

1 BACKGROUND AND RELATED WORK

This section provides background information on time series and ECGs, as well as methods to analyze them.

TODO make this a more complete introduction, give a verbal table of contents with reference to the different sections

1.1 Time Series and Time Series Analysis

This subsection will provide background information on time series and time series analysis methods. A time series is a set of values recorded at specific times. A common form of time series are discrete-time time series (often simply called discrete time series). Discrete time series are time series whose values are recorded at discrete points in time, the most common example of this are time series with values recorded at fixed intervals. Continuous-time time series are time series that are recorded continuously over a certain interval [1]. Time series that contain a single value for each moment in time are called univariate time series, while time series that record multiple values at each moment in time are called multivariate time series [2]. Time series are used in many disciplines to record information on time-dependent processes, e.g. stock prices in economics, the sun's activity in physics, or the heart's activity in medicine. Time series can be recorded digitally, physically, or, if they were recorded physically, can later be digitized. The recorded data can then be used to gain insight into the processes that were studied. To gain insight using a time series, the relevant information needs to be extracted from it—a process that is often called data mining. Data mining of time series is a vast discipline that, among others, includes [3, 4]:

- visualization (graphical representation),
- forecasting (predicting future behavior),
- indexing (finding the most similar time series to a given one),
- clustering (dividing time series into groups of similar ones),
- anomaly detection (detecting parts that are not “normal” or do not fit certain parameters),
- classification (assigning a label based on its features, e.g. “sick” and “not sick”), and
- summarization (reducing the complexity—often length—while preserving important features).

Challenges for time series analysis include the often very large datasets that are difficult for humans to analyze and take up considerable digital storage space. Analyzing very large datasets requires a large amount of computational power because most data mining algorithms become less efficient with larger datasets [3]. To mitigate this issue, time series dimension reduction (also known as dimensionality reduction or time series representation) is used. Dimension reduction transforms a “raw” (unmodified) time series into a representation that is simpler but nonetheless resembles the raw time series. This can be achieved by either using a method that reduces the number of values in a time series, or by extracting only the relevant features from the time series. According to [4, 5], there are four types of dimension reduction methods:

1. data dictated,
2. non-data adaptive,

3. model-based, and
4. data adaptive.

Methods 2–4 have their dimension reduction factors set by user-defined parameters. This means that the user can determine how much the dimension of the data should be reduced [4].

1.1.1 Data dictated representation

Data dictated methods derive their compression ratios from the data automatically, the most common form of this method is the clipped representation [4]. This representation simply transforms the raw time series into a sequence of 1s and 0s. A data points is assigned a 1 if its values is larger than the mean value of the time series, and a 0 otherwise. A sequence of 1s and 0s can be further compressed using various methods from computer science, finally yielding a very large compression ratio of 1057:1 [6].

1.1.2 Non-data adaptive representation

Non-data adaptive methods operate on time series segments with a fixed size to reduce the dimension and they are useful for comparing multiple time series with each other. These methods include the Discrete Wavelet Transform (DWT), the Discrete Fourier Transform (DFT), and the Piecewise Aggregate Approximation (PAA) [4]. The DWT uses wavelets, a limited-duration wave with an average value of 0, which represents both time and frequency information. The DWT is calculated using a series of filters applied to the signal. In [7], the DWT is used to detect beats in ECG signals and achieves a 0.221% detection error rate. The Fast Fourier Transform, an optimized form of the DFT, decomposes the its input signal into many sinus waves of different frequencies. In [8] it is used in conjunction with a machine learning model to achieve a beat classification accuracy of 98.7%. The PAA is part of the process of the SAX representation, thus it will be covered in

TODO refer to the appropriate methods section

1.1.3 Model-based representation

Model based methods use stochastic methods such as Hidden Markov Models (HMM) and the Auto-Regressive Moving Average (ARMA) [4]. A HMM was used in [9] to cluster electroencephalograph recordings (measuring the brain's electrical activity). It was found that their methods was competitive with other established methods in classifying electroencephalograph signals. An auto-regressive model can be used to correctly identify a specific type of arrhythmia in an ECG and to group the occurrences of this arrhythmia together [10].

1.1.4 Data adaptive representation

Data adaptive methods use non-fixed size segments and aim to fit the raw data most closely. Examples of data adaptive methods are the Piecewise Polynomial Approximation (PPA), Piecewise Linear Approximation (PLA), Piecewise Constant Approximation (PCA), and SAX [4]. PPA can be used to compress and ECG by approximating it using polynomials. With second-order poly-

nomials, ECGs can be compressed with a minimal level of distortion [11]. The authors of [12] use a modified PLA representation with adaptive ECG segmentation to successfully reconstruct the 12 standard leads of an ECG from only 3 leads. Using adaptive PCA as the dimension reduction method, the preprocessing and segmentation of ECGs can be significantly sped up while maintaining accuracy comparable to previous methods [13]. The SAX representation will be covered in detail in **TODO** refer to the SAX section and the following subsection 1.1.5 will provide background on the method and its variations.

