Optimal Trajectories

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Zusammenfassung New active driver assistance systems that work at the road- and navigation level as well as automated driving face a challenging task. They have to permanently calculate the vehicle input commands (such as those for the steering, brakes, and the engine/powertrain) in order to realize a desired future vehicle movement, a driving trajectory. This trajectory has to be optimal in terms of some optimization criterion (in general a trade-off between comfort, safety, energy effort, and travelling time), needs to take the vehicular dynamics into account, and must incorporate lane boundaries or the predicted free space amidst (possibly moving) obstacles. This kind of optimization can be mathematically formulated as a so-called optimal control problem. In order to limit the calculation effort, the optimal control problem is usually solved only on a limited prediction interval (starting with the current time) leading to a receding horizon optimization. The chapter illustrates this practically proven approach in detail. Furthermore, the three general principles of dynamic optimization known from control theory and robotics are presented, namely calculus of variations, direct optimization, and dynamic programming. Furthermore, their application to driver assistance systems and automated driving is exemplified and the high practical relevance is supported by the given literature. Finally, the respective advantages and limitations of the optimization principles are discussed in detail proposing their combination for more involved system designs.

1 Introduction

Advanced collision avoidance systems, lane keeping support, traffic jam assistance, and remote valet parking; they all operate on the actuators to relieve the drivers of the lateral and/or longitudinal vehicle control or make it safer for them. Well-defined

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tasks, such as staying in the middle of the marked lane while following the vehicle ahead, can be handled by a set-point controller (Gayko 2012). And yet standard automated parking maneuvers already require a calculated trajectory [1] that has to be adapted to the available parking space (Katzwinkel et al. 2012). As systems need to cover more and more situations, the number of degrees of freedom increase, which makes a trajectory parameterization very complex, especially when the vehicle must take numerous obstacles into account. This calls for a systematic approach based on mathematical optimization (as opposed to heuristic approaches such as potential field and elastic bands methods, see e.g. (Krogh 1984), (Brand 2008), with their inherent limitations, cf. (Koren and Borenstein 1991). In the chapter at hand, we address real-time trajectory optimization[2], a task that an automated vehicle faces when it travels through its environment, also referred to as motion planning in robotics (Latombe 1990; LaValle 2006). The focus will be on methods that engage with the longitudinal and lateral vehicle movement. However, the results can be transferred to novel warning systems that can also benefit from an optimal trajectory prediction, see e.g. (Eichhorn et al. 2013). Speaking most generally, a trajectory optimization method is sought that can handle both structured (e.g. streets) and unstructured environments (parking lots), one that works amongst cluttered static obstacles and in moving traffic as well as exhibits a natural, human-like, anticipatory driving behavior. Using more technical terms, the method should be easy to implement, parameterize, adapt, scale well with the number of vehicle states and the length of the optimization horizon, incorporate nonlinear, high fidelity vehicle models, combine the lateral and longitudinal motion, be complete , allow for both grid maps and object lists representations (with predicted future poses) of the obstacles, be numerically stable, and transparent in its convergence behavior (if applicable). Also, the calculation effort has to be low to allow for short optimization cycles on (low performance) electronic control units so that the vehicle can quickly react to sudden changes in the environment. Unfortunately, there is no such single method that has all these properties. And, most likely, there will never be one. However, different optimization methods can be combined in order to get as close as possible to the above requirements. The next section therefore gives a closer look into the basic principles of trajectory optimization and their application.

2 Dynamic Optimization

When engineers speak about optimization, they usually refer to static optimization, in which the optimization variables p are finite, also called parameters (e.g., finding the most efficient operating point of an engine). Then optimal refers to some well-defined

[1] More precisely: a path, which does not have any time dependency

[2] Notice, that in control theory the term trajectory planning usually implies that there is no feedback of the actual system states on the trajectory. The dynamical system is then only stabilized by a downstream trajectory tracking controller, which is not always advisable. We therefore use the term trajectory optimization instead to be independent of the utilized stabilization concept.

optimization criterion, usually the minimization of a cost function J(p) (e.g., fuel consumption per hour). Trajectory optimization is different in that the optimization variables are functions x(t) of an independent variable t, usually time. It is also called dynamic or infinite-dimensional optimization. Evaluating x(t) therefore requires a cost functional (a "function of a function"), which quantifies the "quality" of the trajectory x(t) by a scalar value. Due to the vehicular focus, a special case will be considered, one that requires the trajectory x(t) to be consistent with some dynamical system model which has an input u. Without such model, the optimization cannot incorporate the inherent properties and physical limitations of the vehicle. This special case of dynamic optimization is called an optimal control problem (e.g., Lewis and Syrmos 1995).

3 Solving the Optimal Control Problem

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4 Comparison of the Approaches

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5 Receding Horizon Optimization

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6 Conclusion

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