

Discovering What Mattered: Answering Reverse Causal Questions by Detecting Unknown Treatment Assignment and Timing as Breaks in Panel Models

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Abstract

Much of empirical research focuses on ‘forward causal’ questions (“Does X cause Y?”). However, ‘reverse causal’ questions (“What causes Y?”) can provide invaluable insights but are difficult to implement in practice. Here we operationalise the modelling of reverse causal questions through the detection of unknown treatment assignment and timing as structural breaks in fixed effects panel models. We show that conventional treatment evaluation of known interventions in a two-way fixed effects panel (often interpreted as difference-in-differences) is equivalent to allowing for heterogeneous structural breaks in the treated units’ fixed effects. Using machine learning, we can thus detect previously unknown heterogeneous treatment effects as structural breaks in individual fixed effects corresponding to unit-specific treatment which can be subsequently attributed to potential causes (such as policy interventions). We demonstrate the feasibility of our approach by detecting the impact of ETA terrorism on Spanish regional GDP per capita without prior knowledge of its occurrence. Our proposed method to detect breaks in panel models can be readily implemented using our open-source R-package ‘gets’ with the ‘getspanel’ update or using the (adaptive) LASSO.

JEL Classification: C21, C23, C52

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1 Introduction

Informed policy decisions require knowledge of which interventions have an impact. To assess the effectiveness of any policy – or the impacts of shocks in general – the wider empirical literature has been primarily concerned with forward causal questions to assess the ‘effects of causes’ (Gelman 2011, Gelman & Imbens 2013, Mill 1843). For example, does terrorism affect GDP per capita, or is a carbon tax successful in reducing emissions? Such forward causal questions (‘Did X have an effect on Y ?’) place specific interventions at the centre of their investigations and attempt to identify the effects of that known specific event on an outcome of choice. Forward causal questions therefore rely on pre-existing knowledge of interventions having taken place. This approach thus risks missing interventions that are *a-priori* unknown.

Policy makers regularly have no prior knowledge of the specific policies, trends, or shocks that have contributed to a certain outcome, prompting them to instead look for answers to *reverse causal questions*: rather than finding the ‘effects of causes’, they attempt to find the ‘causes of effects’ (‘What has affected Y ?’). For example, what affected GDP per capita, or what has reduced emissions? While the introduction of a carbon tax may not have had a strong impact on emissions, a local policy intervention incentivising energy efficiency improvements might have. If this particular intervention is not immediately apparent to policy makers, its contribution will be unaccounted for.

Even though such a ‘reverse causal’ approach is highly relevant to identify potentially unknown impactful policies or interventions, it has not been extensively operationalised in causal empirical modelling. This may be because forward causal questions are comparatively easy to evaluate using the range of available tools for programme evaluation, ranging from matching, difference-in-differences, to synthetic controls. In contrast, it is less obvious how reverse causal questions can be answered in practice. As Gelman & Imbens (2013) put it: “*Reverse causal reasoning is different; it involves asking questions and searching for new variables that might not yet even be in our model*”.

Here we introduce a formal approach to answer reverse causal questions. We expand on the idea of “*searching for new variables*”, and place the concept of reverse causal questions into the domain of variable and model selection, and more specifically break (or anomaly) detection. We propose tackling reverse causal questions by detecting anomalies in the form of structural breaks in the familiar setting of two-way fixed effects (TWFE) panel estimators allowing for heterogeneous treatment effects which are commonly used to evaluate policy. In such a setting, the conventional approach (often interpreted as difference-in-differences) is to include a dummy variables that denote the interaction of treated units with the post-treatment time period. We illustrate that this is equivalent to allowing for step-shift changes in the treated units’ intercepts. If interventions are unknown, treatment assignment and timing can thus be detected as step-shifts conditional on treatment effects being non-zero. In our closely-related papers, Koch et al. (2022) and Pretis (2022) we apply this approach to detect effective climate policies. Here, we provide the theoretical basis formally linking structural breaks to heterogeneous treatment effects.

Specifically, we formulate the detection of structural breaks as a problem of variable selection, where we saturate a TWFE panel model with a full set of step-shift break functions denoting potential treatment of each individual at any point in time. We then apply machine learning selection methods allowing for more candidate variables than observations to identify relevant step-shifts to detect treatment without prior knowledge of its occurrence. Once a break has been identified, it can be interpreted as a treatment dummy for the relevant unit (see Figure 1 for a stylised example). We can then – in a post-estimation analysis – attempt to attribute the treatment dummy to an event that is likely to have affected the treated unit at the detected time.

This operationalises the idea of reverse causal questions – we start with a model commonly used for policy evaluation, but rather than testing a particular policy intervention (in the form of a treatment dummy), we search for structural breaks in the individual fixed effects, which can subsequently be attributed ex-post to events that took place. Thus, rather than assessing effects of causes (as in the forward causal approach), our approach provides a data-driven method to identify effects using break detection which can then be attributed to potential causes. Note that our discussion on reverse causal questions should not be confused with the concept of reverse causality. Reverse causal questions refer to the modelling process of discovering the causes of effects, and do not refer to the direction of causal relations between variables.

1.1 Related Literature & Contribution

Break detection to assess the impact of policy has been commonly used in time series analysis. However, most time series applications do not have control groups, making a clear causal interpretation of any break difficult. A causal interpretation in time series nevertheless is possible where breaks occur in some conditioning variables under super exogeneity (Bazinas & Nielsen, 2015). This has been shown first in Engle et al. (1983) as causal relations invariant to shocks (referred to as super exogeneity). Under such super exogeneity causal identification is possible, see e.g. Martinez (2020), Mukanjari & Sterner (2018), or Pretis (2021) for relevant examples of this.

Where super exogeneity does not hold, has not been tested, or is difficult to establish, however, a causal interpretation of structural breaks is more difficult. Examples in the time series literature range from Perron (1989) detecting breaks in GNP time series attributed to the Great Depression and an oil price shock, Hendry (2020) identifying policy interventions in UK CO₂ per capita emissions, Estrada et al. (2013) quantifying the impact of the Montreal Protocol on CFC emissions and subsequently temperatures, to Apergis & Lau (2015) identifying whether breaks in Australian electricity markets align with policy interventions. Piehl et al. (2003) also use the detection of breaks in time series to assess treatment effectiveness of a youth homicide prevention programme in Boston.

Compared to time series applications, fewer papers tackle structural breaks in a panel setting, and to the best of our knowledge, no paper has formally considered the link between structural breaks and treatment effects in a panel, or the detection of breaks in a TWFE panel to detect treatment. Attributing breaks in panels as treatment was first explored in Pretis (2022) assessing the impact of carbon taxes, but not formally linked to heterogeneous treatment effects. In our related paper, Koch et al. (2022) we apply the break detection approach to EU CO₂ road emissions to identify effective policies in a causal framework, albeit with the focus on the application rather than the econometric theory underlying the approach.

Panel methods for the detection of breaks range from estimating break dates using least-squares to detecting breaks by selecting over break dates using model selection. In the least-squares literature, Chan et al. (2008) extend the Andrews (1993) structural change with unknown change point test (Sup-test) for simple structural breaks to panels in a setting that focuses on detecting changes in coefficients on random variables (rather than on changes in the fixed effect as would be necessary in a policy evaluation framework). De Wachter & Tzavalis (2012) test for common breaks in dynamic panels and Baltagi et al. (2016) study heterogeneous panels with structural breaks using a Bai (1997)-type approach. Recent work has focused on structural breaks in panels using common factors, these include Zhu et al. (2020) who study a single break in dynamic panel with a common factor structure, as well as Cheng et al. (2016) who consider breaks in the form of changing latent factor loadings. Bai et al. (2020) develop a least squares approach to detect breaks in factor loadings in panel factor models.

The above papers use the classical approach of estimating break dates by least squares, while now there is a growing literature using variable selection to identify breaks. In this selection-based literature, Qian & Su (2016b) propose to use the LASSO (Least Absolute Shrinkage and Selection Operator) to detect breaks in simple time series models, with Qian & Su (2016a) extending this approach to detect common breaks in panels. Their method is related to our approach presented here, however, we focus on individual breaks in fixed effects and thus treatment rather than on common breaks. In the factor literature (not focusing on policy evaluation), Li et al. (2016) propose to use the LASSO to detect common breaks in interactive fixed effects models. Conley & Taber (2011) compare inference when the number of treatment groups is small to inference in structural break detection, but they do not explore this link further.

Our paper is perhaps closest in-spirit to Okui & Wang (2021) on the selection side, and Wooldridge (2021) on the treatment estimation side. Okui & Wang (2021) detect heterogeneous (group-specific) structural breaks in coefficients on random variables using the adaptive LASSO. Relative to our approach, Okui & Wang (2021) do not focus on treatment effects and they partial out the individual fixed effects rather than studying breaks in them. Their analysis also focuses on grouped rather than individual structural breaks. We instead concentrate on breaks in unit fixed effects to detect treatment and explore alternative selection methods (in addition to the LASSO we also use the general-to-specific – gets – selection method) which can be embedded in an outlier-robust estimation framework. Nevertheless, the group-specific method of Okui & Wang (2021) may be a promising avenue of future research in the case of multiple (unknown) treated units.

In a standard ‘known-treatment’ setting, Wooldridge (2021) shows that heterogeneous and time-varying treatment effects can be identified and estimated consistently using a TWFE estimator in a common timing and staggered setting using interactions of treatment times and dummies. We show that the starting point of our break detection approach nests Wooldridge’s interacted TWFE specification as a special case where we relax the knowledge around treatment assignment and timing, as well as homogeneity of treatment effects over treated individuals. Thus, the interaction-augmented TWFE estimator proposed in Wooldridge (2021) constitutes the target of model selection in our case, and the final retained models identify heterogeneous treatment effects. Specifically, we show that ‘known’ policy dummy variables in a TWFE panel model are equivalent to step-shifts in the individual fixed effects of the treated units which can be detected using break detection methods. Similarly, we demonstrate that time-varying and heterogeneous treatment effects using interactions are equivalent to allowing for unit-specific impulse dummies which capture single-period structural breaks. In other words, as we show, treatment dummies in a TWFE setting are equivalent to structural breaks taking the form of a step-shift in the individual fixed effects of the panel units. Using this equivalence between step-shifts in the unit-specific intercept (i.e. fixed effect) and known treatment dummies, we therefore propose an alternative estimation approach based on reverse causal questions: rather than simply evaluating a known intervention, we instead estimate a TWFE panel model while searching for potential structural breaks (step-shifts) in the unit-specific intercepts. Notably, this approach identifies unit-heterogeneous fully-time-varying or piece-wise constant treatment effects. The final model (conditional on having identified treatment breaks) corresponds to a heterogeneous treatment effects model where treatment effect heterogeneity is identified using interactions as in Wooldridge (2021) and thus does not suffer from the concerns around heterogeneous treatment effects in staggered interventions (see Goodman-Bacon 2021, Callaway & Sant’Anna 2020, De Chaisemartin & d’Haultfoeuille 2020, 2021, Baker et al. 2021).

Overall, relative to the time series literature using breaks for indicative policy evaluation, we expand the break-detection approach to a panel setting where units without breaks act as a control group against

which a break (i.e. treatment intervention) can be identified. Relative to the existing panel break literature, we focus on machine learning methods to detect breaks in single individual fixed effects and their attribution as treatment interventions. Relative to the vast existing TWFE literature on policy evaluation, our approach implements a reverse-causal estimation strategy detecting previously unknown treatment (or events) while at the same time allowing known interventions to be embedded. Recently the detection of structural breaks has also been applied to detect unknown discontinuities in regression discontinuity design (RDD). For instance, Porter & Yu (2015) use a simple Andrews (1993)-type structural break test to identify a regression discontinuity without prior knowledge of its existence. Similarly to their argument of an unknown discontinuity, we explore the use of break detection to identify previously unknown treatment in TWFEs.

There are a range of nuances to our proposed approach. First, if treatment assignment and timing is known (and happens to have a large effect), then imposing interacted treatment dummies allowing for heterogeneous treatment effects for the known intervention in a TWFE estimator is effectively equivalent to agnostically detecting a break in this fixed effect (if that is the only break retained) and estimating the model post-break detection. The estimated model with an imposed break or a single retained break is *identical*. In other words, if the intervention was known, we could simply run a TWFE estimator, which will be equivalent to having found the one particular intervention and then assessing the estimated model.

Second, our idea is modular with respect to known treatment. If there is a known intervention, we can impose it into the model without selection and estimate its impact, while at the same time searching for additional breaks. This allows us to assess the impact of a known policy while also detecting potentially unknown interventions, effectively implementing the theory-embedding approach described in Hendry & Johansen (2015). It is worth noting that our approach concentrates on causes of effects by first identifying effects. If there are no effects, naturally we cannot find any corresponding cause. Thus, for unknown treatment we cannot distinguish between no treatment or a zero treatment effect. For known treatment, however, this is not a concern as it can easily be embedded as a forced a-priori treatment variable that is introduced into the model independent of the selection. We can also restrict the search for breaks and treatment to a subset of units if we suspect that some units may be treated and are certain that others are not.

Third, our conceptual approach is also modular in terms of the choice of detection method. We can use different machine learning methods of our choice to detect breaks (i.e. treatment), depending on the preferred properties of the selection algorithm. For example, if our main concern is the false-positive detection rate, then we can choose to use methods that control the false discovery rate (such as general-to-specific selection methods, henceforth ‘gets’). If instead we care about computational speed, we could use regularised estimators, such as the (adaptive) LASSO.

There are of course some constraints to our methods. First, when detecting breaks in individual fixed effects, each treated unit will be identified with a separate treatment dummy. While this allows for straightforward heterogeneous treatment effects, it means that we do not gain power if there are multiple units that received the same treatment. Thus, our TWFE break detection approach mirrors the use of interactions to identify known heterogeneous treatment effects (Wooldridge, 2021) and lends itself to panels with longer time series and smaller cross-sectional dimensions with heterogeneous treatment (similar to settings encountered when using synthetic controls).