1.1.5 SAX representation background

A particular dimension reduction method is SAX. Introduced by Lin, Keogh, Lonardi, and Chiu, SAX is a symbolic time series representation method for univariate time series. The authors felt that the symbolic methods available in 2003 did not provide the desired dimension reduction, did not correspond to the raw data accurately enough, and could not be applied to a subset of the total data. SAX uses the averaging of a user-defined number of segments and the labeling of segments with letters to reduce the dimension of the time series data. The number of letters, called the alphabet size, can also be chosen by the user and influences the dimension reduction. The distance between two time series in the SAX representation is guaranteed to resemble the distance between the two raw time series, this is called the distance measure. Since its creation, SAX has found widespread use in data mining and many researchers have attempted to modify and improve it.

The SAX distance measure has been improved to include the standard deviation [14] and a measure of the trend of each averaged segment [15, 16]. Extended SAX modifies SAX to include the minimum and maximum values of each segment for improved representation of the raw data [17] while 1d-SAX incorporates a linear regression over each segment into SAX [18]. A combination of SAX and a polynomial approximation was used to speed up the SAX method [19]. To improve the indexing performance of SAX, iSAX introduced convertible alphabet sizes, allowing SAX representations with different alphabet sizes to be compared with each other and indexed into a tree structure [5]. iSAX 2.0 improves the iSAX index by reducing its computational complexity, enabling it to index a time series that has one billion elements, something that SAX or iSAX cannot do [20]. To perform time series anomaly detection using SAX, Keogh, Lin, and Fu introduced Heuristically Ordered Time series using SAX (HOT SAX) in 2005. Specifically, the authors attempt to detect time series “discords”, a subsequence of a time series that is most different from other segments of the time series. This can theoretically be done by simply comparing all subsequences of the raw time series to all other segments, but this approach is not feasible for long time series because of its complexity. Thus, HOT SAX utilizes SAX to reduce the dimensionality and complexity of the time series and then sorts the resulting SAX segments to speed up the discord detection. The authors suggest further research to investigate the use of HOT SAX on multivariate time series [21]. For an in-depth description of this method, please refer to **TODO** refer to the methodology section of HOT SAX.

TODO mention this in the methodology section as supporting information

SAX and its variants have also been used for the analysis of multivariate time series. SAX-

ARM combines the SAX representation with association rule mining (identifying rules and implications found in the data, i.e. parameter a influences parameter b) to analyze multivariate time series and discover the rules underlying the data [22]. Anacleto, Vinga, and Carvalho introduced MSAX in 2020 and thus expanded the use of SAX to multivariate time series. They utilize multivariate normalization with a covariance matrix and a modified distance measure to achieve this. To analyze their method, the authors use MSAX and SAX in a classification task based on multiple multivariate time series datasets. For these multivariate datasets, SAX was applied to each of their individual time series and those results were combined. Their analysis found that, overall, SAX applied in this way is superior to MSAX when it comes to classification accuracy. In 6 of the 14 tested datasets, SAX was significantly more accurate, in 2 of the MSAX was more accurate, and in the remaining 6 their performance was not significantly different. It should be noted that in the ECG dataset they tested, the accuracy of SAX (~87%) was slightly higher than that of MSAX (~84%), but not significantly so. Anacleto, Vinga, and Carvalho suggest that in future research MSAX should be applied to electronic health records (e.g. ECGs) and that it should be applied to other time series data mining applications besides classification [2]. MSAX will be thoroughly presented in **TODO** refer to methods section. **TODO** mention this in the methodology section as supporting information Another application of SAX to multivariate data used it to visualize multivariate medical test results and enable their analysis [23]. Resource-aware SAX is a SAX variant developed to analyze ECG using a mobile device like a mobile phone. The method takes advantage of the computational efficiency of SAX to perform the ECG analysis on the device and even preserve its battery life. Another application of the SAX method to ECGs is [24], which uses SAX with an added binary measure of the trend of each segment to detect ECG anomalies, achieving a recall value of 98%. The section 1.2 below will elaborate on ECGs and methods of their analysis.

1.2 ECGs and ECG Analysis

The following subsection covers the ECG and methods used in its analysis. Luigi Galvani noted the electrical activity in muscles 1786, but the history of the ECG only started in 1842, when Carlo Matteucci showed the electrical activity of a frog's heartbeat. In the 1870s, it was discovered that each heartbeat is characterized by electrical changes. Then, in 1901-1902, Willem Einthoven created the first ECG recording of a human heartbeat using using 3 leads connected to the limbs of the patient. Einthoven was the first to publish an ECG waveform with the now standard annotations P, Q, R, S, and T for the different features (see Figure 1.1). He would receive the 1924 Nobel Prize in medicine for his invention of the electrocardiograph. As a result of further development, the 12-lead ECG that we know today was created [25, 26]. The 12-lead ECG is comprised of 6 chest leads (measurements of electrodes on the chest) numbered consecutively V1 to V6, as well as 6 limb leads (measurements of electrodes on the limbs) called I, II, III, aVR, aVL, aVF [27].

1.2.1 What is an ECG?

An ECG records the electrical activity that accompanies the contraction and relaxation of the heart muscle. The sinuatrial node, which can spontaneously give off an electrical pulse, initiates the heart

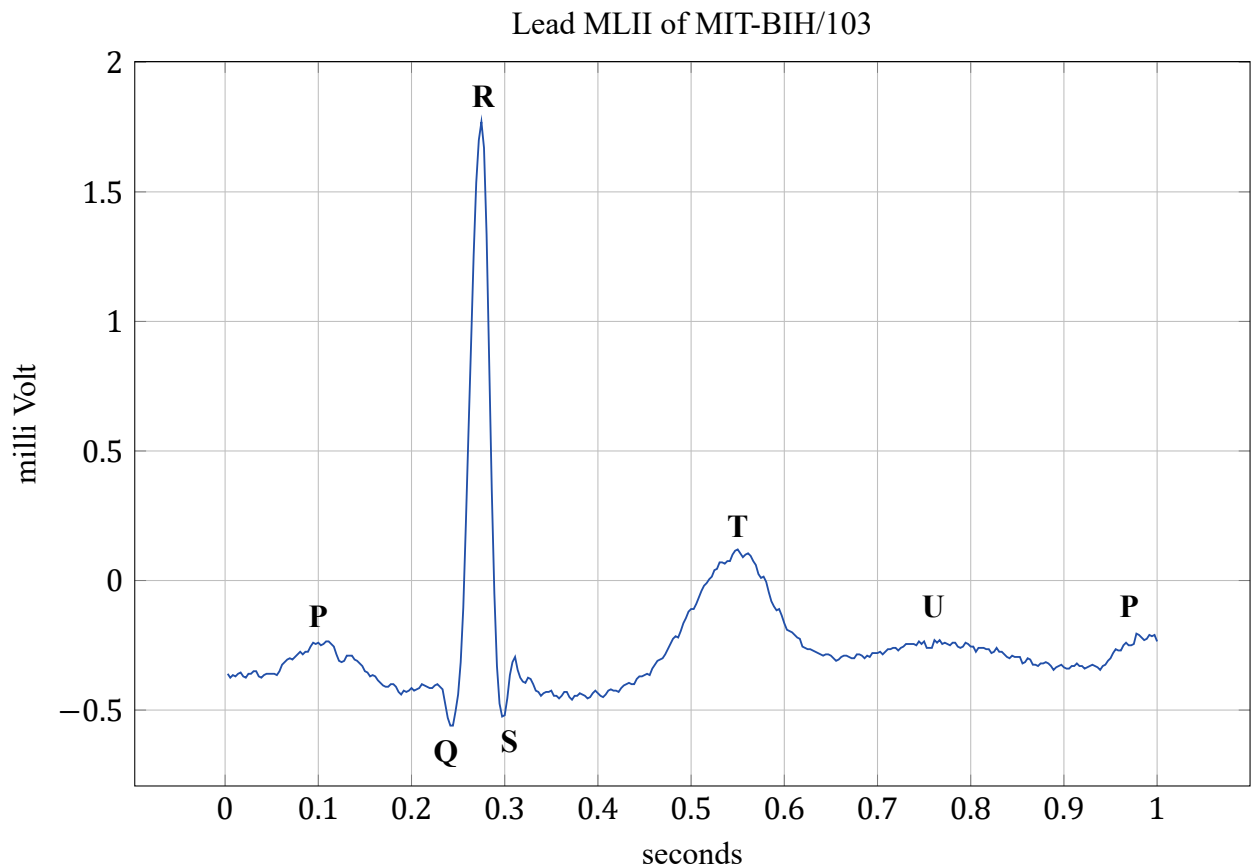


Figure 1.1: Annotated ECG of one heartbeat. This graph is based on lead II, data points 2031–2390 of recording 103 of the MIT-BIH database [29, 30].

beat. Its pulse is conducted through the heart by other specialized fibers, causing the heart to beat. The conduction of electricity is facilitated by Sodium, Calcium, and Potassium ions flowing in and out of cardiac cells [28]. Figure 1.1 shows a ECG wave of a single heartbeat from record 103 of the MIT-BIH database [29, 30] (for more information on the database, see section 1.2.3). The P wave is caused by the depolarization of the atrial node, which allows blood to flow into the heart. The QRS complex, as it is called, is the result of ventricular depolarization and represents the action of pumping blood out of the heart. The T wave is caused by ventricular repolarization in preparation for the next heartbeat. The U wave, only present in about 25% of people, is thought to be caused by mechanical-electric feedback [28, 31]. The last P wave is part of the next heartbeat, which is not shown in Figure 1.1.

The waves and complexes shown in Figure 1.1 are the object of ECG analysis. Changes in their shape, duration, or height can indicate heart conditions. Becker list some of the features relevant for ECG analysis [28]:

- The regularity of the rhythm: are the intervals between the QRS complexes and P waves regular?
- The shape of the QRS complex: do they have similar shape and duration?
- The regularity of the P waves: are the P waves similar and is the interval between P wave and QRS complex similar?
- Is the heart rate regular: measuring the time between QRS complexes can be used to calculate

the heart rate, is this heart rate in the normal range?

- Do the waves and complexes come in the same order each time: each cycle should consist of a P wave, QRS complex, T wave.

Using an ECG to diagnose a cardiac condition is difficult in practice. Small changes in the components of the ECG can be indicators of diseases and those changes can be overlooked, even by trained and specialized physicians. The chance to make a mistake is even higher for non-specialized physicians and trainees [25, 32]. The American Heart Association (AHA) estimates that a physician needs to read at least 500 ECGs with the help of an expert before becoming proficient. One reason for this is that the number of diagnosis that can be performed using an ECG is vast. The AHA lists 88 different conditions and an additional 22 diagnoses related to diseases and conditions that may not directly affect the heart, such as hypothermia or tremors caused by Parkinson's disease [33].

