Mathematical Modeling of Real Time ECG Waveform

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Abstract. Electrocardiogram (ECG) is a digital recording of heart rate variability that is used to detect the cardiac disorders. Often these recordings are affected by physiological and instrumental noises that affects an accurate diagnosis of the disease. An exact understanding of ECG waveform may help in overcoming such issues. Mathematical modeling is efficiently used to understand the pattern of 12-lead ECG and simulate real time ECG's waveform. Real ECG can be taken as a superposition of bounded functions and this property is a defining feature of almost periodic functions (APF). The proposed model has utilized this characteristic of ECG signals to generate the real time ECG waveform with negligibly small error.

Keywords: 12-lead ECG · Almost periodic function
Electrocardiogram

1 Introduction

The electrocardiogram (ECG) waveform helps in diagnosis of heart diseases and their treatment. Despite the inevitable importance of ECG signals, the clinical reliability of some ECG waveforms is questionable and lead to wrong diagnosis of cardiac disease that may result in vital damage. An artificial simulation of the ECG, using mathematical modeling techniques can help in understanding the behavior of ECG signal, and therefore, can be helpful in improving signal quality.

A deeper observation of ECG waveform reveals that it consists of different events and intervals. The event corresponding to the P-wave is associated to the atrial depolarization, while corresponding to T-wave and U-wave is ventricular repolarisation. The waveform of QRS complex is of largest amplitude and shows the depolarization of both the interventricular septum, right ventricle, and left ventricle. These are the main incidents of ECG, however, positions of leads and power conduction may define more intervals. Sinus rhythms can be

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classified by heart rate of the patient, and amplitude of the waves. Other parameters for automated detection of waveform include duration of the event and its location.

The ECG patterns of cardiac activities are recorded by 12-lead by placing electrodes on various limbs of human body as well as on the chest [1]. Sometimes clinical recordings are disturbed by power line fluctuations, instrumental or physiological interferences and the ECG waveform fail to provide an exact diagnosis of the disease. This may lead to vital damage and therefore recording of accurate ECG is one of the biggest challenges of cardiology research. A simulation of real ECG waveform can be of great help in understanding the characteristics and pattern of ECG and hence improving quality of diagnosis.

Mathematical modeling is one of the most efficient techniques of signal simulation, and has done remarkable work in ECG simulation. Researchers have used various techniques to simulate ECG waveform. Remarkable work include the Fourier analysis based quasi-periodic simulator that was proposed by Al-Nashash [2] in 1995, and periodic simulator was presented by Karthik [3] in 2003. Another important contribution is the synthetic ECG model of McSharry [4] that employs iterative algorithms of nonlinear optimization to fit into real ECG. Borsali [5] used curve fitting techniques for matrix alignment of the beats for ECG compression, while Nunes and Nait [6] the Hilbert transform methods. Afterwards Ghaffari [7] developed an algebraic model to artificially generate events of ECG. The ECG simulator, proposed in 2014 [8], was designed for an adaptive noise cancellation model and is based on almost periodic functions. Interference of noise signals is obvious in real ECG and cannot be ignored while modeling a real ECG waveform. Recently Castaño [9] presented autoregressive models of motion artifact contaminated ECG.

For an appropriate frequency of heart beat, the mathematical model of [8] is able to simulate normal as well as abnormal waveforms of the ECG of 12-lead and sinus rhythms, provided that the model parameters of ECG's normal events P, Q, R, S, T and U are suitably chosen. In this paper, this model is improved to simulate real time clinical ECG signal, available on MIT-BIH that is a database of physionet.com [10,11]. The ECG signals of this database are contaminated by noises of various types, like major noises in ECG 101 signals are baseline wander and muscle artifacts. The modeled waveform can help in recognizing exact ECG waveform that may improve the diagnosis of the disease. These real signals are simulated in Matlab by modeling artifacts to be added in the ECG model of [8] to make it comparable with the real ECG waveform. The improved model can produce synthetic waveform of real ECG with negligibly small error.

The rest of the paper includes a description of the mathematical model for ECG simulation in Sect. 2, generation of 12-lead ECG in Sect. 2.1 and modeling of different sinus rhythms in Sect. 2.2. The real ECG waveform is evaluated in Sect. 3, and the concluding remarks fall in Sect. 4.

2 Mathematical Model for Artificial Simulation of ECG Waveform

The whole formation of the mathematical model of [8] depends on number of heartbeats per minute. Overall waveform is artificially generated, following a superposition of waveforms corresponding to each of the events, according to the fact that the amplitude of an event does affect the amplitude of the neighboring event, unless the incident time of consecutive events is almost same.

