1 BACKGROUND AND RELATED WORK

This section provides background information on time series and ECGs, as well as methods to analyze them.

1.1 Time Series and Time Series Analysis

TODO explain what univariate and multivariate time series are This subsection will provide background information on time series and time series analysis methods. A time series is a set of values recorded at specific times. A common form of time series are discrete-time time series (often simply called discrete time series). Discrete time series are time series whose values are recorded at discrete points in time, the most common example of this are time series with values recorded at fixed intervals. Continuous-time time series are time series that are recorded continuously over a certain interval [1]. Time series that contain a single value for each moment in time are called univariate time series, while time series that record multiple values at each moment in time are called multivariate time series [2]. Time series are used in many disciplines to record information on time-dependent processes, e.g. stock prices in economics, the sun's activity in physics, or the heart's activity in medicine. Time series can be recorded digitally, physically, or, if they were recorded physically, can later be digitized. The recorded data can then be used to gain insight into the processes that were studied. To gain insight using a time series, the relevant information needs to be extracted from it–a process that is often called data mining. Data mining of time series is a vast discipline that, among others, includes [3, 4]:

- visualization (graphical representation),
- forecasting (predicting future behavior),
- indexing (finding the most similar time series to a given one),
- clustering (dividing time series into groups of similar ones),
- anomaly detection (detecting parts that are not "normal" or do not fit certain parameters),
- classification (assigning a label based on its features, e.g. "sick" and "not sick"), and
- summarization (reducing the complexity–often length–while preserving important features).

Challenges for time series analysis include the often very large data sets that are difficult for humans to analyze and take up considerable digital storage space. Analyzing very large data sets requires a large amount of computational power because most data mining algorithms become less efficient with larger data sets [3]. To mitigate this issue, time series dimension reduction (also known as dimensionality reduction or time series representation) is used. Dimension reduction transforms a "raw" (unmodified) time series into a representation that is simpler but nonetheless resembles the raw time series. This can be achieved by either using a method that reduces the number of values in a time series, or by extracting only the relevant features from the time series. According to [4], there are four types of dimension reduction methods:

1. data dictated.
2. non-data adaptive,
3. model-based, and

dr-methods:03

4. data adaptive,

Methods 2–4 have their dimension reduction factors set by user-defined parameters. This means that the user can determine how much the dimension of the data should be reduced [4].

TODO fix this next segment, add more citations, maybe short descriptions

TODO turn these into subsections

TODO also cite shieh2008 here, has table and explanations too

TODO maybe just mention the categories, explain what they mean, then do a "in this research the focus is on SAX, a …"

TODO cite all this shit and rephrase

TODO add more information, maybe cut the sax thing short by referring to the methods section

1.1.1 Data dictated representation

Data dictated methods derive their compression ratios from the data automatically, the most common form of this method is the clipped representation [4]. This representation simply transforms the raw time series into a sequence of 1s and 0s. A data points is assigned a 1 if its values is larger than the mean value of the time series, and a 0 otherwise. A sequence of 1s and 0s can be further compressed using various methods from computer science, finally yielding a very large compression ratio of 1057:1 [5].

1.1.2 Non-data adaptive representation

Non-data adaptive methods operate on time series segments with a fixed size to reduce the dimension and they are useful for comparing multiple time series with each other. These methods include the Discrete Wavelet Transform (DWT), the Discrete Fourier Transform (DFT), and the Piecewise Aggregate Approximation (PAA) [4]. The DWT uses wavelets, a limited-duration wave with an average value of 0, which represents both time and frequency information. The DWT is calculated using a series of filters applied to the signal. In [6], the DWT is used to detect beats in ECG signals and achieves a 0.221% detection error rate. The Fast Fourier Transform, an optimized form of the DFT, decomposes the its input signal into many sinus waves of different frequencies. In [7] it is used in conjunction with a machine learning model to achieve a beat classification accuracy of 98.7%. The PAA is part of the process of the SAX representation, thus it will be covered in

TODO refer to the appropriate methods section

1.1.3 Model-based representation

Model based methods use stochastic methods such as Markov Models (MM) and Hidden Markov Models (HMM), and the Auto-Regressive Moving Average (ARMA) [4].

1.1.4 Data adaptive representation

Data adaptive methods use non-fixed size segments and aim to fit the raw data most closely. Examples of data adaptive methods are the Piecewise Polynomial Approximation (PPA), Piecewise Linear Approximation (PLA), Piecewise Constant Approximation (PCA), and SAX [4]. PPA can be used to compress and ECG by approximating it using polynomials. With second-order polynomials, ECGs can be compressed with a minimal level of distortion [8]. The authors of [9] use a modified PLA representation with adaptive ECG segmentation to successfully reconstruct the 12 standard leads of an ECG from only 3 leads. Using adaptive PCA as the dimension reduction method, the preprocessing and segmentation of ECGs can be significantly sped up while maintaining accuracy comparable to precious methods [10]. The SAX representation will be covered in detail in **TODO** refer to the SAX section and the following subsection 1.1.5 will provide background on the method and its variations.

1.1.5 SAX representation background

A particular dimension reduction method is SAX. Introduced by Lin, Keogh, Lonardi, and Chiu, SAX is a symbolic time series representation method for univariate time series. The authors felt that the symbolic methods available in 2003 did not provide the desired dimension reduction, did not correspond to the raw data accurately enough, and could not be applied to a subset of the total data. SAX uses the averaging of a user-defined number of segments and the labeling of segments with letters to reduce the dimension of the time series data. The number of letters, called the alphabet size, can also be chosen by the user and influences the dimension reduction. The distance between two time series in the SAX representation is guaranteed to resemble the distance between the two raw time series, this is called the distance measure. Since its creation, SAX has found widespread use in data mining and many researchers have attempted to modify and improve it.

The SAX distance measure has been improved to include the standard deviation [11] and a measure of the trend of each averaged segment [12, 13]. Extended SAX modifies SAX to include the minimum and maximum values of each segment for improved representation of the raw data [14] while 1d-SAX incorporates a linear regression over each segment into SAX [15]. A combination of SAX and a polynomial approximation was used to speed up the SAX method [16]. To improve the indexing performance of SAX, iSAX introduced convertible alphabet sizes, allowing SAX representations with different alphabet sizes to compared with each other and indexed into a tree structure [17]. iSAX 2.0 improves the iSAX index by reducing its computational complexity, enabling it to index a time series that has one billion elements, something that SAX or iSAX cannot do [18]. To perform time series anomaly detection using SAX, Heuristically Ordered Time series using SAX (HOT SAX) was introduced. HOT SAX sorts segments of a SAX-represented time series by their distance to other segments, effectively identifying the most abnormal segments of a time series [19].

SAX and its variants have also been used for the analysis of multivariate time series. SAX-ARM combines the SAX representation with association rule mining (identifying rules and im-

plications found in the data, i.e. parameter a influences parameter b) to analyze multivariate time series and discover the rules underlying the data [20]. MSAX expanded the use of SAX to multivariate time series by utilizing multivariate normalization with the help of a covariance matrix and a modified distance measure [2]. SAX has also been used to visualize multivariate medical test results and enable their analysis [21]. Resource-aware SAX is a SAX variant developed to analyze ECG using a mobile device like a mobile phone. The method takes advantage of the computational efficiency of SAX to perform the ECG analysis on the device and even preserve its battery life. Another application of the SAX method to ECGs is [22], which uses SAX with an added binary measure of the trend of each segment to detect ECG anomalies, achieving a recall value of 98%. The section 1.2 below will elaborate on ECGs and methods of their analysis.

1.2 ECGs and ECG Analysis

The following subsection covers the ECG and methods used in its analysis. Luigi Galvani noted the electrical activity in muscles 1786, but the history of the ECG only started in 1842, when Carlo Matteucci showed the electrical activity of a frog's heartbeat. In the 1870s, it was discovered that each heartbeat is characterized by electrical changes. Willem Einthoven was the first to publish an ECG waveform with the now standard annotations P, Q, R, S, and T for the different features. Then, in **TODO** refer to the figure of the heart beat here
In 1901-1902, Einthoven created the first ECG recording of a human heartbeat using using 3 leads connected to the limbs of the patient. He would receive the 1924 Nobel Prize in medicine for his invention of the electrocardiograph. As a result of further development, 12-lead ECG that we know today was created [23, 24].

TODO cite becker2006 on how ECGs work

TODO write a section on how the 12 leads work

TODO how does it work

TODO different methods of recording ECGs

TODO annotated ecg for parts, which diseases are apparent

TODO look at which ECG methods were covered in first lit review and take some of those
As mentioned above, SAX has already successfully been applied to ECGs, but there has not been much use of the method in that respect? **TODO** is that true?
MSAX has not yet been applied to ECG analysis.

1.2.1 ECG databases

TODO cite this shit

TODO turn this into at least one full paragraph
ECG data is patient data and thus not freely accessible in most cases. Online databases, most of them on Physionet, are an exception to this rule. Physionet provides databases on many types of medical data. They are all freely available and licensed for research and educational use.

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