Two types of heart conditions that an ECG can detect are cardiac arrhythmias and ischaemic heart disease. Cardiac arrhythmia is a variation of the heart rate or rhythm that does not have a reasonable cause. In other words, heart rate or rhythm variations caused by physical activity could not be considered arrhythmias, while significant variations in a resting state may [34]. In an ECG, arrhythmia is most apparent in changes in the interval between the QRS complexes. Ischaemic heart disease is the main cause of death world-wide [35]. Ischemic heart disease is characterized by restricted blood flow to an area of the heart, causing it to not receive enough blood and oxygen. Blood flow restriction is caused by a blockage (or narrowing) in a blood vessel supplying the heart muscle. An artery can be blocked by a blood clot, but the most common cause is plaque buildup, called atherosclerosis. If the circulation to the heart is completely blocked, the cells in the heart muscle begin to die. This is called myocardial infarction, more commonly known as a heart attack. The deprivation of oxygen the heart experiences leads to the characteristic chest pain commonly associated with heart attacks [36]. In an ECG, ischaemic heart disease can be diagnosed based on changes in the ST segment and the T wave. The diagnosis of ischaemic heart disease and other heart diseases is time sensitive. If a patient has suffers from a heart attack, treatment has to be started as soon as possible. Some forms of treatment are most effective in the first 3 hours after symptom onset and lose most of their effectiveness after 9 to 12 hours. The diagnosis required for treatment to begin should thus be as quick as possible. The real-time information delivery of an ECG is an advantage in this situation, even though there are more time-consuming methods that can deliver more accurate results than an ECG [37].

1.2.2 Computerized ECG analysis

The widespread use of ECGs and the time-sensitive nature of their application as diagnostic tools makes errors, delays, or inconsistencies in their interpretation unacceptable. A recent approach to minimizing this problem is the application of computer technology in ECG recording, storage, and analysis [38]. Time series analysis methods can also be applied to ECGs because ECGs simply represent discrete multivariate time series. As discussed in section , multivariate time series are time series that contain more than one value at each point in time, while discrete time series are time series that are measured at discrete points in time or at set intervals. ECGs fulfill both of

these requirements, as all modern ECGs contain at least 2 leads, most of them 12, and they have set sampling rates, given in samples per second. The common steps of computerized ECG analysis, following [38], are:

1. signal acquisition and filtering,
2. data transformation,
3. waveform recognition,
4. feature extraction, and
5. classification or diagnosis.

Step 1 comprises the digital recording of ECG signals or the digitizing of paper-based ECG records. For either processes, the AHA recommends a sampling frequency of 500 samples per second. ECG filtering is performed to remove noise introduced by patient movements, power line interference, and other factors [38]. This filtering, or denoising, is often performed using digital filters. Their drawbacks are that they only filter out very specific frequencies. Because noisy ECGs contain different types of contaminations, digital filters can be inaccurate. Using wavelet transforms for denoising has the advantage that noise can be more precisely targeted and the clean signal reconstructed afterwards. Choosing appropriate wavelet parameters can be challenging, but methods to optimize this process have been proposed [32]. Step 2 uses the same types of methods for dimension reduction that were discussed in sections 1.1.1–1.1.4 for time series and shall not be repeated here. This includes SAX, which has been successfully applied to ECG analysis [24]. MSAX has, at the time of writing, to the author’s knowledge not been applied to ECG analysis. HOT SAX has been used in [39] to detect anomalies in ECGs. It was found to detect anomalies, but it exhibited a larger amount of false identifications than competing methods.

Steps 3 and 4, the waveform recognition and feature extraction steps, are signified by extracting features that are relevant for diagnosis from the many points of the ECG. This process can also be aided by an appropriate representation chosen in the previous step. The main features targeted for extraction are the PQRST features shown previously in Figure 1.1. The Fast Fourier Transform (see section 1.1.2) provides a way of analysing the frequency domain of the ECG signal, enabling the detection of the QRS complex [8] and other features [40]. The missing time information in the Fast Fourier Transform can lead to difficulties in detecting time-dependent features. The short-time Fourier Transform adds time information to the fast Fourier Transform’s data. This can increase the accuracy of the feature extraction. It has the drawback in the tradeoff between the time and frequency resolutions. Wavelet transforms can also be used for feature extraction [7]. They have the advantage that they are suitable for all frequency ranges. Choosing the right wavelet base for the desired application can be a challenge. The discrete wavelet transform is the most widely used wavelet transform, thanks to its computational efficiency. Statistical methods are also used to extract features from ECGs; those methods are generally less affected by noise in the signal [32].

After the features of the ECG have been extracted, it is often necessary to further reduce the number of features. The reason for this is that a large number of features, despite the high accuracy their analysis may yield, require a high amount of computation to classify. This lengthy computation can negate the advantages gained by high accuracy. Feature reduction sacrifices a

certain amount of information and sometimes precision, but significantly speeds up the classification. There are two approaches to achieve this. First is feature selection, a process that attempts to select a subset of the original data that adequately describes the whole data. Feature selection can be performed by a filter that filters out unnecessary attributes based on some metric. This method is relatively simple, but the filtering process removes data and thus negatively impacts the precision of further steps. The second method, feature extraction, on the other, hand uses dimension reduction methods to keep as much of the original information as possible. Principal component analysis preserves as much of the variance in the original data as it can [7]. Other algorithms focus on separating classes of data, pattern recognition, or retaining the structure of the original data [32]. Here, again, the time series representation methods discussed in sections 1.1.1–1.1.4 can be applied.

