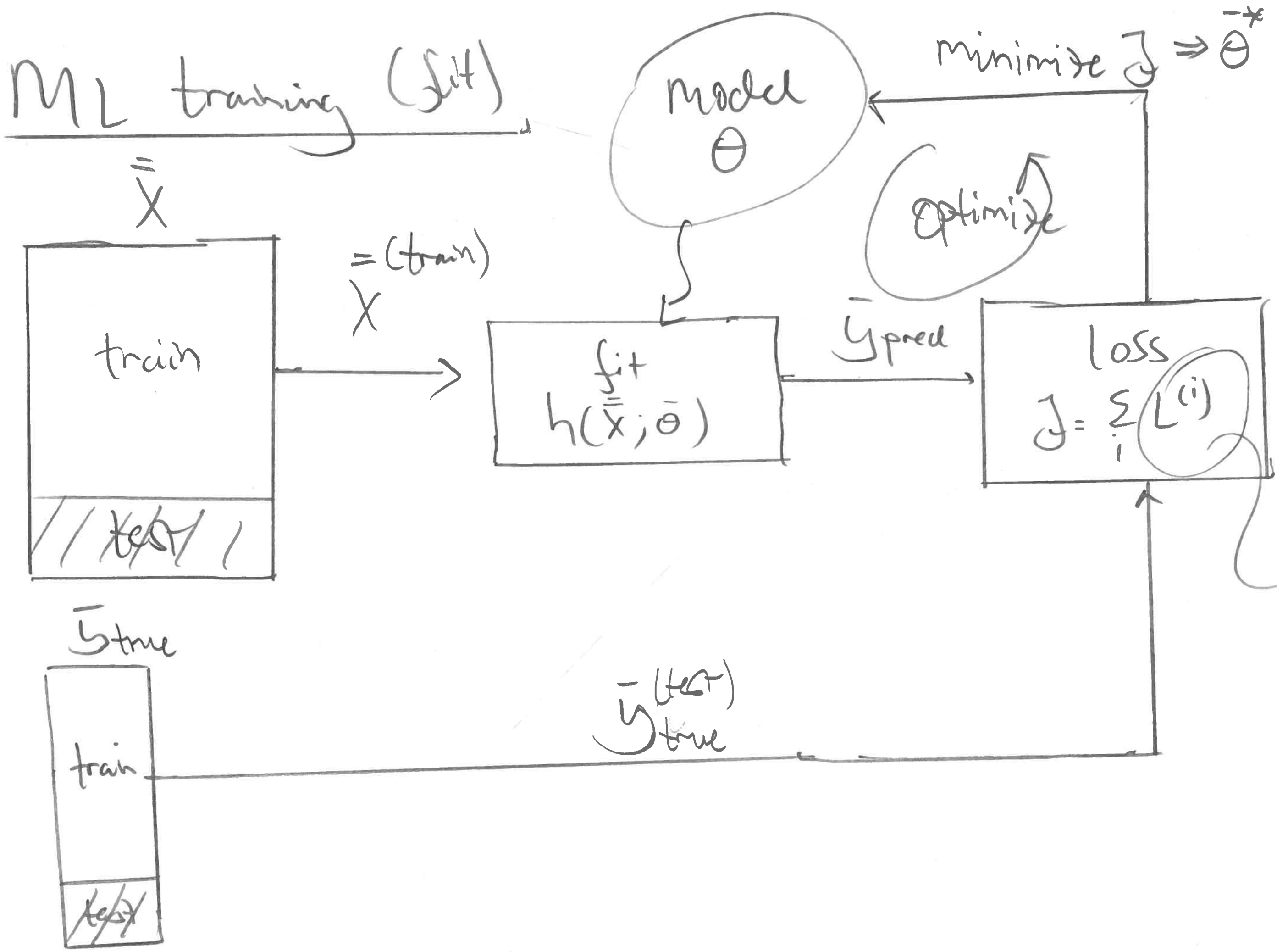
# ITMAL Notes

L02

E20

# ML training (fit)




$J = 42.0$

$J' = 41.0$

$J'' = 39.1$

$\vdots$

$J'''' = 2.42$

training

"difference"

$d(\bar{y}_{true}^{(i)}; y_{pred}^{(i)})$

2

# The Design Matrix

MMLS: $(m \times n)$, 1-indexed

$$\bar{\bar{X}}_{MMLS} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}$$

ML: $(m \times d)$ (Python: 0-indexed)

$$\bar{\bar{X}}_{ML} = \begin{bmatrix} \bar{x}^{(1)T} \\ \bar{x}^{(2)T} \\ \vdots \\ \bar{x}^{(m)T} \end{bmatrix} = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & \cdots & x_d^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \cdots & x_d^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{(m)} & x_2^{(m)} & \cdots & x_d^{(m)} \end{bmatrix}$$

$$\bar{y}_{ML} = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(m)} \end{bmatrix}$$

Design Matrix

Sometimes augmented: $(\bar{\bar{X}}, \bar{y})$

3

# Housing Design Matrix

Sample 1:

$d=4$

$$\vec{x}^{(1)} = \begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \\ \vdots \\ x_d^{(1)} \end{bmatrix} = \begin{bmatrix} -118 \\ 33{,}9 \\ 1416 \\ 38000 \end{bmatrix}$$

$x_1$: long / °

$x_2$: lat / °

$x_3$: inhab. / #

$x_4$: median income / \$

$y^{(1)} = 156\,000$

$y_{true}$: median housing val / \$

(4)

