

## Introduction

Four-center electron repulsion integrals appear in many electronic structure methods, such as many-body perturbation methods like the Random-Phase Approximation or GW and others. To reduce their computational cost, the resolution-of-the-identity (RI) approximation is commonly employed. RI refactors the four-center integrals into three- and two-center integrals, among which the three-center integrals remain a computational bottleneck, particularly when using NAOs. We present a global RI scheme combined with a local metric (attenuated Coulomb) within an all-electron NAO framework, treating molecules and solids on equal footing. Central to the approach is a novel GPU-accelerated algorithm for computing 3-center integrals that relies on domain decomposition to improve load balancing and strong scaling.

## Resolution-of-the-Identity (RI)

**RI factorization [1]:**  $(\mu\nu|\lambda\sigma) = \sum_{PQST} (\mu\nu|_m P) M_{PQ}^{-1} V_{QS} M_{ST}^{-1} (\lambda\sigma|_m T)$

Auxiliary basis:  $\phi_\mu(\mathbf{r})\phi_\nu(\mathbf{r}) = \sum_P \varphi_P(\mathbf{r})$

3-center integrals:  $(\mu\nu|_m P) = \iint \phi_\mu(\mathbf{r})\phi_\nu(\mathbf{r}) m(\mathbf{r}' - \mathbf{r}) \varphi_P(\mathbf{r}') d\mathbf{r}' d\mathbf{r}$

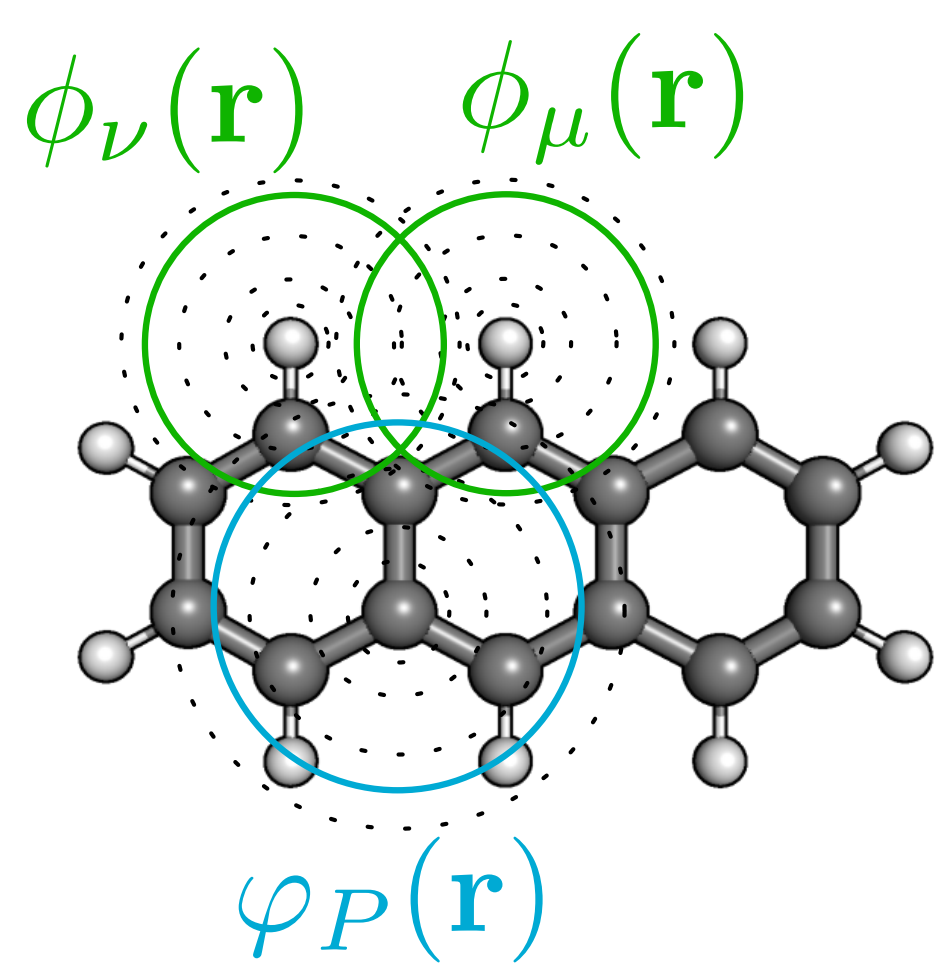
Coulomb metric (RI-V):  $m(\mathbf{r}) = \frac{1}{r}$