Finally, the extracted features can be classified; this is stage 5. In this stage judgements are made based on the prepared input data and the result should be a disease diagnosis. Traditionally, this process is performed by a trained professional, as discussed in section 1.2.1. In the early stages of computerized ECG analysis, classification was performed by algorithms based on human actions when reading an ECG. Those algorithms were basic and not particularly accurate. Currently, the classification at the end of the preparation process is performed by a machine learning algorithm. Such models include the k-nearest-neighbors model which classifies points into groups but which is very expensive to calculate for high-dimensional data. Support vector machines are used for pattern recognition and are able to work with small samples. Artificial neural networks are robust and can work with complex problems, they are generally more accurate than support vector machines [8]. The newest approach is to forego the stages discussed here and use a single neural network to perform all the required tasks “end-to-end”. These networks are fed raw data perform steps 1.1.1–?? internally, as a single model [32]. This approach is relatively new and still actively researched. The previous approach, too, is enjoying active research attention.

1.2.3 ECG databases

A very important element of computerized ECG analysis is the training data. This data is used to train algorithms like neural networks, to manually tweak parameters of methods like SAX, or to validate and test prepared models. To fulfill these criteria, the data must be freely available to other researchers to replicate experiments and it should be fully annotated, meaning that experts determined the diseases that are or are not present as well as annotated the individual heart beats. ECG databases fulfill these requirements. One of the largest repositories of ECG data and physiological data is PhysioNet. PhysioNet was founded in 1999 by the National Institutes of Health (USA) and offers large collections of freely accessible ECG data [30]. These datasets vary in their size from around 10 recordings [41] to over 100 [42]. The QT Database [42] (available at <https://physionet.org/content/qtdb/1.0.0/>) has annotations for all types of ECG waves (P, QRS, T, and U; see Figure 1.1) for 105 two-lead recordings, each 15 minutes long. This database focuses on wave and feature detection as most ECG datasets only have the QRS complex annotated. The St Petersburg INCART 12-lead Arrhythmia Database (available at <https://physionet.org/>

[content/incartdb/1.0.0/](#)) contains 75 30-minute recordings that contain all 12 ECG leads. The significance of this database is that it contains all 12 ECG leads, while most ECG databases only contain 2 (see [29, 42])—this makes it possible to test multivariate detection methods as well as realistic circumstances, where a raw ECG would most likely contain 12 leads. The European ST-T Database [43] (available at <https://physionet.org/content/edb/1.0.0/>) contains 90 recordings of 79 subjects, each being 2 hours long and containing two leads. This database is focused on the ST segment and the T wave (hence the name) and thus focuses on ischaemia detection. One of the most used databases in the literature is the MIT-BIH (Massachusetts Institute of Technology-Beth Israel Hospital) Arrhythmia Database (see [7, 8, 11, 24, 39, 40, 44]). This database is focused on arrhythmia detection and contains 48 two-lead recordings that are each 30 minutes long.

REFERENCES

- [1] P. J. Brockwell and R. A. Davis, *Introduction to Time Series and Forecasting*, en, ser. Springer Texts in Statistics. Cham: Springer International Publishing, 2016, ISBN: 978-3-319-29852-8 978-3-319-29854-2. DOI: [10.1007/978-3-319-29854-2](https://doi.org/10.1007/978-3-319-29854-2).
- [2] M. Anacleto, S. Vinga, and A. M. Carvalho, “MSAX: Multivariate Symbolic Aggregate Approximation for Time Series Classification,” en, in *Computational Intelligence Methods for Bioinformatics and Biostatistics*, P. Cazzaniga, D. Besozzi, I. Merelli, and L. Manzoni, Eds., ser. Lecture Notes in Computer Science, Cham: Springer International Publishing, 2020, pp. 90–97, ISBN: 978-3-030-63061-4. DOI: [10.1007/978-3-030-63061-4_9](https://doi.org/10.1007/978-3-030-63061-4_9).
- [3] J. Lin, E. Keogh, S. Lonardi, and B. Chiu, “A symbolic representation of time series, with implications for streaming algorithms,” en, in *Proceedings of the 8th ACM SIGMOD Workshop on Research Issues in Data Mining and Knowledge Discovery - DMKD '03*, San Diego, California: ACM Press, 2003, pp. 2–11. DOI: [10.1145/882082.882086](https://doi.org/10.1145/882082.882086).
- [4] S. Aghabozorgi, A. Seyed Shirkhorshidi, and T. Ying Wah, “Time-series clustering – A decade review,” en, *Information Systems*, vol. 53, pp. 16–38, Oct. 2015, ISSN: 03064379. DOI: [10.1016/j.is.2015.04.007](https://doi.org/10.1016/j.is.2015.04.007).
- [5] J. Shieh and E. Keogh, “I SAX: Indexing and mining terabyte sized time series,” en, in *Proceeding of the 14th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining - KDD 08*, Las Vegas, Nevada, USA: ACM Press, 2008, p. 623, ISBN: 978-1-60558-193-4. DOI: [10.1145/1401890.1401966](https://doi.org/10.1145/1401890.1401966).
- [6] C. Ratanamahatana, E. Keogh, A. J. Bagnall, and S. Lonardi, “A Novel Bit Level Time Series Representation with Implication of Similarity Search and Clustering,” en, in *Advances in Knowledge Discovery and Data Mining*, T. B. Ho, D. Cheung, and H. Liu, Eds., ser. Lecture Notes in Computer Science, Berlin, Heidelberg: Springer, 2005, pp. 771–777, ISBN: 978-3-540-31935-1. DOI: [10.1007/11430919_90](https://doi.org/10.1007/11430919_90).
- [7] I. Kaur, R. Rajni, and A. Marwaha, “ECG Signal Analysis and Arrhythmia Detection using Wavelet Transform,” en, *Journal of The Institution of Engineers (India): Series B*, vol. 97, no. 4, pp. 499–507, Dec. 2016, ISSN: 2250-2106, 2250-2114. DOI: [10.1007/s40031-016-0247-3](https://doi.org/10.1007/s40031-016-0247-3).
- [8] B. V. P. Prasad and V. Parthasarathy, “Detection and classification of cardiovascular abnormalities using FFT based multi-objective genetic algorithm,” en, *Biotechnology & Biotechnological Equipment*, vol. 32, no. 1, pp. 183–193, Jan. 2018, ISSN: 1310-2818, 1314-3530. DOI: [10.1080/13102818.2017.1389303](https://doi.org/10.1080/13102818.2017.1389303).
- [9] A. Panuccio, M. Bicego, and V. Murino, “A Hidden Markov Model-Based Approach to Sequential Data Clustering,” en, in *Structural, Syntactic, and Statistical Pattern Recognition*, G. Goos *et al.*, Eds., vol. 2396, Berlin, Heidelberg: Springer Berlin Heidelberg, 2002, pp. 734–743, ISBN: 978-3-540-44011-6 978-3-540-70659-5. DOI: [10.1007/3-540-70659-3_77](https://doi.org/10.1007/3-540-70659-3_77).

