
ITD62-124 Data Structure

Sorting Algorithms



Outline:

- Insertion sort
- Bubble sort
- Heap sort
- Shell sort
- Quick sort
- Merge sort
- Radix sort

Introduction

- Input:
A sequence of n numbers a_1, a_2, \dots, a_n
- Output:
A permutation (reordering) a'_1, a'_2, \dots, a'_n of the input sequence such that $a'_1 \leq a'_2 \leq \dots \leq a'_n$



Introduction

▪ Importance of sorting algorithms

There are a variety of situations that we can encounter

- ✓ have **randomly ordered** keys
- ✓ all keys **distinct**
- ✓ How large is the set of keys to be ordered?
- ✓ Need guaranteed **performance**?

Insertion Sort

- insert each item into its proper place in the final array
- moving the current data elements past the already sorted values and repeatedly interchanging it with the preceding value until it is in its correct place
- the array of values to be sorted is divided into two sets (one that stores sorted values and another that contains unsorted values)

Insertion Sort

- Initially, data[0] is the only element in the sorted set
- In Pass 1, data[1] will be placed either before or after data[0], therefore the array is sorted
- In Pass 2, data[2] will be placed either before data[0], in between data[0] and data[1], or after data[1]
- In Pass 3, data[3] will be placed in its proper place
- In Pass n-1, data[n-1] will be placed in its proper place to keep the array sorted

Insertion Sort

- Example: sort a given array by insertion sort

0	1	2	3	4	5
32	6	60	48	30	20

Insertion Sort

- Solution:

- ✓ Initially:

0	1	2	3	4	5
32	6	60	48	30	20

- ✓ Pass 1:

0	1	2	3	4	5
6	32	60	48	30	20

- ✓ Pass 2:

0	1	2	3	4	5
6	32	60	48	30	20

Insertion Sort

- Solution:

✓ Pass 3:

0	1	2	3	4	5
6	32	48	60	30	20

✓ Pass 4:

0	1	2	3	4	5
6	30	32	48	60	20

✓ Pass 5:

0	1	2	3	4	5
6	20	30	32	48	60



Bubble Sort

- ✓ Consecutive adjacent pairs of elements in the array are compared with each other
- ✓ Two adjacent elements are interchanged if they are found to be out of order with respect to each other
- ✓ This process will continue till the list of unsorted elements exhausts

Bubble Sort

- ✓ In Pass 1, $\text{data}[0]$ and $\text{data}[1]$ are compared, then $\text{data}[1]$ is compared with $\text{data}[2]$, $\text{data}[2]$ is compared with $\text{data}[3]$, and so on. Finally, $\text{data}[n-2]$ is compared with $\text{data}[n-1]$. **The biggest element is placed at the highest index of the array.**
- ✓ In Pass 2, $\text{data}[0]$ and $\text{data}[1]$ are compared, then $\text{data}[1]$ is compared with $\text{data}[2]$, $\text{data}[2]$ is compared with $\text{data}[3]$, and so on. Finally, $\text{data}[n-3]$ is compared with $\text{data}[n-2]$. **The second biggest element is placed at the second highest index of the array.**

Bubble Sort

- ✓ In Pass 3, data[0] and data[1] are compared, then data[1] is compared with data[2], data[2] is compared with data[3], and so on. Finally, data[n-4] is compared with data[n-3]. **The third biggest element is placed at the third highest index of the array.**
- ✓ In Pass n-1, data[0] and data[1] are compared so that $\text{data}[0] < \text{data}[1]$. After this step, all the elements of the array are arranged in ascending order.

Bubble Sort

- Example: sort a given array by bubble sort

0	1	2	3	4
6	3	4	9	2

Bubble Sort

■ Solution: Pass 1

✓ Step 1:

0	1	2	3	4
6	3	4	9	2

✓ Step 2:

0	1	2	3	4
3	6	4	9	2

✓ Step 3:

0	1	2	3	4
3	4	6	9	2

✓ Step 4:

0	1	2	3	4
3	4	6	9	2

0	1	2	3	4
3	4	6	2	9



Bubble Sort

▪ Solution: Pass 2

✓ Step 1:

0	1	2	3	4
3	4	6	2	9

✓ Step 2:

0	1	2	3	4
3	4	6	2	9

✓ Step 3:

0	1	2	3	4
3	4	6	2	9

✓ Step 4:

0	1	2	3	4
3	4	2	6	9



Bubble Sort

- Solution: Pass 3

✓ Step 1:

0	1	2	3	4
3	4	2	6	9

✓ Step 2:

0	1	2	3	4
3	4	2	6	9

✓ Step 3:

0	1	2	3	4
3	2	4	6	9

Bubble Sort

- Solution: Pass 4

✓ Step 1:

0	1	2	3	4
3	2	4	6	9

✓ Step 2:

0	1	2	3	4
2	3	4	6	9



Heap Sort

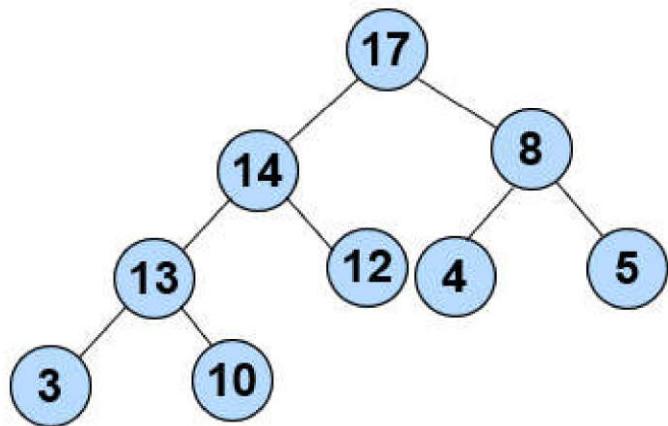
- ✓ Invented by John Williams
- ✓ To sort an array in ascending order, heap sort puts the largest elements at the end of the array, then the second largest in front of it, and so on
- ✓ Start from the end of the array by finding the largest elements
- ✓ Heap sort uses a heap

Heap Sort

- Example: sort the heap using heap sort

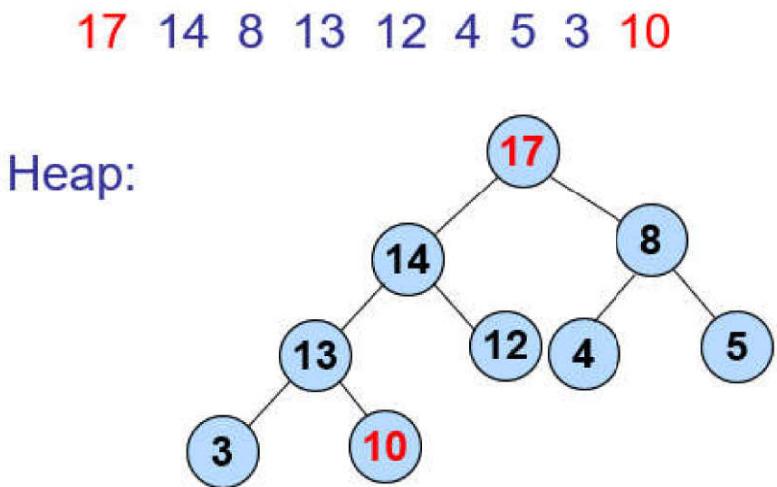
17 14 8 13 12 4 5 3 10

Heap:



Heap Sort

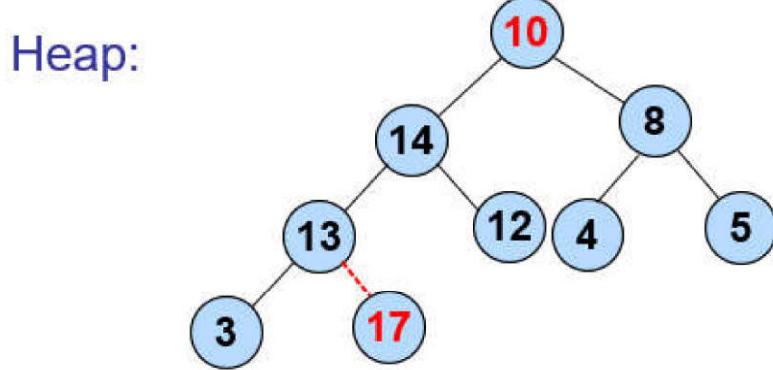
- Solution:



Heap Sort

- Solution:

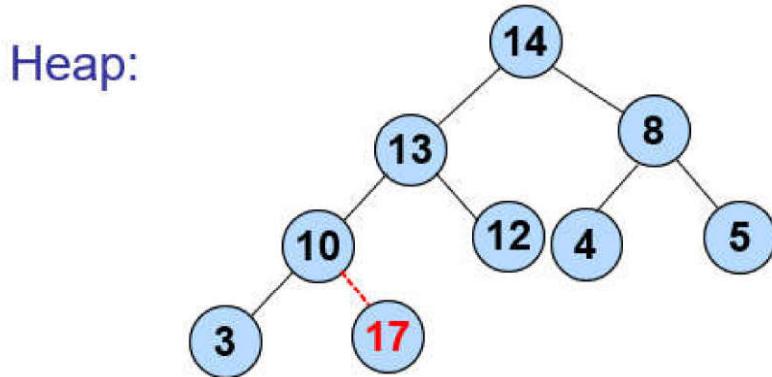
10 14 8 13 12 4 5 3 17



Heap Sort

- Solution:

14 13 8 10 12 4 5 3 17

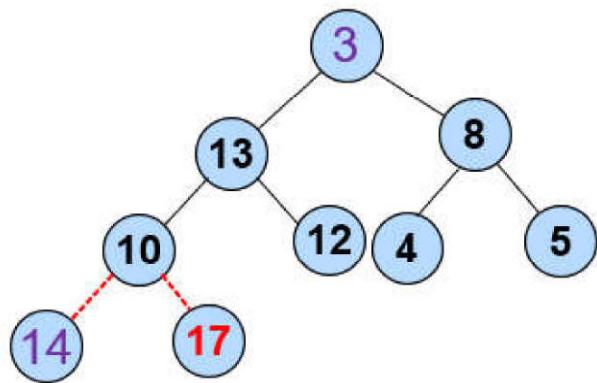


Heap Sort

- Solution:

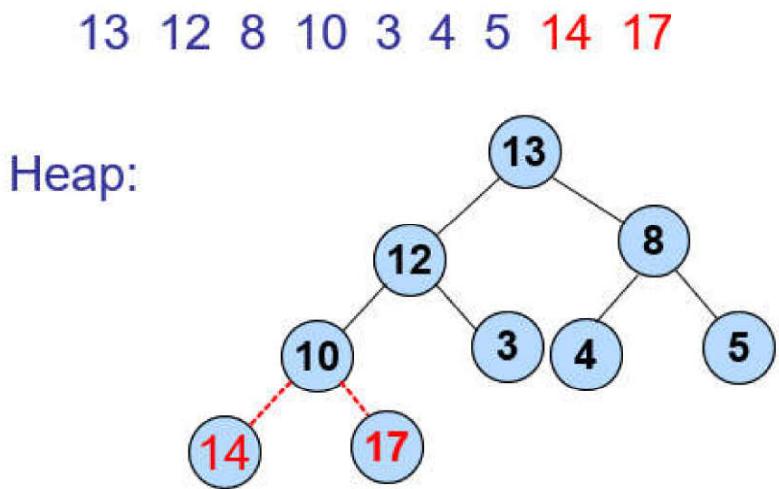
14 13 8 10 12 4 5 3 17

Heap:



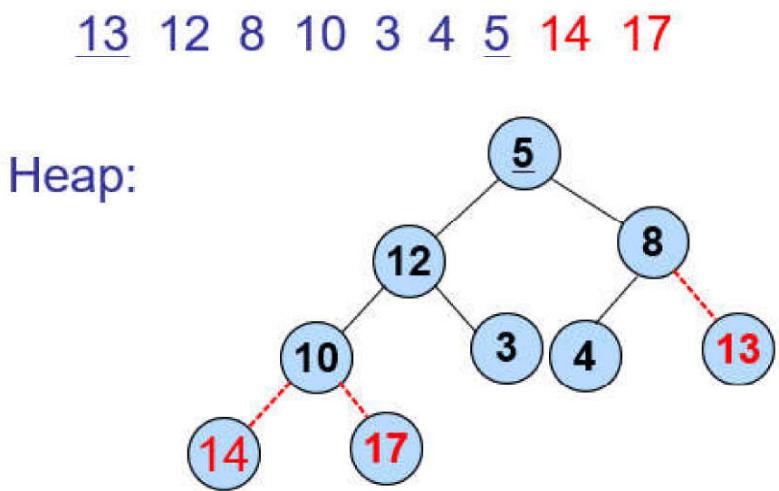
Heap Sort

- Solution:



Heap Sort

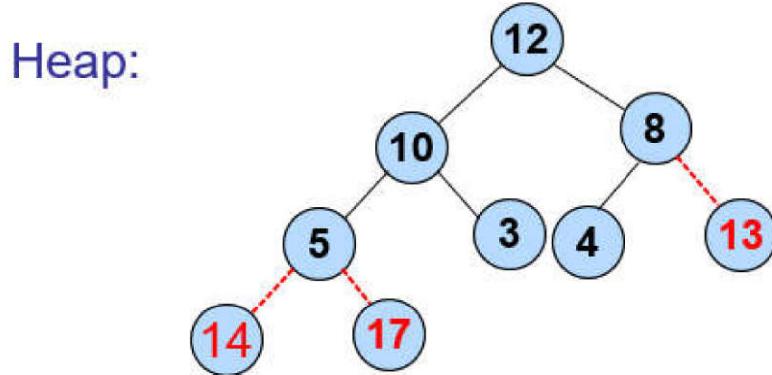
- Solution:



Heap Sort

- Solution:

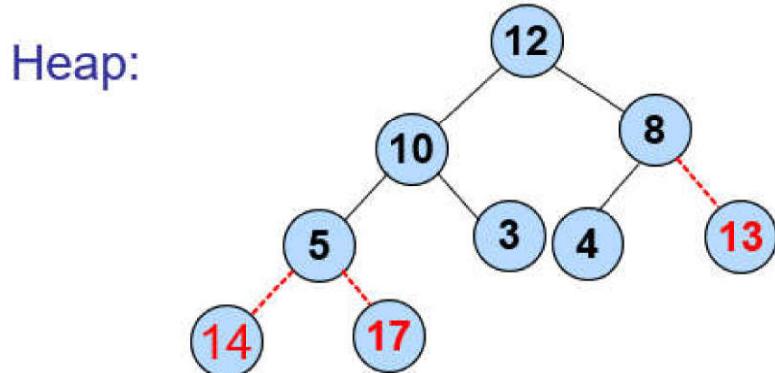
12 10 8 5 3 4 13 14 17



Heap Sort

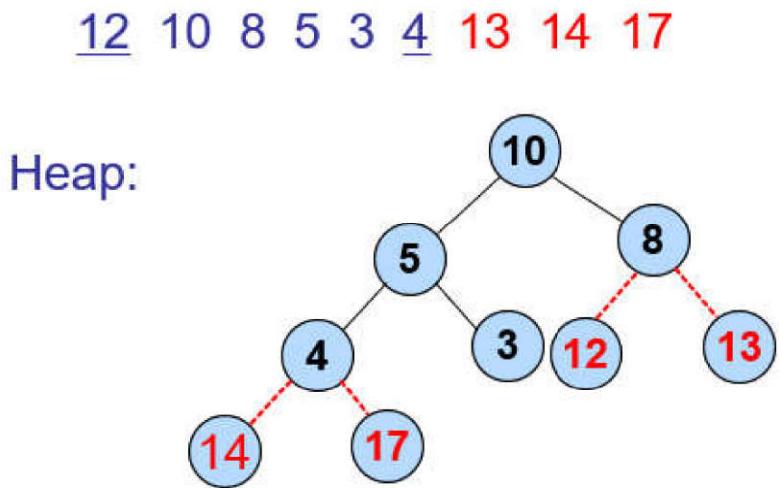
- Solution:

12 10 8 5 3 4 13 14 17



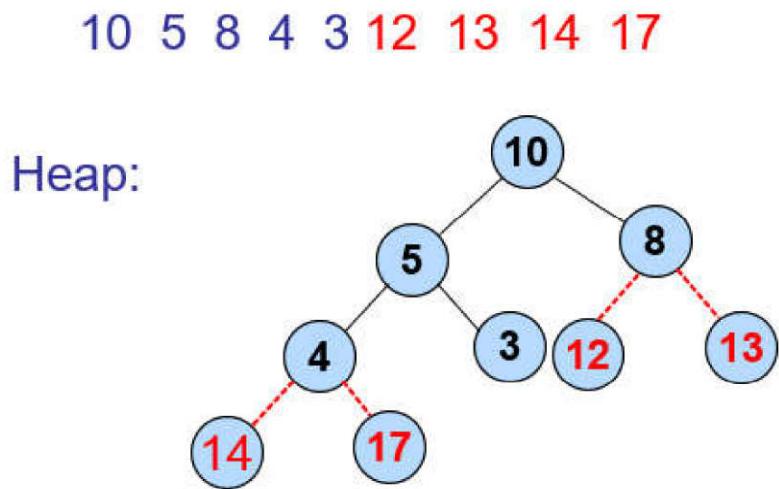
Heap Sort

- Solution:



Heap Sort

- Solution:

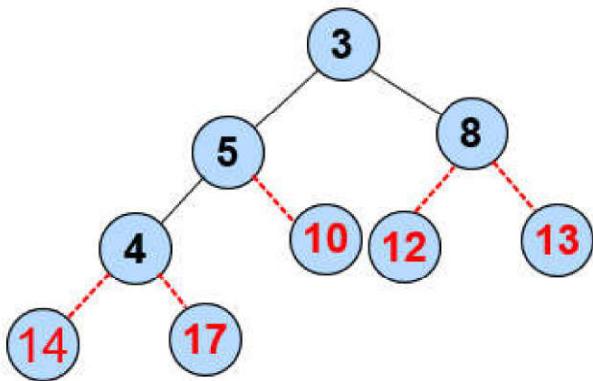


Heap Sort

- Solution:

10 5 8 4 3 12 13 14 17

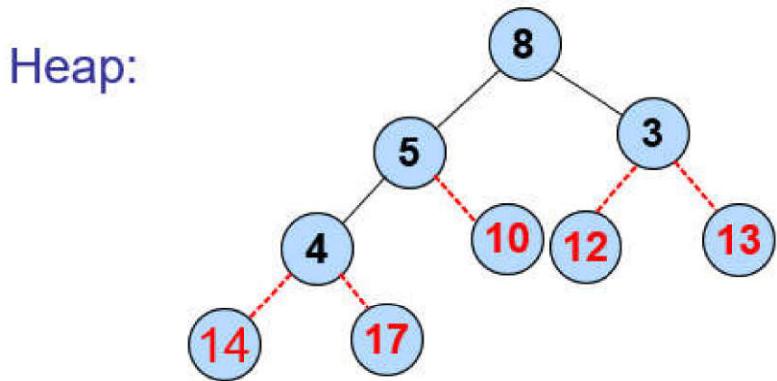
Heap:



Heap Sort

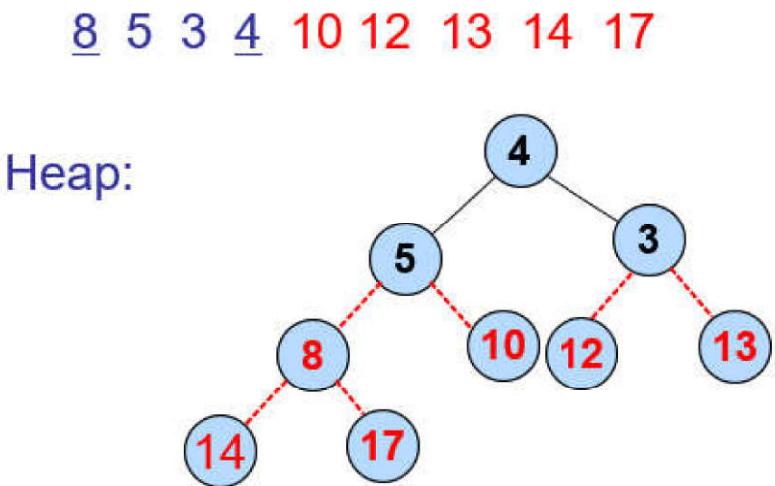
- Solution:

8 5 3 4 10 12 13 14 17



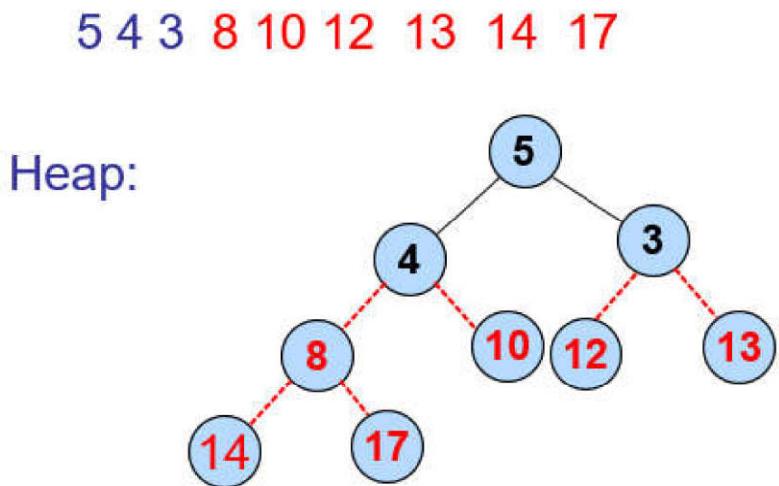
Heap Sort

- Solution:



Heap Sort

- Solution:

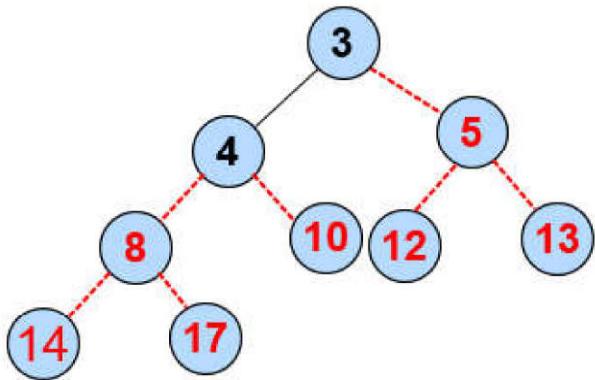


Heap Sort

- Solution:

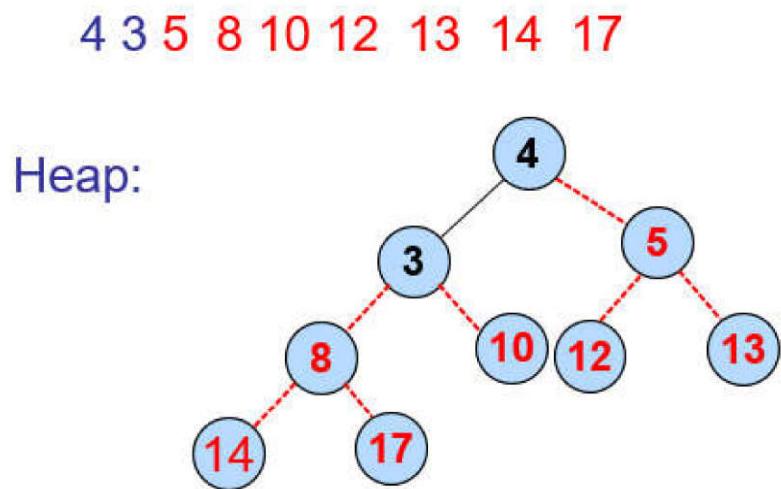
5 4 3 8 10 12 13 14 17

Heap:



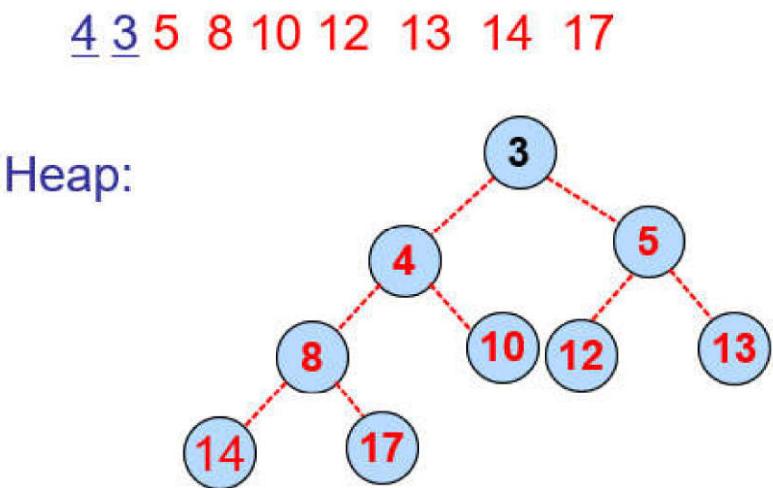
Heap Sort

- Solution:



Heap Sort

- Solution:

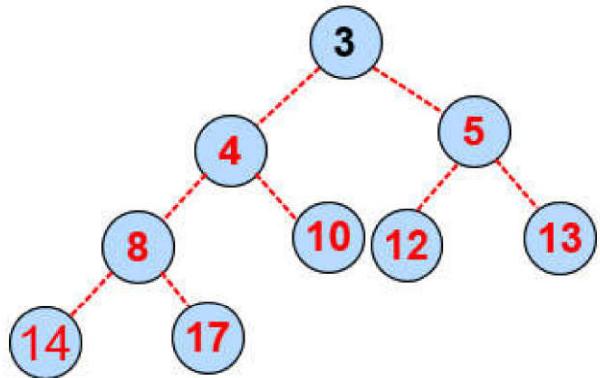


Heap Sort

- Solution:

3 4 5 8 10 12 13 14 17

Heap:



Shell Sort

- ✓ Invented by Donald Shell in 1959
- ✓ Division of an array data into several subarrays
- ✓ Elements spaced farther apart are compared first, then the elements closer to each other are compared, ..., until adjacent elements are compared on the last pass
- ✓ Shell sort improves on the efficiency of insertion sort by quickly shifting values to their destination
- ✓ elements are sorted in multiple passes and in each pass, data are taken with smaller and smaller gap sizes
- ✓ The last step of Shell sort is a plain insertion sort

Shell Sort

```
Shell_Sort(Arr, N)
```

```
Step 1: SET FLAG = 1, GAP_SIZE = N
Step 2: Repeat Steps 3 to 6 while FLAG = 1 OR GAP_SIZE > 1
Step 3:           SET FLAG = 0
Step 4:           SET GAP_SIZE = (GAP_SIZE + 1) / 2
Step 5:           Repeat Step 6 for I = 0 to I < (N - GAP_SIZE)
Step 6:               IF Arr[I + GAP_SIZE] > Arr[I]
                    SWAP Arr[I + GAP_SIZE], Arr[I]
                    SET FLAG = 0
Step 7: END
```

Shell Sort

- Example: sort a given array by shell sort

63, 19, 7, 90, 81, 36, 54, 45, 72, 27, 22, 9, 41, 59, 33

Shell Sort

- Solution:
 - ✓ Step 1:

63	19	7	90	81	36	54	45
72	27	22	9	41	59	33	
63	19	7	9	41	36	33	45
72	27	22	90	81	59	54	

63, 19, 7, 9, 41, 36, 33, 45, 72, 27, 22, 90, 81, 59, 54

Shell Sort

✓ Step 2:

63	19	7	9
41	36	33	45
72	27	22	90
81	59	54	
			
41	19	7	9
63	27	22	45
72	36	33	90
81	59	54	

41, 19, 7, 9, 63, 27, 22, 45, 72, 36, 33, 90, 81, 59, 54

Shell Sort

✓ Step 3:

41	19		7	9
7	9		22	19
63	27	→	33	27
22	45		41	36
72	36		54	45
33	90		63	59
81	59		72	90
54			81	

7, 9, 22, 19, 33, 27, 41, 36, 54, 45, 63, 59, 72, 90, 81

Shell Sort

✓ Step 4:

7	7
9	9
22	19
19	22
33	27
27	33
41	36
36	41
54	45
45	54
63	59
59	63
72	72
90	81
81	90



ANSWER

Quick Sort

▪ Quick sort

- ✓ The original array is divided into two subarrays
 - the first array contains elements less than or equal to a chosen key called **the pivot**
 - the second array contains elements equal to or greater than the pivot
- ✓ The two subarrays can be sorted separately

Quick Sort

- Example: sort a given array by quick sort

27 15 22 37 11 59 18 50 42

Quick Sort

- **Solution:**

27 15 22 37 11 59 18 50 **42**

27 15 22 37 11 59 18 **50** 42

18 15 22 37 11 59 **27** 50 42

18 15 22 37 11 59 **27** 50 42

18 15 22 37 11 59 **27** 50 42

Quick Sort

- Solution:

18 15 22 37 11 59 **27** 50 42

18 15 22 27 11 59 **37** 50 42

18 15 22 27 11 **59** 37 50 42

18 15 22 27 **11** 59 37 50 42

18 15 22 11 **27** 59 37 50 42

18 15 22 11 **27** 59 37 50 42

Pivot = 27, first subarray = 18 15 22 11, second subarray = 59 37 50 42

Quick Sort

- Solution:

18 15 22 11 27 59 37 50 42

- Sort the first subarray

18 15 22 11

11 15 22 18

11 15 22 18

11 15 22 18

11 15 18 22

11 15 18 22

Pivot = 18, first subarray = 11 15, second subarray = 22

Quick Sort

- Solution:

11 15 18 22 27 59 37 50 42

- Sort the first subarray

11 15
11 15

11 15 18 22 27 59 37 50 42

Quick Sort

- Solution:

11 15 18 22 27 59 37 50 42

- Sort the second subarray(59 37 50 42)

59 37 50 42

42 37 50 59

42 37 50 59

42 37 50 59

42 37 50 59

Pivot = 59, first subarray = 42 37 50

Quick Sort

- Solution:

11 15 18 22 27 **42** 37 50 59

- Sort the subarray (42 37 50)

42 37 **50**

42 **37** 50

37 **42** 50

37 42 50

Pivot = 42

11 15 18 22 27 37 **42** 50 59



Merge Sort

▪ Merge sort

- ✓ Merging sorted halves of an array into one sorted array

Merge Sort

- **Example:** sort a given array by merge sort

3 10 8 6 12 7 5 4 24

✓ Solution:

3 10 8 6 12 7 5 4 24

3 10 8 6 12 7 5 4 24

3 10 8 6 12 7 5 4 24

3 10 8 6 12 7 5 4 24

3 10 8 6 12 7 5 4 24

Merge Sort

```
3 8 10      6 12      5 7      4 24  
3 6 8 10 12          4 5 7 24  
3 4 5 6 7 8 10 12 24
```



Radix Sort

▪ Radix sort

- ✓ sorts data with integer keys by grouping keys by the individual digits which share the same significant position and value

Radix Sort

- Example: sort a given array by radix sort

12 1234 9 7234 67 9191 733 197 7 3

Radix Sort

- Solution: 12 1234 9 7234 67 9191 733 197 7 3

Step 1:

bucket 0:

bucket 1: 9191

bucket 2: 12

bucket 3: 733 3

bucket 4: 1234 7234

bucket 5:

bucket 6:

bucket 7: 67 197 7

bucket 8:

bucket 9: 9

9191 12 733 3 1234 7234 67 197 7 9

Radix Sort

- **Solution:** 9191 12 733 3 1234 7234 67 197 7 9

Step 2:

bucket 0: 3 7 9

bucket 1: 12

bucket 2:

bucket 3: 733 1234 7234

bucket 4:

bucket 5:

bucket 6: 67

bucket 7:

bucket 8:

bucket 9: 9191 197

3 7 9 12 733 1234 7234 67 9191 197

Radix Sort

- **Solution:** 3 7 9 12 733 1234 7234 67 9191 197

Step 3:

bucket 0: 3 7 9 12 67

bucket 1: 9191 197

bucket 2: 1234 7234

bucket 3:

bucket 4:

bucket 5:

bucket 6:

bucket 7: 733

bucket 8:

bucket 9:

3 7 9 12 67 9191 197 1234 7234 733

Radix Sort

- **Solution:** 3 7 9 12 67 9191 197 1234 7234 733

Step 4:

bucket 0: 3 7 9 12 67 197 733

bucket 1: 1234

bucket 2:

bucket 3:

bucket 4:

bucket 5:

bucket 6:

bucket 7: 7234

bucket 8:

bucket 9: 9191

3 7 9 12 67 197 733 1234 7234 9191





Class Activity

Q & A



Formative Assessment