Second, all break detection methods evaluate the presence of breaks relative to a specified underlying model. If the model is not well-specified, then breaks that we detect may simply reflect model misspecification. This is of course also a problem in conventional TWFE difference-in-differences settings,

however, can be amplified in our setting if we attempt to attribute a ‘spurious’ break to an event. This effect can be mediated by selecting at tight significance levels to control the false-positive rate (when using gets, see section 3.1.1) or by making use of robust estimators less sensitive to observations falling outside the specified model (such as embedding break detection in a wider outlier-robust estimation framework, e.g. Impulse Indicator Saturation, IIS – see Hendry et al. 2008; Jiao & Pretis 2020, Jiao et al. 2021).

Third, once a treatment effect is detected in the form of a break, it has to be attributed to a potential cause by referring to the existing literature and records. While this can be a challenge and requires subject-specific knowledge, it offers an opportunity to learn from the data. A search for a potential cause of an effect is comparable to arguing that a known intervention was exogenous (or as-if randomly assigned) in a conventional programme evaluation application.

We demonstrate the feasibility of our break-detection approach using a well-known dataset on the economic impacts of terrorism in Spain, where the onset of ETA terrorism in the Basque Country depressed regional GDP per capita relative to unaffected regions. We purposely choose a simple example where the treatment is actually known, in order to demonstrate the feasibility of our method. The dataset was originally analysed by Abadie & Gardeazabal (2003) in their seminal paper introducing synthetic control methods. We show that we can detect the ‘treatment’ of Basque terrorism as breaks in individual fixed effects in models of GDP per capita without prior knowledge of its occurrence, in line with the original results by Abadie & Gardeazabal (2003). The treatment intervention can be detected both in a simple TWFE setting with two regions (where one region is unknowingly treated), as well as in a wider panel model of all of Spain’s mainland regions. Beyond ETA terrorism, we also detect breaks in other regions which we attribute to an industrial crisis in Madrid, and increased regional autonomy in the post-Franco era. Hence, our approach directly operationalises the idea of reverse causal questions – we start with a simple TWFE estimator, use machine-learning selection to identify significant interventions which can be interpreted as treatment effects and subsequently attribute them to events that took place. Our proposed panel break detection approach can be readily implemented using our accompanying R-package ‘gets’ (Pretis et al. 2018) and the ‘getspanel’ update (Schwarz et al. 2021).

The roadmap for the remainder of the paper is as follows. In section 2.2 we first provide a simple illustration of how structural breaks are closely linked to treatment evaluation in two-way fixed effects estimators. We consider the standard case of known treatment assignment and illustrate its equivalence to a step-shift break in the treated units’ intercept. In section 2.3 we then consider the case where treatment assignment and timing is unknown, and we establish that unknown treatment assignment and timing can be identified using impulse dummies in a saturated regression (for fully time-varying effects) and step-dummies (for piece-wise constant effects). Using recent results from Wooldridge (2021), we show how time-varying and unit-heterogeneous treatment effects can be nested in dummy-saturated models with more variables than observations. We further show that if we detect multiple treatment breaks (in multiple different units), they can be interpreted as time-varying treatment effects in a staggered treatment intervention setting. We discuss our approach in a balanced panel without explicitly discussing control variables, however, the results should generalise to the inclusion of other covariates. In the following section 3 we then briefly discuss two estimation approaches using general-to-specific (gets) selection and the adaptive LASSO (and provide some simulations in the Supplementary material). Finally, we apply our methods to models of Spanish regional GDP per capita in section 4.

2 Conceptual Approach: Break Detection to Detect Unknown Treatment Assignment & Timing

We consider the detection of treatment and subsequent estimation of treatment effects when both treatment assignment and treatment timing are unknown. We show that if treatment assignment and timing are unknown, such treatment can be identified by allowing for potential structural breaks at any point in time for any unit in a model including individual and time fixed effects. Applying machine learning methods allowing for model selection for more variables than observations, we then remove all irrelevant treatment dummies and are left with the resulting model that identifies treatment assignment, timing, and estimates treatment effects conditional on the treatment effects being non-zero.

To aid the reader’s understanding, we present a stylised example in Figure 1, which provides guidance for the various aspects of our conceptual approach we touch upon in the following sections.

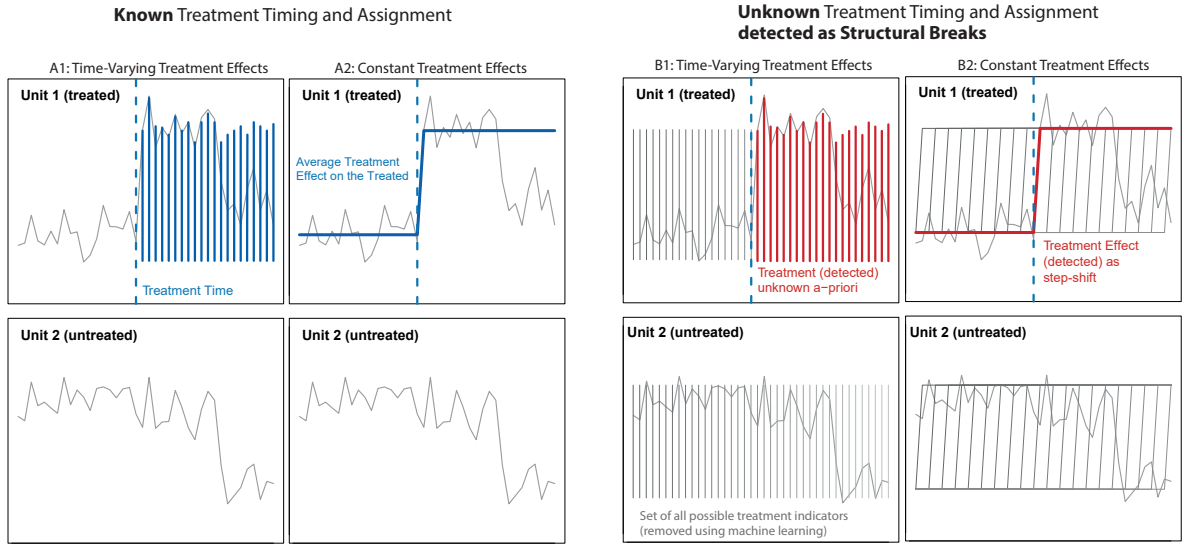


Figure 1: Detecting unknown treatment timing and assignment as structural breaks – a stylised Example using artificial data. Left: ‘Known’ Treatment baseline for time-varying and constant treatment effects. Right: Detecting treatment as breaks using impulses for time-varying and step-shifts. All possible impulse and step-indicators shown in grey, a subset of which (red) identify the true underlying treatment (blue in left panels).

2.1 Setting

To illustrate the overall motivation and the close link between structural breaks and treatment evaluation, consider a panel of N units over T time periods where one group is treated with a single treatment from time $t = q$ onwards. We initially consider the baseline case of known treatment assignment and timing, where the treatment indicator $d = 1$ for the treated group (or individual) and $d = 0$ for the untreated. Using the notation in Wooldridge (2021), we denote by $y_t(0)$ the outcome in the untreated control group, and $y_t(1)$ the outcome in the treated group at time t . The treatment effect at time t due to treatment occurring from time $t = q$ onwards is given by the difference $y_t(1) - y_t(0)$. As is convention in the literature, we focus on the average treatment effect on the treated τ_t :

$$\tau_t = E[y_t(1) - y_t(0) | d = 1] \quad (1)$$

To identify the average treatment effect we assume there is no anticipation of treatment, in other words, the potential outcome for a unit prior to treatment is identical to the untreated units:

$$E[y_t(1) - y_t(0)|d = 1] = 0, \text{ for } t < q \quad (2)$$

Further we rely on the common trend assumption which is standard in much of the treatment effects literature:

$$E[y_t(0) - y_{t=1}(0)|d] = E[y_t(0) - y_{t=1}(0)] = \theta_t, \text{ for } t = 2, \dots, T \quad (3)$$

Finally, we also assume there is at least one untreated unit. We then write the observed outcome as:

$$y_t = y_t(0) + d[y_t(1) - y_t(0)] \quad (4)$$

The expected outcome conditional on treatment is:

$$E[y_t|d] = E[y_t(0)|d] + d \times \tau_t \quad (5)$$

We define the change in y_t over time in absence of treatment as:

$$g_t(0) = y_t(0) - y_{t=1}(0) \quad (6)$$

and under the common trend assumption we have that $E[g_t(0)|d] = E[g_t(0)] = \theta_t$. We thus have that:

$$E[y_{t=1}(0)|d] = \lambda + \xi d \quad (7)$$

where ξ denotes the average pre-treatment difference between the treated and untreated groups and λ denotes the average level of y for the untreated. Combining all above yields the expected value of y_t conditional on treatment as:

$$E[y_t|d] = \lambda + \xi d + \theta_t + d \times \tau_t \quad (8)$$

For illustration purposes, assume the treatment effect is constant over time, $\tau_t = \tau$. Under the assumption of no anticipation we have that:

$$E[y_t|d] = \lambda + \xi d + \theta_t, \text{ for } t < q \quad (9)$$

$$= \lambda + \xi d + \theta_t + d \times \tau, \text{ for } t \geq q \quad (10)$$

This is a standard result in the treatment effects literature and the above model can be consistently estimated using a TWFE estimator (see e.g. Wooldridge 2021):

$$y_{i,t} = c_i + g_t + \tau w_{i,t} + u_{i,t} \quad (11)$$

with $w_{i,t} = d_i \times q_t$ where d_i is an indicator for whether the individual is treated, q_t an indicator for the post-treatment period, c_i denote individual fixed effects, and g_t time fixed effects. Note that Wooldridge (2021) groups the untreated mean into a single intercept, however, the treatment effect estimates are unaffected by whether we include a common intercept or allow for unit-specific intercepts (i.e. fixed effects). Notably, the above model shows the close link to structural breaks as we identify the average

treatment effect as a step-shift of magnitude τ at time q in the treated unit's intercept:

$$\begin{aligned} E[y_{i,t}|d_i = 1] &= c_i + \tau \times 1_{\{t \geq q\}} + g_t \\ &= c_{i,t} + g_t \end{aligned} \tag{12}$$

$$\text{where } c_{i,t} = \begin{cases} c_i & \text{for } t < q \\ c_i + \tau & \text{for } t \geq q \end{cases} \tag{13}$$

Figure 1 (column A2, left) shows a stylised example illustrating how a constant treatment effect corresponds to a simple step-shift in the individual-specific intercept.

2.2 Known Treatment Assignment with Unknown Timing

Now suppose we know which units are treated, but the timing of treatment is unknown. This may be the case when we suspect some intervention or event took place in some regions/countries, but the actual date of the intervention is uncertain. Let H denote the set of treated individuals and $1_{\{i \in H, t \geq q\}}$ an indicator function equal to one when i is part of the treated group and t falls in the post-treatment period. When treatment timing is unknown, we can interpret the identification of treatment effects as a break detection problem where we detect a structural break in the treated unit's specific intercept conditional on there being a non-zero treatment effect:

$$y_{i,t} = c_i + \tau \times 1_{\{i \in H, t \geq q\}} + g_t + u_{i,t} \tag{14}$$

When treatment is known, the above model (14) corresponds to a partial structural change model (see e.g. Perron 2006) with $c_{i,t}$ being allowed to break for treated individuals in the sample, and we estimate the break date q as well as treatment effect τ . If there is only a single treated unit and we detect a structural break in its intercept at the time of treatment, then the resulting model with a structural break is *identical* to the treatment effect model (11). There is thus a close link between break detection and the estimation of treatment effects in TWFE estimators.

However, the above model (14) may be overly restrictive as it assumes a single treatment with known-assignment and unknown-timing. In practice there may exist a myriad of possibly unknown interventions and we may face uncertainty around both treatment assignment as well as timing. In other words, we may not know which (if any) units are treated, and at what time such treatment may have occurred. In addition, treatment effects may also be heterogeneous over treated units as well as over time.

We therefore now turn to the setting where we allow for both treatment assignment and timing to be unknown (see section 2.3), and also relax the assumption of time-constant and homogeneous treatment effects over treated units (Section 2.3.1). Subsequently we consider multiple treatments (which could also be interpreted as staggered adoption) in section 2.3.3.

2.3 Unknown Treatment Assignment & Timing for a Single Treatment

We now consider detecting treatment when treatment assignment and timing are unknown and treatment effects may be heterogeneous over treated units and time. We begin by relaxing the assumption that treatment effects are constant over time in the known treatment setting. Allowing for time-varying

treatment effects τ_t we can write the expected outcome conditional on treatment as:

$$\begin{aligned} E[y_t|d] &= \lambda + \xi d + \theta_t, t < q \\ &= \lambda + \xi d + \theta_t + d \times \tau_t, t \geq q \end{aligned} \quad (15)$$

If treatment assignment and timing is known, the above can be consistently estimated using interactions in a TWFE estimator (see Wooldridge 2021 for the ‘known treatment’ case), where again we here allow for unit-specific intercepts:

$$y_{i,t} = c_i + g_t + \sum_{s=q}^T \tau_s (d_i \cdot 1_{\{t=s\}}) + u_{i,t} \quad (16)$$

This is equivalent to a set of step-shifts of duration 1 (with common coefficient over i in H) at times $q, q+1, \dots, T$, where each time step is denoted as an index s , which ranges from s_1, s_2, \dots, S . We now relax the assumption of having common coefficients over i , in other words, we allow for heterogeneity in treatment effects over i . Let $H = \{m_1, m_2, \dots, m_M\}$ denote the set of M treated units. For example if units $i = 2$ and $i = 3$ are treated, then there are two treated units $M = 2$, and $m_1 = 2$ and $m_2 = 3$. The above model (16) can then be written in a general specification allowing for unit-heterogeneous treatment effects at every time as:

$$y_{i,t} = c_i + g_t + \sum_{j \in H} \sum_{s=q}^T \tau_{j,s} 1_{\{i=j, t=s\}} + u_{i,t} \quad (17)$$

where $1_{\{i=j, t=s\}}$ denotes an indicator function equal to one for all treated i in the set of treated units H and $t = s$ in the post-treatment period $s \geq q$, and zero otherwise. This specification relaxes the restriction of homogeneous treatment effects across treated units. Figure 1 (column A1, left) shows a stylised example of individual impulses capturing a treatment effect. Specifically, each treated post-treatment observation is captured by a single time-period dummy. While these cannot be estimated consistently because they capture single observations, such dummy variables can be estimated unbiasedly (see Hendry & Santos 2005) and, as we showed here, identify unit- and time-specific treatment effects.

To relate the known case to the unknown treatment setting, we further refine our notation. We define an index of the timing of non-zero treatment effects for each treated unit $j \in H$ denoted as $R_j = \{q_{j,s=1}, q_{j,s=2}, \dots, q_{j,S_j}\}$ where S_j denotes the number of treatment indicators for unit j . For example, suppose that in a 3-unit panel with $T = 20$ observations units $i = 2$ and $i = 3$ are treated ($m_1 = 2, m_2 = 3$) with non-zero treatment effects from $t = q, \dots, T$. Then $H = \{2, 3\}$ with corresponding treatment effects at $R_2 = \{q_{2,1} = q, q_{2,2} = q+1, \dots, q_{2,S_2} = T\}$ and $R_3 = \{q_{3,1} = q; q_{3,2} = q+1, \dots, q_{3,S_3} = T\}$. With common treatment timing and effects this implies that $R_2 = R_3$. Thus the known-treatment and known-timing baseline in (17) can be written as:

$$y_{i,t} = c_i + g_t + \sum_{j=m_1}^{m_M} \sum_{s=q_{j,1}}^{q_{j,S_j}} \tau_{j,s} 1_{\{i=j, t=s\}} + u_{i,t} \quad (18)$$

or by simplifying notation as:

$$y_{i,t} = c_i + g_t + \sum_{j \in H} \sum_{s \in R_j} \tau_{j,s} 1_{\{i=j, t=s\}} + u_{i,t} \quad (19)$$

Now what if treatment assignment and timing are unknown? The above model in (19) constitutes the ‘known’ intervention baseline, i.e. the target of model selection/break detection. In 2.3.1 we now consider the detection of treatment assignment and timing allowing for unit-heterogeneous and time-varying treatment effects as in (19) which we will show is matched by a saturating set of unit-time-specific impulse dummies. We then consider treatment detection allowing for unit-heterogeneous but piece-wise constant treatment effects over time, which we will show is nested by a saturating set of unit-specific step-shift breaks.

2.3.1 Detecting Unknown Treatment When Treatment Effects are Fully-Time Varying

If treatment assignment and treatment timing is unknown, we propose that the ‘known’ treatment model (19) can be embedded in a general model allowing for potential treatment of any unit at any point. The most flexible specification that nests the ‘known’ treatment specification (19) as a special case is a fully-saturated model allowing for a treatment dummy for each individual at every point in time:

$$y_{i,t} = c_i + g_t + \sum_{j=1}^N \sum_{s=1}^T \tau_{j,s} 1_{\{i=j, t=s\}} + u_{i,t}. \quad (20)$$

The model in (20), which identifies unit-specific treatment effects for each unit for each time period, however, cannot be estimated as such because the number of parameters matches (or exceeds) the number of observations. Effectively there are NT possible indicator variables added to the balanced panel. Figure 1 (column B1, right) shows the full set of these impulse indicators, a subset of which identify the true treatment effect shown in panels on the right.

The aim is thus to reduce the general model (20) to a sparse model, ideally coinciding with the underlying target of the known baseline (19). Thus, we require the additional assumption that treatment effects are sparse, we have at least one untreated unit and some untreated time-periods for treated units – assumptions that are very common in the wider treatment evaluation literature. Starting with this general model, we then apply machine learning/model selection to remove all but ‘relevant’ dummy variables using selection algorithms capable of handling more variables than observations. We discuss two possible machine learning algorithms in more detail in section 3.1. In fact, the dummy-saturated model in (20) is equivalent to an outlier-robust Huber-skip estimator, where the retained impulse dummies detecting ‘outliers’ relative to the model capture the time-varying unit-specific treatment effects (see e.g. Jiao et al. 2021, Hendry et al. 2008, Johansen & Nielsen 2009, Johansen & Nielsen 2016a). We write the sparse final selected model as:

$$y_{i,t} = \hat{c}_i + \hat{g}_t + \sum_{j \in \hat{H}} \sum_{s \in \hat{R}_j} \hat{\tau}_{j,s} 1_{\{i=j, t=s\}} \quad (21)$$

where we effectively estimate treatment assignment $\hat{H} = \{\hat{m}_1, \hat{m}_2, \dots, \hat{m}_{\hat{M}}\}$, together with the index of treatment occurrence $\hat{R}_j = \{\hat{q}_{j,1}, \hat{q}_{j,2}, \dots, \hat{q}_{j,\hat{S}_j}\}$ where \hat{S}_j denotes the number of treatment indicators for unit j , and the time-varying and unit-specific treatment effects $\hat{\tau}_{j,s}$ conditional on having non-zero treatment effects. Note that we detect treatment when it has an effect, i.e. we detect treatment effects conditional on them being non-zero. Using the resulting estimated treatment effects $\tau_{j,s}$ it is straightforward to compute the average treatment effects for the treated (ATTs) for specific units or time periods. As impulse indicators are orthogonal, it is trivial to compute the standard error for the resulting ATTs. For example, we can compute the estimated ATT for individual j over the entire period of non-zero

detected treatment effects as:

$$\widehat{ATT}_j = \frac{1}{\hat{S}_j} \sum_{s=1}^{\hat{S}_j} \hat{\tau}_{j,s}, \text{ with standard error } se(\widehat{ATT}_j) = \sqrt{\frac{1}{\hat{S}_j} \sum_{s=1}^{\hat{S}_j} se(\hat{\tau}_{j,s})^2} \quad (22)$$

If we are interested in a subset of treated periods we could simply restrict the ATT to those relevant time periods (or units). A remaining issue, however, is that detecting individual impulses may suffer from low power if treatment effects are small and actually constant over some period of time. If we are interested in ATTs over time for some treated units, allowing for piece-wise constant treatment effects may yield higher power of detection which we discuss in the next section 2.3.2.

2.3.2 Detecting Unknown Treatment With Piece-Wise Constant Treatment Effects

While treatment effects may be heterogeneous over individuals i , they may be constant for some time periods. Such constancy over time can lead to higher power to detect treatment. Consider treatment effects in (15) that are constant over time following treatment from $t = q$ onwards, but allowed to vary over treated individuals:

$$y_{i,t} = c_i + g_t + \sum_{j \in H} \tau_j 1_{\{i=j, t \geq q\}} + u_{i,t} \quad (23)$$

Then for each treated unit in H (where $d_i = 1$) with time-invariant treatment effect $\tau_{i,t} = \tau_i$, for all t , the change from pre-treatment to post-treatment is given by a step-shift change in the unit-specific intercept of magnitude τ_i . For example, for treated unit i with time-invariant treatment effect, the expected outcome is given by:

$$\begin{aligned} E[y_{i,t} | d_i = 1] &= c_i + g_t + \tau_i \times 1_{\{t \geq q\}} \\ &= c_{i,t} + g_t \\ \text{where } c_{i,t} &= \begin{cases} c_i & \text{for } t < q \\ c_i + \tau_i & \text{for } t \geq q \end{cases} \end{aligned} \quad (24)$$

which is just a step-shift in the unit-specific intercept (i.e. fixed effect), equal to c_i prior to treatment, and $c_i + \tau_i$ post-treatment. Estimates of τ_i then correspond to the unit-specific average treatment effect over time. If treatment timing and assignment are unknown, we can generalise the impulse-dummy approach to nest known treatment as a special case in a general model now allowing for step-shifts at any point in time as:

$$y_{i,t} = c_i + g_t + \sum_{j=1}^N \sum_{s=2}^T \tau_{j,s} 1_{\{i=j, t \geq s\}} + u_{i,t} \quad (25)$$

where a subset of the step-functions $1_{\{i=j, t \geq s\}}$ correspond to the actual treatment effects model in (23). This allows for any unit to be potentially treated at any point in time – with s starting at 2 rather than 1, so as not to coincide with the fixed effect in c_i . We then aim to remove treatment indicators such that we only retain the subset of truly treated units and time periods. Under sparsity of treatment effects i.e., there remain units and time periods without treatment, we write the final selected model as:

$$y_{i,t} = \hat{c}_i + \hat{g}_t + \sum_{j \in \hat{H}} \sum_{s \in \hat{R}_j} \hat{\tau}_{j,s} 1_{\{i=j, t \geq s\}} \quad (26)$$

where we estimate treatment assignment by detecting those units i that have at least one break indicator retained, and break times are estimated by the starting date of each retained break function. Figure 1 (column B2, right) shows the full set of step-functions, a subset of which identify the true treatment effect.

Note that this setup does not impose that treatment effects have to be strictly constant over time post-treatment, as a linear combination of step-functions can capture time-varying treatment effects.

2.3.3 Unknown Treatment Assignment and Timing For Multiple Treatments

If there is a single underlying treatment and break detection identifies a single intervention then the interpretation and attribution of detected effects is straightforward. However, in practice there may be multiple treatments detected as breaks at different times for multiple different units. What do we identify if we detect multiple such treatment occurrences at different times for different units? Irrespective of the selection algorithm employed (see section 3.1), consider the following final retained model with a range of detected treatment impulse dummies:

$$y_{i,t} = \hat{c}_i + \hat{g}_t + \sum_{j \in \hat{H}} \sum_{s \in \hat{R}_j} \hat{\tau}_{j,s} 1_{\{i=j, t=s\}} \quad (27)$$

or step-functions:

$$y_{i,t} = \hat{c}_i + \hat{g}_t + \sum_{j \in \hat{H}} \sum_{s \in \hat{R}_j} \hat{\tau}_{j,s} 1_{\{i=j, t \geq s\}} \quad (28)$$

What is identified if the detected treatment time varies across units in the panel? For example, what if we find both $j = 1$ and $j = 2$ to be in the treated group, but their treatment timing differs, i.e. $R_{j=1} \neq R_{j=2}$ for both $j = 1$ and $j = 2$? We show that the final retained models with heterogeneous treatment dummy variables (27) and (28) are equivalent to staggered treatment with heterogeneous effects where heterogeneity and staggered adoption are captured through interactions. In other words, the impulse indicator estimator identifies unit and time-specific staggered treatment effects conditional on the treatment effect being non-zero. If treatment effects are constant over time, then a saturating set of step-functions nests the known-treatment assignment and timing model as a special case even when treatment is staggered. To illustrate this equivalence, we follow the discussion in a known-treatment setting by Wooldridge (2021) on how interaction terms identify treatment effects in a staggered treatment setting. Subsequently we show that this is nested by the IIS and SIS break detection estimators in an unknown treatment setting, establishing that detected breaks identify unit- and time-specific treatment effects.

For exposition, consider a staggered treatment DGP where we denote the time of the first intervention by q . We define treatment cohort dummies similar to Wooldridge as d_q, \dots, d_Q where Q denotes the final time of intervention, which would be equal to T when treatment lasts until the end of the sample. We refer to the time of each intervention as $r \in \{q, q+1, \dots, Q\}$. The potential outcome at time t for unit treated at time r is given by $y_t(r)$, with the outcome for the never treated unit referred to as $y_t(\infty)$ i.e. treated at no point in time. The quantities of interest are the treatment effects of each unit first receiving treatment at time r given by the difference in outcomes $y_t(r) - y_t(\infty)$, $r = q, \dots, Q$. In a staggered setting we hope to identify the average treatment effects on the treated ATT for each intervention (given by different cohorts which we will relax to different individuals):

$$\tau_{r,t} = E[y_t(r) - y_t(\infty) | d_r = 1], r = q, \dots, Q; t = r, \dots, T. \quad (29)$$

Under no anticipation and common trends in a standard known-treatment setting, Wooldridge (2021) demonstrates that heterogeneous treatment effects can be consistently estimated in a staggered treatment setting using interactions in a TWFE estimator (we replicate the derivation in Supplementary Section 6.1). Specifically, the expected outcome in a staggered treatment setting can be written as

$$\begin{aligned} E[y_t|\mathbf{d}] &= \eta + \lambda_q d_q + \dots + \lambda_Q d_Q + \theta_t \text{ (pre-treatment } t < q) \\ &= \eta + \lambda_q d_q + \dots + \lambda_Q d_Q + \theta_t + \tau_{q,t} d_q + \dots + \\ &\quad + \tau_{Q,t} d_Q \text{ (post-treatment } t \geq q) \end{aligned} \quad (30)$$

where η is the average level of y for the untreated group and λ_q refers to the average level of y for the treated cohorts pre-treatment. This can be consistently estimated using a TWFE estimator with time-cohort interactions as:

$$y_{i,t} = c_i + g_t + \sum_{r=q}^Q \sum_{s=r}^T \tau_{r,s} (d_{i,r} \cdot 1_{\{t=s\}}) + u_{i,t} \quad (31)$$

In the above equation each cohort has a set of time-varying treatment effect estimates. Now consider each treated unit in the panel being allowed its own treatment effects (i.e. each unit in each cohort receives its own treatment effect or each cohort is of size one). As before, consider $H = \{m_1, m_2, \dots, m_M\}$ as the set of i that are treated at some time, where treatment timing is not exclusive. In other words, m_1 and m_2 may be treated at the same time (but may also be treated at different times). Then relaxing the above assumption that each treatment cohort has the same treatment effect, the above model (31) can be written as:¹

$$y_{i,t} = c_i + g_t + \sum_{j \in H} \sum_{s=r}^T \tau_{j,s} 1_{\{i=j, t=s\}} + u_{i,t} \quad (33)$$

This is identical to the interaction of the treatment dummy $d_{i,r}$ and time dummies $1_{\{t=s\}}$ above, except we disaggregate treatment cohorts into individual units. Using simplifying notation, we can write (33) as:

$$y_{i,t} = c_i + g_t + \sum_{j \in H} \sum_{s \in R_j} \tau_{j,s} 1_{\{i=j, t=s\}} + u_{i,t} \quad (34)$$

This matches the impulse-dummy saturated final specification where treatment assignment and timing is estimated in (27). Similarly, if treatment effects are piece-wise constant over time we can write (33) as:

$$y_{i,t} = c_i + g_t + \sum_{j \in H} \sum_{s \in R_j} \tau_{j,s} 1_{\{i=j, t \geq s\}} + u_{i,t} \quad (35)$$

Estimating this heterogeneous staggered-treatment model matches the post-selection step-function model in (28). Thus, detecting multiple treatments through impulses or step-indicators is equivalent to the estimation of treatment effects in a staggered-intervention setting when heterogeneous treatment effects are identified using interactions. We identify average treatment effects (over time) for each treated unit relative to the never treated cases, conditional on the treatment effect being large enough to be detected. If a single unit experiences more than one treatment – then this can be interpreted as time-varying treatment (where the sum of effects is the treatment effect relative to the never treated), or a separate treatment

¹To recover (31) we could restrict equation (33) as:

$$\tau_{m_l, s} = \tau_{m_k, s} = \tau_{r, s}, \text{ for } k \neq l \text{ and } (m_l, m_k) \in \text{the same treatment cohort } r \quad (32)$$

event relative to treatment received earlier.