- [10] M. Corduas and D. Piccolo, “Time series clustering and classification by the autoregressive metric,” en, *Computational Statistics & Data Analysis*, vol. 52, no. 4, pp. 1860–1872, Jan. 2008, ISSN: 01679473. DOI: [10.1016/j.csda.2007.06.001](https://doi.org/10.1016/j.csda.2007.06.001).
- [11] R. Nygaard and D. Haugland, “Compressing ECG signals by piecewise polynomial approximation,” en, in *Proceedings of the 1998 IEEE International Conference on Acoustics, Speech and Signal Processing, ICASSP '98 (Cat. No.98CH36181)*, vol. 3, Seattle, WA, USA: IEEE, 1998, pp. 1809–1812, ISBN: 978-0-7803-4428-0. DOI: [10.1109/ICASSP.1998.681812](https://doi.org/10.1109/ICASSP.1998.681812).
- [12] H. Zhu, Y. Pan, K.-T. Cheng, and R. Huan, “A lightweight piecewise linear synthesis method for standard 12-lead ECG signals based on adaptive region segmentation,” en, *PLOS ONE*, vol. 13, no. 10, J. Zhao, Ed., e0206170, Oct. 2018, ISSN: 1932-6203. DOI: [10.1371/journal.pone.0206170](https://doi.org/10.1371/journal.pone.0206170).
- [13] A. Zifan, M. H. Moradi, S. Saberi, and F. Towhidkhah, “Automated Segmentation of ECG Signals using Piecewise Derivative Dynamic Time Warping,” en, p. 5, 2006.
- [14] C. T. Zan and H. Yamana, “An improved symbolic aggregate approximation distance measure based on its statistical features,” in *Proceedings of the 18th International Conference on Information Integration and Web-Based Applications and Services*, ser. iiWAS '16, New York, NY, USA: Association for Computing Machinery, Nov. 2016, pp. 72–80, ISBN: 978-1-4503-4807-2. DOI: [10.1145/3011141.3011146](https://doi.org/10.1145/3011141.3011146).
- [15] Y. Sun *et al.*, “An improvement of symbolic aggregate approximation distance measure for time series,” en, *Neurocomputing*, vol. 138, pp. 189–198, Aug. 2014, ISSN: 09252312. DOI: [10.1016/j.neucom.2014.01.045](https://doi.org/10.1016/j.neucom.2014.01.045).
- [16] Y. Yu *et al.*, “A Novel Trend Symbolic Aggregate Approximation for Time Series,” en, vol. abs/1905.00421, p. 9, 2019. [Online]. Available: <http://arxiv.org/abs/1905.00421>.
- [17] B. Lkhagva, Y. Suzuki, and K. Kawagoe, “Extended SAX: Extension of Symbolic Aggregate Approximation for Financial Time Series Data Representation,” en, in *Proceeding of IEICE the 17th Data Engineering Workshop*, Ginowan, Japan, 2006, p. 7. [Online]. Available: https://www.researchgate.net/publication/229046404_Extended_SAX_extension_of_symbolic_aggregate_approximation_for_financial_time_series_data_representation (visited on 02/27/2021).
- [18] S. Malinowski, T. Guyet, R. Quiniou, and R. Tavenard, “1d-SAX: A Novel Symbolic Representation for Time Series,” en, in *Advances in Intelligent Data Analysis XII*, A. Tucker, F. Höppner, A. Siebes, and S. Swift, Eds., ser. Lecture Notes in Computer Science, Berlin, Heidelberg: Springer, 2013, pp. 273–284, ISBN: 978-3-642-41398-8. DOI: [10.1007/978-3-642-41398-8_24](https://doi.org/10.1007/978-3-642-41398-8_24).

