Multivariate Symbolic Aggregate Approximation for ECG Analysis

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May 3, 2021
Bishkek, Kyrgyz Republic

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Outline

1 Introduction

2 Methods

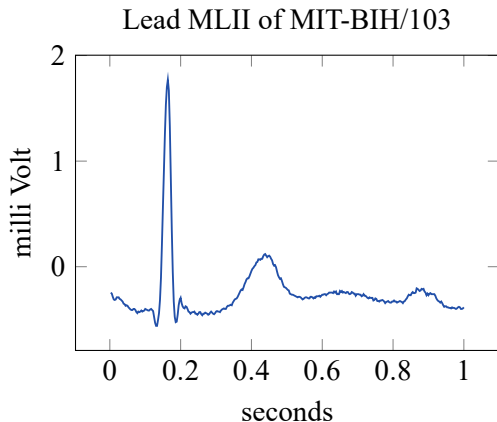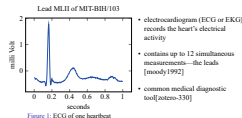3 Preliminary Results

Introduction

What is an ECG?

Figure 1: ECG of one heartbeat

- electrocardiogram (ECG or EKG) records the heart's electrical activity

- contains up to 12 simultaneous measurements—the leads [moody1992]

- common medical diagnostic tool[zotero-330]

MSAX for ECG Analysis
└─Introduction
   └─ECG Basics
      └─What is an ECG?

2021-05-03

Figure 1: ECG of one heartbeat

- muscle contractions caused by electric pulses
- electric pulse can be measured on the skin
- the measuring things are called electrodes
- electrodes form leads (need 2 to measure anything)
- they have specific positions and names
- 12 leads is the modern standard
- most types of heart disease can be detected
- diagnosis and analysis is performed by trained cardiologists
- **datasets available online; contain 2 or more leads (the most significant ones)**
- **I will be using online datasets for my analysis**
- heart diseases are some of the most deadly ones, thus ECG are really important

Figure 2: Annotated ECG of one heartbeat
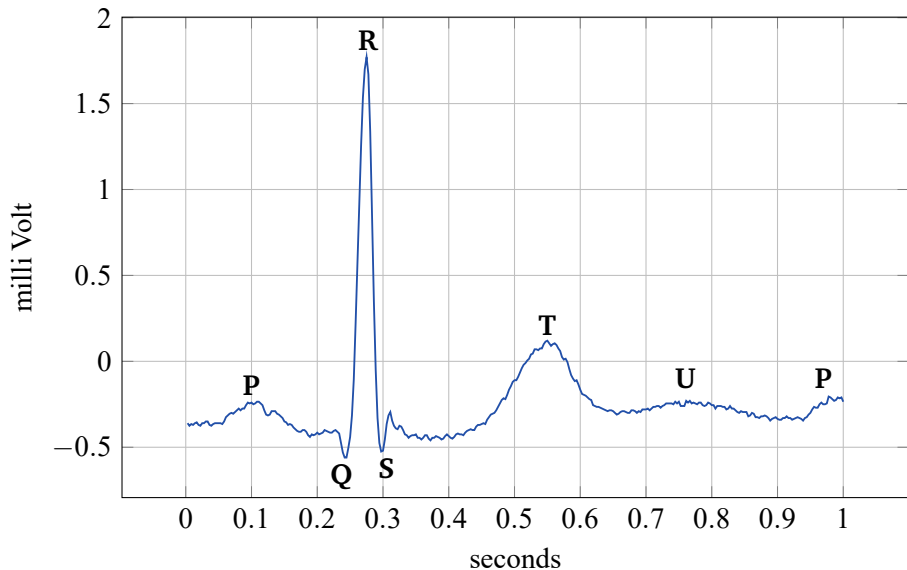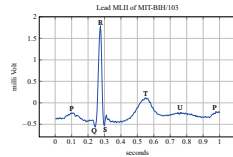
Figure 2: Annotated ECG of one heartbeat

- P wave: atria depolarizing / blood entering the heart
- QRS complex: ventricular depolarization / heart contraction pumping blood
- T: return of ventricle to polarized state
- U: present in 25%, maybe some feedback
- P wave: atria depolarizing / blood entering the heart
- ST-segment: significant, depression, elevation, slope show ischaemia
- R-R interval: shows rhythm and thus arrhytmia etc

ECGs as Time Series

Definition

A discrete time series is an ordered sequence that, at discrete points in time, has n values each. If $n = 1$, the series is univariate and if $n > 1$, it is multivariate.

