7–10] is also very important in ECG wave analysis. The effect of position and number of electrodes during the measurement of ECG waveform on the normal subjects has also been studied by various workers [1,9,11]. Sornmo et al. [12], have proposed a mathematical model for evaluation of QRS-complex of ECG-wave. Feldmenn et al. [13], have related the R-wave amplitude of ECG-wave with the left ventricular chamber size and position where as Rechardson et al. [14] have studied ECG power spectra and the effect of random frequency modulation of that spectra.

In the present paper, a cylindrical model of human body and a harmonic generator model of the heart have been assumed in which the heart electric field vector is assumed to have harmonic components in all directions in space. A mathematical model has been proposed where the heart electric field vector harmonic generator is assumed to be at any presupposed origin inside the human composite dielectric and conducting medium

bounded by a cylindrical surface. The heart electric field generated at this origin is assumed to be propagated through this medium and is available on the cylindrical surface in any physiological plane. A mathematical model has been proposed to estimate the nature of the harmonic component on the surface of the human body. In the present paper, monopolar ECG signals at various locations on a circle in the frontal plane surrounding an arbitrary location as the center near the heart have been recorded. The off-line Fourier analysis of these records has been performed. From this analysis the average value, peak value and phase angle value of each harmonic of ECG wave have been determined. Now the peak harmonic distributions of the monopolar ECG wave at different electrode locations have been plotted. The same peak harmonic distribution has also been plotted from the proposed mathematical model. The experimentally observed data as well as the theoretical data are presented in the paper. It has been observed that the nature of this mathematically predicted pattern almost resembles that of the experimental pattern.

2. Method of approach

The monopolar ECG millivolt signal measured at any electrode location on a normal human body surface with respect to the Wilson's Central Terminal (WCT) is a periodic signal and hence may be represented by the Fourier harmonic components given by,

$$e(t) = C_0 + \sum_{n=1}^{n} A_n \sin n\omega_0 t + B_n \cos n\omega_0 t \qquad (1)$$

where

$e(t)$ = the instantaneous value of ECG voltage.
C_0 = average value of ECG voltage signal.
ω_0 = angular frequency of fundamental component.
T = time period of ECG voltage wave.

$$A_n = (2/T) \int_0^T e(t) \sin(n\omega_0 t) \mathrm{d}t$$

$$B_n = (2/T) \int_0^T e(t) \cos(n\omega_0 t) \mathrm{d}t$$

Assuming $A_n = C_n \cos \theta_n$ and $B_n = C_n \sin \theta_n$, the above Eq. (1) may be written as

$$e(t) = C_0 + \sum_{n=1}^{n} C_n \sin(n\omega_0 t + \theta_n) \qquad (2)$$

where $C_n = [A_n^2 + B_n^2]^{1/2}$ and $\theta_n = \tan^{-1}[B_n/A_n]$.

These sinusoidal harmonic components are generated by the periodic beating of the heart and propagate in all possible directions through the human body. These harmonic component generators may be combined to form the resultant heart voltage or electric field vector generator.

Assume that the human body is equivalent to a uniform dielectric medium bounded by a cylindrical surface and the origin of the heart electric field vector generator is at some point O in this medium inside the body as shown in Fig. 1. The ratio of the harmonic voltage between this origin point and that at any other point of the human body to the distance between them may be assumed to give average component of the harmonic heart electric field vector in that direction. Let the projection of O on the frontal plane of the human body be O′. It may be assumed to be a point on the frontal plane very close to the heart. Let this point be near the left nipple on this plane. Let us assume a reference frame of cylindrical co-ordinates with O as the origin with an arbitrary straight line OX in the plane parallel to the frontal plane of human body as the reference axis and OO′ as the Z-axis in a cylindrical system of co-ordinate axes. Let us

consider a circle of radius ρ with O′ as centre in the frontal plane and an arbitrary line O′X′ as the frontal plane reference axis in this plane parallel to the original reference axis OX. Let the cylindrical co-ordinates of any point P on this circle be (ρ, ϕ, Z) where ϕ is the angle between O′X′ and O′P as shown in Fig. 1.