Sigmoid attenuated coulomb metric (RI-AtC):  $m(\mathbf{r}) = \frac{1/2 \cdot \text{erfc}(\omega(r - r_c))}{r}$

## Integration algorithm $(\mu\nu|_m P)$

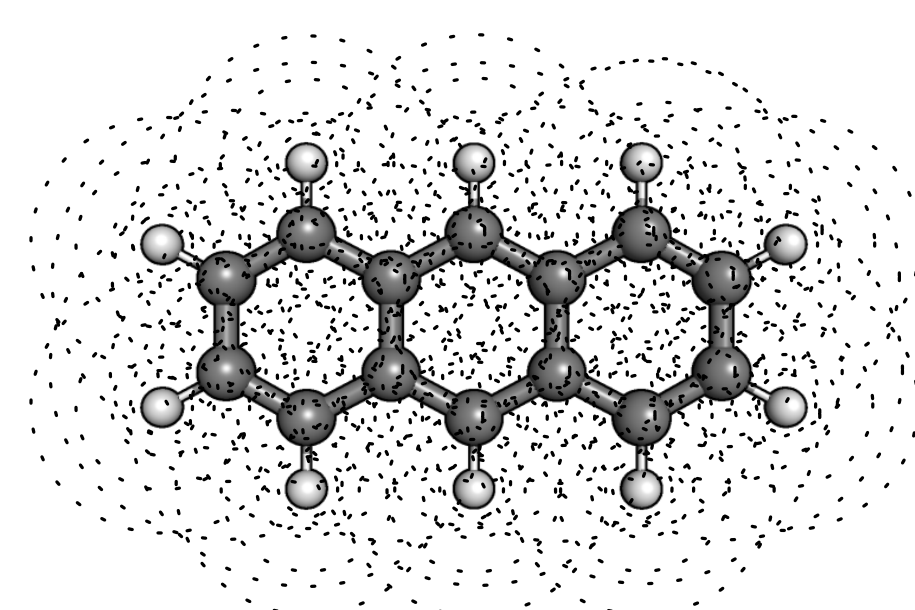
### Old Algorithm [2]:

- distribute over  $P$
- evaluate all  $(\mu\nu|P)$  of local  $P$
- integrations generated from 3 contributing atoms