Sample 2

$\bar{x}^{(2)} = [\ \cdots$

$y^{(2)} =$

$\vdots$

Sample m

$\bar{x}^{(m)} = [\ \vdots$

$y^{(m)}$

Design Matrix

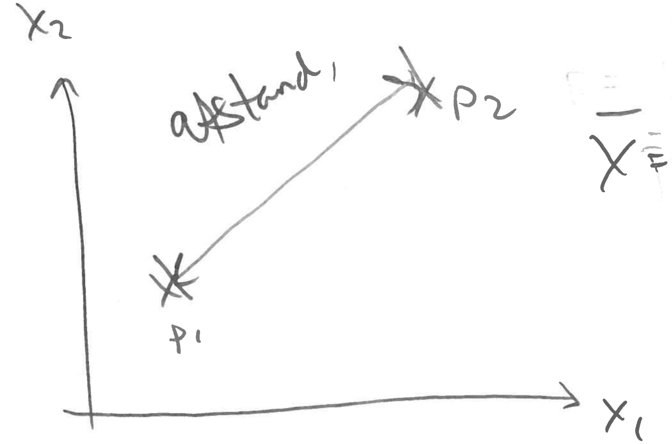
$$\bar{\bar{X}} = \begin{bmatrix} \bar{x}^{(1)T} \\ \bar{x}^{(2)T} \\ \vdots \\ x^{(m)T} \end{bmatrix} = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & \cdots & x_d^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \cdots & x_d^{(2)} \\ \vdots & & \ddots & \vdots \\ x_1^{(m)} & x_2^{(m)} & & x_d^{(m)} \end{bmatrix} \underset{d=4}{\overset{\text{eg}}{=}} \begin{bmatrix} -118 & 33.9 & 1416 & 38000 \\ \vdots & & & \end{bmatrix}$$

$d=4$ (Housing Data)

$$\bar{y}_{true} = \begin{bmatrix} y^{(1)}_{true} \\ y^{(2)}_{true} \\ \vdots \\ y^{(m)}_{true} \end{bmatrix} \underset{d=4}{=} \begin{bmatrix} 156000 \\ \vdots \end{bmatrix}$$

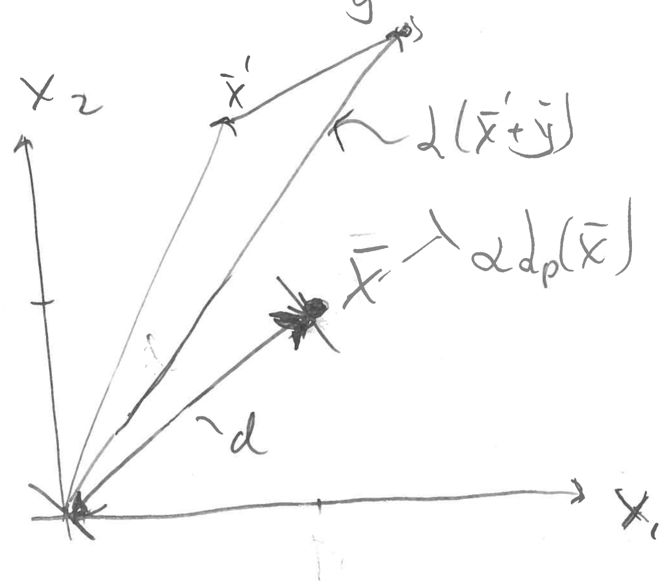
Whole garage augmented fun : $(\bar{\bar{X}} ; \bar{y})$

# Distance / Norms




$\bar{X} = \bar{p}_2 - \bar{p}_1$

$\Rightarrow$




$$d = \| \bar{p}_2 - \bar{p}_1 \|_2 = \| \bar{p} \|_2 = \sqrt{x_1^2 + x_2^2}$$

$\left( d_p : \| \cdot \|_p \right)$

afstands notation $\| \cdot \|_2$

Norm:
1) $d_p(\bar{x}) = \emptyset \Rightarrow \bar{x} = \emptyset$
2) $d_p(\bar{x} + \bar{y}) \leq d_p(\bar{x}) + d_p(y)$
3) $d_p(\alpha \bar{x}) = \alpha \, d_p(\bar{x})$

$\left\{ \begin{array}{l} F_2 \text{, accuracy} \neq \text{norm} \\ R^2 \text{ Coeff of determination} \neq \text{norm} \end{array} \right.$

⑥

# Euclidian Norm Quiz

$L_2 : \|\bar{X}\|_2 = \sqrt{\sum_i |x_i|^2}$

$Q_1 : \bar{X} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad \|\bar{X}\|_2 \stackrel{?}{=} \sqrt{|1|^2 + |3|^2} = \underline{\underline{\sqrt{10}}}$

linear algebra:

$L_2^2 : \|\bar{X}\|_2^2 = \bar{X}^T \cdot \bar{X}$

$Q_2 : \bar{X} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad \|X\|_2^2 = \bar{X}^T \cdot \bar{X} \stackrel{?}{=} \begin{bmatrix} 1 \\ 3 \end{bmatrix}^T \cdot \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$= 1 \cdot 1 + 3 \cdot 3 = \underline{\underline{10}}$

✓

# Binary Classification

Iris Setosa 'S'
Iris Versicolor 'V'

} 2. Classes ("binary")

⇒ Binary Classifier

Setosa ⇒ P
Versicolor != Setosa ⇒ N

## Confusion Matrix

| $y_{pred}$ \ $y_{true}$ | P | N |
|---|---|---|
| P | TP | FP (I) |
| N | FN(II) | TN |

(I) — cost I ; (type I error)

(II) — cost II (type II error)