At any point, the heart vector $\mathbf{E_H}$ may be assumed to have three components $E_{H\rho}$ along the radius vector, $E_{H\theta}$ along the direction perpendicular to the radius vector and E_{HZ} along the direction parallel to the Z-axis, i.e. OO′ axis. Any monopolar harmonic ECG voltage at any point in the frontal plane with respect to WCT may be assumed to be due to the electric field component E_{HZ} of the heart vector. Let the Z-component of the sinusoidal nth harmonic ECG heart electric field vector at the origin O, be $E_{zon} \sin(n\omega_0 t)$, which will propagate, in all-possible directions. Let the average nth harmonic propagation constant along Z direction be γ_{zn} so that at a distance Z from the reference plane it may be measured as

$$E_{zn} = E_{zon} \mathrm{e}^{-\gamma_{zn}} \cdot z \qquad (3)$$

where $\gamma_{zn} = (\alpha + \mathrm{j}\beta)$, α being the attenuation constant and β being the phase shift constant.

Now assuming the human body as a uniform propagating medium of conductivity σ and permittivity ε the wave equation of the electric field vector may be given by

$$\nabla^2 \mathbf{E} = -n^2 \omega_0^2 \mu \varepsilon \mathbf{E} + \mathrm{j} n \omega_0 \mu \sigma \mathbf{E} \qquad (4)$$

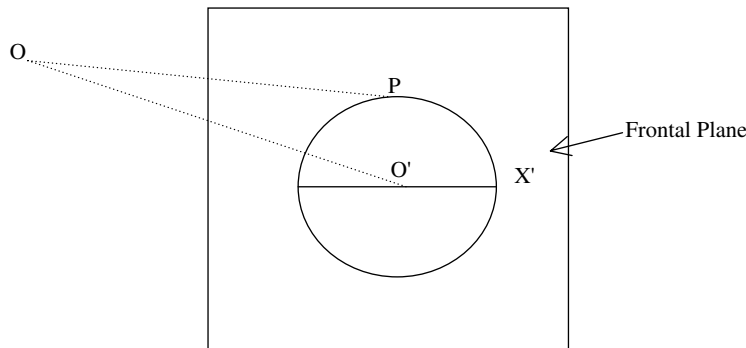Equating the nth harmonic Z-component on both sides in cylindrical co-ordinates we have,

Fig. 1. Mathematical model of ECG wave.

$$\delta^2 E_{zn}/\delta\rho^2 + (1/\rho^2)\delta^2 E_{zn}/\delta\phi^2 + \rho\delta^2 E_{zn}/\delta z^2$$
$$+ (1/\rho)\delta E_{zn}/\delta\rho$$
$$= -(n^2\omega_0^2\mu\varepsilon - jn\omega_0\mu\sigma)E_{zn} \tag{5}$$

Let $\quad E_{zn} = P(\rho)Q(\phi)e^{-\gamma_{zn}} \cdot z \tag{6}$

where $P(\rho)$ is a function of ρ only and $Q(\phi)$ is a function of ϕ only.

Putting this in the above Eq. (5) we get,

$$Q\delta^2 P/\delta\rho^2 + (P/\rho^2)\delta^2 Q/\delta\phi^2 + \gamma^2 PQ + (Q/\rho)\delta P/\delta\rho$$
$$= -(n^2\omega_0^2\mu\varepsilon - jn\omega_0\mu\sigma)PQ$$

Or $\quad (\rho/P)\delta^2 P/\delta\rho^2 + (1/\rho)Q\delta^2 Q/\delta\phi^2 + h^2\rho$
$\quad + (1/P)\delta P/\delta\rho = 0$

where

$$h^2 = \gamma^2 + n^2\omega_0^2\mu\varepsilon - jn\omega_0\mu\sigma \tag{7}$$

Or $\quad (\rho^2/P)\delta^2 P/\delta\rho^2 + (\rho/P)\delta P/\delta\rho + h^2\rho^2$
$\quad + (1/Q)\delta^2 Q/\delta\phi^2 = 0$

Now $(\rho^2/P)\delta^2 P/\delta\rho^2 + (\rho/P)\delta P/\delta\rho + h^2\rho^2$ is a function of ρ only and $(1/Q)\delta^2 Q/\delta\phi^2$ is a function of ϕ only, So the above equation is possible only when each of these parts of the equation is equal to a constant, say m^2.

Hence

$$(\rho^2/P)\delta^2 P/\delta\rho^2 + (\rho/P)\delta P/\delta\rho + h^2\rho^2 = m^2 \tag{8}$$

and

$$(1/Q)\delta^2 Q/\delta\phi^2 = m^2 \tag{9}$$

From Eq. (8), we get

$$\delta^2 P/\delta\rho^2 + (1/\rho)\delta P/\delta\rho + (h^2 - m^2/\rho^2)P = 0 \tag{10}$$

This is the standard form of Bessel's equation in terms of ρh. Hence the solution of this equation is given by

$$P(\rho) = J_m(\rho h) \tag{11}$$

The solution of Eq. (9) is given by

$$Q = (C_m \cos m\phi + D_m \sin m\phi) \tag{12}$$

By suitable orientation of the reference axes D_m may be reduced to zero.

Hence from Eq. (6) we get,

$$E_{zn} = P(\rho)Q(\phi)e^{-\gamma_{zn}} \cdot z$$
$$\text{or,} \quad E_{zn} = J_m(\rho h)C_m \cos(m\phi)e^{-\gamma_{zn}} \cdot z \tag{13}$$

Now from Eq. (7) we get,

$$h^2 = \gamma_{zn}^2 + n^2\omega_0^2\mu\varepsilon - jn\omega_0\mu\sigma$$

Or, $\quad h^2 = (\alpha + j\beta)^2 + n^2\omega_0^2\mu\varepsilon - jn\omega_0\mu\sigma$
$\quad\quad = (\alpha^2 - \beta^2 + n^2\omega_0^2\mu\varepsilon) + j(2\alpha\beta - n\omega_0\mu\sigma)$

or, $\quad |h^2| = [(\alpha^2 - \beta^2 + n^2\omega_0^2\mu\varepsilon)^2 + (2\alpha\beta - n\omega_0\mu\sigma)^2]^{1/2}$
$\quad\quad = [\omega_0^4\mu^2\varepsilon^2 n^4 + \{2(\alpha^2 - \beta^2)\omega_0^2\mu\varepsilon$
$\quad\quad\quad + \omega_0^2\mu^2\sigma^2\}n^2 - 4\alpha\beta n\omega_0\mu\sigma + (\alpha^2 - \beta^2)^2]^{1/2}$
$$\tag{14}$$

or, $\quad |h^2| = [a_1 n^4 + a_2 n^2 - a_3 n + a_4]^{1/2} \tag{15}$

where a_1, a_2, a_3 and a_4 are constants and

$$a_1 = w_0^2\mu^2\varepsilon^2, \quad a_2 = 2(\alpha^2 - \beta^2)\omega_0^2\mu\varepsilon + \omega_0^2\mu^2\sigma^2,$$
$$a_3 = 4\alpha\beta\omega_0\mu\sigma, \quad a_4 = (\alpha^2 - \beta^2)^2 \tag{16}$$

or $\quad |h| = [a_1 n^4 + a_2 n^2 + a_3 n + a_4]^{1/4} \tag{17}$

Assuming $a_2 = 0 = a_3 = a_4$ and $a_1 = k^4$ we get

$$|h| = kn \tag{18}$$

Hence from Eq. (13) we have,

$$E_{zn}(t) = C_m J_m(\rho kn) \cos(m\phi)e^{-\alpha z}e^{j(n\omega_0 t - \beta z)} \tag{19}$$

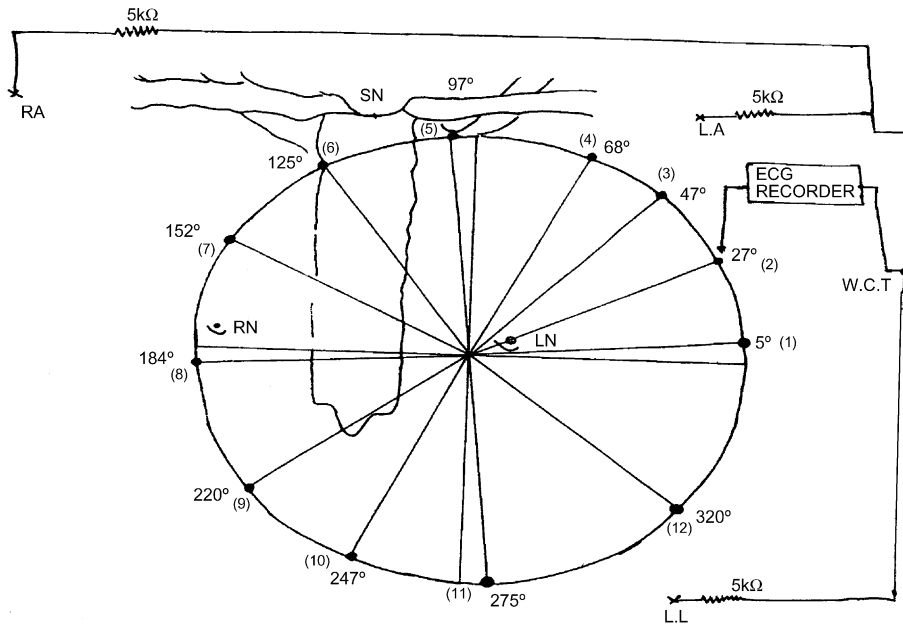Hence the monopolar ECG voltage on the frontal surface of the body may be given by,

$$V_{zn}(t) = zE_{zn}(t) = zC_m J_m(\rho kn) \cos(m\phi)e^{-\alpha z}e^{j(n\omega_0 t - \beta z)} \tag{20}$$

Hence the resultant monopolar ECG voltage on the frontal plane of human body for a particular non-zero value of m may be given by,

$$V_z(t) = \sum_{n=0}^{n} V_{zn} = \sum_{n=0}^{n} zC_m J_m(\rho kn) \cos(m\phi)e^{-\alpha z}e^{j(n\omega_0 t - \beta z)} \tag{21}$$

where the values of C_m, k, α and β depends on the physiological condition of the human body.

For a particular electrode location on a particular human body, ρ, z, m, ϕ, α and β may be assumed to be constants. Hence Eqs. (20) and (21) may be respectively given by the following Eqs. (22) and (23).

SCHEME FOR ELECTRODE PLACEMENT IN POLAR CO ORDINATE

● REPRESENTS ELECTRODE LOCATION

SN = SUPRASTERNAL NOTCH ;   RA = RIGHT ARM.
RN = RIGHT NIPPLE   ;   LA = LEFT ARM.
LN = LEFT NIPPLE   ;   LL = LEFT LEG.

[CENTRE FIXED AT 15 C.M. FROM SUPRASTERNAL NOTCH AND
2 C.M. FROM LEFT NIPPLE]

Fig. 2. Circular array of ECG electrode locations in the frontal plane of human body.
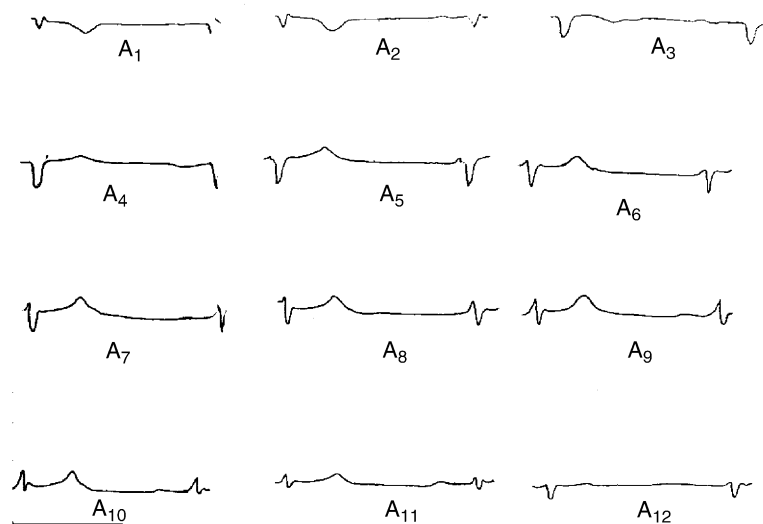
Fig. 3. Frontal plane ECG records in a circular array of electrode locations of a normal subject.

$$V_{zn}(t) = J_m(\rho kn) \qquad (22)$$

$$V_z(t) = \sum_{n=0}^{n} J_m(\rho kn)\cos(n\omega_0 t - \theta) \qquad (23)$$

where $\theta = \beta z$.

3. Experiment

In the present project an empirical circle is se-lected on the frontal surface of the human body with the center at an arbitrary location about 2 cm from the left nipple and about 15 cm from suprasternal notch as shown in Fig. 2 with the RA, LA and LL electrodes connect to the WCT through 5 kΩ resistance each.

The radius of the circle is arbitrarily selected to be about 12 cm. Now the test electrode of the ECG recorder with its common point connected to WCT was placed in different locations along the circumferences of the circle at angular positions of 5°, 27°, 47°, 68°, 125°, 152°, 184°, 220°, 247°, 275°, 320° with respect to some reference axis as shown in Fig. 2. The recorded ECG waves for a normal subject are shown in Fig. 3.

Now the Fourier harmonic amplitudes of each of the recorded waves are obtained by using BA-SIC software program of Fourier analysis. Now for each recorded ECG wave of each electrode, a graph is drawn by plotting spectral harmonic amplitude (C_n) against the order (n) of harmonics. This spectral amplitude pattern for all the elec-
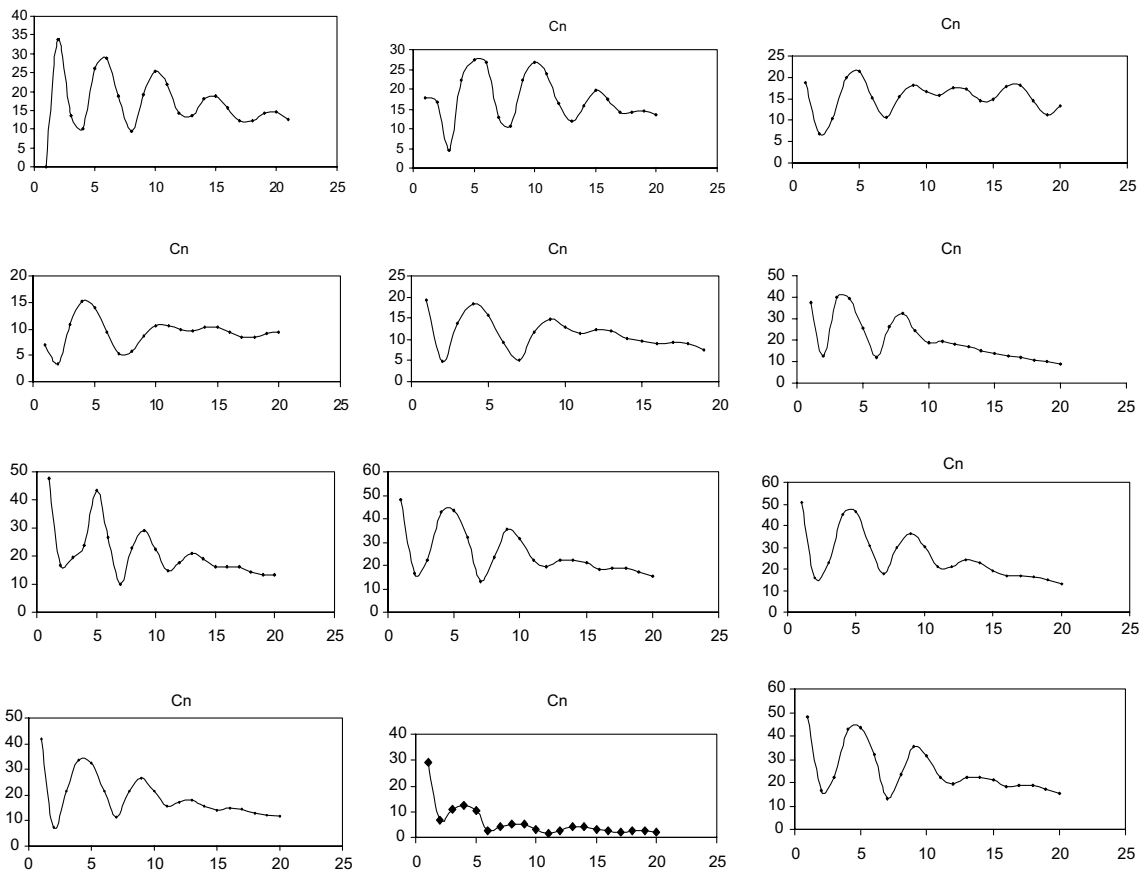

Fig. 4. Frontal plane spectral amplitude pattern of ECG records in a circular array of electrode locations of a normal subject for 20 harmonics.
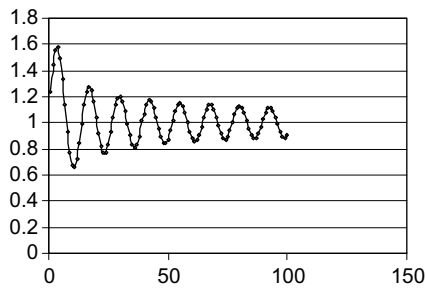
Fig. 5. Spectral amplitude pattern of ECG wave according to the proposed mathematical model for 20 harmonics.

trodes on the circle in the frontal plane for a normal subject are shown in Fig. 4.

Now according to the proposed mathematical model as explained by Eq. (22), the harmonic pattern of the ECG wave is drawn for some arbitrarily selected values of ρ, k, m and θ and the pattern is as shown in Fig. 5. Again the resultant ECG voltage wave according to Eq. (23) is also drawn for the same values of ρ, k, m and θ and for 10 harmonics. The resultant ECG wave pattern in time plane is as shown in Fig. 6.

4. Discussions

The spectral ECG amplitude pattern, for 20 harmonics and 12 electrode locations arranged in a circle on the frontal plane of a normal subject, shown in Fig. 4, reveals that all the patterns have almost similar nature and this explains the need for a mathematical model to explain this pattern. The theoretical ECG wave spectral amplitude pattern as shown in Fig. 5 is found to have almost similar waveform as the experimental pattern

shown in Fig. 4. The apparent deviations between the two types of pattern may be due to the effect of the physiological constants C_m, a_1, a_2, a_3, a_4, k, α, and β as shown in Eqs. (16), (18), (20) and (21). The experiment has been repeated on a number of subjects in the presence of a physiologist and the similar results have been obtained. The resultant ECG wave shown in Fig. 6 is drawn following Eq. (23) for 10 harmonics and appears to give an ECG wave pattern found in normal subjects. In this case also the apparent deviations of the pattern from the practical normal pattern may be due to the same physiological constants and number of harmonics.

The order m of the Bessel's function may be assumed to be different for different human bodies for the same electrode location or for different electrode locations for the same body. The value of m may be selected by trial and error method until the mathematically predicted pattern may have identical nature with the practically observed pattern for a particular subject at a particular electrode location. The values of the other constants in Eqs. (20) and (21) depend on the average conductivity and permittivity of the living organs, order of harmonics and many unknown physiological parameters. Hence the exact values of the constants may not be known but an approximate expected ECG wave pattern may be predicted by trial and error method assuming some arbitrary values of the constants. Hence the clinical abnormalities present in a subject may be estimated by comparing the experimental ECG pattern with the expected pattern for a particular subject. Here it may be noted that the conventional ECG models [1,2] are primarily based on spherical model of human torso which deviates from the actual
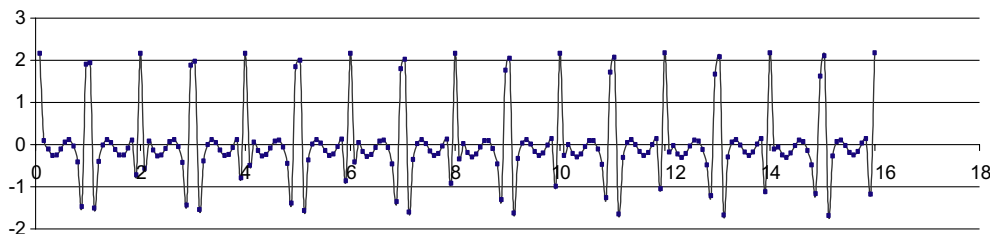

Fig. 6. Resultant ECG wave pattern for 10 harmonics at a particular electrode location from the proposed mathematical model.

cylindrical shape of the torso and hence the proposed cylindrical model of human body may be treated to have more accuracy then the classical models. However much more rigorous experimental works and analysis is required in order to find the actual advantage of the proposed model. The detailed research work in this line is under progress and the observation from the preliminary work is reported in the present paper.

Acknowledgements

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