2.1 Mathematical Model

Let us denote the number of heartbeats per minute by N, and J be the set of all the events of ECG waveform corresponding to P-wave, Q-wave, R-wave, S-wave, T-wave and U-wave. At time n, if $\Theta_j(n)$ is a function of time period $\tau(n) = \frac{60}{N}$ with the property $\Theta_j(n) \approx \Theta_j(n + \tau(n))$, then for event $j \in J$ that is incident at t_j, [8] yields

$$\Theta_j(n) = A_j + \sum_{n=1}^{N} \alpha_j(n) cos\{2\pi n \int_{t_j}^{n} f(n)dn\} \tag{1}$$

where $f(n) = 1/\tau(n)$ is the fundamental frequency of heartbeat whereas A_j and $\alpha_j(n)$ represent the Fourier coefficients. Simplifying (1) and then substituting $\omega = 2\pi f(n) = \frac{2\pi}{\tau(n)}$,

$$\Theta_j(n) = A_j + \sum_{n=1}^{N} \alpha_j(n) cos\{\omega(n - t_j)n\} \tag{2}$$

Occurrence of each event j can be taken as cyclic with period $\tau_j = \frac{\tau(n)}{d_j}$ and frequency $f_j = \frac{1}{\tau_j}$. For given amplitude m_j, if duration d_j and location t_j is known for each $j \in J$, then the coefficients A_j & $\alpha_j(n)$ are given by:

$$A_j = \begin{cases} \frac{2m_j}{\pi\tau_j} & , \quad j \in \{P, T, U\}; \\ \frac{m_j}{2\tau_j} & , \quad j \in \{R\}; \\ \frac{-m_j}{2\tau_j} & , \quad j \in \{Q, S\}; \end{cases} \tag{3}$$

and,

$$\alpha_j(n) = \begin{cases} \frac{4fm_jd_j}{\pi\{1-(2nfd_j)^2\}} cos\{\omega_j n\} & , j \in \{P, T, U\}; \\ \frac{2m_j}{fd_j(n\pi)^2}(1 - cos\{\omega_j n\}) , & j \in \{R\}; \\ \frac{-2m_j}{fd_j(n\pi)^2}(1 - cos\{\omega_j n\}) , & j \in \{Q, S\}; \end{cases} \tag{4}$$

where $\omega_j = \pi f_j = \frac{\pi}{\tau_j}$. The superposition of events Θ_j yield an ECG waveform of a single lead for appropriate values of model parameters (m_j, d_j, t_j). For known N, the ECG waveform is approximated as:

$$\begin{aligned} ECG(n) &= \sum_{j \in J} \Theta_j(n) \\ &= A_o + \sum_{j \in J} \sum_{n=1}^{N} \alpha_j(n) cos\{\omega(n - t_j)n\} \end{aligned} \tag{5}$$

where a nonzero value of $A_o = \sum_{j \in J} A_j$ represent ECG waveform from the line of zero voltage. Since occurrence of an event is independent of N, (5) can be rewritten as:

$$ECG(n) = A_o + \sum_{n=1}^{N} \sum_{j \in J} \alpha_j(n) cos\{\omega(n - t_j)n\} \tag{6}$$

For a given set of model parameters, (6) can generate an ECG waveform artificially. In the following sections, this mathematical model will be used to generate ECG waveforms of different leads and sinus rhythms. However, a real ECG may contains several artifacts, such as baseline wander (BW), power line interference (PLI) and EMG noise etc. Therefore, in order to model a waveform of real ECG signal, these artifacts should also be modeled artificially.

2.2 Generating ECG Waveforms for 12-leads

ECG signals may either be recorded as a standard 12-lead ECG waveform that looks at different parts of heart, or as individual rhythm strips that look at heart rate variability (HRV). In this section, model (6) is used to simulate online available images of 12-lead ECG waveforms of Fig. 1. Setting Number of hear beat per minute as $N = 72$, and choosing appropriate values of model parameters, Fig. 2 shows a short segment of the recording of each of the 12-leads. These segments resemble well with the online available derivations of Fig. 1.

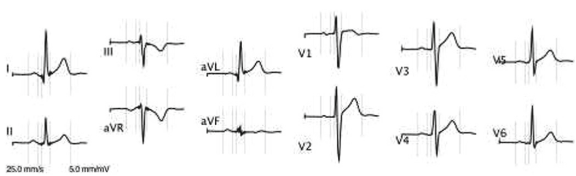

Fig. 1. Online available 12-lead ECG waveform (http://en.wikipedia.org/wiki/File:ECG_12derivations.png).

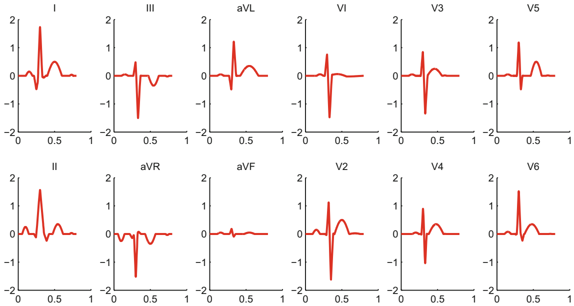

Fig. 2. 12-lead synthetic ECG waveform.

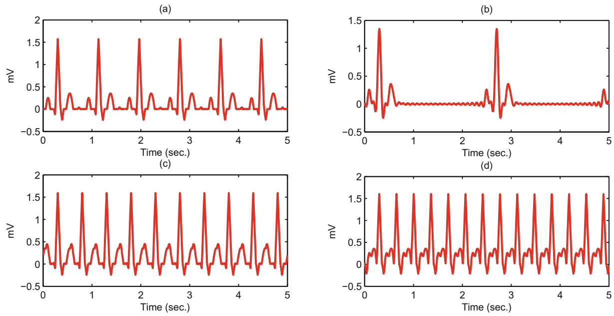

Fig. 3. Heart rate variability (HRV) components: (a) Normal sinus arrhythmia, with 72 beats per minute, (b) Sinus bradycardia, with 25 beats per minute, (c) Sinus Tachycardia, with 120 beats per minute, and (d) Supraventricular Tachycardia, with 170 beats per minute.

2.3 Modeling ECG Waveforms for Different Sinus Rhythms

An important thing about the mathematical model (5) is that frequencies of all the events are controlled by the heart beat frequency f. This action is quite visible in the ECG waveforms of different rhythm stripes which determine heart rate variability (HRV). HRV is a measure of alterations in heart rate and is composed of two major components: high frequency respiratory sinus arrhythmia (RSA) and low frequency sympatric components. Some important HRV components are shown in Fig. 3 according to the following categories:

(a) Normal Sinus Arrhythmia: Frequency of Normal ECG is 60 to 100 beats per minutes, i.e., $60 \leq N \leq 100$, see Fig. 3(a).
(b) Sinus Bradycardia: A sinus rhythm of less than 60 beats per minute, as shown in Fig. 3(b).
(c) Sinus Tachycardia: The sinus node sends out electrical signals faster than usual, speeding up the rate and generating a sinus rhythm of more than 100 beats per minute, Fig. 3(c).

(d) Supraventricular Tachycardia: Usually caused by re-entry of currents within the atria or between ventricles and atria producing higher heart rates of $150 - 250$. Sinus rhythm at 170 BPM is shown in Fig. 3(d).

Out of all the 12-leads, clinical ECG machines mostly record Lead I, II and III only that cause missing information and hence wrong diagnosis. However, to understand heart's rhythm, only one lead is enough and for this Lead II is a trivial choice. Figure 3 demonstrates the simulated waveforms of sinus rhythms of four types, described above. Starting with event P, the signals are sampled with 10^{-4} sec resolution for 5 s, with time: $n = 0.001 : 0.0001 : 5$. The values of model parameters taken form Table 1 of [8].

3 Evaluation of a Real ECG Waveform of MIT-BIH

In this section, proposed mathematical model is modified to simulate real ECG signals. The real time ECG waveform of MIT-BIH arrhythmia database [10] will be used for this purpose. This arrhythmia data base contains 48 clinical ECG recordings of patients from different age groups and are labeled as 101, 102, etc. These ECG signals consist of ambulatory recordings of two channel ECGs, obtained from different patients who were facing heart problems of different types. Out of 48 patients, there were 25 men whose ages were between $32 - 89$ years and 23 were women between $23 - 89$ years of age. Although the clinical machines generate an analogue data, but the available recordings were digitized by taking 360 samples per second per channel and resolution of 11-bit. Out of these recordings, a one second waveform of ECG 101 is chosen to be modeled here. Being real time signals, these recordings are contaminated by different kinds of artifacts and instrumental noises. The ECG 101 has contamination of baseline wander (BW) and muscle artifacts (EMG). Furthermore, the power line interference (PLI) is present in almost all the recording that is often because of the perturbations of machine's power line, and can be avoided by using fine power supply and better quality machines. To have a better modeling of real signals, these artifacts are generated artificially, and then added in modeled ECG waveform. The simulations are made for 1 s duration, with a sampling of 360 signals. The heartbeat frequency N is chosen according to the patient's age, and frequency range provided on database.

3.1 MIT-BIH 101

The patient is a female with age $= 75$ years, and we have taken $N = 68$ for simulation of its one second digitized waveform. As described earlier, real time signals are mostly contaminated by artifacts, and therefore simulated noises are required to be added in the modeled ECG.

Starting with a suitable choice of A_o, the model parameters are chosen carefully to make the modeled waveform comparable with the real one. Table 1 gives the values of amplitude, duration and location that are used to generate the modeled ECG101. Figure 4 shows a comparison of the waveforms of modeled ECG101 and clinical ECG MIT-BIH101. This modeled waveform has contamination of two modeled noises in it: the BW noise and EMG noise.

The BW noise, modeled by

$$A_{BW} sin(2\pi f_{BW} n) + cos(2\pi f_{BW} n)$$

of frequency $f_{BW} = 0.25$ is added in modeled ECG waveform. Amplitude A_{BW} of this baseline wander is taken as a fraction of amplitude of event R that is largest in most of the leads.

The EMG noise is due to movements of muscles and varies randomly as the patients muscles move. For this reason, ECG noise can be generated by perturbing a sinusoidal signal with a random frequency noise.

Addition of these artifacts to the model of Sect. 2.1 generate modeled ECG 101, provided that the value of A_o is suitably chosen. Figure 4 shows a comparison of the modeled ECG 101 and MIT-BIH 101 for the first second. The two waveforms are looking similar, showing ECG 101 to be a fairly good approximation of the real signal. The differences are because of the present of powerline interference in the real ECG. To view them separately, waveform of modeled ECG 101 and the real MIT-BIH 101 are shown in Fig. 5. The two waveforms resemble to a great extent, showing minor error in Fig. 5(c) that shows the need of recording machines with fine powerline. These comparisons and error curve of Fig. 5(c) show the accuracy of modeled ECG waveform with negligibly small error as compared to the range of the signals in the waveform.

Table 1. Parameters for Modeled ECG to Simulate a 1-s Waveform of MIT-BIH 101

Model Parameters	P-Wave	Q-Wave	R-Wave	S-Wave	T-Wave	U-Wave
Amplitude (mV)	32	−8	328	18	42	0.1
Duration (sec.)	0.15	0.06	0.035	0.35	0.13	0.15
Location (sec.)	0.1	0.2	0.23	0.27	0.5	0.51

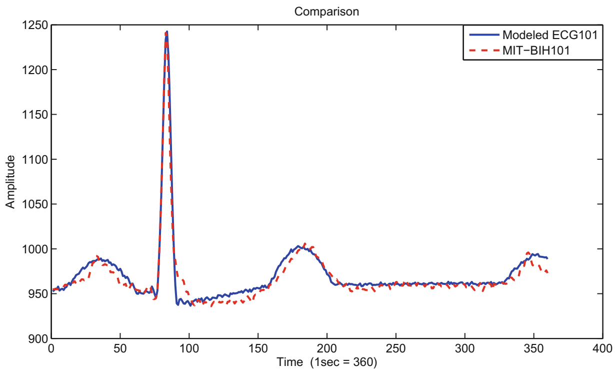

Fig. 4. Comparison of 1-s waveforms of Modeled ECG 101 and MIT-BIH 101.

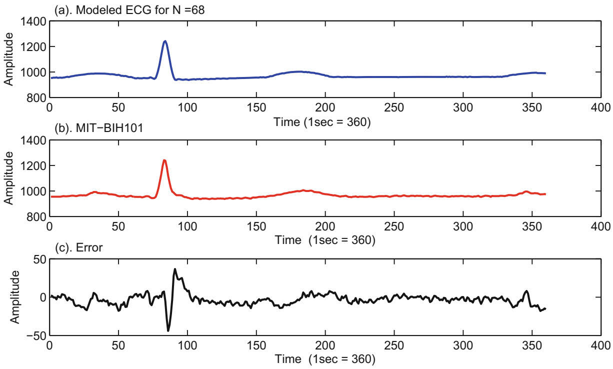

Fig. 5. Sinus Rhythms: (a) Modeled ECG 101 with 68 beats per minute, (b) MIT-BIH 101, (c) Error.

4 Conclusion

The simulations of this paper have shown that the proposed ECG model is able to generate real ECG waveforms of various frequencies and leads. Furthermore, with suitable modeling of artifacts, this model can simulate real time signals successfully with high accuracy. An important characteristic of the proposed model is that it is designed in Matlab, and requires superposition of almost periodic functions only. The simulated applications are done by employing periodic nature of the functions, however future work can be done to extend this model to general almost periodic functions. This extension may be a great help in minimizing the error between real and modeled signals.

The proposed modeling of real time ECG waveform can help in better understanding of clinical ECG recordings that will lead to better diagnosis of the heart problem. This work can further be extended to model waveform of the signals of longer time.

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