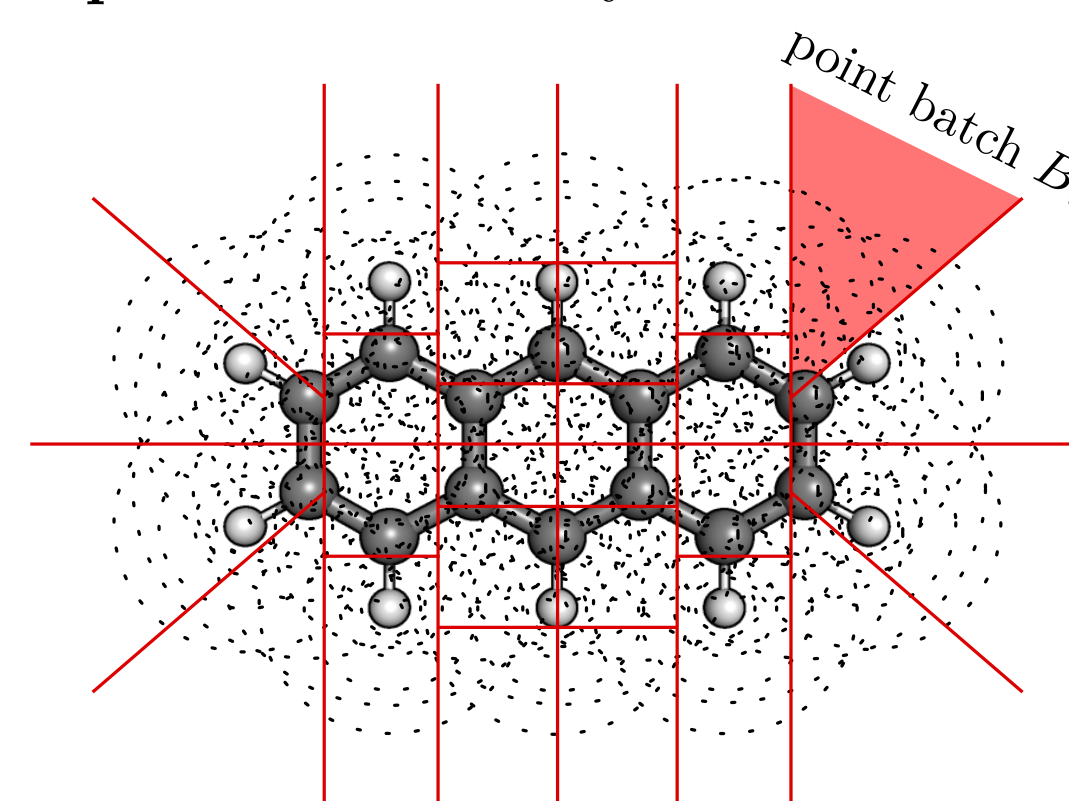


### New Algorithm:

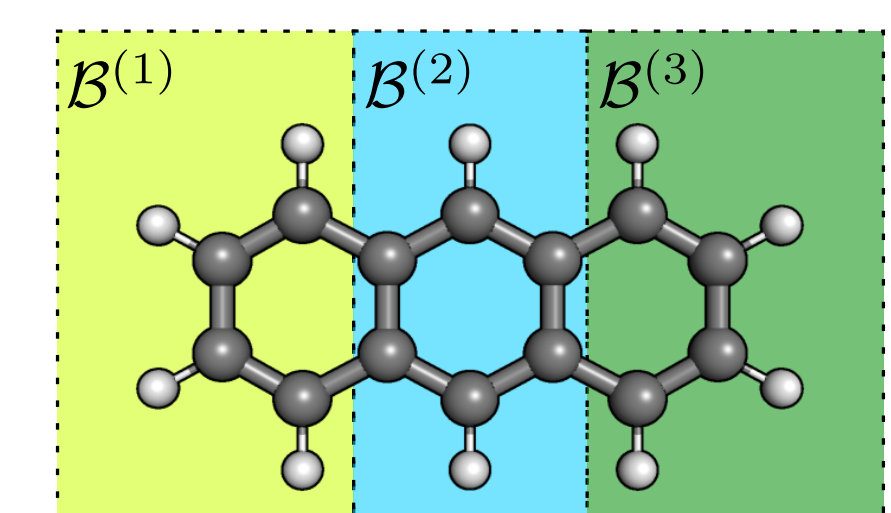
a) overlapping atom-centered integration grid



