

# Time Complexity

Mohammed Rizin  
Unemployed

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## 1 Time Complexity

### 1.1 Simple For Loop

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**Algorithm 1** Simple for loop

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```
for  $i \leftarrow 0$  to  $n - 1$  do
    ....STMT....
end for
```

---

Time complexity:  $O(n)$ .

### 1.2 Simple Reverse Loop

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**Algorithm 2** Simple reverse loop

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```
for  $i \leftarrow n$  down to 1 do
    ....STMT....
end for
```

---

Time complexity:  $O(n)$ .

### 1.3 For Loop with Step

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**Algorithm 3** Simple for loop with step

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```
for  $i \leftarrow 0$  to  $n - 1$  step 2 do
    ....STMT....
end for
```

---

Time complexity:  $O(n)$  (as  $n/2$  is asymptotically  $O(n)$ ).

### 1.4 Nested Loops

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**Algorithm 4** Nested loops

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```
for  $i \leftarrow 0$  to  $n - 1$  do
    for  $j \leftarrow 0$  to  $i - 1$  do
        ....STMT....
    end for
end for
```

---

Time complexity:  $O(n^2)$ .

$i$	$j$	STMT	Total STMT	Total Time	Time Complexity
0	0	1	1	1	1
1	0	1	2	3	3
1	1	1	3	6	6
2	0	1	4	10	10
2	1	1	5	15	15
2	2	1	6	21	21

The total number of executions is  $n(n+1)/2$ , so the time complexity is  $O(n^2)$ .

## 1.5 Summation Loop

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**Algorithm 5** Summation loop

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```

 $p \leftarrow 0$ 
for  $i \leftarrow 1$  while  $p \leq n$  do
     $p \leftarrow p + i$ 
end for

```

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$i$	$p$	STMT	Total STMT	Total Time	Time Complexity
1	1	1	1	1	1
2	3	1	2	3	3
3	6	1	3	6	6
4	10	1	4	10	10
5	15	1	5	15	15
6	21	1	6	21	21
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$k$	$\frac{k(k+1)}{2}$	1	$k$	$\frac{k(k+1)}{2}$	$\frac{k(k+1)}{2}$

Assuming  $p \leq n$ :

$$p = \frac{k(k+1)}{2},$$

$$\frac{k(k+1)}{2} > n,$$

$$k^2 > n,$$

$$k > \sqrt{n}.$$

Time complexity:  $O(\sqrt{n})$ .