2.4 Challenges

Naturally there are challenges to our proposed approach to detect treatment, and properties of the final identified model will depend on the machine learning/model selection algorithms employed. In section 3, we briefly discuss the general properties of using ‘gets’ (through impulse- and step-indicator saturation, IIS and SIS respectively) and the (adaptive) LASSO (Tibshirani 1996, Zou 2006) and how they relate to the power of identifying treatment correctly, controlling the false-positive rate of retained break variables, and conducting valid inference.

First, we may miss that treatment occurred (a relevant break variable is not retained). However, our approach allows a researcher to embed a known or suspected treatment just like in a difference-in-difference treatment evaluation setting (see section 3.2). Additionally, varying the acceptable false-positive rate can also result in identifying more potential treatments.

Second, we may detect spurious treatment as false-positives by retaining irrelevant break variables. Though, the ‘gets’ approach described in section 3.1.1 allows an explicit control of the false-positive rate.

Third, we may face challenges with post-selection inference (effects may be biased as large breaks are more likely to be retained than small ones). Some of these concerns can be mitigated through bias-correction and adjustment for post-selection inference.²

Fourth, if treatment affected all units under analysis, then the treatment effect will be subsumed into the year fixed effects g_t and not detectable as such – but again such a treatment would also not be identified with comparable treatment evaluation methods.

3 Operationalising the Detection of Treatment Assignment and Timing

3.1 Detection Methods and Their Approximate Properties

The idea of detecting structural breaks to identify treatment can be operationalised by applying break-detection in a panel setting, starting with the general saturated models (20) or (25) for fully time-varying and piece-wise constant treatment effects respectively. We emphasise that the idea of detecting treatment by detecting breaks is separate from the method of implementation – there are numerous possible machine learning detection/selection methods available and their properties will determine the effectiveness of detecting previously unknown treatment. Here we briefly consider two model selection approaches: general-to-specific selection using impulse-indicators (interpreted as an outlier-robust estimator) or step-indicators (for piece-wise constant treatment), and the (adaptive) LASSO, though we are not limited to these in practice. For example, the group-specific break detection approach in Okui and Wang (2021) could be a promising future avenue of detecting treated groups rather than individuals. Note that in the outlier-robust/general-to-specific setting, the problem of variable selection is generally studied under the null of no treatment – i.e. we focus on the false positive rate of detection which is also the main calibration parameter. In turn, in the shrinkage-based model selection literature (e.g. the LASSO) the focus has been on consistent selection, with less attention being paid to the false positive rate. We also consider the performance of each in a simulation study reported in Supplementary Section 6.2.

²See e.g., the coefficient bias correction function in the ‘gets’ package.

3.1.1 Treatment Detection using ‘gets’ and Impulse- or Step- Indicator Saturation

The impulse-indicator saturated model (20) is equivalent to impulse indicator saturation (IIS, see Hendry, Johansen and Santos, 2008) in a panel and can be interpreted as a Huber-skip outlier-robust estimator (see e.g. Jiao et al. 2021, Johansen & Nielsen 2009, Johansen & Nielsen 2016a). Coefficients on dummies that are used to determine outliers correspond thus to individual- and time-specific treatment effects. IIS has well-established properties under the null of no outliers (here interpreted as no treatment/zero treatment effects) where the false positive rate can be easily controlled by specifying the relevant tuning parameter. IIS corresponds to a robust Huber-skip estimator and targets the false-positive rate of detection by removing impulse indicators up to the chosen level of significance γ_c . For example, under a normal reference distribution, choosing $c = 1.96$ would correspond to a target level of significance of $\gamma_{1.96} = 0.05$. We denote the observed false positive rate $\hat{\gamma}_c$ as the proportion of spuriously retained indicators at the chosen cut-off c out of all possible break variables considered:

$$\hat{\gamma}_c = \frac{L_c}{L} \quad (36)$$

where $L_c = \sum_{j=1}^{\hat{M}} \hat{S}_j$ is the number of retained indicator variables at cut-off c and L denotes the total number of potential treatment variables selected over, usually equal to the total sample size $L = n = NT$ in a balanced panel allowing for treatment at any point in time for every unit. The asymptotic properties of IIS under the null of no breaks as the total sample size $n \rightarrow \infty$ are explored in Hendry et al. (2008) and Johansen and Nielsen (2009; 2016b), who show that when there are no breaks (and accounting for multiple testing), the false positive rate of retained breaks (i.e. the number of retained indicator dummies L_c relative to all possible indicators L) converges to the chosen nominal level of significance of selection γ_c :

$$\hat{\gamma}_c = \frac{L_c}{L} \rightarrow \gamma_c, \text{ as } n \rightarrow \infty \quad (37)$$

where n denotes the sample size (in a balanced panel $n = NT$). In other words, if there is no treatment effect (i.e. if there are no true underlying breaks), then the proportion of spuriously detected indicators converges to the chosen level of significance, e.g. 1% for $\gamma_c = 0.01$. In the present context of detecting treatment at any point in time for any unit in a balanced panel, selecting at $\gamma_c = 0.01$ yields an expected number of $0.01 \times NT$ spuriously retained indicators. Thus, IIS in a Huber-skip robust interpretation makes it straightforward to control the false discovery rate of breaks (and thereby treated units) by varying γ_c .

We can estimate the set of treated units \hat{H} as those that have at least one treatment indicator (i.e. impulse dummy) retained:

$$i \in \hat{H} \text{ if } \hat{Q}_i > 0 \quad (38)$$

For practical purposes, this definition could also be made more stringent to differentiate between ‘outliers’ and actual treatment that can be attributed to potential causes. In other words, we could restrict identification of treatment to some minimum of consecutive impulse dummies. The number of estimated treatment breaks for unit j is given by \hat{S}_j (with $E[\hat{S}_j] = \gamma_c \times T$). The probability of a particular unit in the panel being falsely-classified as ever-treated depends on the number of time series observations for each unit. Consider a panel of N individuals over T time periods. IIS adds NT dummies, with an expected number of retained dummies of $\gamma_c \times NT$. The probability of at least one break per individual will depend on the number of time periods in the sample and the cut-off γ_c . The probability of a particular unit i being spuriously classified as ever-treated is given the probability of at least one observation

of unit i being falsely-classified as treated:

$$P(i \in \hat{H} | d_i = 0) = 1 - (1 - \gamma_c)^T \quad (39)$$

which increases with T because for larger samples (and fixed γ_c) the probability of retaining an indicator spuriously increases as the number of indicators increases with T . Under the null of no treatment (when in fact no unit is treated), the expected number of falsely detected treated units is then given by:

$$E[\hat{M}] = P(i \in \hat{H} | d_i = 0) \times N = (1 - (1 - \gamma_c)^T)N \quad (40)$$

If we are worried about the false-positive *rate* of treated units specifically (rather than the false-positive rate γ_c of treatment at any point in time for any unit), it is possible to scale γ_c to ensure a stable false-positive rate of classifying the treatment group. Let p_H denote the target false positive rate of a unit being incorrectly-classified as treated. Then for any target false positive rate p_H , we can choose γ_c as:

$$\gamma_c = 1 - (1 - p_H)^{\frac{1}{T}} \quad (41)$$

This controls the false positive rate of being assigned to the ever treated group to p_H in expectation. For example if $T = 50$, and we aim for a false-positive rate of a single unit incorrectly being classified as treated of 5% i.e. $p_H = 0.05$, then we should set the nominal level of selection to $\gamma_c = 0.001 = 1 - (1 - 0.05)^{\frac{1}{50}}$. Similarly, we could set the target level of significance to maintain a stable *number* of false-positive treated units. If on average we are willing to accept a total of $N_0 = E[\hat{M}]$ false-positive treated units in expectation (where \hat{M} is the estimated number of treated units), the above results imply that:

$$N_0 = (1 - (1 - \gamma_c)^T)N \quad (42)$$

which can be targeted by setting γ_c to:

$$\gamma_c = 1 - \left(1 - \frac{N_0}{N}\right)^{\frac{1}{T}} \quad (43)$$

and which will yield N_0 expected treated units in expectation when there are in fact no treated units in the true underlying DGP. For example, if we have a panel of $N = 20$, $T = 50$, and we are willing to accept one unit to be falsely-classified as treated on average ($N_0 = 1$), then we can set $\gamma_c = 0.001 \approx 1 - (1 - \frac{1}{20})^{\frac{1}{50}}$. Thus, if we are concerned about the false positive rate of treatment classification, then treatment detection in a panel perhaps warrants tighter target significance levels γ_c than conventionally used in the selection/break detection literature.

If we consider the piece-wise constant treatment effects model matched by step indicators (25), then selecting over treatment variables using the tree search ‘gets’ is equivalent to applying step-indicator saturation (SIS, Castle et al., 2015) in a fixed effects panel where blocks of steps are included for each individual. SIS uses a near exhaustive tree-search based on a specified level of significance γ_c up to which individual step-functions are removed. The properties of SIS are reasonably well-understood (see Castle et al., 2015; Nielsen & Qian, 2018), and transfer to the panel setting when interpreted as a least-squares dummy variable estimator. The asymptotic properties of SIS under the null of no breaks as $n \rightarrow \infty$ are explored in Nielsen & Qian (2018), who show that when there are no breaks (and accounting for multiple testing), the false positive rate of retained breaks (i.e. the number of detected break indicators L_c relative to all possible break variables L) converges to the chosen nominal level of significance of selection γ_c :

$$\hat{\gamma}_c = \frac{L_c}{L} \rightarrow \gamma_c, \text{ as } n \rightarrow \infty \quad (44)$$

Specifically, if we allow for possible treatment of each unit at every point in time, then – in absence of treatment – the expected value of detected breaks is $\gamma_c \times N(T - 1)$ in a balanced panel.³ Which again translates into a probability of being classified as treated as above, with an exponent of $(T-1)$:

$$P(i \in \hat{H} | d_i = 0) = 1 - (1 - \gamma_c)^{(T-1)} \quad (45)$$

Then for any target false positive rate of being classified as treated p_H , we could choose γ_c as:

$$\gamma_c = 1 - (1 - p_H)^{(1/(T-1))} \quad (46)$$

This matches the properties of IIS except we are searching over $N(T - 1)$ rather than NT possible indicators in an exhaustive search – of course the first indicator would coincide with the fixed effects.

Under the alternative (i.e. in the presence of actual treatment), for simple cases (where the number of variables does not exceed the number of observations), ‘gets’ has been shown to be a consistent model selection procedure retaining all relevant variables with probability equal to one as $n \rightarrow \infty$ (see e.g. Campos et al., 2003). In our setting where the number of variables can exceed the number of observations, we investigate the performance under the alternative (in the presence of structural breaks/treatment) using a range of simulations (see section 6.2). As the selection rule is pre-specified, coefficients on impulse and step-indicators could be bias-corrected to address concerns about post selection inference (see Pretis et al. 2018 for an implementation of bias correction in SIS). The ‘gets’ selection approach using IIS or SIS can be readily implemented to detect treatment breaks in panels using the R-package ‘gets’ with the ‘getspanel’ update.

3.1.2 Treatment Detection using the (adaptive) LASSO

As a second possible selection approach we briefly consider one variant of the LASSO to detect unknown treatment in the TWFE panel model. Unlike ‘gets’, the LASSO does not target the false-positive rate, instead penalising the L1-norm of possible coefficients. The simple LASSO itself is not a consistent model selection method, however, the adaptive LASSO which modifies the weights on coefficients is consistent and exhibits oracle properties (Huang et al., 2008). In particular, Huang et al. (2008) show the oracle properties of adaptive LASSO in high-dimensional problems where the number of regressors increases with the sample size.

To implement the LASSO to identify treatment we require different weights v on the coefficients that will be penalised. We specify the weights on control variables such that these are never removed from the model (e.g. the individual and time fixed effects), while the potential treatment variables will receive penalty weights that allow them to be dropped from the model. Since the base models with impulses (20) or steps (25) may contain more variables than observations we cannot use conventional OLS as an initial estimator to determine the penalty weights for the adaptive LASSO. Instead, we follow the fixed effects panel approach of Kock (2013) and Kock (2016) to use the conventional LASSO as an initial estimator, and subsequently take the inverse of the initial LASSO coefficients as the penalty weights in the second step of the adaptive LASSO. The least squares objective function for the adaptive LASSO implementation in our setting is given by:

$$\arg \min_{c, g, \tau} \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T (y_{i,t} - c_i - g_t - \sum_{j=1}^N \sum_{s=2}^T \tau_{j,s} 1_{\{i=j, t \geq s\}})^2 + \lambda \sum_{j=1}^N \sum_{s=2}^T v_{j,s} |\tau_{j,s}| \quad (47)$$

³Albeit simulation results show a higher false-positive rate for SIS in small samples, warranting perhaps a more conservative choice of γ_c .

where the second term denotes the penalty term on the break coefficients τ with tuning parameter λ and penalty weights v corresponding to the inverse of the coefficients in an initial LASSO estimator. The tuning parameter λ can be chosen using cross-validation or information criteria. Closely related to our work, albeit not focused on fixed effects, Qian & Su (2016a) use the adaptive LASSO to estimate common breaks across individuals. Okui & Wang (2021) show that the adaptive LASSO – albeit using a fused structure – can further be used to estimate breaks that are heterogeneous across groups. However, they do not focus on breaks in fixed effects or treatment evaluation. Larger breaks, i.e. larger treatment effects, are more likely to be retained in the final model – akin to the gets approach in section 3.1.1 – potentially complicating inference on the final retained model. Post-selection inference has received a fair amount of attention in the LASSO literature. Simple data-splitting approaches (such as Cox 1975) are not feasible in our setting as the treatment variables only apply to a subset of observations. Lee et al. (2016) propose a post-selection inference correction for the LASSO. Alternatively, Zhao et al. (2021) show that the naive approach of re-estimating an OLS model post-selection can perform surprisingly well in many settings.