- [19] M. M. M. Fuad and P.-F. Marteau, "TOWARDS A FASTER SYMBOLIC AGGREGATE APPROXIMATION METHOD:" en, in *Proceedings of the 5th International Conference on Software and Data Technologies*, University of Piraeus, Greece: SciTePress - Science and Technology Publications, 2010, pp. 305–310, ISBN: 978-989-8425-22-5 978-989-8425-23-2. DOI: [10.5220/0003006703050310](https://doi.org/10.5220/0003006703050310).
- [20] A. Camerra, T. Palpanas, J. Shieh, and E. Keogh, "iSAX 2.0: Indexing and Mining One Billion Time Series," in *Proceedings - IEEE International Conference on Data Mining, ICDM*, Dec. 2010, pp. 58–67. DOI: [10.1109/ICDM.2010.124](https://doi.org/10.1109/ICDM.2010.124).
- [21] E. Keogh, J. Lin, and A. Fu, "HOT SAX: Efficiently Finding the Most Unusual Time Series Subsequence," en, in *Fifth IEEE International Conference on Data Mining (ICDM'05)*, Houston, TX, USA: IEEE, 2005, pp. 226–233, ISBN: 978-0-7695-2278-4. DOI: [10.1109/ICDM.2005.79](https://doi.org/10.1109/ICDM.2005.79).
- [22] H. Park and J.-Y. Jung, "SAX-ARM: Deviant event pattern discovery from multivariate time series using symbolic aggregate approximation and association rule mining," en, *Expert Systems with Applications*, vol. 141, p. 112 950, Mar. 2020, ISSN: 0957-4174. DOI: [10.1016/j.eswa.2019.112950](https://doi.org/10.1016/j.eswa.2019.112950).
- [23] P. Ordóñez *et al.*, "Visualizing Multivariate Time Series Data to Detect Specific Medical Conditions," *AMIA Annual Symposium Proceedings*, vol. 2008, pp. 530–534, 2008, ISSN: 1942-597X. [Online]. Available: <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2656052/> (visited on 03/30/2021).
- [24] C. Zhang *et al.*, "Anomaly detection in ECG based on trend symbolic aggregate approximation," en, *Mathematical Biosciences and Engineering*, vol. 16, no. 4, pp. 2154–2167, 2019, ISSN: 1547-1063. DOI: [10.3934/mbe.2019105](https://doi.org/10.3934/mbe.2019105).
- [25] M. AlGhatrif and J. Lindsay, "A brief review: History to understand fundamentals of electrocardiography," en, *Journal of Community Hospital Internal Medicine Perspectives*, vol. 2, no. 1, p. 14 383, Jan. 2012, ISSN: 2000-9666. DOI: [10.3402/jchimp.v2i1.14383](https://doi.org/10.3402/jchimp.v2i1.14383).
- [26] W. Fye, "A History of the origin, evolution, and impact of electrocardiography," en, *The American Journal of Cardiology*, vol. 73, no. 13, pp. 937–949, May 1994, ISSN: 00029149. DOI: [10.1016/0002-9149\(94\)90135-X](https://doi.org/10.1016/0002-9149(94)90135-X).
- [27] S. Meek and F. Morris, "Introduction. I—Leads, rate, rhythm, and cardiac axis," *BMJ : British Medical Journal*, vol. 324, no. 7334, pp. 415–418, Feb. 2002, ISSN: 0959-8138. [Online]. Available: <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC1122339/> (visited on 05/21/2021).
- [28] D. E. Becker, "Fundamentals of Electrocardiography Interpretation," *Anesthesia Progress*, vol. 53, no. 2, pp. 53–64, 2006, ISSN: 0003-3006. DOI: [10.2344/0003-3006\(2006\)53\[53:FOEI\]2.0.CO;2](https://doi.org/10.2344/0003-3006(2006)53[53:FOEI]2.0.CO;2).