- digital ECGs are discrete multivariate time series: [anacleto2020]
    - have > 1 value at each point, often $n = 12$

    - recorded at discrete, evenly spaced time points

- time series analysis methods can be applied to ECGs

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- time series analysis methods can be applied to ECGs

- modern ECGs have at least 2, most have 12
- digital ones have set sampling frequencies, even the machines have set frequencies
- multivariate: measure more than 1 lead per time point
- discrete: set sample frequency in the machines
- discrete: because measured at discrete moments in time
- time series: they are data measured at equal time intervals
- n measurements per point in time (i.e. leads)
- $n = 1$ is univariate, $n > 1$ is multivariate

ECG Analysis

- standard method: manual analysis by cardiologist

- recently: automated or computer-assisted ECG analysis

- multiple stages: [kligfieldpaul2007](1) signal acquisition; (2) data transformation, processing, filtering; (3) waveform recognition, feature extraction; (4) classification

- current research focus: artificial neural networks…[xie202]

- relatively new methods are SAX, MSAX, and HOTSAX

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- current research focus: artificial neural networks…[xie202]
- relatively new methods are SAX, MSAX, and HOTSAX

- is relatively slow; time is of the essence
- lots of training required
- error prone
- maybe not feasible for long ECGs
- can speed up process
- can pick up details humans miss
- digitizing paper ECGs or recording digital ones
- filtering to remove various types of noise
- reduce complexity of the data
- select important features and neglect irrelevant ones to ease analysis
- often added, figure out if there is some disease present or not
- balance between accuracy and complexity needed
- ann: hand all the steps discussed to a NN; use as good classifier too

SAX, MSAX, and HOTSAX

- Lin *et al.* (2003): Symbolic Aggregate Approximation (SAX)—simplified, symbolic representation [lin2003]   [zhang2019]

- Anacleto *et al.* (2020): Multivariate SAX (MSAX)—expands SAX to multivariate time series [anacleto2020]

- Keogh *et al.* (2005): Heuristically Ordered Time series using Symbolic Aggregate Approximation (HOTSAX)—discord discovery algorithm for SAX

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- ecg as letters that mean same thing as original
- guaranteed to behave like the original data
- works on univariate time series
- has been used on ECGs
- takes the correlation between ecg leads into account
- cov mat: covariance between each lead and variance on diag
- uses sax representation to make the finding of discords easier
- can use MSAX just as well

Time Series Discords

Definition

A time series discord is the subsequence of a time series that is most different from all other subsequences.

k time series discords are the k most different subsequences.

- discords represent anomalies in an ECG

- can be found by comparing all subsequences to all other subsequences; does not scale well [keogh2005]

- HOTSAX makes this process faster

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- these can be diseases, noise, etc
- the discord does not discern
- this is not feasible because of complexity

Hypothesis

HOTSAX with MSAX will increase the number of relevant discords detected compared to HOTSAX with SAX.
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- mention that MSAX to ECGs in particular is new
- mention that HOTSAX with MSAX is new
- results will not be great as HOTSAX is not a real classifier; this is about finding out if MSAX adds more useful information
- THIS METHOD WILL NOT BE SUPER ACCURATE; MANY ECG changes are relatively small and would get lost in the SAX process
- THE METHOD HAS NO AWARENESS OF MEDICAL RELEVANCE OR ANY OF THAT
- NOVEL: MSAX IS SUPER NEW; HAS NOT BEEN APPLIED TO ECGs IN THIS WAY; ALSO NOT USED WITH HOTSAX

Methods

Step 1: Z-Normalization

Assumption

The time series values are normally distributed.

SAX

- normalize univariate time series

- uses scalar mean and variance

MSAX

- normalize multivariate time series

- uses vector mean and covariance matrix

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The time series values are normally distributed.

SAX	MSAX
• normalize univariate time series	• normalize multivariate time series
• uses scalar mean and variance	• uses vector mean and covariance matrix

- say that the process is the same as MSAX based on SAX
- this is assumed and this worked for other people who applied SAX to ECGs
- to compare time series, normalization is the accepted step
- what is this
- takes into account the correlation between leads

Step 2: Dimensionality Reduction

PAA

Piecewise Aggregate Approximation (PAA) takes T time series points, splits it into w ($w < T$) segments, and averages each of them.
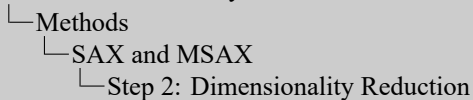
<table>
<tr><td>

SAX

- apply PAA to time series

</td><td>

MSAX

- apply PAA to each of the time series individually
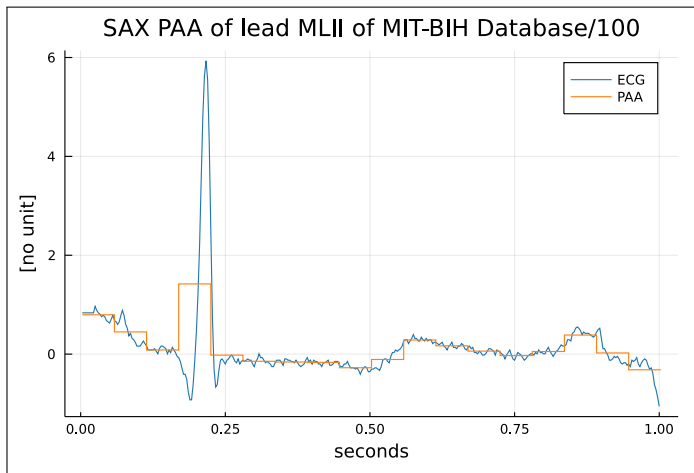
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SAX	MSAX
• apply PAA to time series	• apply PAA to each of the time series individually

- this reduces complexity
- PAA form of time series is shorter and simpler
- it still somewhat corresponds to the original

Figure 3: ECG with PAA (MITBIH/100, $w = 18$, $T = 360$)

Step 3: Discretization

SAX Discretization

Find breakpoints splitting $\mathcal{N}(0, 1)$ into B equiprobable segments.
Assign a letter to each area: a to most-negative, b to the next biggest…
PAA segments get letters based on which area they are in.

SAX

- discretize the time series

- results in one word

MSAX

- discretize each time series individually

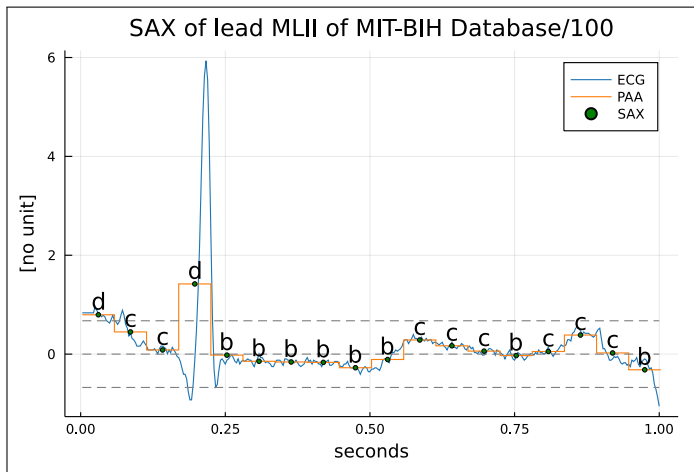- results in one word, one letter per time series

- result is called word
- N is the alphabet size
- big thing here is that this gives defined probability to each letter; makes no sense for real numbers (like PAA values)
- simplifies time series even more
- creates discrete categories, can be more useful

Figure 4: SAX (MITBIH/100, $w = 18$, $T = 360$, alphabet size 4)

Step 4: Distance Measure

MINDIST

A distance measure is defined to compare two SAX words. Distance is defined for a pair of letters: 0 if they are neighbors; absolute difference of breakpoint values otherwise.

SAX

$$\sqrt{\frac{T}{w}}\sqrt{\sum_{i=1}^{w}\left(dist(\hat{q}[i], \hat{c}[i])\right)^2}$$

MSAX

$$\sqrt{\frac{T}{w}}\sqrt{\sum_{i=1}^{w}\left(\sum_{j=1}^{n}\left(dist(\hat{q}_j[i], \hat{c}_j[i])\right)^2\right)}$$
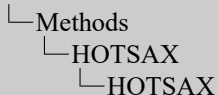
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SAX

$$\sqrt{\frac{n}{w}}\sqrt{\sum_{i=1}^{w}(dist(\hat{q}[i],\hat{c}[i]))^2}$$

MSAX

$$\sqrt{\frac{n}{w}}\sqrt{\sum_{i=1}^{w}\left(\sum_{j=1}^{s}(dist(\hat{q}_j[i],\hat{c}_j[i]))^2\right)}$$

- n – length of original time series
- w – length of word
- this lower-bounds the euclidean distance, meaning that results in SAX should hold true for the real data too

HOTSAX

- "brute-force" discord discovery is slow, needs T^2 operations

- HOTSAX speeds up discord discovery by considering that
    - discords are rare, start with rarest segment

    - similar segments have similar distances, consider together

- HOTSAX detects anomalies, it is not a classifier

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- HOTSAX detects anomalies, it is not a classifier
- it uses SAX and MSAX for dimensionality reduction

- this is the basic idea that can speed up the process
- it is not guaranteed to do so, but it does not decrease efficiency
- this speeds up the process even more as we have fewer elements
- because of lower bounding, it still gives accurate results

Results

Implementation

- SAX, MSAX, HOTSAX implemented in Julia, a scientific programming language

- used annotated digital ECGs from the MIT-BIH arrhythmia database [moody1992]

- HOTSAX performed for different w, B, subsequence lengths

- results exported to CSV file and analyzed using the R programming language

- fast, type support, great libraries, JIT compilation
- ecgs have all heart beats annotated
- know which are normal, diseases, noise, etc
- 48 recordings of 30 minutes
- w - paa segments; B - alphabet size; subsequence length for HOTSAX

Preliminary Results

- focus on comparing SAX and MSAX with the top $k = 80$ discords

- to analyze the relevance of results, recall (sensitivity) is used

- analyzed total of 816 results for different parameters (SAX and MSAX for each)

- recall for MSAX is higher compared to SAX

- if SAX is applied to 2 leads and the results combined, it outperforms MSAX

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- recall for MSAX is higher compared to SAX
- if SAX is applied to 2 leads and the results combined, it outperforms MSAX

- how many relevant items are selected
- recall = true positive / (true positive + false negative)
- this is done because for medical things it is more useful to look at a couple too many segments than not enough

Outlook

- perform statistical tests for significance of the result

- analyze the outliers visible in the boxplots

- more tests with different sets of parameters

- explore the influence of parameters on the result

- use the 12-lead INCART ECG database to investigate the influence of larger numbers of leads

- for example t-test, biserieal correlation

Thank You!

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[2] M. Anacleto, S. Vinga, and A. M. Carvalho, "MSAX: Multivariate Symbolic Aggregate Approximation for Time Series Classification," en, in *Computational Intelligence Methods for Bioinformatics and Biostatistics*, P. Cazzaniga, D. Besozzi, I. Merelli, and L. Manzoni, Eds., ser. Lecture Notes in Computer Science, Cham: Springer International Publishing, 2020, pp. 90–97, ISBN: 978-3-030-63061-4. DOI: 10.1007/978-3-030-63061-4_9.

[3] E. Keogh, J. Lin, and A. Fu, "HOT SAX: Efficiently Finding the Most Unusual Time Series Subsequence," en, in *Fifth IEEE International Conference on Data Mining (ICDM'05)*, Houston, TX, USA: IEEE, 2005, pp. 226–233, ISBN: 978-0-7695-2278-4. DOI: 10.1109/ICDM.2005.79.