3.2 Embedding Known Interventions

There are two ways in which break detection to identify treatment can be implemented: either as an agnostic way to detect fully unknown treatment assignment and timing, or as a robustness check embedding known treatment and searching for additional previously-unknown interventions. Above we outlined the case where we detect treatment as a purely agnostic data-driven approach to identify interventions without any prior knowledge of their occurrence. While the approach is agnostic and any unit may be treated at any point in time, a potential downside is a loss in power if treatment assignment and timing is known and there are multiple treated units with a homogeneous treatment effects, since each treated unit would have to be identified individually.

If treatment assignment and timing is known for a particular intervention then break detection can be adapted as a robustness check for additional unknown treatment in conventional TWFE difference-in-differences models. In this case we force the known treatment dummy (or dummies for interactions) to be included in the model, and select over additional treatment indicators. This corresponds to the Hendry & Johansen (2015) theory-embedding approach where fixed regressors are embedded in a wider information set that we select over.

Then selection takes place over the break variables to detect additional treatment (omitting the break variables perfectly coinciding with known treatment dummies), known treatment dummies remains in the model without being selected over. This allows additional unknown treatment to be detected, while the coefficient on the forced (not-selected-over) break variable yields an estimate of the conventional treatment effect in a TWFE panel. It is worth highlighting that we do not necessarily need to allow for a break at every point in time (or individual). If there is a strong reason that the break should be localised in particular time periods (or among particular individuals), then only those could be included in the candidate set of break variables selected-over.

3.3 Ex-Post Attribution of Detected Events

Having identified treatment as structural breaks the remaining challenge is then to attribute the detected effects to possible causes. Much of the difference-in-differences TWFE literature is dedicated to justifying that specific known interventions were exogenous (or as-if randomly assigned). Similar to such subject-specific justifications, the ex-post attribution of events to possible causes will require subject-specific knowledge. Ultimately, in absence of a randomised experiment, making the case for a known

intervention to be exogenous is comparable to searching for a potential cause of a detected effect. Particularly, once a potential cause of a detected effect has been identified, we could have simply estimated a conventional difference-in-differences model using the ‘known’ intervention. Thus, in the proposed reverse causal approach we expect that much of the discussion will be dedicated to arguing that a particular detected break coincides with a particular event that was discovered after the effect was observed. Naturally there may be many such events that took place and it can be difficult to attribute the observed effect to that single event. However, the same challenge applies in ‘known’ treatment evaluation – treatment has to occur in isolation without other events taking place at the same time affecting the treated units. So while the search for causes is different than arguing that a cause was unique, subject-specific knowledge will be necessary in both settings.

4 Illustrative Application: Detecting the Impacts of Terrorism on GDP per Capita

We demonstrate our break detection approach to identify unknown treatment assignment and timing using a well-known dataset on Spanish regional GDP per capita (see Abadie & Gardeazabal, 2003). We purposely choose a well-known example to illustrate our methods. We also provide a policy-focused application with novel data in our closely-related paper in Koch et al. (2022). The dataset for our illustrative application here spans all of mainland Spain’s 15 regions (where we exclude the Canary and Balearic Islands) over 31 years from 1965 to 1995 for a total of 465 region-year observations. In their seminal paper, Abadie & Gardeazabal (2003) used a forward causal approach to study the effect of ETA terrorism on regional economic output. The authors find a substantial reduction in regional GDP in response to local terrorism introducing synthetic control methods. Here we ask the reverse causal question: what affected regional GDP per capita in the Basque Country (or wider Spain)? We show that the “treatment” taking the form of ETA terrorism (alongside a number of other previously unidentified treatments) can be detected without prior knowledge of its occurrence using our proposed break detection approach.

To illustrate our methods, we first consider a simple TWFE panel setting with two regions (the Basque Country and Madrid) where we search for breaks to detect treatment in GDP per capita.⁴ We then expand this into a multi-region panel of mainland Spain to assess breaks in a wider context. Our results show that we can detect the effect of ETA terrorism without prior knowledge of its occurrence and obtain treatment effect estimates that are near-identical to a known-treatment model. Our break detection approach also provides evidence that the treatment effects of GDP impacts of ETA terrorism were transitory and are no longer detectable post-1990. In addition, in the panel with more than two regions we also detect breaks which we attribute to an industrial crisis and increased autonomy following the Franco era in other regions.

4.1 Detecting Treatment in a Panel with Two Regions

We first consider a simple panel with two regions: the Basque country and Madrid ($N = 2, T = 31, NT = 62$). For comparison, we initially estimate the forward causal ‘infeasible’ model of log GDP per capita (controlling for log investments Inv similar to Abadie & Gardeazabal 2003) using a TWFE estimator with a known intervention of Basque terrorism to provide a baseline relative to our break

⁴For completeness we also show that a simple time series model of Basque GDP per capita is unable to identify ETA terrorism impacts due to the lack of control groups – see Supplementary Material 6.3).

detection approach. We then demonstrate that we can directly detect the terrorism ‘treatment’ without prior knowledge using our reverse causal approach.

As a baseline, consider a TWFE estimator with a ‘known’ intervention of ETA terrorism. We estimate baseline models first allowing for time-varying treatment effects using interactions in (48), then assuming time-invariant treatment effects in (49) specified as a dummy variable for the Basque region in the ‘post-treatment’ period, defined here as 1979 onwards, as Abadie & Gardeazabal (2003) found that the impact of terrorism was notable in GDP per capita from the end of the 1970s.

‘Known’ Treatment (fully time-varying treatment effects):

$$\log(GDPpc_{i,t}) = \alpha_i + \phi_t + \sum_{s=1979}^{1995} d_i \tau_s 1_{\{t=s\}} + \beta_1 \log(Inv)_{i,t} + u_{i,t} \quad (48)$$

where $d_i = 1_{\{i=\text{Basque}\}}$

‘Known’ Treatment (time-constant treatment effects):

$$\log(GDPpc_{i,t}) = \alpha_i + \phi_t + d_{i,t} \tau + \beta_1 \log(Inv)_{i,t} + u_{i,t} \quad (49)$$

where $d_{i,t} = 1_{\{i=\text{Basque}, t \geq 1979\}}$

Estimation results for the ‘known’ baseline models are shown in Tables 1 and 2, under the columns “Known TWFE”. The results of the known baseline show an approximate 5% reduction in GDP per capita in the Basque country relative to Madrid in response to ETA terrorism in this simple two-region model. This result is similar across the time-varying model (see equation 20) (where the estimated ATT is given by the average of the impulse coefficients) as well as the piece-wise constant treatment effects model (see equation 25). Specifically, the ATT across impulses in the known baseline in (48) is -0.0496 (se=0.0197), and the time-constant estimate given by the coefficient in (49) on the known step-function is -0.0495 (se=0.006).

Now suppose the “treatment” of ETA terrorism in the Basque country was unknown, and we approached the data with our reverse causal question of ‘what affected GDP per capita? We demonstrate how treatment interventions can be detected without prior knowledge of their occurrence.

4.1.1 Unknown Treatment with Fully Time-Varying Effects

We now estimate a model allowing for the potential treatment of any unit at any point in time first using impulse dummies capturing time-varying treatment and select over them using the ‘gets’ selection algorithm (we consider the LASSO for the piece-wise constant setting below). The model is saturated with a full set of impulse dummies in (50) which are selected over at a target level of significance γ_c . We consider three different target significance levels, $\gamma_c = 0.05$ as well as 0.025 and 0.01 to illustrate the impact of the calibration choice on treatment detection.

‘Unknown’ Treatment:

$$\log(GDPpc_{i,t}) = c_i + g_t + \sum_{j=1}^2 \sum_{s=1966}^{1995} \tau_{j,s} 1_{\{i=j, t=s\}} + \beta_1 \log(Inv)_{i,t} + u_{i,t} \quad (50)$$

The resulting detected impulses, which we interpret as unit-specific time-varying treatment effects, are shown in Figure 2 (for $\gamma_c = 0.05$) and Table 1 (for all three values of γ_c). We detect the treatment of

Basque terrorism without prior knowledge of its occurrence as individual impulses in the Basque region from 1980 to 1990. Each coefficient provides an estimate of the unit- and time-specific treatment effect. We can easily compute our estimates of the ATT by taking the mean of the impulses over time. Standard errors for the ATT are also straight-forward to compute as impulses are orthogonal. Computing the ATT over the time period from 1980 to 1990 from the model with $\gamma_c = 0.05$ yields an estimate of the ATT of -0.059 (se=0.016) which is nearly identical (and not significantly different) to the known-treatment baseline estimate of -0.0496 (se=0.0197). The fact that the ATT using detected impulses is marginally larger than the known baseline ATT can be explained by the fact that the impulses are only retained up to 1990 while the ‘known’ baseline time-varying treatment considers treatment effects up until the end of the sample in 1995. Indeed, we only detect treatment breaks up until 1990, suggesting that the impacts of ETA terrorism on GDP were transitory and no longer detectable post-1990. This is consistent with the known-treatment baseline which finds predominantly insignificant time-varying treatment effects after 1990.

Varying γ_c , we successfully detect the intervention at relatively loose levels of significance $\gamma_c = 0.05$ or $\gamma_c = 0.025$. The loss of power for more conservative levels of the target false positive rate becomes apparent when we set $\gamma_c = 0.01$, where we do not detect any treatment as impulse dummies coinciding with ETA terrorism. However, this reduction in power can be tackled by specifying piece-wise constant treatment effects using step functions as we demonstrate in the following section 4.1.2. Note that in this N=2 panel, the treatment effects are relative to the single control region and one could achieve the same detected treatment if Basque country was selected as the ‘control’, in which case the treatment effects would be detected for Madrid and opposite-signed. We would then interpret them as the effect of the absence of terrorism.

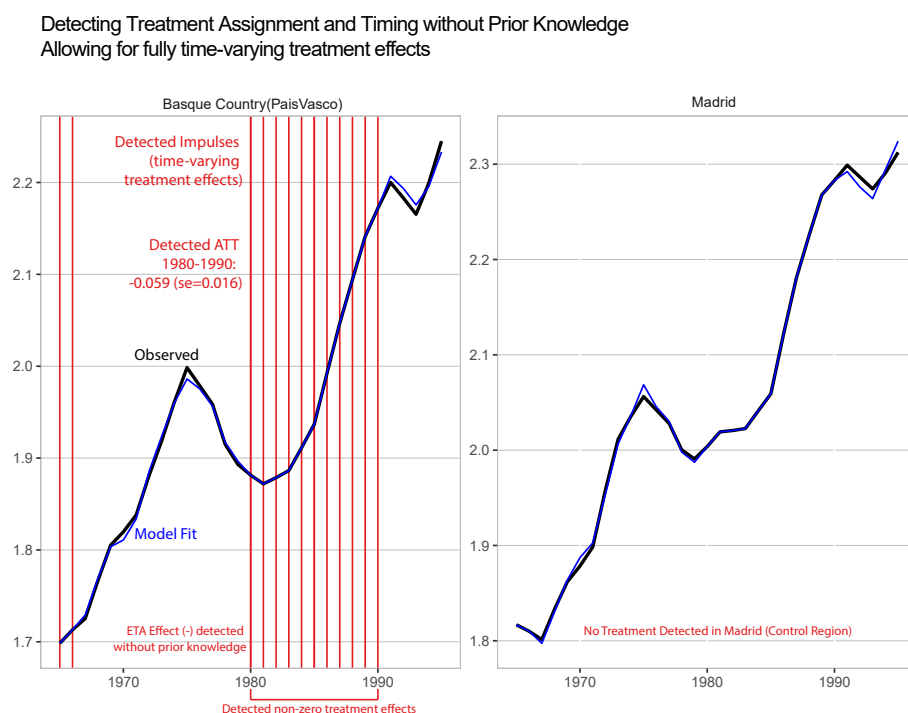


Figure 2: TWFE Panel: GDP per Capita, Basque Country & Madrid – Time-Varying Treatment detected using IIS in ‘gets’ for a target significance level of $\gamma_c = 0.05$. Red vertical lines denote detected impulses (identifying time-varying treatment effects).