- [29] G. Moody and R. Mark, "The impact of the MIT-BIH Arrhythmia Database," en, *IEEE Engineering in Medicine and Biology Magazine*, vol. 20, no. 3, pp. 45–50, May-June/2001, ISSN: 07395175. DOI: [10.1109/51.932724](https://doi.org/10.1109/51.932724).
- [30] A. L. Goldberger *et al.*, "PhysioBank, PhysioToolkit, and PhysioNet: Components of a New Research Resource for Complex Physiologic Signals," en, *Circulation*, vol. 101, no. 23, Jun. 2000, ISSN: 0009-7322, 1524-4539. DOI: [10.1161/01.CIR.101.23.e215](https://doi.org/10.1161/01.CIR.101.23.e215).
- [31] J. Wasilewski and L. Poloński, "An Introduction to ECG Interpretation," en, in *ECG Signal Processing, Classification and Interpretation*, A. Gacek and W. Pedrycz, Eds., London: Springer London, 2012, pp. 1–20, ISBN: 978-0-85729-867-6 978-0-85729-868-3. DOI: [10.1007/978-0-85729-868-3_1](https://doi.org/10.1007/978-0-85729-868-3_1).
- [32] L. Xie *et al.*, "Computational Diagnostic Techniques for Electrocardiogram Signal Analysis," en, *Sensors*, vol. 20, no. 21, p. 6318, Nov. 2020, ISSN: 1424-8220. DOI: [10.3390/s20216318](https://doi.org/10.3390/s20216318).
- [33] A. H. Kadish *et al.*, "ACC/AHA Clinical Competence Statement on Electrocardiography and Ambulatory Electrocardiography," *Circulation*, vol. 104, no. 25, pp. 3169–3178, Dec. 2001. DOI: [10.1161/circ.104.25.3169](https://doi.org/10.1161/circ.104.25.3169).
- [34] C. Antzelevitch and A. Burashnikov, "Overview of Basic Mechanisms of Cardiac Arrhythmia," *Cardiac electrophysiology clinics*, vol. 3, no. 1, pp. 23–45, Mar. 2011, ISSN: 1877-9182. DOI: [10.1016/j.j.ccep.2010.10.012](https://doi.org/10.1016/j.j.ccep.2010.10.012).
- [35] A. N. Nowbar *et al.*, "Mortality From Ischemic Heart Disease: Analysis of Data From the World Health Organization and Coronary Artery Disease Risk Factors From NCD Risk Factor Collaboration," en, *Circulation: Cardiovascular Quality and Outcomes*, vol. 12, no. 6, Jun. 2019, ISSN: 1941-7713, 1941-7705. DOI: [10.1161/CIRCOUTCOMES.118.005375](https://doi.org/10.1161/CIRCOUTCOMES.118.005375).
- [36] Institute of Medicine (US) Committee on Social Security Cardiovascular Disability Criteria, "Ischemic Heart Disease," en, in *Cardiovascular Disability: Updating the Social Security Listings*, Washington, DC: National Academies Press (US), 2010. [Online]. Available: <https://www.ncbi.nlm.nih.gov/books/NBK209964/> (visited on 05/21/2021).
- [37] N. Herring, "ECG diagnosis of acute ischaemia and infarction: Past, present and future," en, *QJM*, vol. 99, no. 4, pp. 219–230, Feb. 2006, ISSN: 1460-2725, 1460-2393. DOI: [10.1093/qjmed/hcl025](https://doi.org/10.1093/qjmed/hcl025).
- [38] P. Kligfield *et al.*, "Recommendations for the Standardization and Interpretation of the Electrocardiogram: Part I: The Electrocardiogram and Its Technology: A Scientific Statement From the American Heart Association Electrocardiography and Arrhythmias Committee, Council on Clinical Cardiology; the American College of Cardiology Foundation; and the Heart Rhythm Society Endorsed by the International Society for Computerized Electrocardiology," en, *Circulation*, vol. 115, no. 10, pp. 1306–1324, Mar. 2007, ISSN: 0009-7322, 1524-4539. DOI: [10.1161/CIRCULATIONAHA.106.180200](https://doi.org/10.1161/CIRCULATIONAHA.106.180200).

- [39] H. Sivaraks and C. A. Ratanamahatana, “Robust and Accurate Anomaly Detection in ECG Artifacts Using Time Series Motif Discovery,” en, *Computational and Mathematical Methods in Medicine*, vol. 2015, pp. 1–20, 2015, ISSN: 1748-670X, 1748-6718. DOI: [10.1155/2015/453214](https://doi.org/10.1155/2015/453214).
- [40] R. Valupadasu and B. R. R. Chunduri, “Identification of Cardiac Ischemia Using Spectral Domain Analysis of Electrocardiogram,” en, in *2012 UKSim 14th International Conference on Computer Modelling and Simulation*, Cambridge, United Kingdom: IEEE, Mar. 2012, pp. 92–96, ISBN: 978-1-4673-1366-7 978-0-7695-4682-7. DOI: [10.1109/UKSim.2012.22](https://doi.org/10.1109/UKSim.2012.22).
- [41] D. S. Baim *et al.*, “Survival of patients with severe congestive heart failure treated with oral milrinone,” en, *Journal of the American College of Cardiology*, vol. 7, no. 3, pp. 661–670, Mar. 1986, ISSN: 07351097. DOI: [10.1016/S0735-1097\(86\)80478-8](https://doi.org/10.1016/S0735-1097(86)80478-8).
- [42] P. Laguna, R. Mark, A. Goldberg, and G. Moody, “A database for evaluation of algorithms for measurement of QT and other waveform intervals in the ECG,” en, in *Computers in Cardiology 1997*, Lund, Sweden: IEEE, 1997, pp. 673–676, ISBN: 978-0-7803-4445-7. DOI: [10.1109/CIC.1997.648140](https://doi.org/10.1109/CIC.1997.648140).
- [43] A. Taddei *et al.*, “The European ST-T database: Standard for evaluating systems for the analysis of ST-T changes in ambulatory electrocardiography,” en, *European Heart Journal*, vol. 13, no. 9, pp. 1164–1172, Sep. 1992, ISSN: 1522-9645, 0195-668X. DOI: [10.1093/oxfordjournals.eurheartj.a060332](https://doi.org/10.1093/oxfordjournals.eurheartj.a060332).
- [44] P. Kanani and M. Padole, “ECG Heartbeat Arrhythmia Classification Using Time-Series Augmented Signals and Deep Learning Approach,” en, *Procedia Computer Science*, Third International Conference on Computing and Network Communications (CoCoNet’19), vol. 171, pp. 524–531, Jan. 2020, ISSN: 1877-0509. DOI: [10.1016/j.procs.2020.04.056](https://doi.org/10.1016/j.procs.2020.04.056).