b) split grid into point batches  $B_i$

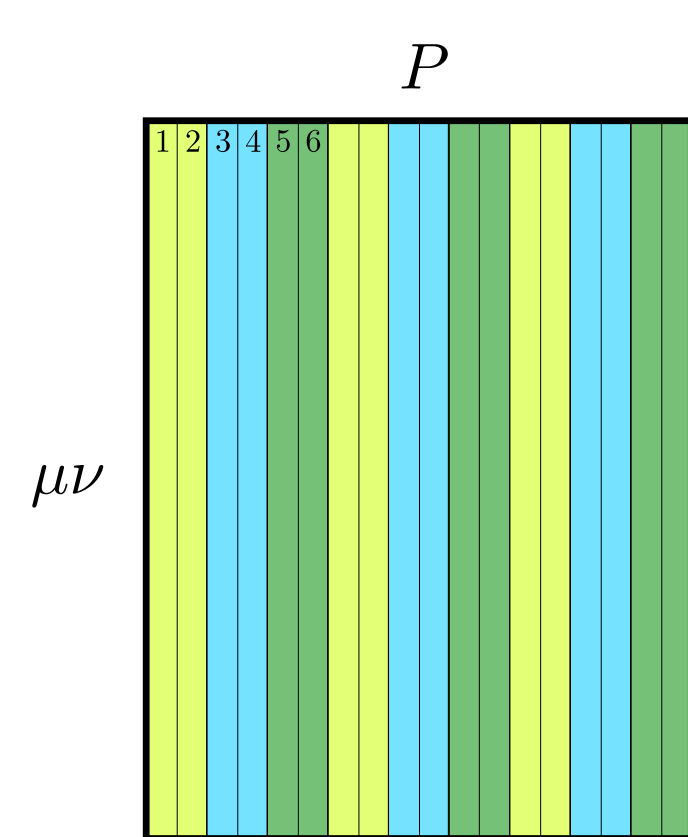


c) point batch distribution

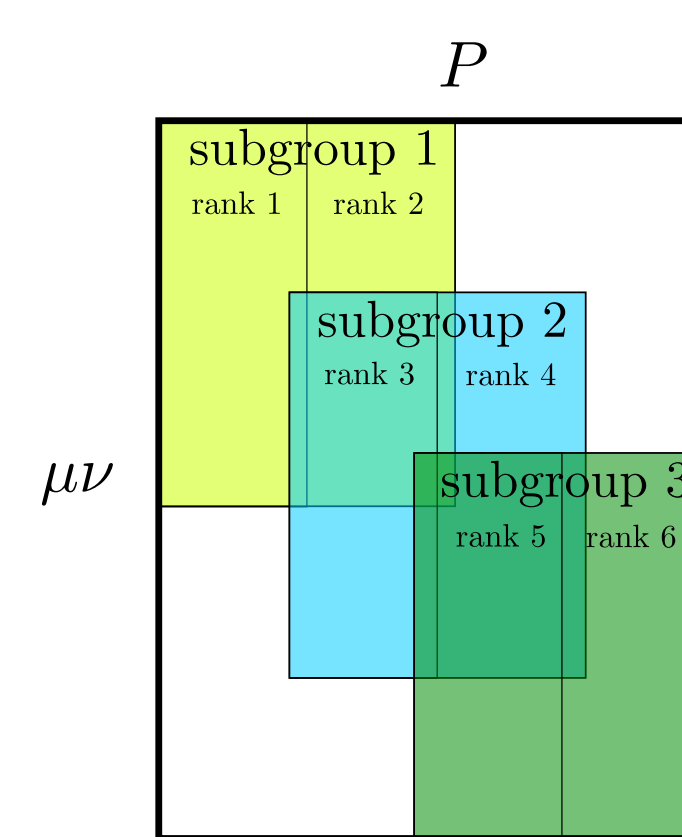


1	3	5
2	4	6
subgroup 1    subgroup 2    subgroup 3		

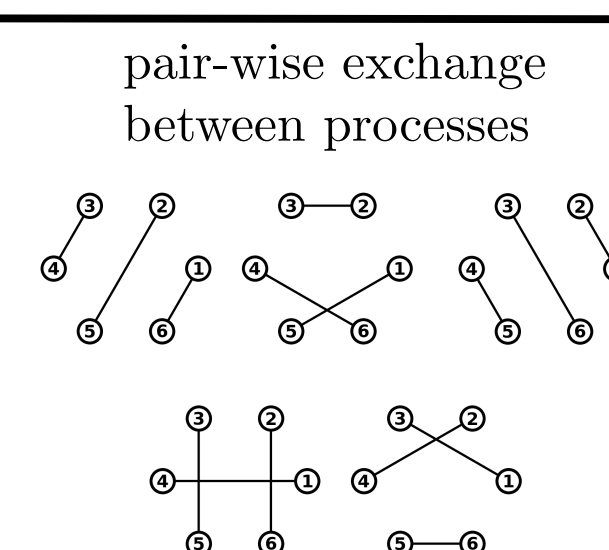
h) global matrix  $(\mu\nu|_m P)$   
1D block-cyclic distribution



f) global matrix  $(\mu\nu|_m P)$

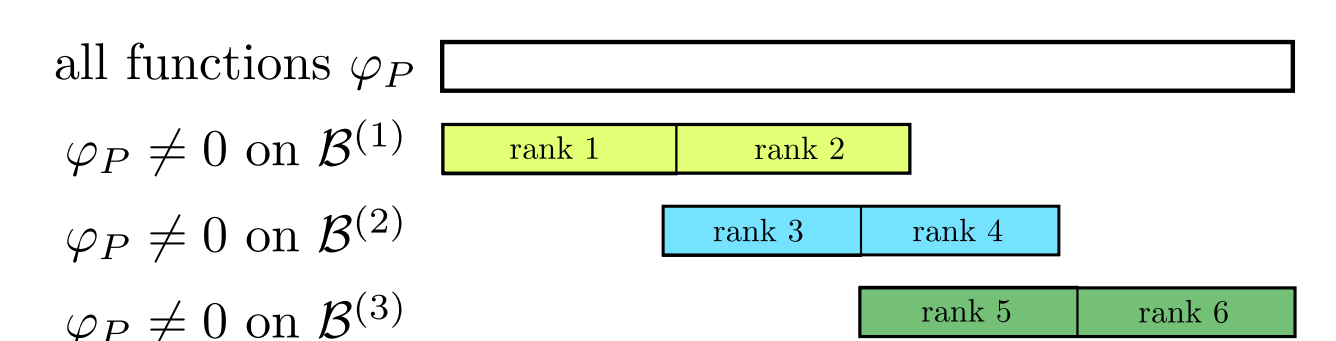


g) sum and redistribute



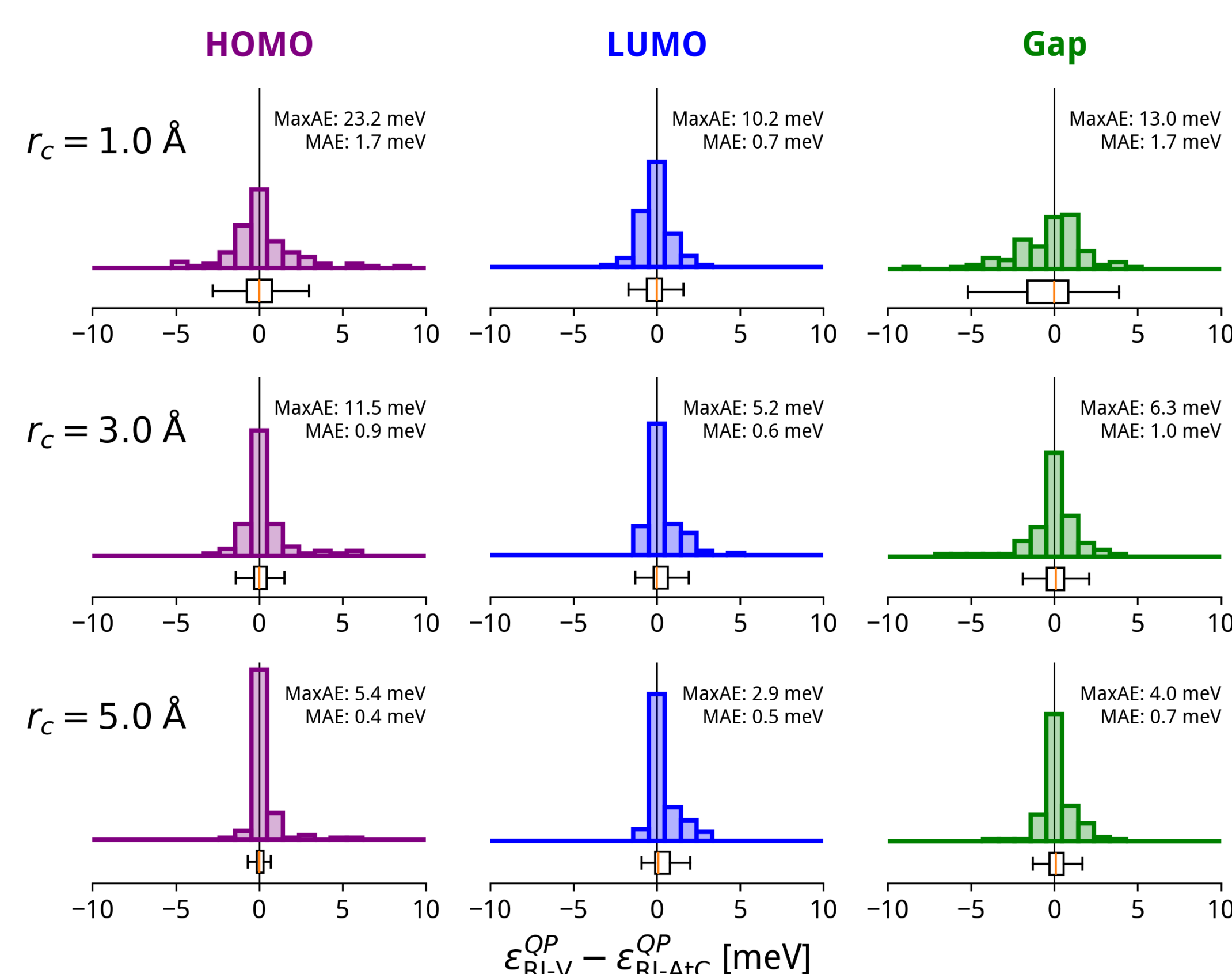
e) integrate all  $(\mu\nu|_m P)$   
- only local  $P$   
- on all points in  $B^{(i)}$

d) auxiliary basis function distribution

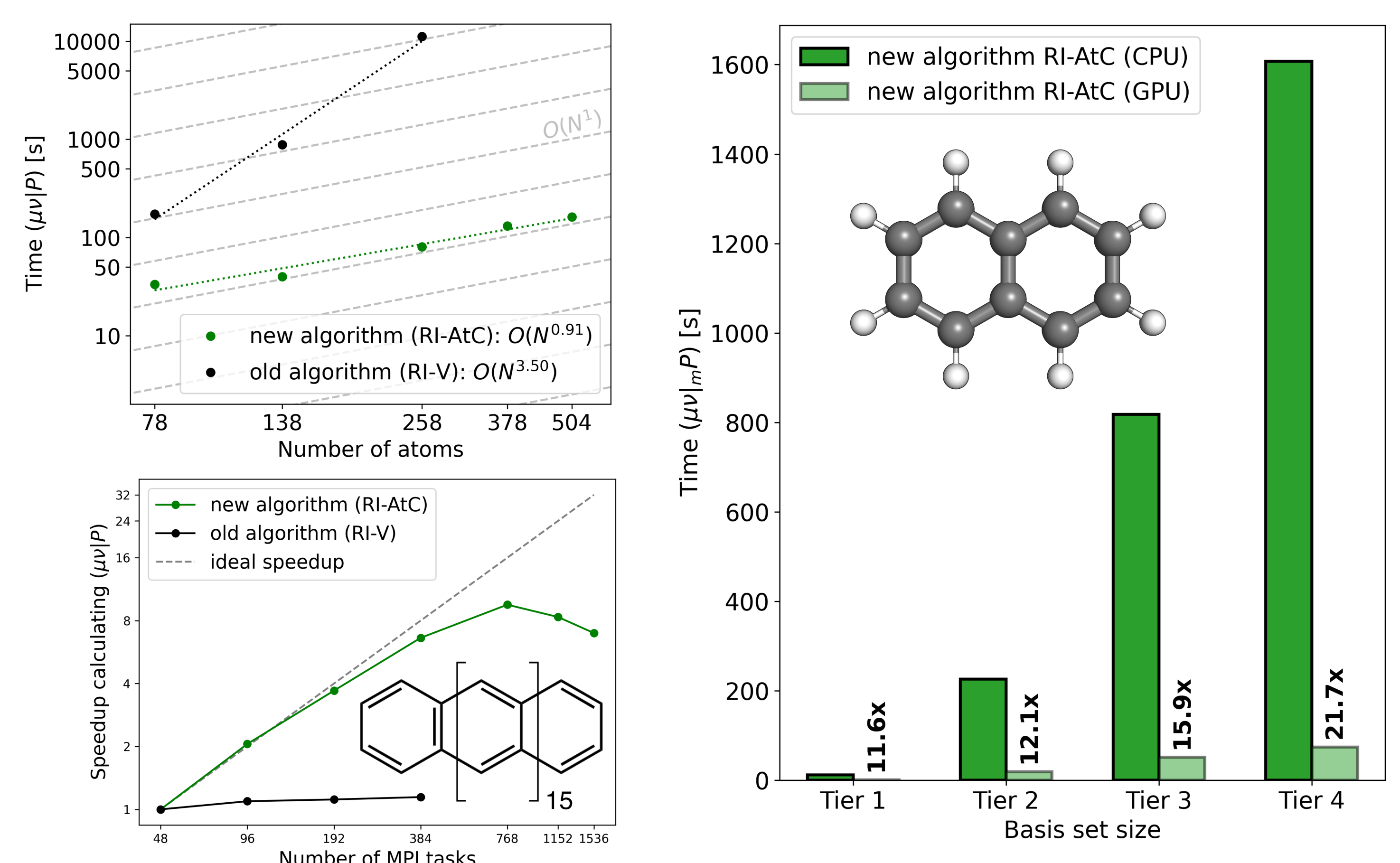


## Accuracy

### GW100 Benchmark:



## Performance



## References

[1] O. Vahtras, et al., Chem. Physics. Lett. **1993**, 213 (5), 514.

[2] X. Ren, et al., New J. Phys. **2012**, 14 (5), 053020.

Founded by:



Computing time:



Implemented inside:



Online Poster:

