Disjoint Sets

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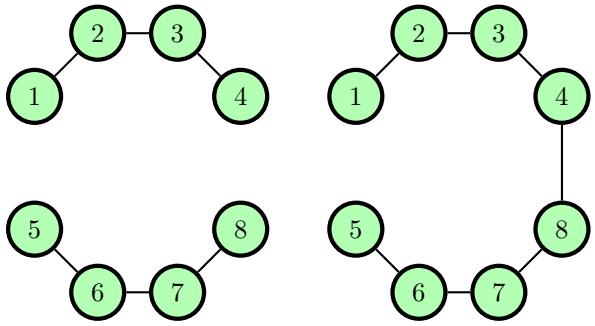
1 Disjoint sets & operations

This is similar to the concept from sets in Mathematics but not exactly same. Disjoint set is modified for its proper usage in creating algorithms. The famous algorithm based on disjoint set is criskell's algorithm.

There are two main operation in a disjoint set:

1. Find
2. Join

Lets us find How we can do



Step 1 Step 2

Figure 1: Sample Graph

First we can represent them as two sets:

$$S_1 = \{1, 2, 3, 4\}$$
$$S_2 = \{5, 6, 7, 8\}$$

Now we connect the edge (4, 8)

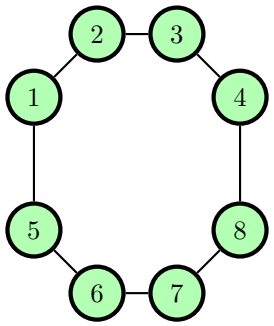
$$S_1 = \{1, 2, 3, \underbrace{4}_{\cdot}\}$$

$$S_2 = \{5, 6, 7, \underbrace{8}_{\cdot}\}$$

Both are in different set Create the Union of two sets

$$S_3 = S_1 \cup S_2 = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

2 Detecting a cycle



Step 3

Figure 2: Sample Graph

In Figure, we considered one more edge between 1 and 5. When we search for those elements and find which set it belong to, we end up finding same set for both the values. Hence, It's a cycle. If both the elements of an edge is found to in same set, then We say that the graph has a cycle.

Confusion : Most of them would be confused we Union operation
In mathematics, we do union between sets just as operation. But in here we have a purpose of doing the operation. Let me explain it!
If there is an edge between $(u, v)|u, v \in \mathcal{G}$, check which set does u and v belong to different set S_1 and S_2 will connect each other. Once they are connected, we perform union operation. Still Daunting actually!
Lets me explain more simply.

Initially we have two sets $S_1 = \{1, 2, 3, 4\}$ and $S_2 = \{5, 6, 7, 8\}$

Now we connect the edge (4, 8)

$S_1 = \{1, 2, 3, \underbrace{4}_{\cdot}\}$

$S_2 = \{5, 6, 7, \underbrace{8}_{\cdot}\}$

Both are in different set Create the Union of two sets

$S_3 = S_1 \cup S_2 = \{1, 2, 3, 4, 5, 6, 7, 8\}$

When we have an edge between u and v, Why do we perform union, why not any other operation?
Hmm...! Let me put it in another way to make it more clear.
One of the most plausible reason I could come up is. Once we join the two sets, every elements of set S_1 can be accessed by the elements of S_2 and vice versa. Maybe I should get okay with this explanation. Haha!

3 Graphical Representation

Now let create another graph and see how we can represent it in a disjoint set.

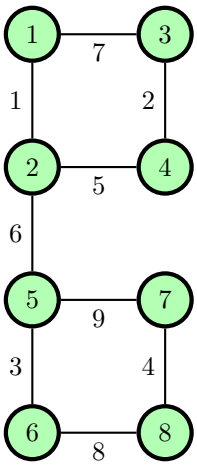


Figure 3: Sample Graph

1. At first we have n singletons sets: $\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \{7\}, \{8\}$
2. Now we connect the edge 1 (1, 2) i.e., sets $\{1\}$ and $\{2\}$, so we perform Union $\{1, 2\}$. 1 is the parent of 2.
3. Then we connect the edge 2 (3, 4) i.e., sets $\{3\}$ and $\{4\}$, so we perform Union $\{3, 4\}$. 3 is the parent of 4.
4. Then we connect the edge 3 (5, 6) i.e., sets $\{5\}$ and $\{6\}$, so we perform Union $\{5, 6\}$. 5 is the parent of 6.
5. Then we connect the edge 4 (7, 8) i.e., sets $\{7\}$ and $\{8\}$, so we perform Union $\{7, 8\}$. 7 is the parent of 8.
6. Then we connect the edge 5 (2, 4) i.e., sets $\{1, 2\}$ and $\{3, 4\}$, so we perform Union $\{1, 2, 3, 4\}$.
   1 is the parent of 2 and 3 is the parent of 4 or 3 is the parent of 4 and 1 is the parent of 2. We choose 1 as root in this case.
7. Then we connect the edge 6 (2, 5) i.e., sets $\{1, 2, 3, 4\}$ and $\{5, 6\}$, so we perform Union $\{1, 2, 3, 4, 5, 6\}$. 1 is the parent of 2 and 3 is the parent of 4 and 5 is the parent of 6.
8. Finally, we connect the edge 7 (1, 3). Both are in the same set. Hence, a cycle is detected.

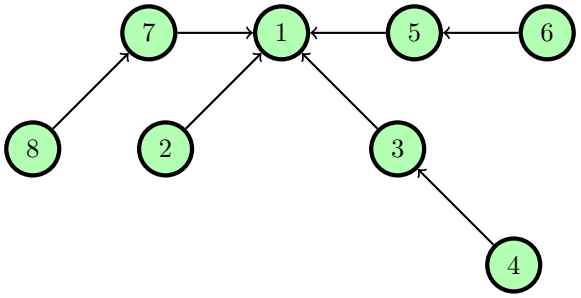


Figure 4: Graphical Representation

4 Array Representation

-1	-1	-1	-1	-1	-1	-1	-1
1	2	3	4	5	6	7	8

Initial Array:

1. **Union(1, 2):**

-2	1	-1	-1	-1	-1	-1	-1
1	2	3	4	5	6	7	8

2. **Union(3, 4):**

-2	1	-2	3	-1	-1	-1	-1
1	2	3	4	5	6	7	8

3. **Union(5, 6):**

-2	1	-2	3	-2	5	-1	-1
1	2	3	4	5	6	7	8

4. **Union(7, 8):**

-2	1	-2	3	-2	5	-2	7
1	2	3	4	5	6	7	8

5. **Union(2, 4):**

-4	1	1	3	-2	5	-2	7
1	2	3	4	5	6	7	8

6. **Union(2, 5):**

-6	1	1	3	1	5	-2	7
1	2	3	4	5	6	7	8

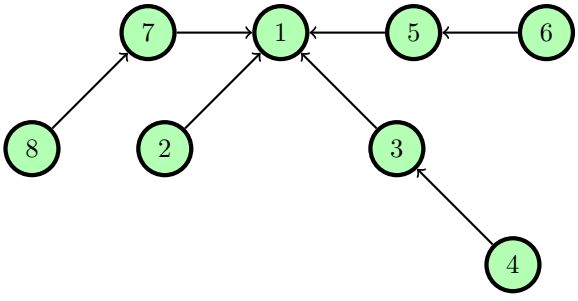
7. **Union(1, 3):**

-6	1	1	3	1	5	-2	7
1	2	3	4	5	6	7	8

Cycle Detected

8. **Union(6, 8):**

-8	1	1	3	1	5	1	7
1	2	3	4	5	6	7	8



9. **Union(5, 7):**

-8	1	1	3	1	5	1	7
1	2	3	4	5	6	7	8

Cycle Detected

5 Weighted Union and collapsing Find

Weighted Union and collapsing Find are the two techniques used to optimize the disjoint set operations.

1. **Weighted Union:** In this technique, we connect the smaller set to the larger set. This helps in reducing the height of the tree.

2. **Collapsing Find:** In this technique, we make the parent of the node as the root of the tree. This helps in reducing the height of the tree.

After applying the Weighted Union and collapsing Find techniques, the array representation will be as follows:

-8	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8

and the graphical representation will be as follows: