Time Complexity

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1 Time Complexity

1.1 Simple For Loop

Algorithm 1 Simple for loop

```
for $i \leftarrow 0$ to $n - 1$ do
    ....$STMT$....
end for
```

Time complexity: $O(n)$.

1.2 Simple Reverse Loop

Algorithm 2 Simple reverse loop

```
for $i \leftarrow n$ down to $1$ do
    ....$STMT$....
end for
```

Time complexity: $O(n)$.

1.3 For Loop with Step

Algorithm 3 Simple for loop with step

```
for $i \leftarrow 0$ to $n - 1$ step $2$ do
    ....$STMT$....
end for
```

Time complexity: $O(n)$ (as $n/2$ is asymptotically $O(n)$).

1.4 Nested Loops

Algorithm 4 Nested loops

```
for $i \leftarrow 0$ to $n - 1$ do
    for $j \leftarrow 0$ to $i - 1$ do
        ....$STMT$....
    end for
end for
```

Time complexity: $O(n^2)$.

i	j	STMT	Total STMT	Total Time	Time Complexity
0	0	1	1	1	1
1	0	1	2	3	3
1	1	1	3	6	6
2	0	1	4	10	10
2	1	1	5	15	15
2	2	1	6	21	21

The total number of executions is $n(n+1)/2$, so the time complexity is $O(n^2)$.

1.5 Summation Loop

Algorithm 5 Summation loop

```
  $p \leftarrow 0$
  for $i \leftarrow 1$ while $p \leq n$ do
      $p \leftarrow p + i$
  end for
```

i	p	STMT	Total STMT	Total Time	Time Complexity
1	1	1	1	1	1
2	3	1	2	3	3
3	6	1	3	6	6
4	10	1	4	10	10
5	15	1	5	15	15
6	21	1	6	21	21
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
k	$\frac{k(k+1)}{2}$	1	k	$\frac{k(k+1)}{2}$	$\frac{k(k+1)}{2}$

Assuming $p \leq n$:

$$\begin{aligned} p &= \frac{k(k+1)}{2}, \\ \frac{k(k+1)}{2} &> n, \\ k^2 &> n, \\ k &> \sqrt{n}. \end{aligned}$$

Time complexity: $O(\sqrt{n})$.

1.6 Multiplication Loop

Algorithm 6 Multiplication loop

```
  for $i \leftarrow 1$ while $i \leq n$ step $2 \cdot i$ do
      ...Statement...
  end for
```

step	i
1	2
2	4
3	8
4	16
5	32
$\vdots$	$\vdots$
k	2^k

$$\begin{aligned} \text{This stops when } i > n : \\ \text{i.e. } 2^k > n, \\ \log_2 2^k > \log_2 n \\ k \cdot \log_2 2 > \log_2 n \\ k > \log_2 n \end{aligned}$$

Time complexity: $O(\log_2 n)$.