Table 1: Detecting Fully-Time-Varying Treatment: Two-Region Panel Model

| Dependent Variable: | | log(GDPpc) (Basque & Madrid) | | | Known Treatment 'Known' TWFE |
|--------------------------------|-----------|------------------------------|--|----------------------------|---------------------------------|
| Model: | | gets ($\gamma_c = 0.05$) | Unknown Treatment gets ($\gamma_c = 0.025$) | gets ($\gamma_c = 0.01$) | |
| <i>Variables</i> | | | | | |
| log(Invest) | | 0.1048*** (0.0329) | 0.0777* (0.0376) | -0.0234 (0.0534) | -0.0638 (0.0563) |
| $\tau: i=\text{Basq.}, t=1965$ | | -0.0657*** (0.0163) | -0.0559*** (0.0193) | -0.0156 (0.0320) | |
| $\tau: i=\text{Basq.}, t=1966$ | | -0.0352** (0.0151) | | | |
| $\tau: i=\text{Basq.}, t=1979$ | | | | | -0.0348 (0.0213) |
| $\tau: i=\text{Basq.}, t=1980$ | | -0.0514*** (0.0143) | -0.0463** (0.0173) | | -0.0474** (0.0183) |
| $\tau: i=\text{Basq.}, t=1981$ | | -0.0859*** (0.0151) | -0.0783*** (0.0181) | | -0.0664*** (0.0189) |
| $\tau: i=\text{Basq.}, t=1982$ | | -0.0661*** (0.0142) | -0.0622*** (0.0172) | | -0.0699*** (0.0185) |
| $\tau: i=\text{Basq.}, t=1983$ | | -0.0679*** (0.0145) | -0.0621*** (0.0175) | | -0.0598*** (0.0183) |
| $\tau: i=\text{Basq.}, t=1984$ | | -0.0743*** (0.0158) | -0.0652*** (0.0189) | | -0.0455** (0.0199) |
| $\tau: i=\text{Basq.}, t=1985$ | | -0.0630*** (0.0154) | -0.0549*** (0.0184) | | -0.0401* (0.0193) |
| $\tau: i=\text{Basq.}, t=1986$ | | -0.0615*** (0.0145) | -0.0557*** (0.0175) | | -0.0530** (0.0183) |
| $\tau: i=\text{Basq.}, t=1987$ | | -0.0514*** (0.0142) | -0.0492** (0.0173) | | -0.0655*** (0.0194) |
| $\tau: i=\text{Basq.}, t=1988$ | | -0.0475*** (0.0142) | -0.0456** (0.0173) | | -0.0632*** (0.0196) |
| $\tau: i=\text{Basq.}, t=1989$ | | -0.0483*** (0.0142) | -0.0451** (0.0172) | | -0.0561** (0.0188) |
| $\tau: i=\text{Basq.}, t=1990$ | | -0.0342** (0.0142) | | | -0.0395* (0.0186) |
| $\tau: i=\text{Basq.}, t=1991$ | | | | | -0.0328 (0.0200) |
| $\tau: i=\text{Basq.}, t=1992$ | | | | | -0.0354* (0.0194) |
| $\tau: i=\text{Basq.}, t=1993$ | | | | | -0.0443* (0.0207) |
| $\tau: i=\text{Basq.}, t=1994$ | | | | | -0.0332 (0.0237) |
| $\tau: i=\text{Basq.}, t=1995$ | | | | | -0.0046 (0.0214) |
| <i>Fixed-effects</i> | | | | | |
| Region | Yes | Yes | Yes | Yes | Yes |
| Year | Yes | Yes | Yes | Yes | Yes |
| <i>Fit statistics</i> | | | | | |
| Observations | N=2, T=31 | N=2, T=31 | N=2, T=31 | N=2, T=31 | N=2, T=31 |
| Within R ² | 0.87 | 0.79 | 0.023 | | 0.84 |

Standard-errors in parentheses

Signif. Codes: ***: 0.01, **: 0.05, *: 0.1

4.1.2 Unknown Treatment with Piece-Wise Constant Effects

Treatment effects may be piece-wise constant and thus detected with greater likelihood (due to the higher power of step-functions). To illustrate this, we estimate a TWFE panel (51) saturated with a full set of step-functions denoting potential treatment in either region (Basque or Madrid) at any point in time:

‘Unknown’ Treatment:

$$\log(GDPpc_{i,t}) = c_i + g_t + \sum_{j=1}^2 \sum_{s=1966}^{1995} \tau_{j,s} 1_{\{i=j, t \geq s\}} + \beta_1 \log(Inv)_{i,t} + \epsilon_{i,t} \quad (51)$$

We select over treatment functions using ‘gets’ at two target levels of $\gamma_c = 0.001$ and $\gamma_c = 0.01$ as well as using the adaptive LASSO where we penalise the possible treatment coefficients τ , with penalty weights chosen using the simple LASSO as an initial estimator, and the tuning parameter selected using cross-validation.

Table 2 and Figure 3 shows the results of break detection. The adaptive LASSO estimates are reported using the ‘naive’ approach of re-estimating the selected model using OLS (see e.g. Zhao et al., 2021). Detecting treatment using ‘gets’ at $\gamma_c = 0.001$ results in a single treatment indicator being retained for the Basque Country from 1979 onwards. The resulting selected model is *identical* (in absence of any bias-correction due to selection) to the TWFE estimator with known treatment intervention imposed, with the estimated coefficient on the retained break variable of -0.0496 (se=0.0197) matching the estimated treatment effect in the TWFE difference-in-differences model. In other words – without knowing that treatment occurred – we are able to detect the treatment intervention and estimate a model effectively *identical* to the known intervention panel. Similarly, the adaptive LASSO is able to identify treatment (detecting a negative intervention in Basque country in 1980), with the estimated ‘naive’ post-LASSO treatment effect near identical to the ‘known’ imposed intervention in 1979. The adaptive LASSO further detects additional earlier breaks which is unsurprising as it can be often less conservative than ‘gets’ with low levels of γ_c . Relaxing the target level of γ_c to a less conservative level of 0.01 results in additional breaks being detected which can be interpreted as time-varying treatment effects: the negative break in 1981 suggests that the initial impact of ETA terrorism became larger in the early 1980s, however, the opposite-signed break in 1990 provides evidence of the transitory nature of the impact. Consistent with our results from the fully-time-varying specification (and known baseline), treatment effects post-1990 are closer to zero (see section 4.1.1).

Overall, both ‘gets’ and the adaptive LASSO implementation of our proposed break detection approach detect the ‘treatment’ without prior knowledge of its occurrence. Break detection estimates to detect treatment suggest a roughly 5% reduction in GDP per capita in response to terrorism in the Basque region relative to Madrid as the control region, which is identical to the known intervention TWFE estimator. Further, it is worth noting that the break detection approach suggests a reduction in GDP per capita from around 1979/1980 onwards, which is consistent with Abadie and Gardeazabal’s finding that GDP per capita reductions occurred with a lag relative to the onset of terrorism in the mid 1970s.

Thus, not only are we able to detect treatment without prior knowledge on which regions were treated and when treatment occurred, but the estimated break dates also provide insights into the lagged onset of the economic impacts of terrorism.

Detecting Treatment Assignment and Timing without Prior Knowledge
Allowing for time-constant treatment effects

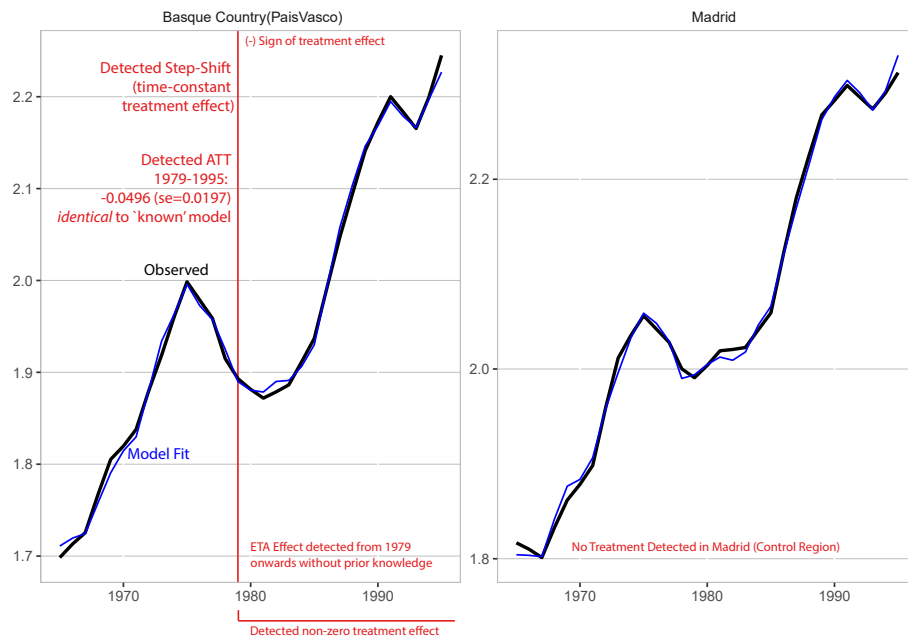


Figure 3: TWFE Panel: GDP per Capita, Basque Country & Madrid – Treatment detected using SIS in ‘gets’ at $\gamma_c = 0.001$. Red vertical lines denote detected step-shifts (identifying treatment effects).

Table 2: Detecting Piece-Wise Constant Treatment: Two-Region Panel Model

| Dependent Variable: | | log(GDPpc) (Basque & Madrid) | | | Known Treatment 'Known' TWFE |
|--|--|-------------------------------|-------------------------------|-------------------------------|---------------------------------|
| Model: | | Unknown Treatment | | Adapt. LASSO | |
| | | gets ($\gamma_c = 0.001$) | gets ($\gamma_c = 0.01$) | | |
| <i>Variables</i> | | | | | |
| log(Invest) | | -0.1065*** (0.0294) | -0.0540* (0.0314) | -0.0624* (0.0320) | -0.1065*** (0.0294) |
| τ : Break ($i=Basq, t \geq 1966$) | | | | 0.0324 (0.0190) | |
| ----- | | | | | |
| τ : Known ETA ($i=Basq, t \geq 1979$) | | | | | -0.0495*** (0.0063) |
| τ : Break ($i=Basq, t \geq 1979$) | | -0.0495*** (0.0063) | -0.0401*** (0.0115) | | |
| τ : Break ($i=Basq, t \geq 1980$) | | | | -0.0471*** (0.0065) | |
| τ : Break ($i=Basq, t \geq 1981$) | | | -0.0176 (0.0119) | | |
| τ : Break ($i=Basq, t \geq 1990$) | | | 0.0277*** (0.0092) | | |
| <i>Fixed-effects</i> | | | | | |
| Region | | Yes | Yes | Yes | Yes |
| Year | | Yes | Yes | Yes | Yes |
| <i>Fit statistics</i> | | | | | |
| Observations | | N=2, T=31 | N=2, T=31 | N=2, T=31 | N=2, T=31 |
| Within R ² | | 0.69 | 0.77 | 0.671 | 0.69 |

Standard-errors in parentheses

Signif. Codes: ***: 0.01, **: 0.05, *: 0.1

4.2 Detecting Treatment in a Panel with Multiple Regions

We repeat the above analysis for a panel covering all of mainland Spain using ‘gets’. We now include all $N = 15$ regions of mainland Spain over $T = 31$ years (for a total sample size of $NT = 465$). Just as before, we compare the detected treatment in this larger panel to the benchmark of a known intervention by imposing the ‘treatment’ as a dummy variable for the Basque region from 1979 onwards in a TWFE estimator. The ‘known treatment’ baseline yields an estimated treatment effect of -0.155 (se=0.018) relative to the control regions in wider Spain (see Table 3).⁵

Our break detection results using gets at $\gamma_c = 0.001$ show that even in this more general setting we are able to detect the treatment of ETA terrorism through the impacts on GDP in the Basque Country without prior knowledge of its occurrence (see Figure 4 and Table 3). The ETA treatment is detected in 1978 (close to the imposed intervention in the known TWFE estimator in 1979) with an estimated treatment effect of -0.156 (se=0.012) which is *near identical* to the ‘known treatment’ benchmark.

In addition to the ETA break in 1978, we also detect a small number of possible treatment effects through breaks in the fixed effects of other regions.⁶ It is worth noting though that the break associated with the ETA ‘treatment’ is the single largest break in magnitude compared to all detected breaks. Given the set of detected effects (captured through breaks) for some of the regions, the next step of our approach (see section 3.3) is to investigate the relevant literature for potential causes.

A brief review of the literature on Spanish economic history suggests that the positive breaks (i.e. positive treatment effects on GDP per capita) in Extremadura, Galicia, and Rioja, may correspond to the increased autonomy of the regions awarded in the post-Franco era. The negative break in Madrid in 1970 coincides with an industrial crisis that hit Madrid disproportionately relative to other regions (Rodríguez-Pose & Hardy 2021, and Tobío 1989).

The fact that ex-post attribution is not always straightforward is highlighted by the fact that we have yet to identify likely causes for the positive break in Castilla-La Mancha in 1972 (though it is worth noting that the film adaptation of the highly popular musical “Man of la Mancha” was released in that year), and the negative breaks in Asturias (in 1986) and Madrid (in 1990).

⁵This estimate in the known benchmark and the detected break setting is larger than the two-region panel because the control group is different. The two-region panel only included Madrid as a control region)

⁶To control for outlying observations we also combine our selection over step functions with selection over impulse dummies, where impulses could capture outliers or can also be interpreted as single-period time-varying treatment indicators. Only a single outlying observation is identified: Madrid, 1965.

Table 3: Detecting Piece-Wise Constant Treatment: 15-Region Panel Model

| Dependent Variable: | | log(GDPpc) | |
|--|--|---|---------------------------------|
| Model: | | Unknown Treatment gets ($\gamma_c = 0.0001$) | Known Treatment 'Known' TWFE |
| <i>Variables</i> | | | |
| log(Invest) | | 0.1171*** (0.0121) | 0.1377*** (0.0175) |
| τ : Break ($i=\text{Basq}, t \geq 1978$) | | -0.1560*** (0.0120) | |
| τ : Known ETA ($i=\text{Basq}, t \geq 1979$) | | | -0.1553*** (0.0182) |
| τ : Break: ($i=\text{Castilla-La Mancha}, t \geq 1972$) | | 0.1169*** (0.0143) | |
| τ : Break: ($i=\text{Extremadura}, t \geq 1987$) | | 0.1350*** (0.0127) | |
| τ : Break: ($i=\text{Galicia}, t \geq 1976$) | | 0.0980*** (0.0121) | |
| τ : Break: ($i=\text{Madrid}, t \geq 1970$) | | -0.1256*** (0.0176) | |
| τ : Break: ($i=\text{Madrid}, t \geq 1990$) | | -0.0903*** (0.0150) | |
| τ : Break: ($i=\text{Princip. De Asturias}, t \geq 1986$) | | -0.1220*** (0.0123) | |
| τ : Break: ($i=\text{La Rioja}, t \geq 1981$) | | 0.0796*** (0.0117) | |
| τ : Impulse: ($i=\text{Madrid}, t = 1965$) | | 0.0914** (0.0356) | |
| <i>Fixed-effects</i> | | | |
| Region | | Yes | Yes |
| Year | | Yes | Yes |
| Observations | | N=15, T=31, NT=465 | N=15, T=31, NT=465 |
| Within R ² | | 0.71 | 0.29 |

Standard-errors in parentheses

Signif. Codes: ***: 0.01, **: 0.05, *: 0.1

Detecting Treatment Assignment and Timing without Prior Knowledge
Allowing for time-varying and time-constant treatment effects

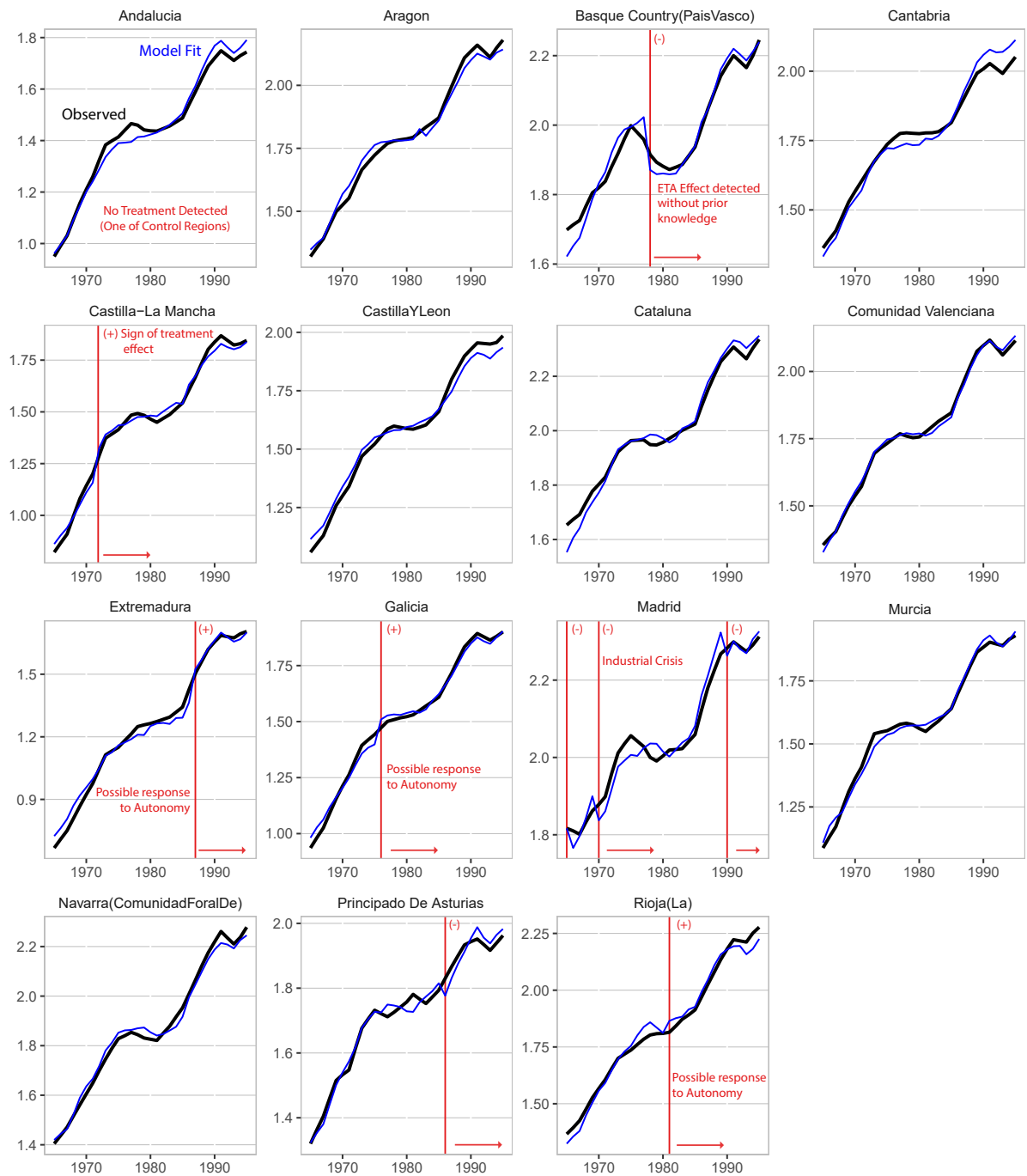


Figure 4: TWFE Panel: GDP per Capita in 15 Regions of Mainland Spain – Treatment detected using ‘gets’ and $\gamma_c = 0.001$. Red vertical lines denote detected step-shifts (identifying treatment effects).

5 Conclusion

We operationalise the modelling of reverse causal questions by searching for structural breaks in fixed effects panel models identifying previously unknown treatment effects which can subsequently be attributed to potential causes. We show that the two-way fixed effects estimator, which identifies heterogeneous treatment effects through interactions, can be nested as a special case of impulse- or step-dummy saturated models – a subset of which identifies underlying treatment effects.

We demonstrate the feasibility of detecting previously unknown treatment assignment and timing by using two machine learning methods suitable for selection over more candidate variables than observations (here using ‘gets’ and the adaptive LASSO, though many other approaches such as bayesian model selection would also be feasible).

Our application to the economic impacts of terrorism in Spain demonstrates that we can detect the effects of ‘treatment’ (taking the form of terrorist activity) on GDP per capita without prior knowledge of its occurrence. The estimated treatment effects, when the assignment of treatment and its timing is unknown are near identical to imposing the same treatment as a known intervention *a-priori*. More broadly, our proposed approach is modular and allows for the detection of structural breaks in fixed effects panels with flexible choices for the machine learning algorithms employed. Crucially, using machine learning this allows for the detection of effective policies without prior knowledge of their occurrence or effectiveness. When using gets or the adaptive LASSO, the approach can be readily applied using our freely-available open-source R-packages ‘gets’ and ‘getspanel’.

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6 Supplementary Material

6.1 Identifying Heterogeneous Treatment Effects in Staggered Treatment

Here we briefly summarise the results from Wooldridge (2021), deriving equation (30) to identify treatment effects when treatment is staggered. We assume there is no anticipation of treatment for each $r = q, q + 1, \dots, Q$:

$$E[y_t(r) - y_t(\infty)|\mathbf{d}] = 0, \text{ for } t < r. \quad (52)$$

We also require a common trend assumption that the trend in absence of treatment is common regardless of state of treatment:

$$E[y_t(\infty) - y_1(\infty)|d_q, \dots, d_Q] = E[y_t(\infty) - y_1(\infty)] = \theta_t, \text{ for } t = 2, \dots, T \quad (53)$$

and we assume at least one untreated group. The observed outcome in any period is given by:

$$y_t = y_t(\infty) + d_q te_t(q) + \dots + d_Q te_t(Q) \quad (54)$$

where no anticipation implies that for the pre-treatment period ($t < q$):

$$E[y_t|\mathbf{d}] = E[y_t(\infty)|\mathbf{d}] \quad (55)$$

and for $t \geq q$:

$$E[y_t|\mathbf{d}] = E[y_t(\infty)|\mathbf{d}] + d_q \tau_{q,t} + \dots + d_Q \tau_{Q,t} \quad (56)$$

We then write the never treated outcome $y_t(\infty)$ as an initial outcome and change relative to the initial period:

$$y_t(\infty) = y_1(\infty) + g_t(\infty) \quad (57)$$

By the common trend assumption $E[g_t(\infty)|\mathbf{d}] = \theta_t$:

$$E[y_t(\infty)|\mathbf{d}] = E[y_1(\infty)|\mathbf{d}] + E[g_t(\infty)|\mathbf{d}] = \eta + \lambda_q d_q + \dots + \lambda_Q d_Q + \theta_t \quad (58)$$

which subsequently allows us to write the expected outcome as equation (30).

If we are interested in treatment effects of units treated at one point relative to those treated at a different point in time, as in Wooldridge (2021), we can define a sub-group ATT for those treated at r compared to for example one period later at $r + 1$ as

$$\tau_{(r:r+1)} = E[y_t(r) - y_t(r + 1)|d_r = 1] \quad (59)$$

This can be expressed as the difference in treatment effects relative to the untreated group:

$$y_t(r) - y_t(r + 1) = [y_t(r) - y_t(\infty)] - [y_t(r + 1) - y_t(\infty)] \quad (60)$$

Thus

$$\tau_{(r:r+1)} = \tau_{r,t} - E[y_t(r + 1) - y_t(\infty)|d_r = 1] \quad (61)$$

which under no anticipation and parallel trends simplifies to:

$$\tau_{(r:r+1)} = \tau_{r,t} - \tau_{r+1,t} \quad (62)$$

and which is matched by the difference in coefficients $\hat{\tau}$ obtained post-break detection on treatment dummies (step-functions or impulses).

6.2 Simulation Study

Here we investigate the properties of detecting treatment in our reverse causal setting using ‘gets’ and the adaptive LASSO. For the simulations we focus on detecting piece-wise constant treatment in the form of step-functions. Future work will expand simulations to also include fully-time-varying effects through impulse indicators.

We vary the treatment effect size σ as well as the number of treated units n . We compare the detection of unknown treatments against the ‘known treatment’ standard TWFE estimator for a single treated unit as well as multiple treated units. We then consider the case where we impose a known treatment while searching for additional treatment as described in section 3.2.

We simulate the DGP in (23) with errors drawn from the standard normal distribution and evaluate the performance of treatment detection as follows. For ‘gets’ we select over the full set of break functions using varying target levels of significance (γ_c). We use cross-validation to determine the penalty level for the adaptive LASSO. To measure the false positive rate of detection we compute the proportion of spuriously retained breaks (out of all possible spurious breaks). To measure whether we correctly identify treatment, we classify the proportion of correctly identified treated observations as those for which the detected breaks include the true treatment effect within a $(1 - \gamma_c)$ confidence interval.

Figure 5 shows the false positive rate together with the correctly classified proportion of treated observations for a single treated unit when varying the treatment magnitude (as a function of the standard deviation of the error term). Note that for a treatment effect size of 0 no treatment is present and hence no treatment should be identified – in this case therefore the rejection frequency yields a measure of the false-positive rate. Results show that treatment detection using ‘gets’ (red, solid) is close to the benchmark of a known treatment estimated using a conventional TWFE estimator (blue solid). The false positive rate is stable around the chosen level of significance of selection (red dashed). The adaptive LASSO (green solid) using cross-validation to choose the penalty factor also achieves a high level of accurate classification, however, is consistently lower than ‘gets’ for all significance levels considered. The adaptive LASSO using cross-validation also exhibits an erratic false positive rate (green dashed).

We increase the number of treated units from one to two and then five in our simulations (with identical treatment timing and homogeneous treatment effects) with results shown in Figures 6 and 7. As expected, as we increase the number of treated units, the correct classification (detection of treatment) falls relative to the known treatment case (using a single dummy variable) as our treatment detection approach has to identify a separate treatment dummy per treated unit. Nevertheless, the correct rejection frequency remains high given that no prior information about treatment assignment or timing was used.

Finally, we consider the costs of searching for additional treatment when there is a single known treatment that has been imposed from the outset (i.e. forced in the model and not selected over; see section 3.2).

Figure 8 shows the root-mean-squared error (RMSE) of the estimated treatment effect on the known treatment dummy when selecting over additional break variables relative to the simple TWFE estimator (without selection), together with the false-positive rate of detected treatment (gauge). The DGP only contains the single known treatment, with no other unknown treatment occurring. Thus this provides an assessment of the costs of searching for additional breaks when a known treatment is embedded. The results in Figure 8 show that searching for additional treatment when a known treatment is imposed, increases the RMSE on the estimated treatment effect for known treatment, however, for increasingly conservative selection significance levels this cost shrinks close to zero. This can be seen as an insurance cost – controlling for possible treatment (or breaks) that have been omitted from a standard model increases the RMSE of the known treatment indicator while providing robustness against omitted breaks. In other words, searching for additional breaks (i.e. treatment) lowers the precision on a known forced treatment somewhat, but the degree to which the RMSE increases can be easily controlled by choosing conservative levels of selection when using ‘gets’. For ‘gets’, as Figure 8 shows, the false positive rate

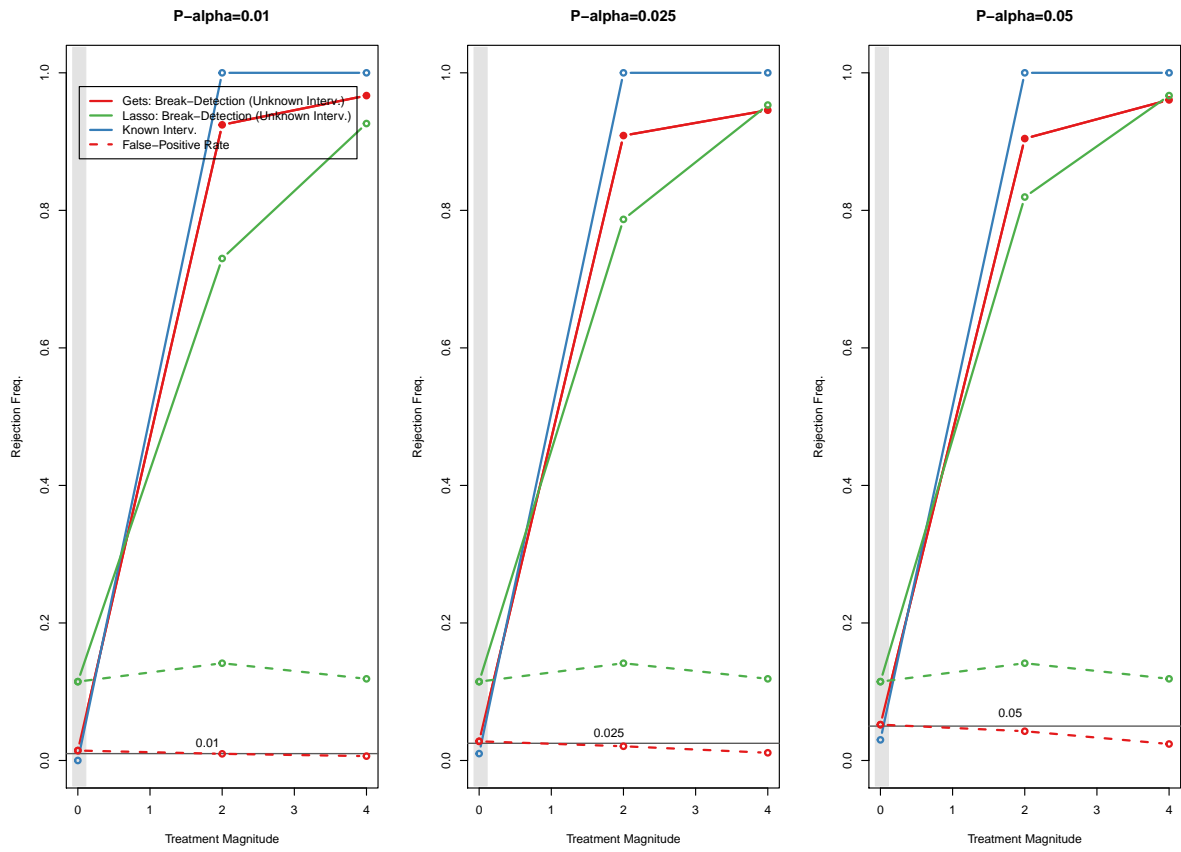


Figure 5: Simulation: Detecting treatment with **one** unknown treated units using ‘gets’ (SIS) and the adaptive LASSO compared to a ‘known’ treatment with $N = 10$

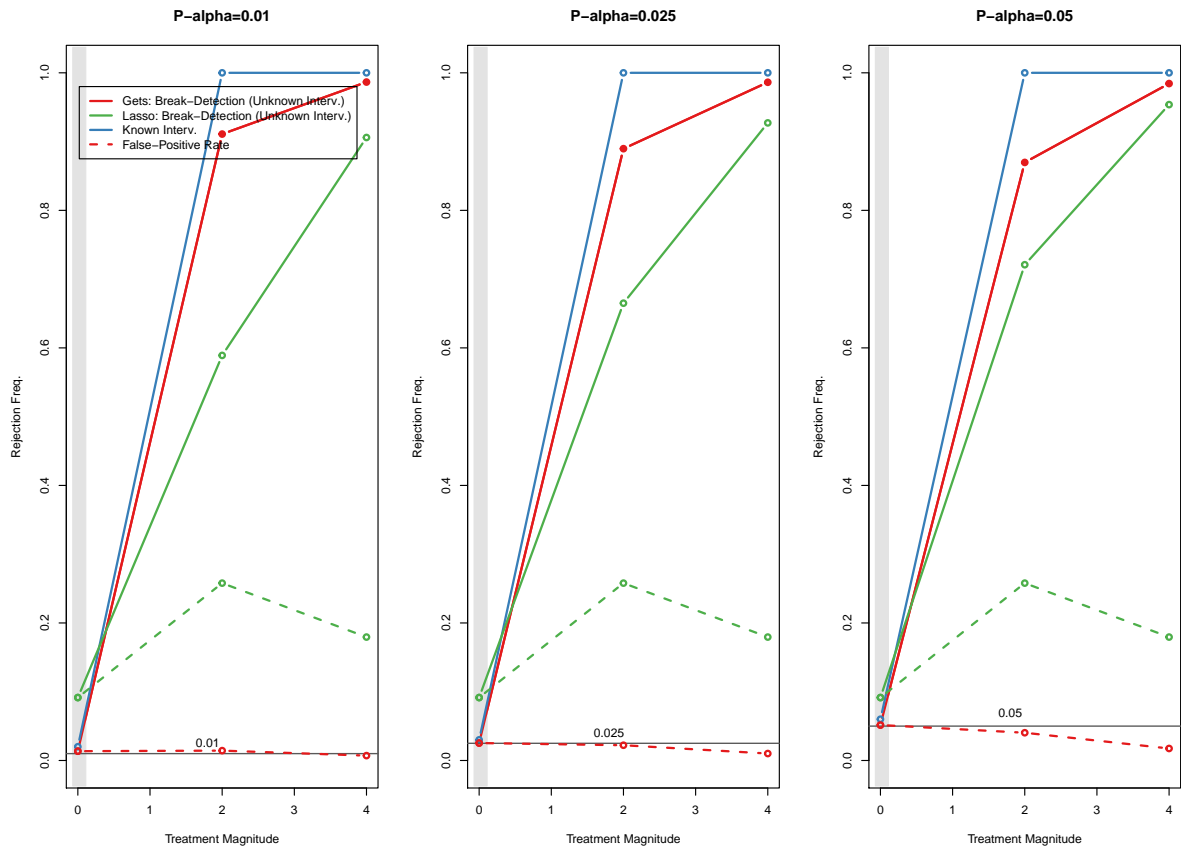


Figure 6: Simulation: Detecting treatment with **two** unknown treated units using ‘gets’ (SIS) and the adaptive LASSO compared to a ‘known’ treatment with $N = 10$

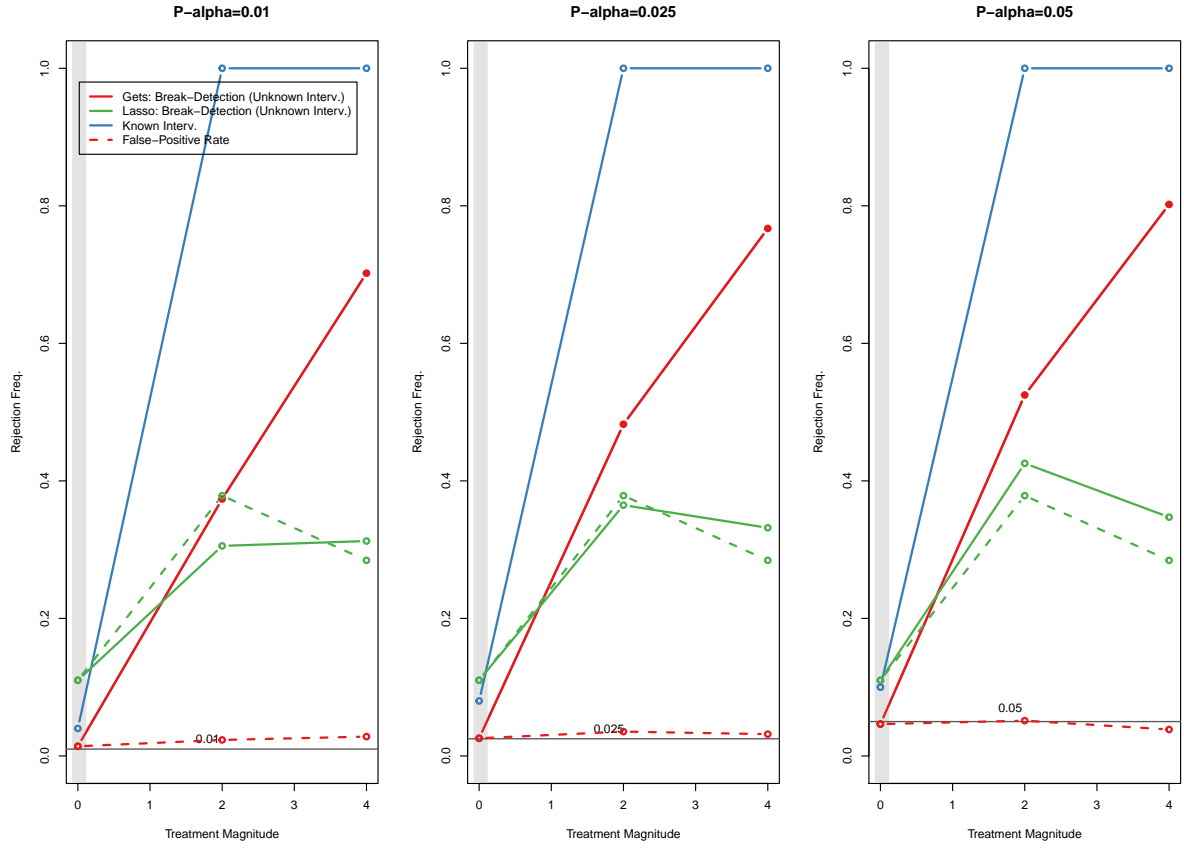


Figure 7: Simulation: Detecting treatment with **five** unknown treated units using ‘gets’ (SIS) and the adaptive LASSO compared to a ‘known’ treatment with $N = 10$

(gauge) again is stable around the specified nominal level of significance. Such control is more difficult to achieve when using the LASSO due to not targeting the false-positive rate.

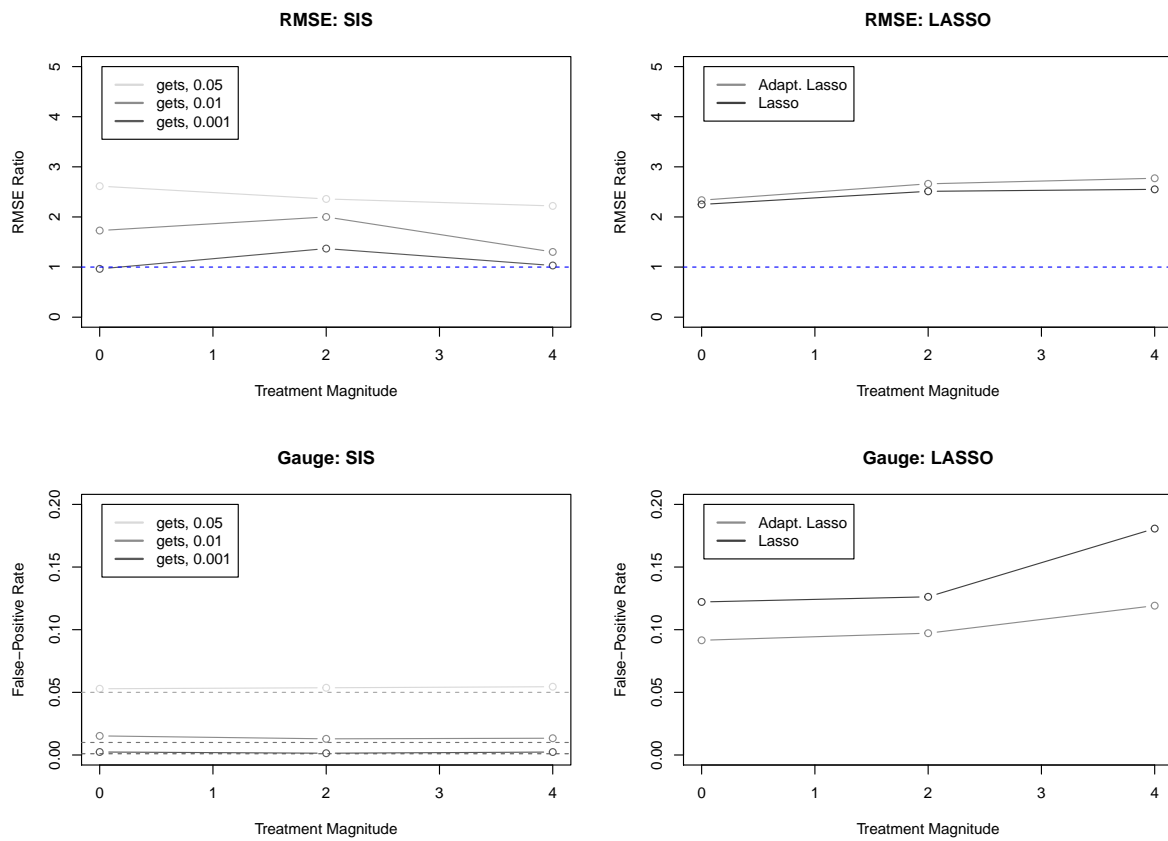


Figure 8: Top: RMSE of estimated ‘known’ single treatment effect when searching for additional treatment relative to known TWFE estimator. Bottom: False positive rate of detected breaks.

6.3 Simple Time Series Approach

We estimate a simple time series model of Basque GDP per capita in (63) and demonstrate that in absence of control groups, we are unable to detect the impact of ETA Basque terrorism *a-priori*. We model the log of GDP per capita as a function of log investment (one of the original control variables in Abadie and Gardeazabal), while searching for structural breaks in the intercept using step-indicators with ‘gets’ at a conservative target significance level of $\gamma_c = 0.001$:

SIS – Time Series for Basque Country only:

$$\log(GDPpc)_t = \beta_0 + \beta_1 \log(Inv)_t + \sum_{s=1966}^{1995} \tau_s 1_{\{t \geq s\}} + \epsilon_t \quad (63)$$

Estimation results of this time series model are shown in Table 4 and Figure 9. While multiple breaks are found, the negative impact of ETA terrorism on GDP per capita in the Basque region is not detectable due to the lack of control regions. There are no detected breaks with negative coefficients during the period that ETA was active.

Time Series Model (no Control Regions)
Allowing for step-shifts

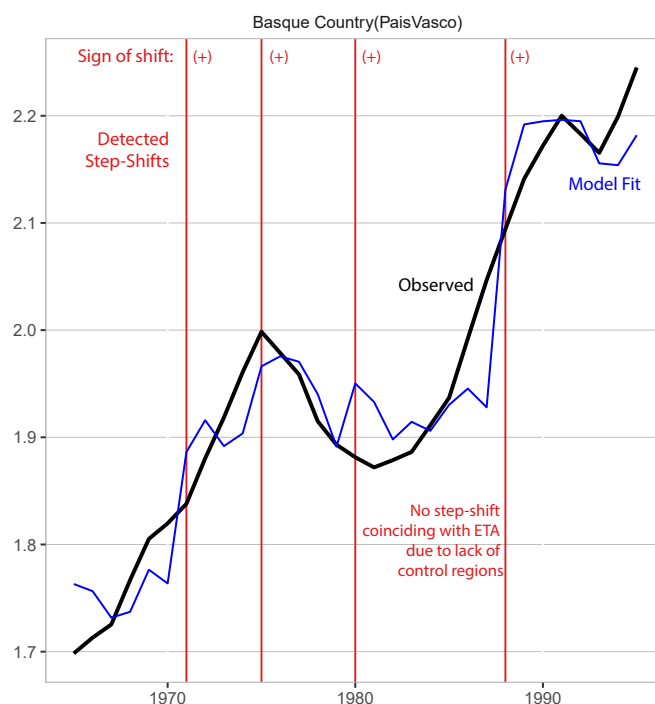


Figure 9: Simple Time Series Model of Basque GDP per Capita – Breaks detected using ‘gets’ and $\gamma_c = 0.001$.

Table 4: Detecting Breaks in a Simple Time Series Model (Basque Country)

| Dependent Variable: Model: | log(GDPpc) Time Series |
|---|---------------------------|
| <i>Variables</i> | |
| Constant | 0.5367 (0.5008) |
| log(Invest) | 0.3788** (0.1556) |
| Break ($i=\text{Basq.}, t \geq 1971$) | 0.1663*** (0.0321) |
| Break ($i=\text{Basq.}, t \geq 1975$) | 0.1308*** (0.0463) |
| Break ($i=\text{Basq.}, t \geq 1980$) | 0.0536 (0.0417) |
| Break ($i=\text{Basq.}, t \geq 1988$) | 0.1737*** (0.0392) |
| <i>Fit statistics</i> | |
| Observations | 31 |
| R ² | 0.92 |
| <i>Standard-errors in parentheses</i> | |
| <i>Signif. Codes: ***: 0.01, **: 0.05, *: 0.1</i> | |