



LORDS INSTITUTE OF ENGINEERING TECHNOLOGY

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Department of Science and Humanities

Academic Year: 2023-24

I-B.E. I-SEMESTER

QUESTION BANK

[SEE]

ENGINEERING PHYSICS

[U23PH101]

[Common for CSE, CSD, CIVIL and MECH]

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Note: A question bank is versatile and flexible FAQs that cover the entire syllabus of a subject. It is used by students and teachers for learning and assessment purposes only.



LORDS INSTITUTE OF ENGINEERING AND TECHNOLOGY
(UGC Autonomous)
SEE QUESTION BANK
ACADEMIC YEAR: 2023-24

Programme:	I- B.E. I-Semester	Branch:	CSE, CSD, MECH, CIVIL
Name of the Course:	ENGINEERING PHYSICS	Subject Code:	U23PH101

S. No.	UNIT-I SAQs	CO	BTL
1.	Summarize space lattice with a diagram.	CO1	BTL2
2.	What is primitive unit cell and non-primitive unit cell?	CO1	BTL1
3.	Draw SC and BCC unit cells.	CO1	BTL1
4.	What are Miller indices?	CO1	BTL1
5.	Define lattice plane with a diagram.	CO1	BTL1
6.	Sketch (111) plane.	CO1	BTL1
7.	State Bragg's law.	CO1	BTL1
8.	Write a short note on basis.	CO1	BTL1
9.	Calculate the longest wavelength that can be analyzed by rocksalt crystal of interplanar spacing $d=2.82 \text{ \AA}^0$ in the first order diffraction.	CO1	BTL3
10.	Classify the point defects.	CO1	BTL2
11.	When X-rays are diffracted by crystal planes whose inter planar spacing is 2 \AA and produces second order diffraction at 30° . Find wavelength of X-rays.	CO1	BTL1
12.	The length of a cubic crystal system (a) is 2.5 \AA . Find inter planar spacing (d) between (110) planes.	CO1	BTL1
13.	Distinguish between Schottky and Frenkel defects.	CO1	BTL2
14.	Recall lattice parameters.	CO1	BTL1
15.	Define inter planar spacing with the neat diagram.	CO1	BTL1
S. No.	UNIT-I LAQs	CO	BTL
16.	Derive an expression for inter planar spacing for cubic crystal systems.	CO1	BTL3
17.	Derive an expression for concentration of Schottky defects in the case of ionic crystals.	CO1	BTL3
18.	Explain 7 types of crystal systems.	CO1	BTL2
19.	Discuss the powder diffraction method to understand the structure of the crystals.	CO1	BTL2

20.	Explain the procedure to obtain the Miller indices of a given plane with one example.	CO1	BTL2
21.	Derive an expression for concentration of Frenkel defects in the case of ionic crystals.	CO1	BTL3
22.	Explain the experimental procedure of the powder diffraction method to understand the structure of the crystals.	CO1	BTL2
23.	Deduce an expression for inter planar spacing for cubic crystal systems.	CO1	BTL3
24.	Define point defect. Elaborate point defects with neat diagrams.	CO1	BTL2
25.	Derive an expression for concentration of Schottky defects in the case of ionic crystals.	CO1	BTL2
26.	Define the following: a) Space lattice b) Unit cell c) Lattice parameters d) Bravais lattices e) Lattice planes	CO1	BTL1
27.	Explain Bragg's law. Find the wavelength of X-rays when diffracted by $(1\ 1\ 1)$ planes of a cubic crystal whose lattice constant is $3.5\ \text{\AA}$ and produces second order diffraction at 45° .	CO1	BTL2
28.	What are Miller indices and write the procedure to obtain Miller indices of a given plane.	CO1	BTL1
29.	What is inter planar spacing, explain with a diagram. Find the inter planar spacing of a cubic crystal whose lattice constant is $1.74\ \text{\AA}$ and Miller indices are $(1\ 1\ 0)$.	CO1	BTL2
30.	Draw the planes (111) , (101) , (120) , (100) , (112) and $(\bar{1}, 1, 0)$	CO1	BTL1
S. No.	UNIT-II SAQs	CO	BTL
31.	Explain the spontaneous and stimulated emissions.	CO2	BTL2
32.	Write the characteristics of Laser.	CO2	BTL1
33.	Explain the population inversion mechanism.	CO2	BTL2
34.	The efficiency of Ruby laser is poor than He-Ne laser. Give the justification.	CO2	BTL2
35.	Find the wavelength of the GaAs semiconductor laser whose energygap is $1.44\ \text{eV}$.	CO2	BTL2
36.	Write the applications of lasers.	CO2	BTL1
37.	Explain the total internal reflection phenomena.	CO2	BTL2
38.	Outline the Numerical Aperture in optical fiber.	CO2	BTL1
39.	Explain the losses in optical fibers.	CO2	BTL2

40.	Write the applications of optical fibers.	CO2	BTL1
41.	Find the numerical aperture of an optical fiber whose refractive index of core and cladding are 1.55 and 1.50 respectively.	CO2	BTL2
42.	List the various pumping mechanisms.	CO2	BTL1
43.	Name the types of optical fiber based on refractive index profile.	CO2	BTL1
44.	Find the acceptance angle of an optical fiber whose refractive index of core and cladding are 1.55 and 1.50 respectively.	CO2	BTL2
45.	Define the acceptance angle in optical fiber.	CO2	BTL1
S. No.	UNIT-II LAQs	CO	BTL
46.	Describe the construction and working of Ruby laser.	CO2	BTL2
47.	Explain the construction and working of He-Ne laser.	CO2	BTL2
48.	Discuss the construction and working of semiconductor laser.	CO2	BTL2
49.	Derive an expression for the relations of Einstein's coefficients.	CO2	BTL3
50.	Distinguish between spontaneous and stimulated emissions.	CO2	BTL2
51.	Define population inversion. Write the characteristics of laser and mention the various applications of laser.	CO2	BTL2
52.	Explain the classification of optical fibers in detail.	CO2	BTL2
53.	Derive an expression for the Numerical Aperture of an optical fiber.	CO2	BTL2
54.	What is the principle of optical fibers? Explain various attenuation mechanisms in optical fibers.	CO2	BTL2
55.	Explain types of optical fibers based on refractive index profiles.	CO2	BTL2
56.	Explain the propagation of light through optical fiber by total internal reflection.	CO2	BTL2
57.	Derive an expression for the acceptance angle in optical fibers.	CO2	BTL3
58.	Discuss the principle of an optical fiber. Write applications of optical fibers.	CO2	BTL2
59.	Elaborate absorption, spontaneous emission and stimulated emissions with diagrams.	CO2	BTL2
60.	Discuss the losses in optical fibers.	CO2	BTL2
S. No.	UNIT-III SAQs	CO	BTL
61.	Write any two properties of wave function.	CO3	BTL1
62.	List any two properties of matter waves.	CO3	BTL1
63.	What is the physical significance of wave function?	CO3	BTL1

64.	Explain de-Broglie's wavelength.	CO3	BTL2
65.	Find the wavelength of an electron when accelerated by a potential of 1600 V.	CO3	BTL2
66.	What is the objective of Davisson-Germer experiment with electrons and which hypothesis is supported by this experiment.	CO3	BTL1
67.	Matter waves are not electromagnetic waves. Justify?	CO3	BTL2
68.	Find the wavelength of a proton moving with a velocity of 2200 m/s. (mass of the proton = 1.67×10^{-27} kg)	CO3	BTL2
69.	State Bloch's theorem.	CO3	BTL1
70.	Sketch energy band structures for conductors, semiconductors and insulators.	CO3	BTL1
71.	Write any two differences between intrinsic and extrinsic semiconductors.	CO3	BTL1
72.	Draw the I-V characteristics of P-N junction diode.	CO3	BTL1
73.	List any two applications of Hall effect.	CO3	BTL2
74.	Define Solar cell and mention the applications of solar energy.	CO3	BTL1
75.	Write down the various applications of LED.	CO3	BTL1
S. No.	UNIT-III LAQs	CO	BTL
76.	Derive an expression for the Schrodinger's time independent wave equation.	CO3	BTL3
77.	What are Matter waves? Write properties of matter waves. Explain the physical significance of wavefunction.	CO3	BTL2
78.	Derive an expression for the Schrodinger's time dependent wave equation.	CO3	BTL3
79.	Derive an expression for the energy values for a particle in 1-D box.	CO3	BTL3
80.	Derive an expression for the energy values and wavefunctions for a particle in 1-D box.	CO3	BTL3
81.	Apply Schrodinger's time independent wave equation to obtain energy values of a particle in 1-D box.	CO3	BTL3
82.	What is de-Broglie's wavelength? Find the velocity and kinetic energy of an electron whose wavelength is 1.66×10^{-10} m.	CO3	BTL2
83.	What are salient features of Kronig-Penney model? Explain the formation of allowed and forbidden energy bands based on Kronig-Penney model.	CO3	BTL2
84.	Classify the materials into conductors, semiconductors and insulators based on the formation of energy bands and write any four properties for each.	CO3	BTL2
85.	Derive an expression for intrinsic carrier concentration.	CO3	BTL3
86.	Discuss intrinsic and extrinsic semiconductors with neat diagrams.	CO3	BTL2

87.	Explain the formation of p-n junction diode along with its I-V characteristics.	CO3	BTL2
88.	What is thermistor, explain its characteristics with neat diagrams.	CO3	BTL2
89.	Explain Hall effect and calculate the Hall coefficient. List few applications of Hall effect.	CO3	BTL2
90.	Define Hall coefficient, Carrier concentration and mobility. The Hall coefficient of certain silicon specimen was found to be $-7.35 \times 10^{-5} \text{ m}^3 \text{C}^{-1}$ from 100 K to 400 K. Determine the nature of the semiconductor. Further, the electrical conductivity was found to be $200 \Omega^{-1} \text{m}^{-1}$. Calculate the concentration and mobility of charge carriers.	CO3	BTL2
S. No.		CO	BTL
91.	List the types of magnetic materials.	CO4	BTL1
92.	Summarize magnetic domains.	CO4	BTL2
93.	Sketch Hysteresis curve.	CO4	BTL1
94.	Give any two differences between soft and hard ferromagnetic materials.	CO4	BTL2
95.	Write any two applications of ferrites.	CO4	BTL1
96.	List any two general properties of superconductors.	CO4	BTL1
97.	Define relative permeability.	CO4	BTL1
98.	Explain high Tc superconductors.	CO4	BTL2
99.	Define SQUIDs with suitable example.	CO4	BTL1
100.	What are the critical conditions of a superconductor?	CO4	BTL1
101.	Explain Meissner's effect.	CO4	BTL2
102.	Write any two reasons to show superconductors are dielectric materials.	CO4	BTL1
103.	A superconducting Tin has critical temperature of 3.7 K and critical magnetic field of 0.306 T. Find the critical magnetic field at 2K.	CO4	BTL2
104.	Write any four applications of superconductors.	CO4	BTL1
105.	Give the relation between relative permeability and magnetic susceptibility.	CO4	BTL2
S. No.		CO	BTL
106.	Classify magnetic materials in to dia, para, ferro, antiferro and ferri magnetic materials.	CO4	BTL2
107.	Explain the formation of domains. Discuss the Hysteresis curve.	CO4	BTL2
108.	Distinguish between hard and soft magnetic materials.	CO4	BTL2
109.	What is superconductivity. Discuss the general properties of superconductors. Write applications of superconductors.	CO4	BTL2

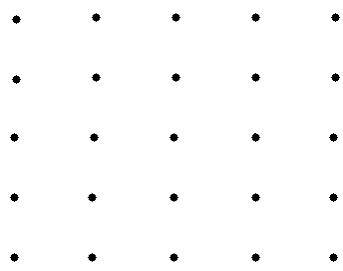
110.	Explain Ferrites and write applications of Ferrites.	CO4	BTL2
111.	Explain Meissner's effect. Distinguish Type-I and Type-II superconductors.	CO4	BTL2
112.	Describe BCS Theory.	CO4	BTL2
113.	Explain high Tc superconductors. Write applications of superconductors.	CO4	BTL2
114.	Discuss Meissner's effect. Write general properties of superconductors.	CO4	BTL2
115.	Elaborate BCS theory to explain superconductivity.	CO4	BTL2
116.	Discuss Meissner's effect. Write applications of superconductors	CO4	BTL2
117.	What are the critical conditions of a superconductors. Explain high Tc superconductors.	CO4	BTL2
118.	Explain concept of domains. Discuss the Hysteresis in ferromagnetic materials.	CO4	BTL2
119.	Distinguish between dia, para and ferro magnetic materials.	CO4	BTL2
120.	Write any six general properties of superconductors. Explain Meissner's effect.	CO4	BTL2
S. No.	UNIT-V SAQs	CO	BTL
121.	Name the types of polarizations.	CO5	BTL1
122.	Define dielectric constant.	CO5	BTL1
123.	Define space-charge polarization with neat diagram.	CO5	BTL1
124.	Draw the structure of Barium titanate.	CO5	BTL1
125.	Define orientational polarization with neat diagram.	CO5	BTL1
126.	Explain the polarization mechanism in dielectrics.	CO5	BTL2
127.	Calculate the electronic polarizability of neon. The radius of neon atom is 0.158 nm. (Given, Permittivity of free space = $8.854 \times 10^{-12} \text{ Fm}^{-1}$)	CO5	BTL2
128.	What are nanomaterials?	CO5	BTL2
129.	Mention the names of the techniques used for the preparation of Nanomaterials.	CO5	BTL1
130.	Write a short note on Top-down method.	CO5	BTL1
131.	Mention some effects of size reduction of nanoparticles.	CO5	BTL1
132.	Define quantum well.	CO5	BTL1
133.	Write any two applications of Ferroelectrics.	CO5	BTL1
134.	What is meant by quantum confinement?	CO5	BTL2
135.	Write the applications of nanomaterials.	CO5	BTL1
S. No.	UNIT-V LAQs	CO	BTL

136.	Obtain an expression for electronic polarizability in terms of radius of atom.	CO5	BTL3
137.	Elaborate Ferroelectricity with its properties. Mention applications of Ferroelectrics.	CO5	BTL2
138.	Derive an expression for ionic polarizability.	CO5	BTL3
139.	Explain the types of polarizations with their polarizability equations.	CO5	BTL2
140.	Define the following terms a) Dielectric constant b) Polarization c) Permittivity d) Ferro electricity	CO5	BTL1
141.	Define the types of polarization and explain Barium titanate.	CO5	BTL2
142.	Derive an expression for electronic polarizability in terms of radius of atom.	CO5	BTL3
143.	Explain the main factors which cause the properties of nanomaterials to differ significantly from other materials.	CO5	BTL2
144.	Illustrate Top-down Ball milling approach for the preparation of nanomaterials.	CO5	BTL2
145.	Explain Sol-gel Bottom-up approach for the preparation of nanomaterials.	CO5	BTL2
146.	Describe the CVD Bottom-up approach for the preparation of nanoparticles.	CO5	BTL2
147.	Explain how oxide nanoparticles can be obtained by Bottom-up Sol-gel method.	CO5	BTL2
148.	Explain Ball milling Top-down approach for the preparation of nanomaterials.	CO5	BTL2
149.	Discuss the various physical, electronic, magnetic, mechanical and chemical properties of nanomaterials.	CO5	BTL2
150.	Write a note on nanomaterials and discuss their applications in various fields.	CO5	BTL2

UNIT-I (SAQs)

Question(1): Summarize space lattice with a diagram.

Solution: A lattice is a translationally periodic set of points. We can have 1D (1-dimension), 2D and 3D (or even higher dimensional) lattices. Usually the 3D lattice is called a **space lattice**. A lattice is a series of points that are arranged in a distinct pattern.



Question(2): What is a primitive unit cell and non-primitive unit cell?

Solution: Primitive unit cell: All fundamental unit cell which contain one lattice point is called a primitive unit cell. In a primitive unit cell, constituents particles are present only at the corners of the unit cell.

Non Primitive unit cell:

The unit cell associated with more than one lattice point are referred as non primitive unit cel. In non primitive unit cell, the constituent particles are present not only at the corner, but also in centers of a face and body of the unit cell.

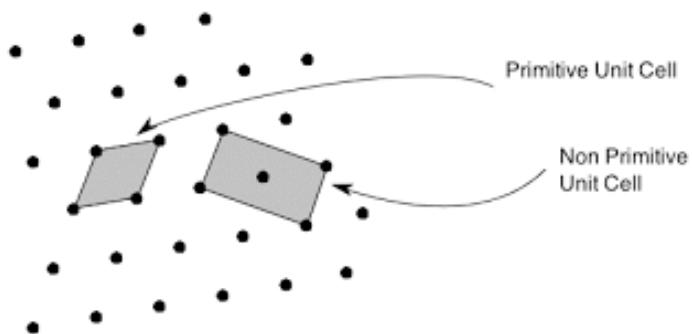


Figure 1: Primitive and Non Primitive unit cells.

Question(3): Draw SC and BCC unit cells.

Solution:

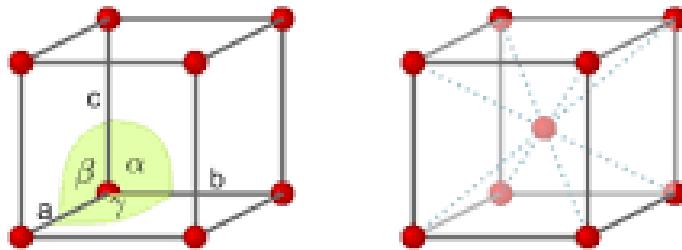


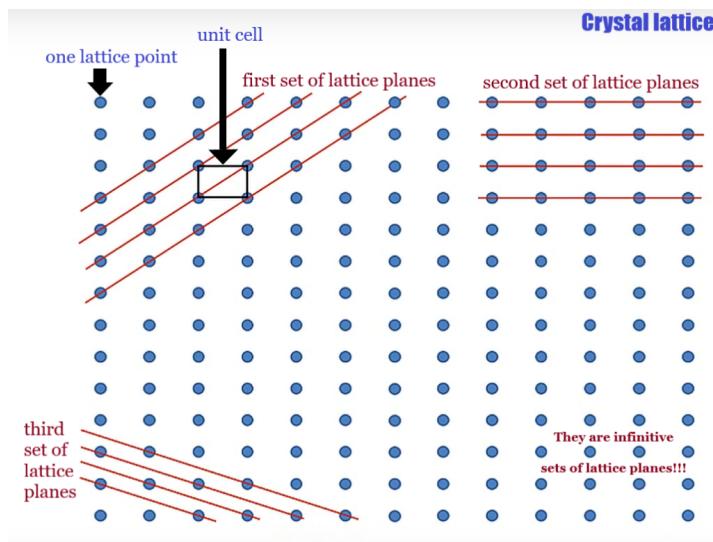
Figure 2: SC and BCC

Question(4): What are Miller indices?

Solution: In order to indicate lattice planes for crystal analysis. **Miller** suggested a method of indicating the orientation of a plane. Miller indices is a group of three numbers that indicates the orientation of a plane or set of parallel planes of atoms in a crystal.

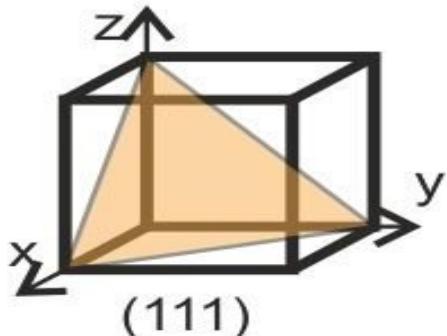
Question(5): Define lattice plane with a diagram.

Solution: A plane passing through various lattice points in a crystal is known as the lattice plane. On in other words, planes which contain a large concentration of atoms are known as lattice planes. Various types of lattice planes in a crystal lattice are drawn.



Question(6): Sketch (111) plane.

Solution:



Miller indices : (1 1 1)

Reciprocal : $\frac{1}{1} \frac{1}{1} \frac{1}{1}$

Intercept : 1 1 1

So the plane (1 1 1) will cut all the axes x y and z .

Question(7): State Bragg's law

Solution: According to the Bragg's law the path difference should be an integral multiple of wavelength of incident X-ray

$$2d \sin \theta = n\lambda$$

- d : Interplanar spacing (perpendicular distance between two planes)
- λ : Wavelength of incident X-ray
- θ : Bragg angle/Glancing angle/Incident angle
- n : Order of diffraction

Question(8): Write a short note on basis.

Solution: The crystal structure is formed by associating every lattice point with an assembly of atoms or molecules or ions, which are identical in composition, arrangement and orientation, is called as the basis. Basis is a collection

of atoms in particular fixed arrangement in space. We could have a basis of a single atom as well as a basis of a complicated but fixed arrangement of hundreds of atoms.

Question(9): Calculate the longest wavelength that can be analyzed by rocksalt crystal of interplanar spacing $d = 2.82 \text{ \AA}$ in the first order diffraction.

Solution: Given

$$\begin{aligned}d &= 2.82 \text{ \AA} \\n &= 1 \text{ (first order)}\end{aligned}$$

Using Bragg's law ($2d \sin \theta = n\lambda$)

$$\begin{aligned}\lambda &= \frac{2d \sin \theta}{n} \\&\lambda = 2d \sin \theta\end{aligned}$$

For longest wavelength ($\sin \theta)_{\max} = 1$

$$\begin{aligned}\lambda_{\max} &= 2d \\&\lambda_{\max} = 2 \times 2.82 \text{ \AA} \\&\lambda_{\max} = 5.64 \text{ \AA}\end{aligned}$$

Question(10): Classify point defects.

Solution: We can classify the point defects as follows:

- (i) Vacancy defect
- (ii) Interstitial defect
- (iii) Schottky defect
- (iv) Frenkel defect

Question(11): When X-rays are diffracted by crystal planes whose interplanar spacing is 2 \AA and produces second order diffraction at 30° . Find wavelength of X-rays.

Solution: Given

$$\begin{aligned}\theta &= 30^\circ \\ d &= 2 \text{ \AA} \\ n &= 2 \text{ (second order)}\end{aligned}$$

Using Bragg's law ($2d \sin \theta = n\lambda$)

$$\begin{aligned}2d \sin 30 &= 2\lambda \\ 2 \times d \times \frac{1}{2} &= 2\lambda \\ \lambda &= \frac{d}{2}\end{aligned}$$

Thus, the wavelength of X-rays will be $\lambda = 1 \text{ \AA}$

Question (12): The length of a cubic crystal (a) is 2.5 \AA . Find inter planar spacing (d) between $(1 \ 1 \ 0)$ planes.

Solution:

$$\begin{aligned}\text{Given : length of cubic crystal } a &= 2.5 \text{ \AA} \\ (h \ k \ l) &= (1 \ 1 \ 0)\end{aligned}$$

$$\begin{aligned}d &= \frac{a}{\sqrt{h^2 + k^2 + l^2}} \\ &= \frac{a}{\sqrt{2}} \\ &= \frac{2.5}{\sqrt{2}} \\ d &= 1.76 \text{ \AA}\end{aligned}$$

Question(13): Distinguish between Schottky and Frenkel defects.

Solution:

Schottky defect

1. A pair of vacancy defects is known as schottky defect.
2. Density of the crystal will decrease.

Frenkel defect

1. A pair of vacancy and interstitial defect is known as Frenkel defect.
2. Density of the crystal remains the same.

Question(14): Recall lattice parameters.

Solution: Lattice parameters are defined as a set of 6 parameters (in 3D)

- The lengths of the unit cell along the x , y , z directions are defined as a , b , c , respectively.
- The angles between these axes are defined by α , β and γ .

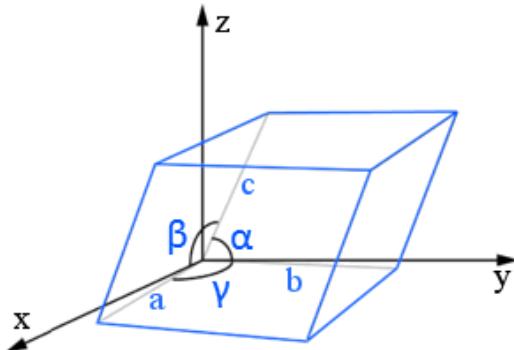


Figure 3: A unit cell with lattice parameters

Question(15): Define inter planar spacing with neat diagram.

Solution: The interplanar spacing d between adjacent planes having Miller indices (hkl) is defined as the distance between first such plane from a parallel plane passing through the origin. The interplanar spacing is given by

$$d = \frac{1}{\sqrt{\left(\frac{h}{a}\right)^2 + \left(\frac{k}{b}\right)^2 + \left(\frac{l}{c}\right)^2}}$$

where $(h k l)$ are the Miller indices of the plane ABC and a , b and c are the primitive vectors of x , y and z axis, respectively.

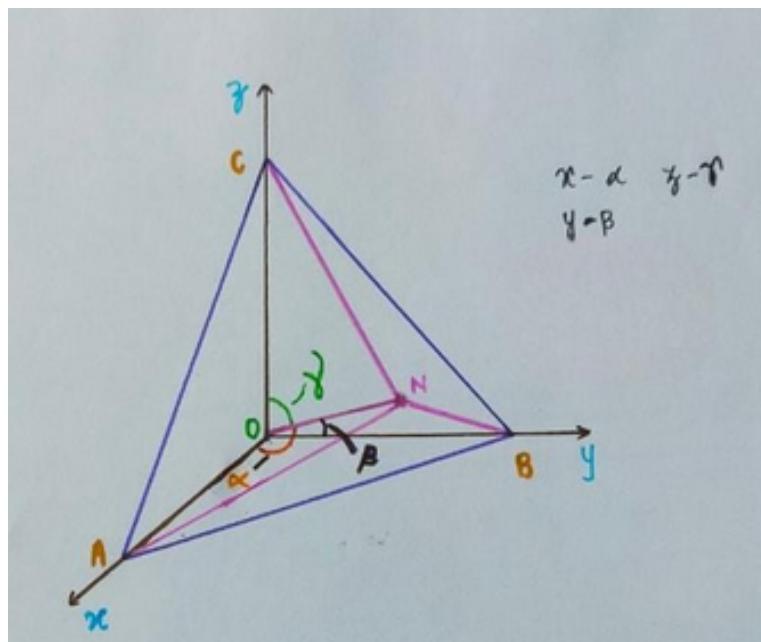


Figure 4: Lattice plane in a crystal with x, y and z coordinates system.

UNIT-I (LAQs)

Question(16): Derive an expression for inter planar spacing for cubic crystal systems.

Solution: Let $(h k l)$ be the Miller indices of the plane ABC . If the a , b and c are the primitive vectors of x , y and z axis, respectively, then the plane ABC intersects the x -axis at a/h , y -axis at b/k , z -axis at c/l .

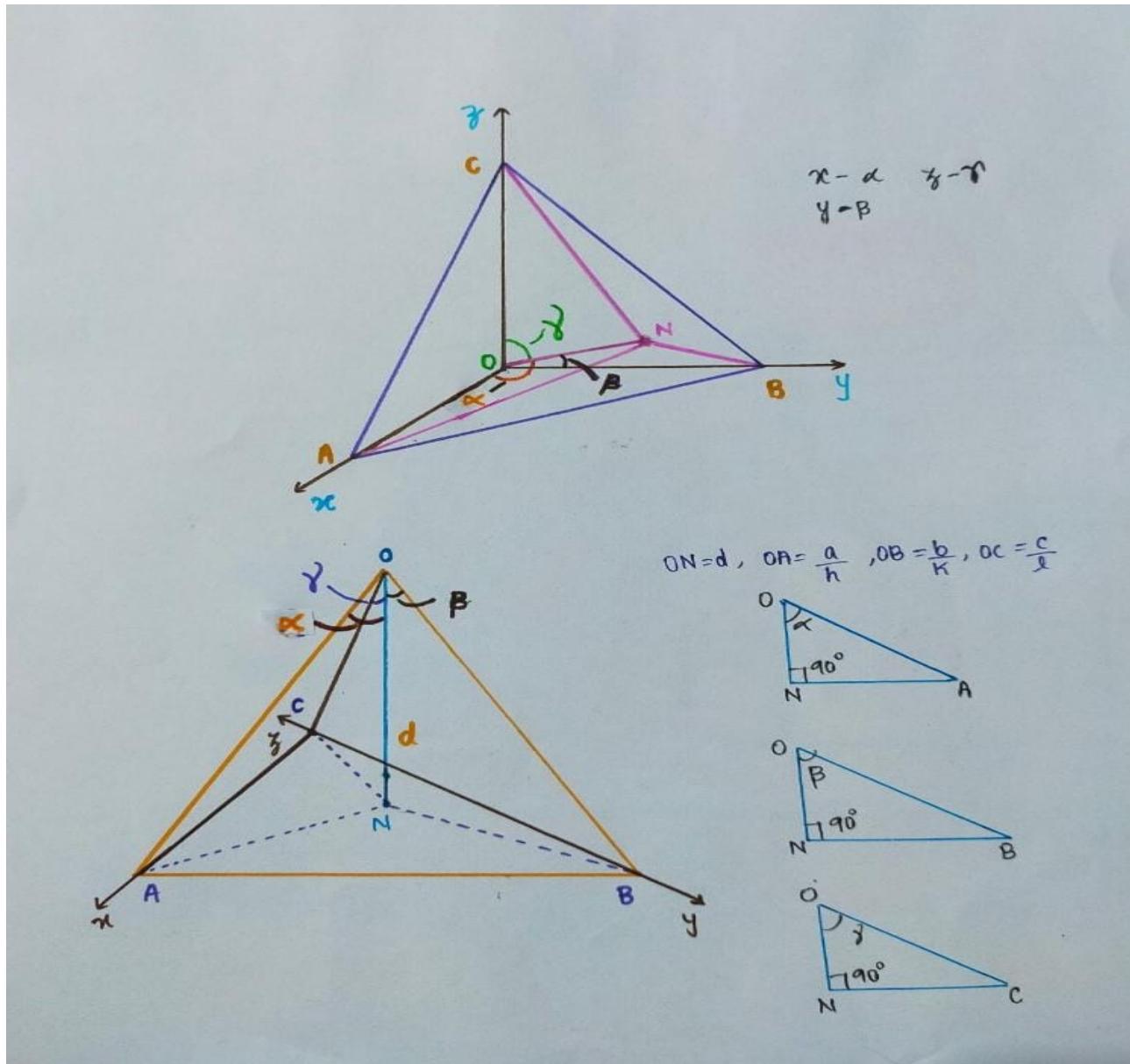


Figure 5: Lattice plane in a crystal with x , y and z coordinates system.

To derive the interplanar spacing, consider a reference plane at origin (O), par-

parallel to plane ABC. Draw a normal ON from the reference plane on the plane ABC, then the perpendicular distance d will be interplanar distance.

In $\triangle ONA$, we can write

$$\begin{aligned}\cos \alpha &= \frac{ON}{OA} \\ &= \frac{d}{a/h} \\ &= \frac{dh}{a}\end{aligned}\tag{1}$$

In $\triangle ONB$, we can write

$$\begin{aligned}\cos \beta &= \frac{ON}{OB} \\ &= \frac{d}{b/k} \\ &= \frac{dk}{b}\end{aligned}\tag{2}$$

In $\triangle ONC$, we can write

$$\begin{aligned}\cos \gamma &= \frac{ON}{OC} \\ &= \frac{d}{c/l} \\ &= \frac{dl}{c}\end{aligned}\tag{3}$$

Using the property of directional cosine, we get

$$\begin{aligned}\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma &= 1 \\ \left(\frac{dh}{a}\right)^2 + \left(\frac{dk}{b}\right)^2 + \left(\frac{dl}{c}\right)^2 &= 1 \\ d^2 \left[\left(\frac{h}{a}\right)^2 + \left(\frac{k}{b}\right)^2 + \left(\frac{l}{c}\right)^2 \right] &= 1 \\ d^2 &= \frac{1}{\left[\left(\frac{h}{a}\right)^2 + \left(\frac{k}{b}\right)^2 + \left(\frac{l}{c}\right)^2\right]} \\ d &= \frac{1}{\sqrt{\left(\frac{h}{a}\right)^2 + \left(\frac{k}{b}\right)^2 + \left(\frac{l}{c}\right)^2}}\end{aligned}\tag{4}$$

Finally, we can write the expression of perpendicular distance between two parallel planes as follows

$$d = \frac{1}{\sqrt{\left(\frac{h}{a}\right)^2 + \left(\frac{k}{b}\right)^2 + \left(\frac{l}{c}\right)^2}} \quad (5)$$

For cubic system $a = b = c$, the above equation becomes

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \quad (6)$$

This expression is interplanar spacing for cubic system with the given value of Miller indices $(h k l)$ and lattice constant a .

Question(17): Derive an expression for concentration of Schottky defects in the case of ionic crystals.

Solution: In ionic crystal, when an anion is completely displaced from its regular position, it creates a vacancy. To get the charge neutrality, a cation is also displaced from its regular position and hence a pair of vacancies is created.

The number of ways in which the n -pair of vacancies are formed in the regular lattice sites of N ions are given by

$$\begin{aligned} w &= \frac{N!}{n!(N-n)!} \frac{N!}{n!(N-n)!} \\ &= \left[\frac{N!}{n!(N-n)!} \right]^2 \end{aligned} \quad (7)$$

We can now write the change in entropy as

$$\Delta S = k_B \ln w \quad (8)$$

Where k_B is Boltzmann constant. Now, we can write

$$\begin{aligned} \Delta S &= k_B \ln \left[\frac{N!}{n!(N-n)!} \right]^2 \\ &= 2k_B \ln \left[\frac{N!}{n!(N-n)!} \right] \\ &= 2k_B [\ln N! - \ln n!(N-n)!] \\ &= 2k_B [\ln N! - \{\ln n! + \ln(N-n)!\}] \\ &= 2k_B [\ln N! - \ln n! - \ln(N-n)!] \end{aligned} \quad (9)$$

Where we have used the following formulas $[\ln a^2 = 2 \ln a, \ln(\frac{a}{b}) = \ln a - \ln b, \ln(ab) = \ln a + \ln b]$. Using Stirling formula

$$\ln x! = x \ln x - x \quad (10)$$

The above Eq.(9) can be written as

$$\begin{aligned} \Delta S &= 2k_B [\{N \ln N - N\} - \{n \ln n - n\} - \{(N-n) \ln(N-n) - (N-n)\}] \\ &= 2k_B [N \ln N - N - n \ln n + n - (N-n) \ln(N-n) + (N-n)] \\ &= 2k_B [N \ln N - n \ln n - (N-n) \ln(N-n)] \end{aligned} \quad (11)$$

Let E_s be the energy required to form a pair of vacancy. Then the change in the internal energy of the crystal when n -pair of vacancies are formed within it, is given by

$$\Delta U = nE_s \quad (12)$$

Now, we can define the free energy of the crystal as follows:

$$F = \Delta U - T\Delta S \quad (13)$$

After substituting the values of ΔU and ΔS , the free energy of the crystal can be written as

$$F = nE_s - 2k_B T [N \ln N - n \ln n - (N-n) \ln(N-n)] \quad (14)$$

Now, we will calculate the partial derivative of F with respect to n

$$\begin{aligned} \frac{\partial F}{\partial n} &= E_s - 2k_B T \left[0 - \left\{ n \frac{1}{n} + \ln n \times 1 \right\} \right. \\ &\quad \left. - \left\{ (N-n) \frac{1}{(N-n)} (-1) + \ln(N-n)(0-1) \right\} \right] \\ &= E_s - 2k_B T [-1 - \ln n + 1 + \ln(N-n)] \\ &= E_s - 2k_B T [\ln(N-n) - \ln n] \\ &= E_s - 2k_B T \ln \left(\frac{N-n}{n} \right) \end{aligned} \quad (15)$$

Here, we have used $\frac{\partial(uv)}{\partial x} = u \frac{\partial v}{\partial x} + v \frac{\partial u}{\partial x}$. Now, as we know that at equilibrium temperature the free energy should be minimum. Thus, for free energy to be minimum, the first derivative of free energy should be zero

$$\frac{\partial F}{\partial n} = 0 \quad (16)$$

$$\begin{aligned}
E_s - 2k_B T \ln \left(\frac{N-n}{n} \right) &= 0 \\
E_s &= 2k_B T \ln \left(\frac{N-n}{n} \right) \\
\frac{E_s}{2k_B T} &= \ln \left(\frac{N-n}{n} \right) \\
-\frac{E_s}{2k_B T} &= \ln \left(\frac{n}{N-n} \right) \\
\frac{n}{N-n} &= e^{-\frac{E_s}{2k_B T}} \\
n &= (N-n)e^{-\frac{E_s}{2k_B T}}
\end{aligned} \tag{17}$$

Now, we can take $(N-n) \approx N$, because $N \gg n$. Under this condition, the above equation becomes

$$n = Ne^{-\frac{E_s}{2k_B T}} \tag{18}$$

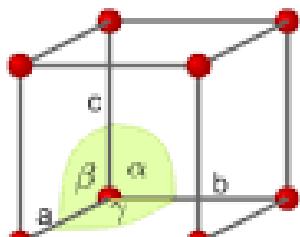
This is the concentration of Schottky defect or number of Schottky defects in ionic crystal.

Question(18): Explain 7 types of crystal systems.

Solution: Depending upon the geometric considerations, we can group the crystal into seven crystal systems. In these 7 crystal systems, each system is characterized by different values of a, b, c and α, β, γ .

1. Cubic Crystal System:

- In this system $a = b = c$ and $\alpha = \beta = \gamma = 90^\circ$.
- **Examples:** Diamond, Si, Au, Cu, Ag, NaCl etc.



primitive

Figure 6: Cubic crystal system

2. Tetragonal Crystal System:

- In this crystal system $a = b \neq c$ and $\alpha = \beta = \gamma = 90^\circ$.
- **Examples:** White tin, SnO_2 , TiO_2 etc.

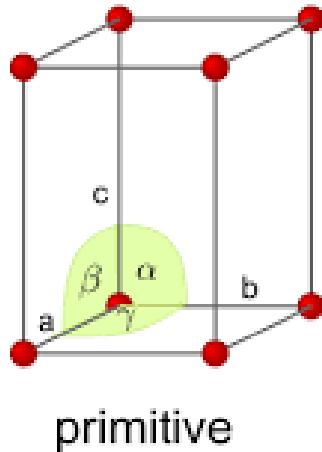


Figure 7: Tetragonal crystal system

3. Orthorhombic Crystal System:

- In this crystal system $a \neq b \neq c$ and $\alpha = \beta = \gamma = 90^\circ$.
- **Examples:** Cerussite (PbCO_3), Celestine (SrSO_4), Carnalite ($\text{KCl} \cdot \text{MgCl}_2 \cdot 6\text{H}_2\text{O}$) etc.

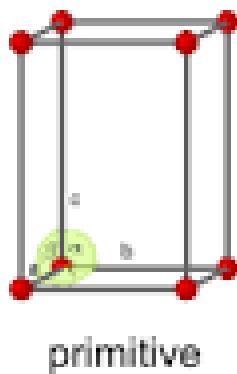


Figure 8: Orthorhombic crystal system

4. Monoclinic Crystal System:

- In this crystal system $a \neq b \neq c$ and $\alpha = \gamma = 90^\circ \neq \beta$.
- **Examples:** $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$, $\text{K}_2\text{MgSO}_4 \cdot 6\text{H}_2\text{O}$ etc.

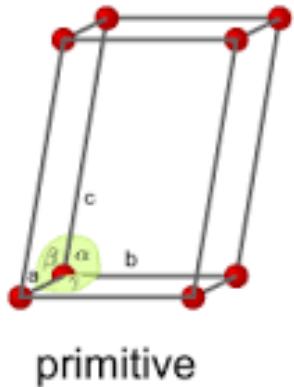


Figure 9: Monoclinic crystal system.

5. Triclinic Crystal System:

- In this crystal system $a \neq b \neq c$ and $\alpha \neq \beta \neq \gamma \neq 90^\circ$.
- **Examples:** Potassium dichromate ($\text{K}_2\text{Cr}_2\text{O}_7$), Albite ($\text{NaAlSi}_3\text{O}_8$), Copper Sulfate Pentahydrate ($\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$) etc.

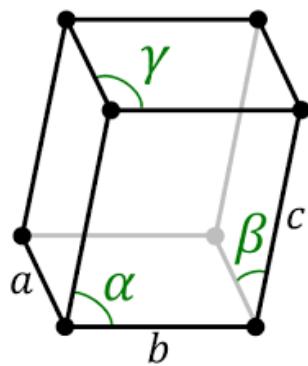


Figure 10: Triclinic crystal system.

6. Trigonal (Rhombohedral) Crystal System:

- In this crystal system $a = b = c$ and $\alpha = \beta = \gamma \neq 90^\circ < 120^\circ$.

- Examples: Calcite (CaCO_3), As, Sb, Bi etc.

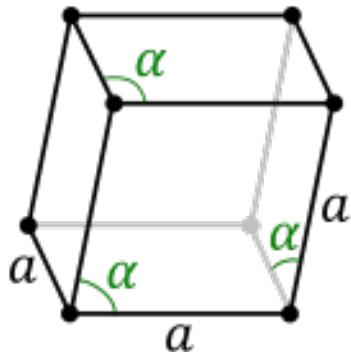


Figure 11: Trigonal crystal system.

7. Hexagonal Crystal System:

- In this crystal system $a = b \neq c$ and $\alpha = \beta = 90^\circ, \gamma = 120^\circ$.
- Examples: Quartz, Mg, Cd, Zn etc.

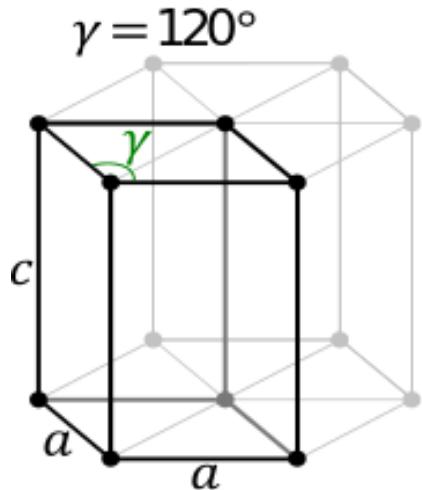


Figure 12: Hexagonal crystal system.

Question(19): Discuss the powder diffraction method to understand the structure of crystals.

Solution:

This is the experimental X-ray diffraction method which is used to study the crystal structures of polycrystalline type. This method is also known as **Debye-Scherrer method**.

Experimental setup of powder diffraction method:

- The experimental setup consists of cylindrical strip having a photographic film in its inner side. This will work as a camera and this camera is known as the Debye-Scherrer camera.
- The sample in the form of the powder taken in a fine glass tube is kept at the center of the cylinder.
- A monochromatic beam of X-ray is allowed to incident on the powder crystal as shown in Fig.

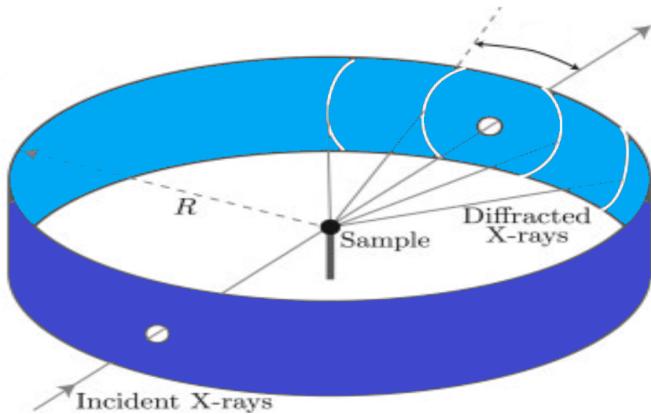


Figure 13: Experimental setup of powder diffraction method.

- The lattice planes in the crystal will have the different orientations.
- When monochromatic X-ray of one fixed wavelength (λ) is incident on the powder (sample), the crystals having orientation angles θ , which are satisfying the Bragg's condition ($2d \sin \theta = n\lambda$) for a particular value of d , will lie in a cone.
- The crystals having orientation angles θ for different sets of d form cones of different radii.
- The photographic film consists of concentric circular arcs as shown in Fig.

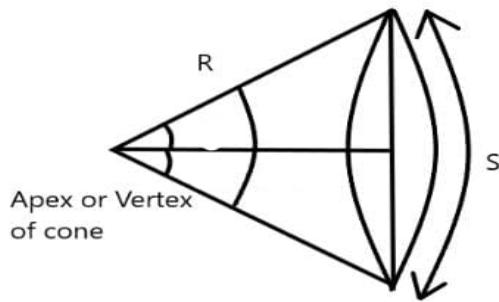
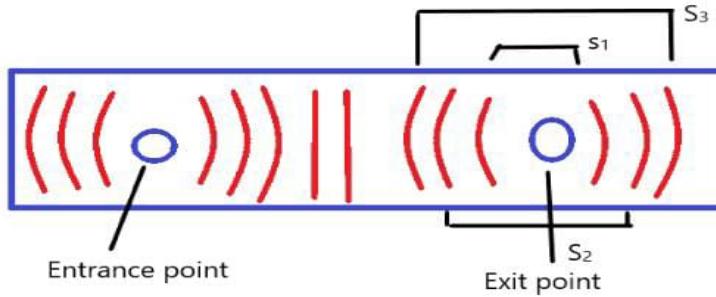


Figure 14: (a) Diffraction pattern on photographic film (b) Geometry of cone

- From above Fig, we can write

$$4\theta(\text{in Radian}) = \frac{S}{R} \quad (19)$$

For different cones, we can write $4\theta_1 = \frac{S_1}{R}$, $4\theta_2 = \frac{S_2}{R}$, $4\theta_3 = \frac{S_3}{R}$ and so on

We can rewrite Eq.(19) as

$$\begin{aligned}\theta(\text{in Degree}) &= \frac{S}{4R} \times \frac{180}{\pi} \\ \theta(\text{in Degree}) &= \frac{S}{4R} \times 57.3\end{aligned}$$

If we take radius of the cylinder $R = 57.3 \text{ nm}$, then

$$\theta(\text{in Degree}) = \frac{S}{4} \quad (20)$$

- Thus, we can obtain the θ value from the measured value of S . Since the wavelength λ is known. So with the help of all these, we can calculate the value of interplanar spacing d . The interplanar spacing d will be helpful to estimate the crystal structure.

Question(20): Explain the procedure to obtain the Miller indices of a given plane with one example.

Solution: In order to indicate lattice planes for crystal analysis. Miller suggested a method of indicating the orientation of a plane.

Miller indices is a group of three numbers that indicates the orientation of a plane or a set of parallel planes of atoms in a crystal.

Procedure of finding Miller indices:

- Find the intercepts of the desired planes on the three coordinates axes (means x , y , z axes). Let these intercepts be (pa, qb, rc) .
- Divide these intercepts with their respective primitive vectors $(\frac{pa}{a}, \frac{qb}{b}, \frac{rc}{c}) = (p, q, r)$.
- Take the ratio of the reciprocals of these numbers:
 $\frac{1}{p} : \frac{1}{q} : \frac{1}{r}$.
- Convert these reciprocals into whole numbers by multiplying each with their L.C.M. to get the smallest whole number.
- These smallest whole numbers give the Miller indices $(h k l)$.

Example: Given intercepts on x , y z axes are 2, 3, 2, respectively. The miller indices is calculated by doing the following steps:

$$\begin{aligned}
 \text{Intercepts} &= 2 \ 3 \ 2 \\
 \text{Reciprocal} &= \frac{1}{2} \ \frac{1}{3} \ \frac{1}{2} \\
 \text{Ratio of reciprocal} &= \frac{1}{2} : \frac{1}{3} : \frac{1}{2} \\
 \text{Multiply with L.C.M.} &= \frac{6}{2} : \frac{6}{3} : \frac{6}{2} \\
 \text{Miller indices} &= (3 \ 2 \ 3)
 \end{aligned}$$

Question(21): Derive an expression for concentration of concentration of Frenkel defects in the case of ionic crystals.

Solution: When an atom (or) ion is completely displaced from its regular position, it creates a vacancy in that position. To get charge neutrality, this atom (or) ion occupies interstitial space (or) void space in the regular lattice, hence a vacancy and interstitial are formed.

The number of ways in which the n -pair of vacancies are formed in the regular lattice sites of N ions having N_i interstitial sites can be written as

$$w = \frac{N!}{(N-n)!n!} \frac{N_i!}{(N_i-n)!n!} \quad (21)$$

We can now write the entropy

$$\Delta S = k_B \ln w \quad (22)$$

Where k_B is Boltzmann constant ($k_B = 1.38 \times 10^{-23}$ Joule/Kelvin). Using above equations, we can write

$$\begin{aligned} \Delta S &= k_B \ln \left[\frac{N!}{(N-n)!n!} \frac{N_i!}{(N_i-n)!n!} \right] \\ &= k_B \left[\ln \frac{N!}{(N-n)!n!} + \ln \frac{N_i!}{(N_i-n)!n!} \right] \end{aligned} \quad (23)$$

We can write

$$\begin{aligned} \ln \frac{N!}{(N-n)!n!} &= [\ln N! - \ln (N-n)! n!] \\ &= [\ln N! - \{\ln(N-n)! + \ln n!\}] \\ &= [\ln N! - \ln(N-n)! - \ln n!] \end{aligned} \quad (24)$$

Using stirling formula $\ln x! = x \ln x - x$, the above equation becomes

$$\begin{aligned} \ln \frac{N!}{(N-n)!n!} &= [(N \ln N - N) - \{(N-n) \ln(N-n) - (N-n)\} - \{n \ln n - n\}] \\ &= [N \ln N - N - (N-n) \ln(N-n) + N - n - n \ln n + n] \\ &= [N \ln N - (N-n) \ln(N-n) - n \ln n] \end{aligned} \quad (25)$$

Similarly, we can write

$$\begin{aligned}
 \ln \frac{N_i!}{(N_i - n)!n!} &= [\ln N_i! - \ln (N_i - n)! n!] \\
 &= [\ln N_i! - \{\ln(N_i - n)! + \ln n!\}] \\
 &= [\ln N_i! - \ln(N_i - n)! - \ln n!]
 \end{aligned} \tag{26}$$

Again using striling formula, we can write

$$\begin{aligned}
 \ln \frac{N_i!}{(N_i - n)!n!} &= [(N_i \ln N_i - N_i) - \{(N_i - n) \ln(N_i - n) - (N_i - n)\} \\
 &\quad - \{n \ln n - n\}] \\
 &= [N_i \ln N_i - N_i - (N_i - n) \ln(N_i - n) + N_i - n - n \ln n + n] \\
 &= [N_i \ln N_i - (N_i - n) \ln(N_i - n) - n \ln n]
 \end{aligned} \tag{27}$$

Now, the ΔS can be written as

$$\begin{aligned}
 \Delta S &= k_B [N \ln N + N_i \ln N_i - (N - n) \ln(N - n) - (N_i - n) \ln(N_i - n) \\
 &\quad - 2n \ln n]
 \end{aligned}$$

Let E_f be the energy required to form a vacancy. Then the change in the internal energy of the crystal when n -vacancies are formed within it, is given by

$$\Delta U = nE_f \tag{28}$$

Now, we can define the free energy of the crystal as follows:

$$F = \Delta U - T\Delta S \tag{29}$$

Which can be written as

$$\begin{aligned}
 F &= nE_f - k_B T [N \ln N + N_i \ln N_i - (N - n) \ln(N - n) \\
 &\quad - (N_i - n) \ln(N_i - n) - 2n \ln n]
 \end{aligned}$$

Now, we will calculate the partial derivative of F with respect to n

$$\begin{aligned}
\frac{\partial F}{\partial n} &= E_f - k_B T \left[0 + 0 - \left\{ (N-n) \frac{1}{(N-n)} (-1) + \ln(N-n)(0-1) \right\} \right. \\
&\quad \left. - \left\{ (N_i-n) \frac{1}{(N_i-n)} (-1) + \ln(N_i-n)(0-1) \right\} - 2 \left\{ n \frac{1}{n} + \ln n \times 1 \right\} \right] \\
&= E_f - k_B T [0 - \{-1 - \ln(N-n)\} - \{-1 - \ln(N_i-n)\} - 2 \{1 + \ln n\}] \\
&= E_f - k_B T [1 + \ln(N-n) + 1 + \ln(N_i-n) - 2 - 2 \ln n] \\
&= E_f - k_B T [\ln(N-n) + \ln(N_i-n) - \ln n^2] \\
&= E_f - k_B T \ln \left[\frac{(N-n)(N_i-n)}{n^2} \right]
\end{aligned} \tag{30}$$

Now, as we know that at equilibrium temperature the free energy should be minimum. Thus, for free energy to be minimum, the first derivative of free energy should be zero

$$\begin{aligned}
\frac{\partial F}{\partial n} &= 0 \\
E_f - k_B T \ln \left[\frac{(N-n)(N_i-n)}{n^2} \right] &= 0 \\
E_f &= k_B T \ln \left[\frac{(N-n)(N_i-n)}{n^2} \right] \\
\frac{E_f}{k_B T} &= \ln \left[\frac{(N-n)(N_i-n)}{n^2} \right] \\
-\frac{E_f}{k_B T} &= \ln \left(\frac{n^2}{(N-n)(N_i-n)} \right) \\
\frac{n^2}{(N-n)(N_i-n)} &= e^{-\frac{E_f}{k_B T}} \\
n^2 &= (N-n)(N_i-n)e^{-\frac{E_f}{k_B T}} \\
n &= \sqrt{(N-n)(N_i-n)} e^{-\frac{E_f}{2k_B T}}
\end{aligned} \tag{31}$$

Now, we can take $(N-n) \approx N$, $(N_i-n) \approx N_i$, because $N \gg n$, $N_i \gg n$

$$n = \sqrt{(NN_i)} e^{-\frac{E_f}{2k_B T}} \tag{32}$$

This is the concentration of Frenkel defect or number of Frenkel defects in ionic crystal.

Question(22): Explain the experimental procedure of the powder diffraction method to understand the structure of the crystals.

Solution:

This is the experimental X-ray diffraction method which is used to study the crystal structures of polycrystalline type. This method is also known as **Debye-Scherrer method**.

Experimental setup of powder diffraction method:

- The experimental setup consists of cylindrical strip having a photographic film in its inner side. This will work as a camera and this camera is known as the Debye-Scherrer camera.
- The sample in the form of the powder taken in a fine glass tube is kept at the center of the cylinder.
- A monochromatic beam of X-ray is allowed to incident on the powder crystal as shown in Fig.

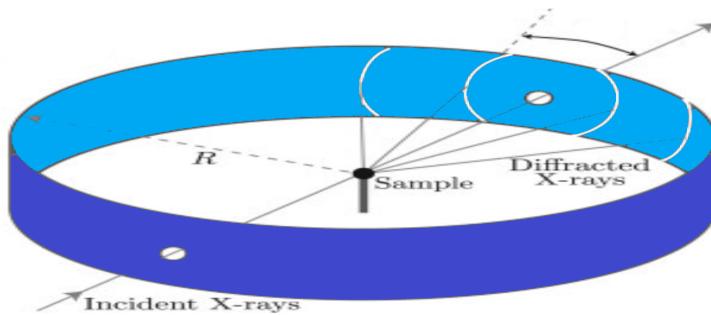


Figure 15: Experimental setup of powder diffraction method.

- The lattice planes in the crystal will have the different orientations.
- When monochromatic X-ray of one fixed wavelength (λ) is incident on the powder (sample), the crystals having orientation angles θ , which are satisfying the Bragg's condition ($2d \sin \theta = n\lambda$) for a particular value of d , will lie in a cone.

- The crystals having orientation angles θ for different sets of d form cones of different radii.
- The photographic film consists of concentric circular arcs as shown in Fig.

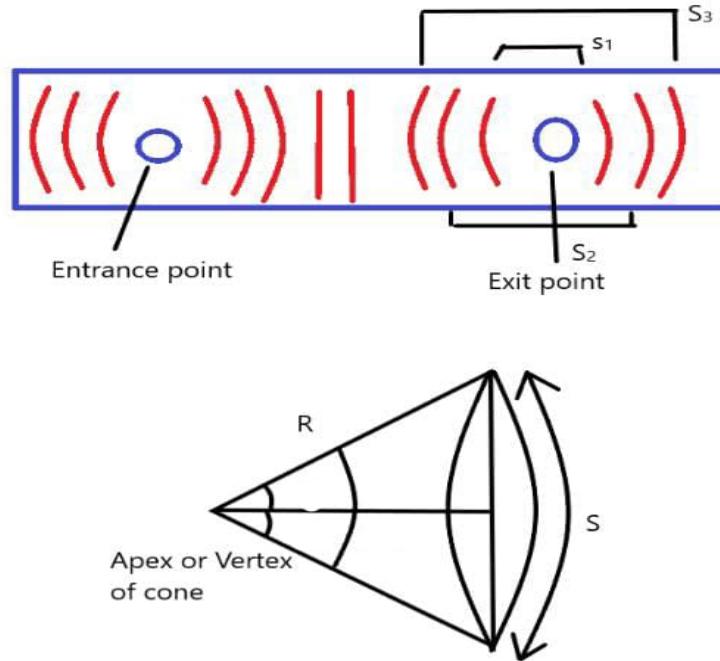


Figure 16: (a) Diffraction pattern on photographic film (b) Geometry of cone

- From above Fig, we can write

$$4\theta(\text{in Radian}) = \frac{S}{R} \quad (33)$$

For different cones, we can write $4\theta_1 = \frac{S_1}{R}$, $4\theta_2 = \frac{S_2}{R}$, $4\theta_3 = \frac{S_3}{R}$ and so on

We can rewrite the above equation as

$$\begin{aligned}\theta(\text{in Degree}) &= \frac{S}{4R} \times \frac{180}{\pi} \\ \theta(\text{in Degree}) &= \frac{S}{4R} \times 57.3\end{aligned}$$

If we take radius of the cylinder $R = 57.3$ nm, then

$$\theta(\text{in Degree}) = \frac{S}{4} \quad (34)$$

- Thus, we can obtain the θ value from the measured value of S . Since the wavelength λ is known. So with the help of all these, we can calculate the value of interplanar spacing d . The interplanar spacing d will be helpful to estimate the crystal structure.

Question(23): Deduce an expression for inter planar spacing for cubic crystal systems.

Solution: Let $(h k l)$ be the Miller indices of the plane ABC . If the a , b and c are the primitive vectors of x , y and z axis, respectively, then the plane ABC intersects the x -axis at a/h , y -axis at b/k , z -axis at c/l .

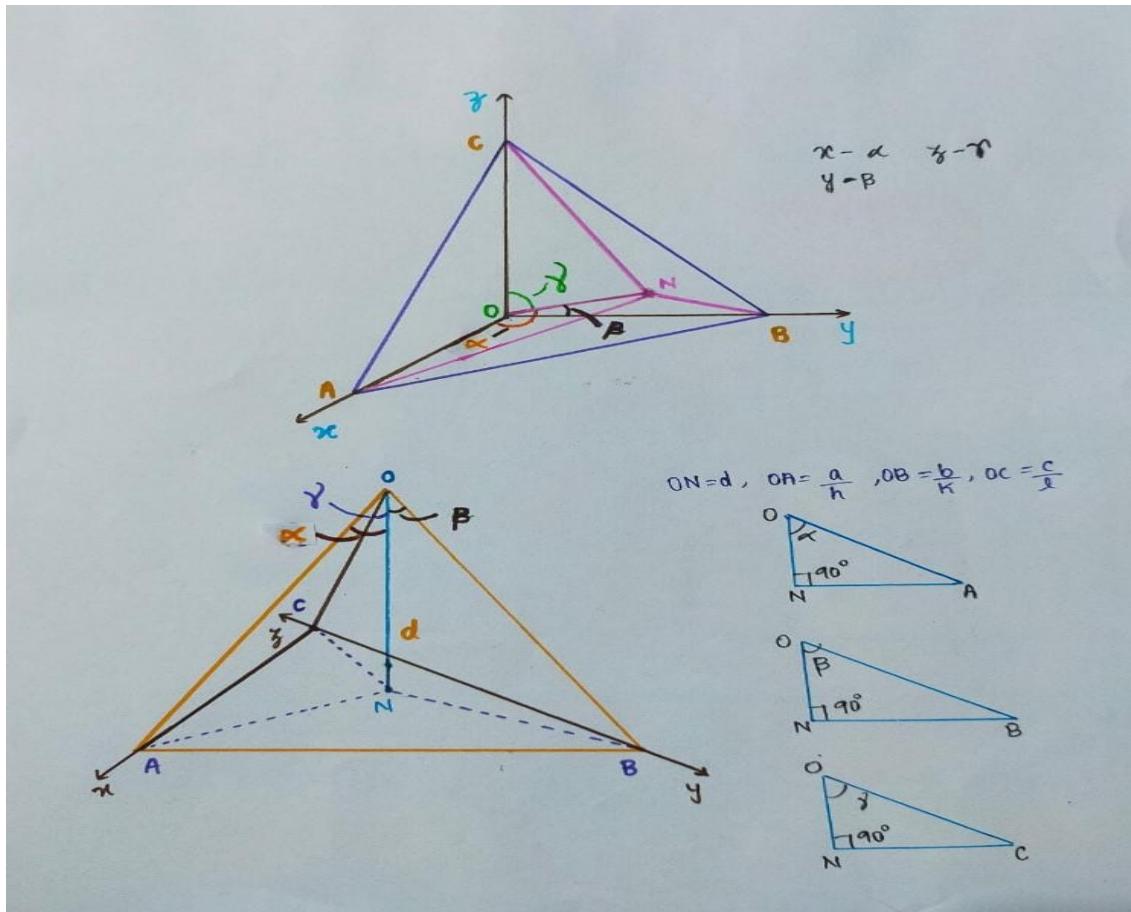


Figure 17: Lattice plane in a crystal with x, y and z coordinates system.

To derive the interplanar spacing, consider a reference plane at origin (O), parallel to plane ABC. Draw a normal ON from the reference plane on the plane

ABC, then the perpendicular distance d will be interplanar distance.

In $\triangle ONA$, we can write

$$\begin{aligned}\cos \alpha &= \frac{ON}{OA} \\ &= \frac{d}{a/h} \\ &= \frac{dh}{a}\end{aligned}\tag{35}$$

In $\triangle ONB$, we can write

$$\begin{aligned}\cos \beta &= \frac{ON}{OB} \\ &= \frac{d}{b/k} \\ &= \frac{dk}{b}\end{aligned}\tag{36}$$

In $\triangle ONC$, we can write

$$\begin{aligned}\cos \gamma &= \frac{ON}{OC} \\ &= \frac{d}{c/l} \\ &= \frac{dl}{c}\end{aligned}\tag{37}$$

Using the property of directional cosine, we get

$$\begin{aligned}\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma &= 1 \\ \left(\frac{dh}{a}\right)^2 + \left(\frac{dk}{b}\right)^2 + \left(\frac{dl}{c}\right)^2 &= 1 \\ d^2 \left[\left(\frac{h}{a}\right)^2 + \left(\frac{k}{b}\right)^2 + \left(\frac{l}{c}\right)^2 \right] &= 1 \\ d^2 &= \frac{1}{\left[\left(\frac{h}{a}\right)^2 + \left(\frac{k}{b}\right)^2 + \left(\frac{l}{c}\right)^2\right]} \\ d &= \frac{1}{\sqrt{\left(\frac{h}{a}\right)^2 + \left(\frac{k}{b}\right)^2 + \left(\frac{l}{c}\right)^2}}\end{aligned}\tag{38}$$

Finally, we can write the expression of perpendicular distance between two parallel planes as follows

$$d = \frac{1}{\sqrt{\left(\frac{h}{a}\right)^2 + \left(\frac{k}{b}\right)^2 + \left(\frac{l}{c}\right)^2}} \quad (39)$$

For cubic system $a = b = c$, the above equation becomes

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \quad (40)$$

This expression is interplanar spacing for cubic system with the given value of Miller indices $(h k l)$ and lattice constant a .

Question(24): Define point defect. Elaborate point defects with neat diagram.

Solution:

point defects Those defects in the crystal which occur around an atom or particle are called point defects. These defects occur only at or around a single lattice point. They do not extend in space in any dimension. That's why they are also called zero dimensional (0-D) defects. These are the smallest possible defects in any crystalline solid material.

Types of point defects We can classify the point defects as follows:

- (i) Vacancy defect
- (ii) Interstitial defect
- (iii) Schottky defect
- (iv) Frenkel defect

Vacancy defect: The simplest defect is vacancy. The absence of an atom in the regular periodic lattice creates a vacancy. The diagram for vacancy formation is shown in Fig.

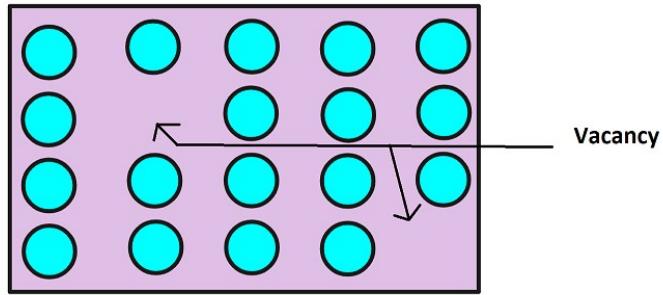


Figure 18: Vacancy defect.

Interstitial defect: The arrangement of atoms in the regular lattice form void (empty) spaces. The missed atom from its regular lattice may occupy the void space in the lattice known as interstitial. The diagram for interstitial defect is shown in Fig.

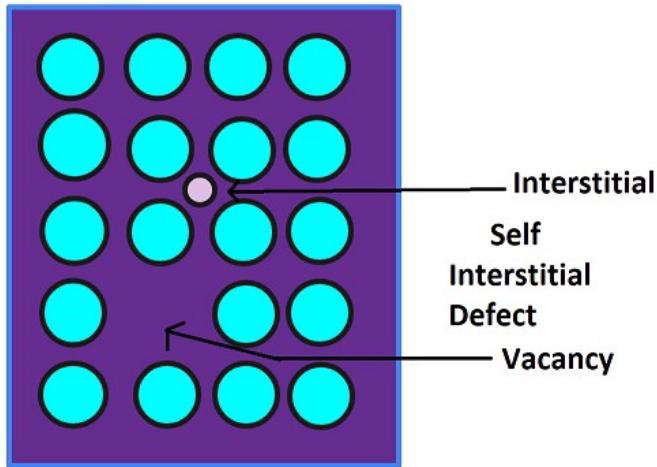
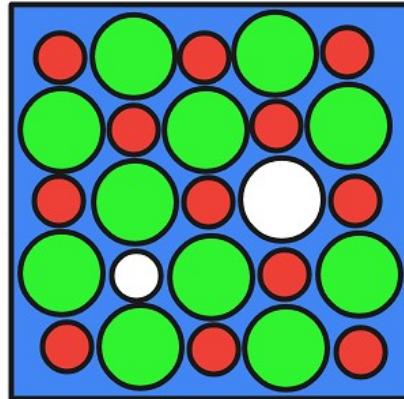


Figure 19: Interstitial defect.

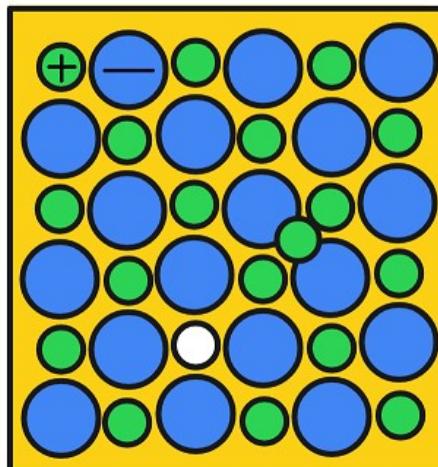
Schottky defect: In ionic crystal when an anion is completely displaced from its regular position, a vacancy is created in that position. To get charge neutrality, a cation is also displaced from its regular position and hence a pair of vacancy is created. The type of point defect in which a pair of vacancy is created, is called Schottky defect. The diagram for Schottky defect is shown in Fig.



Schottky Defect

Figure 20: Schottky defect.

Frenkel defect: In crystal when an atom (or) ion is completely displaced from its regular position, it creates a vacancy in that position. To get charge neutrality, this atom (or) ion occupies interstitial space (or) void space in the regular lattice, hence a vacancy and interstitial are formed. The type of point defect in which a vacancy and interstitial are formed is known as Frenkel defect. The diagram for Frenkel defect is shown in Fig.



Frenkel Defect

Figure 21: Frenkel defect.

Question(25): Derive an expression for concentration of Schottky defects in the case of ionic crystals.

Solution: In ionic crystal, when an anion is completely displaced from its regular position, it creates a vacancy. To get the charge neutrality, a cation is also displaced from its regular position and hence a pair of vacancies is created.

The number of ways in which the n -pair of vacancies are formed in the regular lattice sites of N ions are given by

$$\begin{aligned} w &= \frac{N!}{n!(N-n)!} \frac{N!}{n!(N-n)!} \\ &= \left[\frac{N!}{n!(N-n)!} \right]^2 \end{aligned} \quad (41)$$

We can now write the change in entropy as

$$\Delta S = k_B \ln w \quad (42)$$

Where k_B is Boltzmann constant. Now, we can write

$$\begin{aligned} \Delta S &= k_B \ln \left[\frac{N!}{n!(N-n)!} \right]^2 \\ &= 2k_B \ln \left[\frac{N!}{n!(N-n)!} \right] \\ &= 2k_B [\ln N! - \ln n!(N-n)!] \\ &= 2k_B [\ln N! - \{\ln n! + \ln(N-n)!\}] \\ &= 2k_B [\ln N! - \ln n! - \ln(N-n)!] \end{aligned} \quad (43)$$

Where we have used the following formulas $[\ln a^2 = 2 \ln a, \ln(\frac{a}{b}) = \ln a - \ln b, \ln(ab) = \ln a + \ln b]$. Using Stirling formula

$$\ln x! = x \ln x - x \quad (44)$$

The above equation can be written as

$$\begin{aligned} \Delta S &= 2k_B [\{N \ln N - N\} - \{n \ln n - n\} - \{(N-n) \ln(N-n) - (N-n)\}] \\ &= 2k_B [N \ln N - N - n \ln n + n - (N-n) \ln(N-n) + (N-n)] \\ &= 2k_B [N \ln N - n \ln n - (N-n) \ln(N-n)] \end{aligned} \quad (45)$$

Let E_s be the energy required to form a pair of vacancy. Then the change in the internal energy of the crystal when n -pair of vacancies are formed within it, is given by

$$\Delta U = nE_s \quad (46)$$

Now, we can define the free energy of the crystal as follows:

$$F = \Delta U - T\Delta S \quad (47)$$

After substituting the values of ΔU and ΔS , the free energy of the crystal can be written as

$$F = nE_s - 2k_B T [N \ln N - n \ln n - (N - n) \ln(N - n)] \quad (48)$$

Now, we will calculate the partial derivative of F with respect to n

$$\begin{aligned} \frac{\partial F}{\partial n} &= E_s - 2k_B T \left[0 - \left\{ n \frac{1}{n} + \ln n \times 1 \right\} \right. \\ &\quad \left. - \left\{ (N - n) \frac{1}{(N - n)} (-1) + \ln(N - n)(0 - 1) \right\} \right] \\ &= E_s - 2k_B T [-1 - \ln n + 1 + \ln(N - n)] \\ &= E_s - 2k_B T [\ln(N - n) - \ln n] \\ &= E_s - 2k_B T \ln \left(\frac{N - n}{n} \right) \end{aligned} \quad (49)$$

Here, we have used $\frac{\partial(uv)}{\partial x} = u \frac{\partial v}{\partial x} + v \frac{\partial u}{\partial x}$. Now, as we know that at equilibrium temperature the free energy should be minimum. Thus, for free energy to be minimum, the first derivative of free energy should be zero

$$\frac{\partial F}{\partial n} = 0 \quad (50)$$

$$\begin{aligned} E_s - 2k_B T \ln \left(\frac{N - n}{n} \right) &= 0 \\ E_s &= 2k_B T \ln \left(\frac{N - n}{n} \right) \\ \frac{E_s}{2k_B T} &= \ln \left(\frac{N - n}{n} \right) \\ -\frac{E_s}{2k_B T} &= \ln \left(\frac{n}{N - n} \right) \\ \frac{n}{N - n} &= e^{-\frac{E_s}{2k_B T}} \\ n &= (N - n)e^{-\frac{E_s}{2k_B T}} \end{aligned} \quad (51)$$

Now, we can take $(N - n) \approx N$, because $N \gg n$. Under this condition, the above equation becomes

$$n = Ne^{-\frac{E_s}{2k_B T}} \quad (52)$$

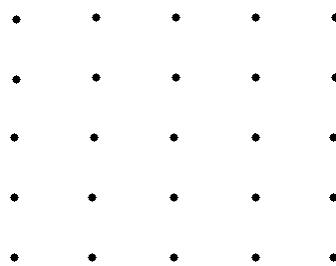
This is the concentration of Schottky defect or number of Schottky defects in ionic crystal.

Question(26): Define the following terms.

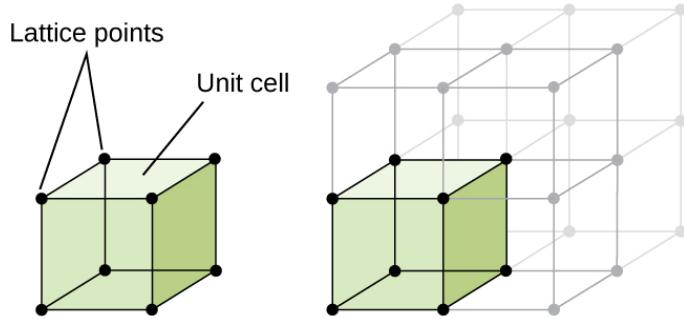
- (a) Space lattice
- (b) Unit cell
- (c) Lattice parameters
- (d) Bravais lattices
- (e) Lattice Planes

Solution:

- **Space lattice:** A lattice is a translationally periodic set of points. We can have 1D (1-dimension), 2D and 3D (or even higher dimensional) lattices. Usually the 3D lattice is called a **space lattice**. A lattice is a series of points that are arranged in a distinct pattern.



- **Unit cell:** Unit cell is the smallest part (portion) of a crystal lattice. It is the simplest repeating unit in a crystal structure. The entire lattice is generated by the repetition of the unit cell in different directions.



- **Lattice parameters:** Lattice parameters are defined as a set of 6 parameters (in 3D)

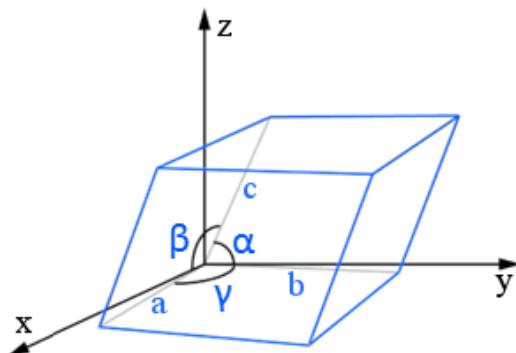


Figure 22: A unit cell with lattice parameters

- (i) The lengths of the unit cell along the x , y , z directions are defined as a , b , c , respectively.
- (ii) The angles between these axes are defined by α , β and γ .

These a , b , c , α , β , γ are collectively known as the **lattice parameters** in 3-dimensions (3D).

- **Bravais lattice:** Depending upon the geometric considerations, we can group the crystal into seven crystal systems. In these 7 crystal systems, each system is characterized by different values of a , b , c and α , β , γ . Each crystal system can further classified into following types of lattices:

Types of Lattices:

- (i) Simple or Primitive Lattice (P): It has atoms only at the corners of a unit cell.

- (ii) Body Centered Lattice (I): It has extra atom at the center of the unit cell.
- (iii) Face Centered Lattice (F): It has extra atom at the center of the each face.
- (iv) Base Centered Lattice (C): It has extra atom at the center of the base.

Thus, our seven crystal systems with their corresponding type of lattices will result in 14 Bravais lattices in 3D.

- **Lattice Planes:** A plane passing through various lattice points in a crystal is known as the lattice plane. On in other words, planes which contain a large concentration of atoms are known as lattice planes. Various types of lattice planes in a crystal lattice are drawn.

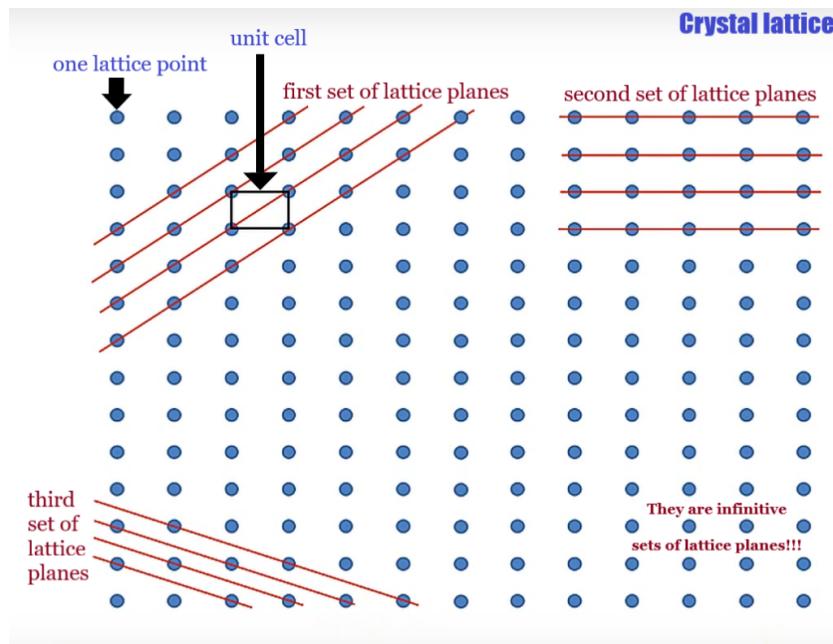


Figure 23: Various types of lattice plane.

Question(27): Explain Bragg's law. Find the wavelength of X-rays when diffracted by (111) planes of a cubic crystal whose lattice constant is 3.5 Å and produces second order diffraction at 45° .

Solution: As we know the diffraction is the phenomena of bending of light after falling on an obstacle. For diffraction to occur the size of the obstacle must be comparable with the wavelength of radiation. Since the interatomic

spacing is only 2 to 3 Å, X-rays whose wavelength is in the same range can be used to used for crystal diffraction studies. W. L. Bragg used the X-rays diffraction phenomena to derive the well known Bragg's law.

Derivation of Bragg's law:

- Let us consider a crystal made up of equidistant parallel planes of atoms with the interplanar spacing d as shown in Fig. The incident monochromatic X-rays of wavelength λ fall on these planes. In Fig, the incident rays are represented by AO and $A'O'$, both the incident rays fall at angle θ .

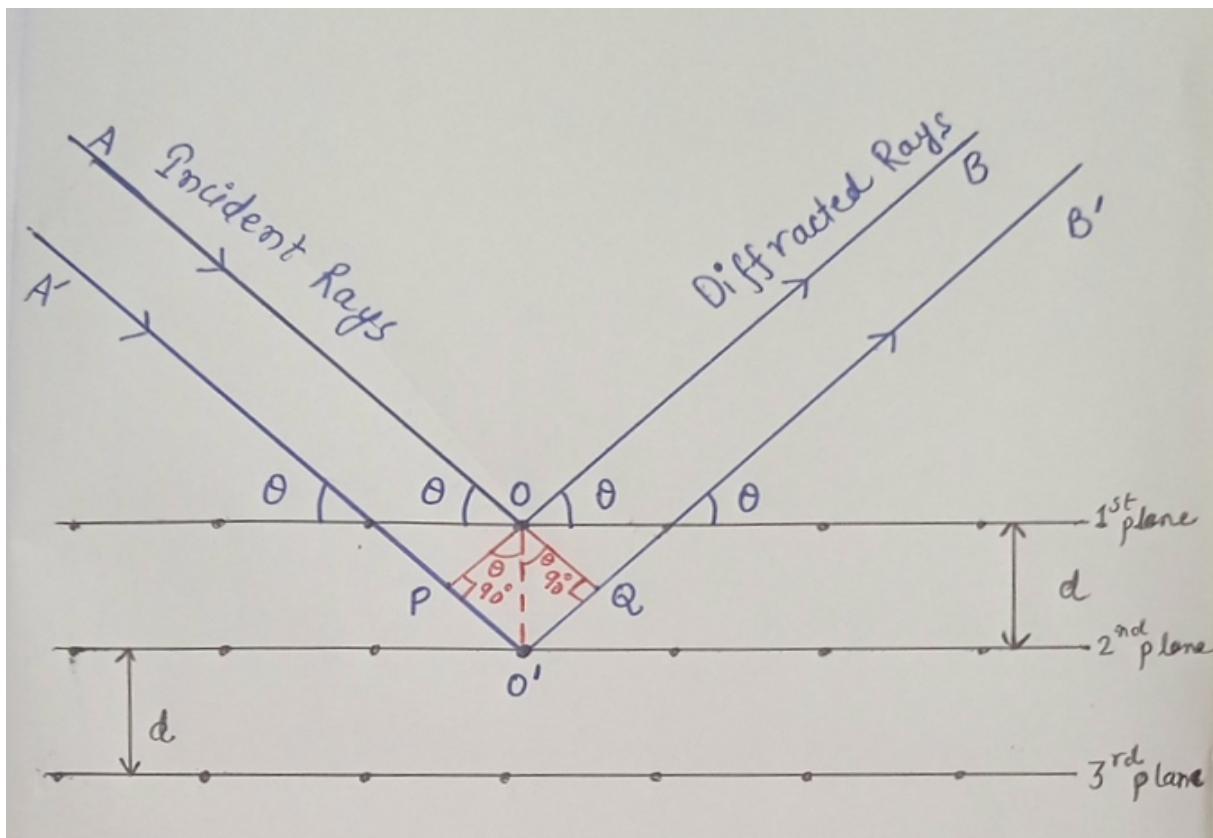


Figure 24: Diffraction of X-rays by crystal planes.

- These incident rays get diffracted by the planes (means by the atoms on the planes). In Fig, the diffracted rays are represented by OB and $O'B'$.
- Draw the normals (perpendicular) OP and OQ as drawn in Fig.
- It is clear from Fig, that the path difference between incident and diffracted

waves is given by

$$\text{Path difference} = PO' + O'Q \quad (53)$$

In $\triangle OPO'$

$$\sin \theta = \frac{PO'}{OO'} = \frac{PO'}{d}$$

$$PO' = d \sin \theta$$

Similarly in $\triangle OQO'$

$$\sin \theta = \frac{O'Q}{OO'} = \frac{O'Q}{d}$$

$$O'Q = d \sin \theta$$

After substituting the values of PO' and $O'Q$, the path difference becomes

$$\text{Path difference} = 2d \sin \theta \quad (54)$$

- According to Bragg's law the path difference should be an integral multiple of wavelength

$$\text{Path difference} = n\lambda \quad (55)$$

where $n=1,2,3,\dots$

- After Combining, we get

$$2d \sin \theta = n\lambda \quad (56)$$

This condition is known as the Bragg's law.

- d : Interplanar spacing (perpendicular distance between two planes)
- λ : Wavelength of incident X-ray
- θ : Bragg angle/Glancing angle/Incident angle
- n : Order of diffraction [Example: $n=1$ (first order), $n=2$ (second order)]

- As we know the maximum value of $\sin \theta = 1$, thus from above equation, we can write

$$\frac{n\lambda}{2d} \leq 1$$

This sets the limitation on wavelength. The wavelength λ should not exceed twice the interplanar spacing for diffraction to occur.

Solution of numerical problem:

Given

$$\begin{aligned}(h, k, l) &= (1, 1, 1) \\ \theta &= 45^\circ \\ a &= 3.5 \text{ \AA} \\ n &= 2 \text{ (second order)}\end{aligned}$$

Using Bragg's law ($2d \sin \theta = n\lambda$)

$$\begin{aligned}2d \sin 45^\circ &= 2\lambda \\ 2 \times d \times \frac{1}{\sqrt{2}} &= 2\lambda \\ \lambda &= \frac{d}{\sqrt{2}}\end{aligned}$$

Using

$$\begin{aligned}d &= \frac{a}{\sqrt{h^2 + k^2 + l^2}} \\ &= \frac{a}{\sqrt{3}} \\ &= \frac{3.5}{\sqrt{3}} \\ d &= 2.02 \text{ \AA}\end{aligned}$$

Thus, the wavelength of X-rays will be

$$\begin{aligned}\lambda &= \frac{2.02}{1.414} \text{ \AA} \\ &= 1.43 \text{ \AA}\end{aligned}$$

Question(28): What are Miller indices and write the procedure to obtain Miller indices of a given plane.

Solution:

Miller Indices: In order to indicate lattice planes for crystal analysis. Miller suggested a method of indicating the orientation of a plane. Miller indices is a group of three numbers that indicates the orientation of a plane or set of parallel planes of atoms in a crystal.

Procedure of finding Miller indices:

- Find the intercepts of the desired planes on the three coordinates axes (means x, y, z axes). Let these intercepts be (pa, qb, rc) .
- Divide these intercepts with their respective primitive vectors $(\frac{pa}{a}, \frac{qb}{b}, \frac{rc}{c}) = (p, q, r)$.
- Take the ratio of the reciprocals of these numbers:
 $\frac{1}{p} : \frac{1}{q} : \frac{1}{r}$.
- Convert these reciprocals into whole numbers by multiplying each with their L.C.M. to get the smallest whole number.
- These smallest whole numbers give the Miller indices $(h k l)$.

Example: Given intercepts on x, y, z axes are 2, 3, 2, respectively. Find the Miller indices.

Solution:

$$\begin{aligned}\text{Intercepts} &= 2 \ 3 \ 2 \\ \text{Reciprocal} &= \frac{1}{2} \ \frac{1}{3} \ \frac{1}{2} \\ \text{Ratio of reciprocal} &= \frac{1}{2} : \frac{1}{3} : \frac{1}{2} \\ \text{Multiply with L.C.M.} &= \frac{6}{2} : \frac{6}{3} : \frac{6}{2} \\ \text{Miller indices} &= (3 \ 2 \ 3)\end{aligned}$$

Question(29): What is interplanar spacing, explain with a diagram. Find the interplanar spacing of a cubic crystal whose lattice constant is 1.74\AA and Miller indices are (110) .

Solution: The interplanar spacing d between adjacent planes having Miller indices (hkl) is defined as the distance between first such plane from a parallel plane passing through the origin. The interplanar spacing is given by

$$d = \frac{1}{\sqrt{\left(\frac{h}{a}\right)^2 + \left(\frac{k}{b}\right)^2 + \left(\frac{l}{c}\right)^2}}$$

Where $(h k l)$ are the Miller indices of the plane ABC . If the a, b and c are the primitive vectors of x, y and z axis, respectively.

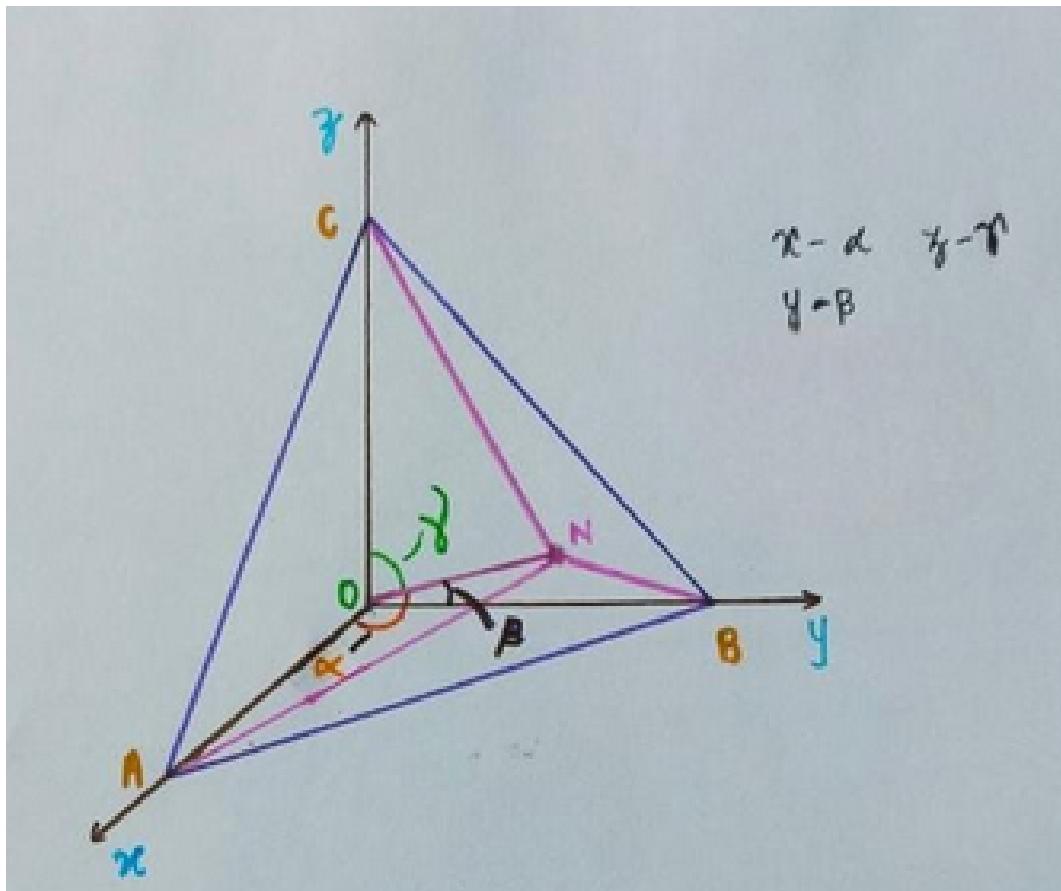


Figure 25: Lattice plane in a crystal with x, y and z coordinates system.

Solution of numerical problem

Given : lattice constant $a = 1.74 \text{ \AA}$
 $(h k l) = (1 1 0)$

Using

$$\begin{aligned} d &= \frac{a}{\sqrt{h^2 + k^2 + l^2}} \\ &= \frac{a}{\sqrt{2}} \\ &= \frac{1.74}{\sqrt{2}} \\ d &= \frac{1.74}{1.414} \text{ \AA} \\ d &= 1.23 \text{ \AA} \end{aligned}$$

Question(30): Draw (111), (101), (120), (100), (112) and ($\bar{1}10$).

Solution:

- Plane (1 1 1)

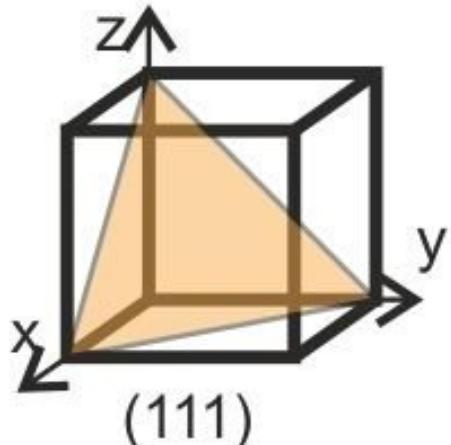


Figure 26: (1 1 1) plane

Miller indices : (1 1 1)

Reciprocal : $\frac{1}{1} \frac{1}{1} \frac{1}{1}$

Intercept : 1 1 1

So the plane (1 1 1) will cut all the axes x y and z .

- Plane (1 0 1)

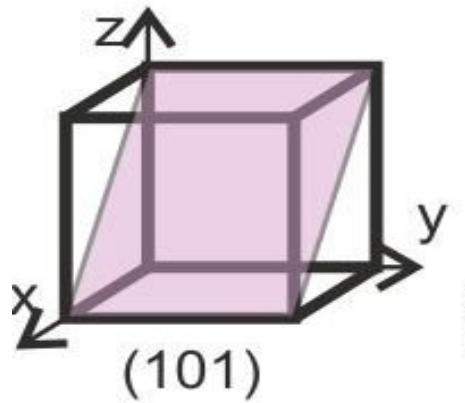


Figure 27: (1 0 1) plane.

Miller indices : (1 0 1)

Reciprocal : $\frac{1}{1} \frac{1}{0} \frac{1}{1}$

Intercept : $1 \infty 1$

So the plane (1 0 1) will cut the x and z axes and it will be parallel to y axis because the intercepts on y axis is ∞ .

- Plane (1 2 0)

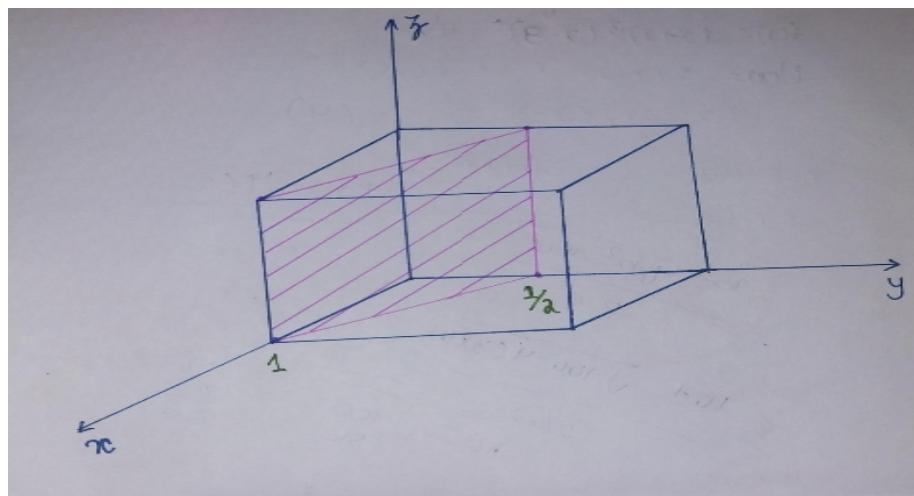


Figure 28: (1 2 0) plane

Miller indices : (1 2 0)

Reciprocal : $\frac{1}{1} \frac{1}{2} \frac{1}{0}$

Intercept : $1 \frac{1}{2} \infty$

So the plane (1 2 0) will cut the x and y axes at 1 and $\frac{1}{2}$, respectively, and will not cut the z -axis.

- **Plane (1 0 0)**

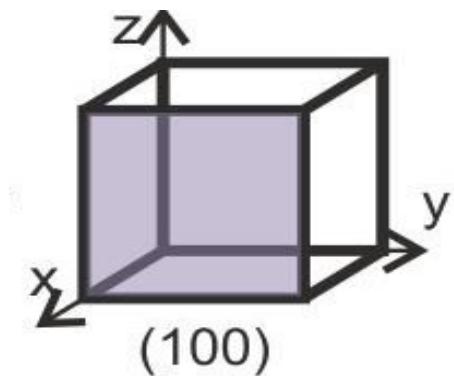


Figure 29: (1 0 0) plane

Miller indices : (1 0 0)

Reciprocal : $\frac{1}{1} \frac{1}{0} \frac{1}{0}$

Intercept : $1 \infty \infty$

So the plane (1 0 0) will only cut the x -axis and it will be parallel to y and z axes because the intercepts on y and z are ∞ .

- **Plane (1 1 2)**

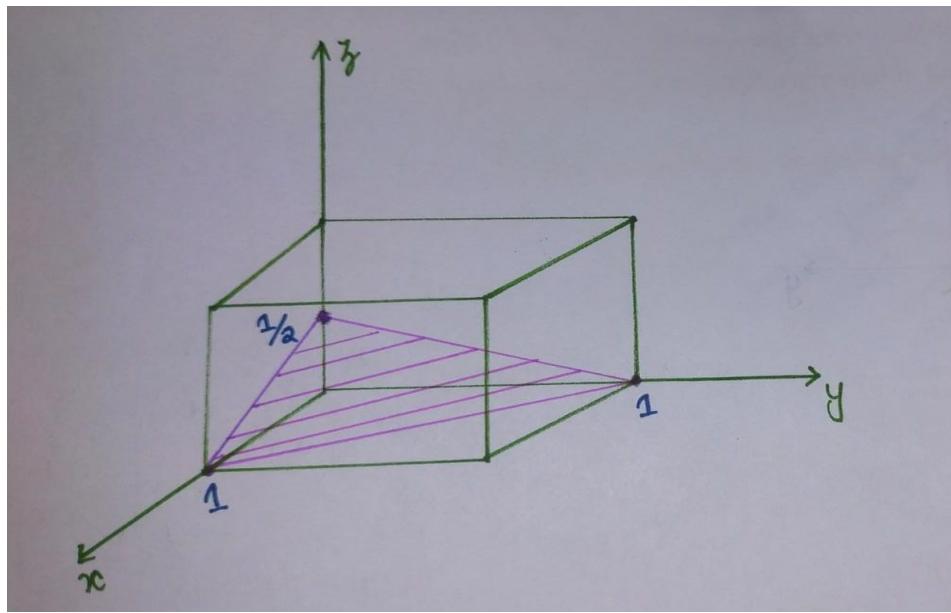


Figure 30: (1 1 2) plane

Miller indices : (1 1 2)

Reciprocal : $\frac{1}{1} \frac{1}{1} \frac{1}{2}$

Intercept : $1 \ 1 \ \frac{1}{2}$

So the plane (1 1 2) will cut all the axes.

- Plane ($\bar{1} \ 1 \ 0$)

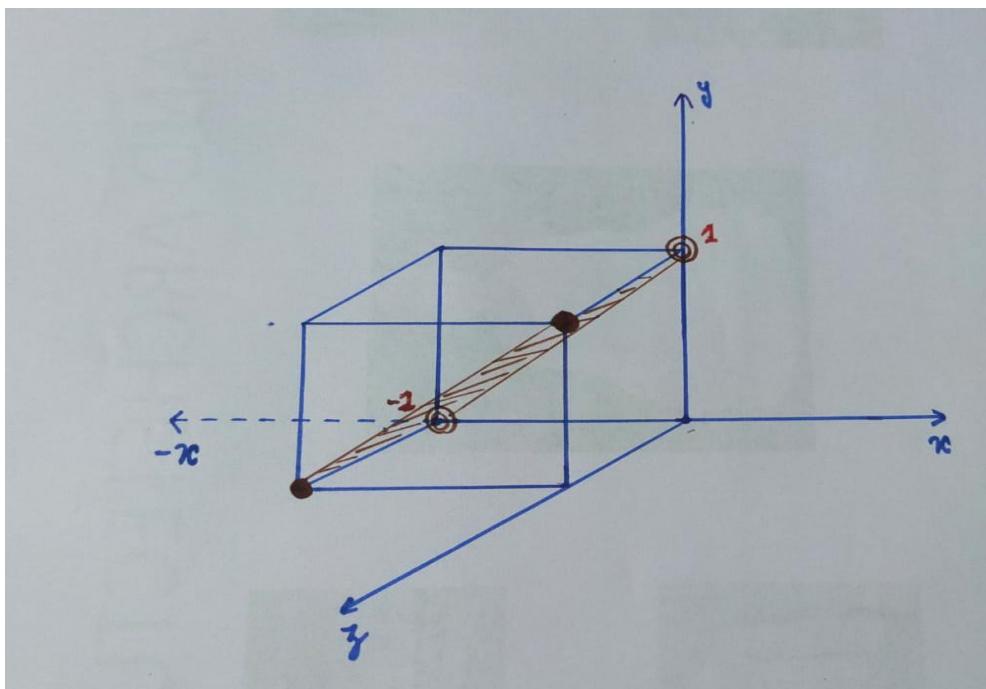


Figure 31: $(\bar{1} \ 1 \ 0)$ plane

Miller indices : $(-1 \ 1 \ 0)$

Reciprocal : $\frac{1}{-1} \ \frac{1}{1} \ \frac{1}{0}$

Intercept : $-1 \ 1 \ \infty$

So the plane $(\bar{1} \ 1 \ 0)$ will cut x and y axes.

UNIT-II (SAQs)

Question(31) Explain the spontaneous and stimulated emissions.

Solution:

Spontaneous emission: An atom in an upper level can decay spontaneously to the lower level and emit a photon of energy $h\nu = E_2 - E_1$ if the transition between E_2 and E_1 is radiative. This photon has a random direction and phase.

Stimulated emission: An incident photon causes an upper level atom to decay, emitting a “stimulated” photon whose properties are identical to those of the incident photon. The term “stimulated” underlines the fact that this kind of radiation only occurs if an incident photon is present.



Question(32) Write the characteristics of Laser.

Solution: Laser has certain unique properties when compared to ordinary sources of light. They are as follows

1. High Monochromaticity
2. High Directionality
3. High Intensity
4. High Coherence

Question(33): Explain the population inversion mechanism.

Solution: In normal (general) condition, more number of atoms present in lower energy level than higher energy level as shown in fig.

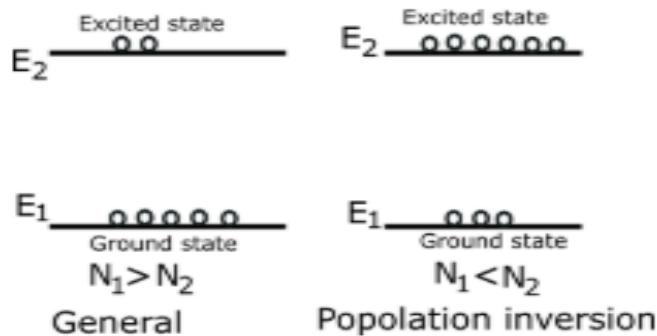


Figure 32: Energy Level system.

When energy applied, atoms move to the higher energy level makes more number of atoms in higher energy level than lower energy level. This process is called population inversion. The process of achieving population inversion is called pumping process.

Question(34): The efficiency of Ruby laser is poor than He-Ne laser. Give the justification.

Solution: The efficiency of Ruby laser is poor than the He-Ne laser because Ruby laser is a solid (i.e. crystals have defects) laser due to which defects are present in it, whereas He-Ne laser is gas state laser so there are no defects present in it.

Question(35): Find the wavelength of the GaAs semiconductor laser whose energy gap is 1.44 eV.

Solution: Given $E_g = 1.44 \text{ eV} = 1.44 \times 1.6 \times 10^{-19} \text{ J}$
laser wavelength is given by

$$\begin{aligned}
 \lambda &= \frac{hc}{E_g} \\
 &= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{1.44 \times 1.6 \times 10^{-19}} \\
 &= 8.627 \times 10^{-7} \text{ m} \\
 \lambda &= 8627 \text{ Å}
 \end{aligned}$$

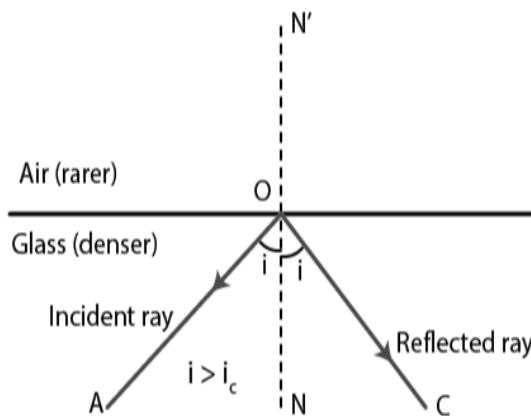
Question(36): Write the applications of lasers.

Solution: Lasers are used in

1. Industry
2. Engineering
3. Research
4. Medicine
5. Army

Question(37): Explain the total internal reflection phenomena.

Solution: When a light ray incident with an angle (i) greater than the critical angle (i_c) then the ray totally reflected back into the same medium, this phenomena is known as total internal reflection.



Question(38): Outline the Numerical Aperture in optical fiber.

Solution: Numerical Aperture gives the light gathering capacity of an optical fiber. It is defined as the sin of the acceptance angle (θ_A)

$$\begin{aligned} \text{NA} &= \sin \theta_A \\ \text{NA} &= \sin[\sin^{-1}(\sqrt{n_1^2 - n_2^2})] \\ \text{NA} &= \sqrt{n_1^2 - n_2^2} \end{aligned}$$

where n_1 and n_2 are the refractive indices of core and cladding materials.

Question(39): Explain the losses in optical fiber.

Solution: There are three types of losses in optical fiber namely:

1. Absorption losses
2. Scattering losses
3. Bending losses

Question(40): Write the applications of optical fibers.

Solution: Optical fibers are used in

- Communication
- Defence
- Industries
- Broadcasting
- Lighting and Decorations.

Question(41): Find the numerical aperture of an optical fiber whose refractive index of core and cladding are 1.55 and 1.50, respectively.

Solution: Given $n_1 = 1.55$ and $n_2 = 1.50$

NA can be calculated as

$$\begin{aligned}
 \text{NA} &= \sqrt{n_1^2 - n_2^2} \\
 &= \sqrt{1.55^2 - 1.50^2} \\
 &= \sqrt{2.40 - 2.25} \\
 &= 0.39
 \end{aligned}$$

Question(42): List the various pumping mechanisms.

Solution: There are 5 pumping mechanisms:

1. Optical Pumping
2. Electrical Pumping
3. Chemical Pumping
4. Inelastic collision of atoms
5. Direct Pumping

Question(43): Name the types of optical fiber based on refractive index profile.

Solution: Optical fibers are classified into two types based on refractive index profile, namely

1. Step index optical fibers
2. Graded index optical fibers

Question(44): Find the acceptance angle of an optical fiber whose refractive index of core and cladding are 1.55 and 1.50, respectively.

Solution: Given $n_1 = 1.55$ and $n_2 = 1.50$

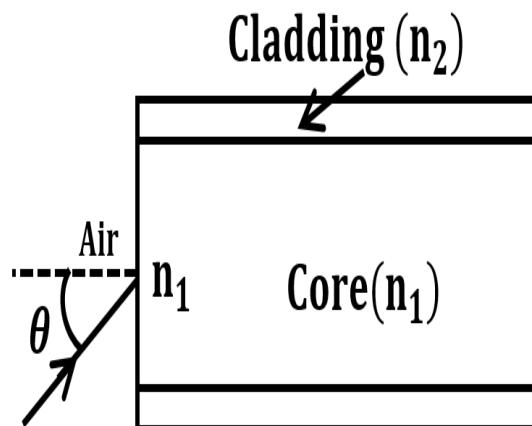
Acceptance angle can be calculated as

$$\begin{aligned}\theta_A &= \sin^{-1}(\sqrt{n_1^2 - n_2^2}) \\ &= \sin^{-1}(\sqrt{1.55^2 - 1.50^2}) \\ &= \sin^{-1}(\sqrt{2.40 - 2.25}) \\ &= \sin^{-1}(0.39) \\ &= 22.9^\circ\end{aligned}$$

Question(45): Define the acceptance angle in optical fiber.

Solution: Acceptance angle (θ_A) for an optical fiber is defined as the maximum angle of incidence at the interface of medium (air usually) and core (n_1) for which light ray enters and travel along the optical fiber

$$\theta_A = \sin^{-1}(\sqrt{n_1^2 - n_2^2})$$



UNIT-II (LAQs)

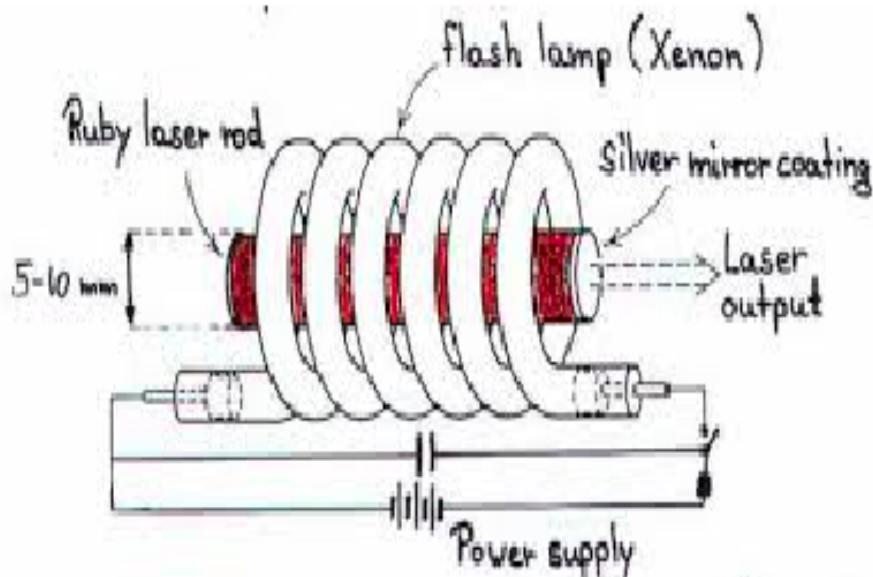
Question(46): Describe the construction and working of Ruby laser.

Solution: Introduction:

- Ruby laser is a solid state laser. It was developed by T. H. Maiman in 1960.
- It is a 3 level state laser.
- It is a high power laser.
- It is a pulsed laser.

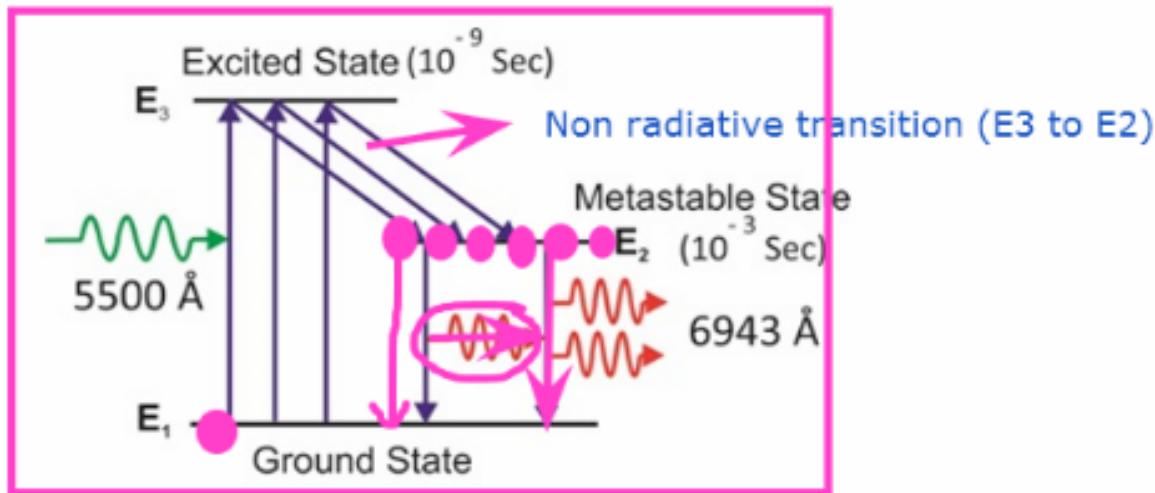
Construction:

- It consists of a Ruby rod which is a crystal of Al_2O_3 doped with Cr_2O_3 such that rod gets pink color.
- The two ends of the rod are highly grinded and polished such that one end acts as fully reflecting and the other with partially.
- The rod surrounded with Xenon Flash lamp to create population inversion through optical pumping.
- The external power connected to the Xenon flash lamp to start the laser action.



Working:

- The working of Ruby laser can be explained with the help of chromium ions (Cr_3^+) energy level diagram as shown in Fig.
- Initially the (Cr_3^+) ions present in the lower energy level (E_1).



Energy Level Diagram of Ruby LASER

- When the power is switched ON, these ions absorb 5500 \AA wavelength and move to the energy levels (E_3)
- The life time of E_3 level is around 10^{-9} sec. After the lifetime majority of Cr_3^+ ions drop to E_2 state by emitting heat. This transition is known as very fast non-radiative transition.
- The life time of E_2 level is about 10^{-3} to 10^{-5} sec and hence Cr_3^+ ions spend more time.
- The Cr_3^+ ions, which complete their life time drops to E_1 state and emits a non-directional photon.
- Among these non-directional photons, many photon emit along the axis of Ruby rod act as an external energy to the remaining chromium ions whose lifetime not yet complete, which leads to stimulated emission.
- Finally, a laser of wavelength 6943 \AA emitted from the partially reflected face as shown in Fig.

Question(47): Explain the construction and working of He-Ne laser.

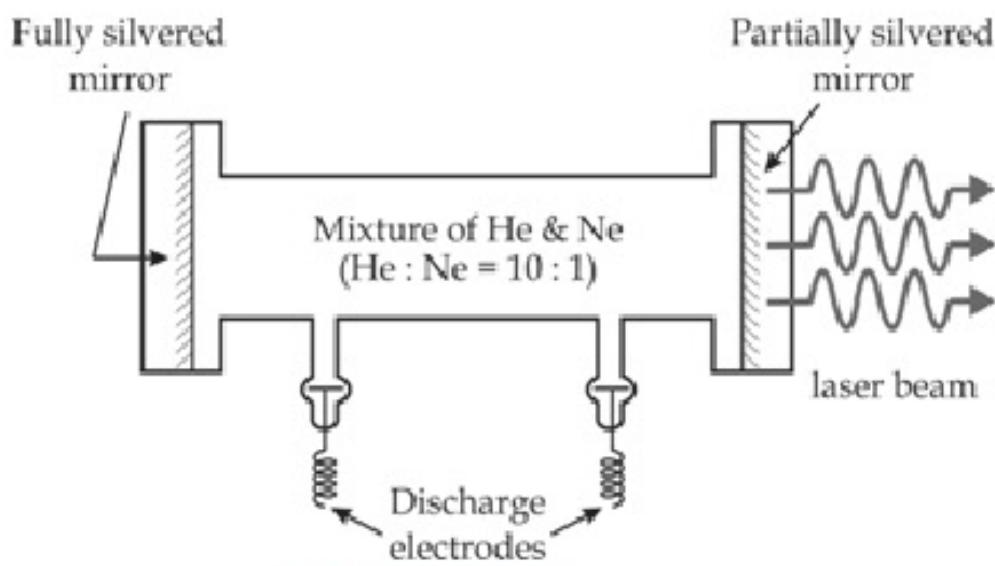
Solution:

Introduction:

- It is a first gas laser.
- It was discovered by Ali Javan in the year 1961.
- It is a 4 level laser.
- It is a low power laser. (mW)
- It is a continuous laser.

Construction:

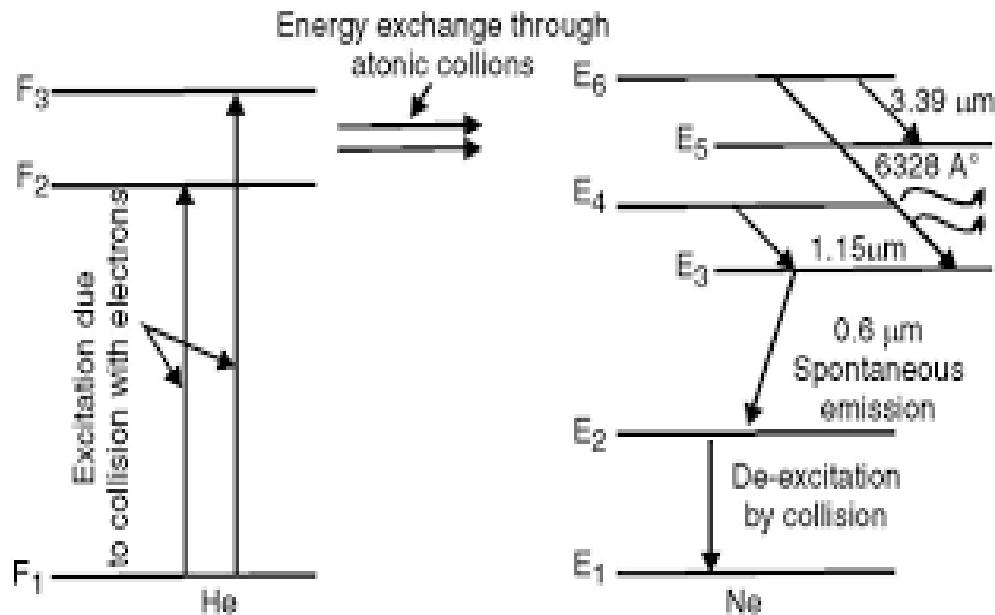
- It consists of a gas discharge tube filled with He and Ne gas at different pressures and different ratios ($10 : 1$).
- The two ends of gas discharge tube enclosed with two quartz crystals polished such that one acts as fully reflecting and the other with partially.
- The electrical pumping is used to achieve the population inversion in Ne atoms by connecting electrodes to the discharge tube.
- The Rf power are connected to the electrodes to start the laser action.



He-Ne LASER

Working:

- The working of $He - Ne$ laser can be explained with the help of energy level diagrams of both He and Ne atoms as shown in Fig.



- Initially, both atoms are in lower energy level.
- When the power is switched ON, the electrons exited He atoms by electron excitation and then He atoms exited the Ne atoms by collisional transfer energy method, $e^- + He \Rightarrow He^*$
- For Ne atoms E_6 and E_4 states are metastable states, Hence,
 - The transitions, E_6 to E_5 gives 3.39 μm wavelength laser.
 - E_6 to E_3 gives 6328 \AA laser.
 - E_4 to E_3 gives 1.15 μm laser.
 - The remaining are spontaneous transitions and few are not allowed.
 - Using proper filters we can generate 6328 \AA wavelength of laser which is in visible region.

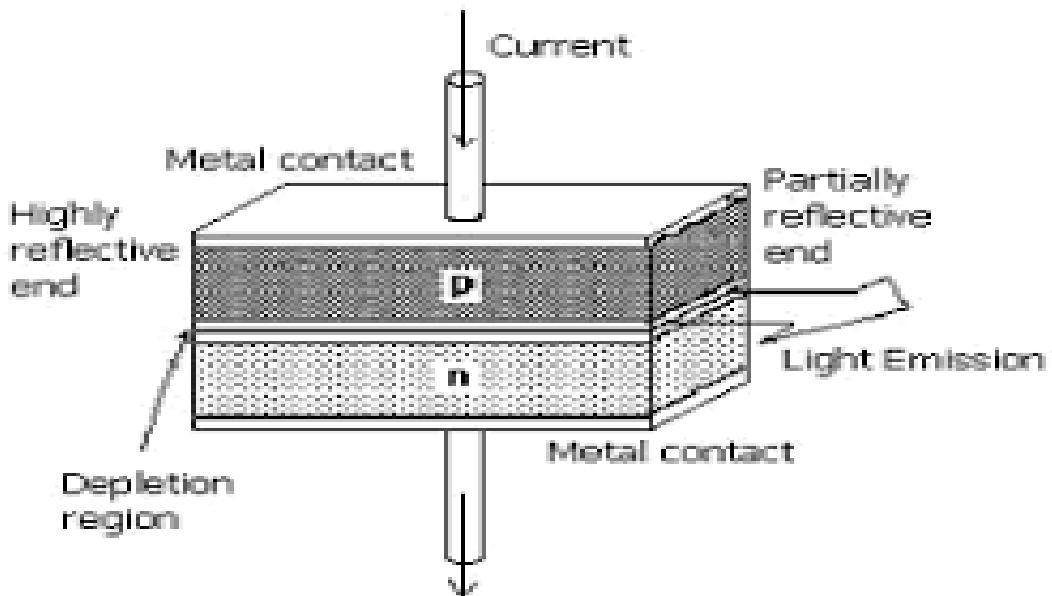
Question(48): Discuss the construction and working of semiconductor laser.

Introduction:

- Semiconductor lasers are more advantageous than any other laser.
 - (a) They are easy to fabricate.
 - (b) Less complex circuitry.
 - (c) Low cost
 - (d) Portable
- In semiconductor laser, a direct band gap semiconductor is used where recombination of electron and hole emit energy in the form of light.
- The wavelength of emitted light depends on the energy gap of semiconductor.

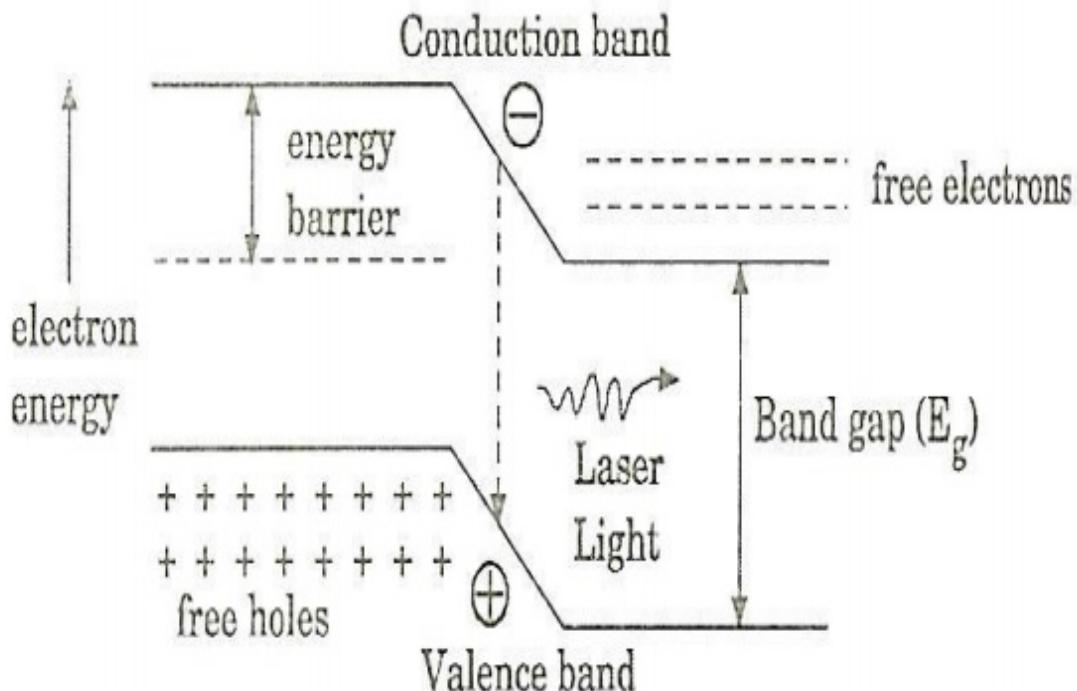
Construction:

- A Semiconductor laser contain a p-n Junction formed from direct band gap semiconductor which act as an active medium.
- The sides of the p-n junction are polished properly in such a way that one side act as fully reflective and the other as partially reflective.
- Direct pumping is used to achieve the population inversion
- Forward bias is applied to start the laser action.



Working:

- The working of semiconductor laser can be explained with the help of energy level diagram of a $p - n$ Junction as shown in Fig.



- Due to the presence of depletion region, there is no current across the junction.

- When Forward voltage is applied, the population inversion is achieved and electrons recombine with hole results light emission.
- The emitted light wavelength, $\lambda = \frac{hc}{E_g}$, where E_g is the energy band gap.
- For $GaAs$ semiconductor, $E_g = 1.44eV$, then the laser wavelength is 8626\AA .

Question(49): Derive an expression for the relations of Einstein's coefficients.

Solution: According to Einstein when radiation interact with matter three processes take place

- Absorption
- Spontaneous Emission
- Stimulated Emission

Let us assume that the number of atoms in energy level E_2 is N_2 and number of atoms in energy level E_1 is N_1 .

The rate of absorption depends on N_1 and incident energy density ρ

$$\begin{aligned} R_{ab(12)} &\propto N_1 \rho \\ R_{ab(12)} &= B_{12} N_1 \rho \end{aligned}$$

The rate of spontaneous emission depends on N_2

$$\begin{aligned} R_{SP(21)} &\propto N_2 \\ R_{SP(21)} &= A_{21} N_2 \end{aligned}$$

The rate of stimulated emission depends on N_2 and incident energy density ρ

$$\begin{aligned} R_{St(21)} &\propto N_2 \rho \\ R_{St(21)} &= B_{21} N_2 \rho \end{aligned}$$

At equilibrium, the rate of adsorption is equal to the rate of emission

$$\begin{aligned}
 R_{ab} &= R_{em} \\
 R_{ab} &= R_{SP} + R_{St} \\
 B_{12}N_1\rho &= A_{21}N_2 + B_{21}N_2\rho \\
 B_{12}N_1\rho - B_{21}N_2\rho &= A_{21}N_2 \\
 \rho[B_{12}N_1 - B_{21}N_2] &= A_{21}N_2 \\
 \rho &= \frac{A_{21}N_2}{[B_{12}N_1 - B_{21}N_2]} \\
 \rho &= \frac{A_{21}}{[B_{12}\left(\frac{N_1}{N_2}\right) - B_{21}]}
 \end{aligned}$$

From Boltzmann distribution law, we can write $N_1 = N_0 e^{-\frac{E_1}{KT}}$ and $N_2 = N_0 e^{-\frac{E_2}{KT}}$, it will give

$$\begin{aligned}
 \frac{N_1}{N_2} &= \frac{N_0 e^{-\frac{E_1}{KT}}}{N_0 e^{-\frac{E_2}{KT}}} \\
 \frac{N_1}{N_2} &= e^{\frac{(E_2-E_1)}{KT}} \\
 \frac{N_1}{N_2} &= e^{\frac{h\nu}{KT}} \quad (E_2 - E_1 = h\nu)
 \end{aligned}$$

Substituting this, we will get

$$\begin{aligned}
 \rho &= \frac{A_{21}}{[B_{12}\left(e^{\frac{h\nu}{KT}}\right) - B_{21}]} \\
 \rho &= \frac{A_{21}}{B_{12}\left[e^{\frac{h\nu}{KT}} - \frac{B_{21}}{B_{12}}\right]}
 \end{aligned}$$

According to Planck's, the incident radiation density is given by

$$\rho = \frac{8\pi h\nu^3}{c^3} \frac{1}{\left[e^{\frac{h\nu}{KT}} - 1\right]}$$

Comparing both the equations, we will get

$$\begin{aligned}
 B_{12} &= B_{21} \\
 \frac{A_{21}}{B_{21}} &= \frac{8\pi h\nu^3}{c^3}
 \end{aligned}$$

These two relations are Einstein A and B coefficients relations. It is observed that

$$A_{21} \ll B_{21}$$

This shows that the stimulated emission must dominate over spontaneous emission to produce high efficient laser.

Question(50): Distinguish between spontaneous and stimulated emissions.

Solution:

Spontaneous Emission	Stimulated Emission
<ul style="list-style-type: none"> After spending life time atom come to lower level and emit one photon known as spontaneous emission. 	<ul style="list-style-type: none"> Before life time, by applying energy atom comes to lower energy level and emit two photons known as stimulated emission.
<ul style="list-style-type: none"> Non-directional photons emitted in spontaneous emission. 	<ul style="list-style-type: none"> Highly directional photons are emitted.
<ul style="list-style-type: none"> The emitted photons have different wave length. 	<ul style="list-style-type: none"> The emitted photons have same wave length.
<ul style="list-style-type: none"> The emitted photons are incoherent. 	<ul style="list-style-type: none"> The emitted photons are highly coherent.
<ul style="list-style-type: none"> The emitted photons have low intensity. 	<ul style="list-style-type: none"> The emitted photons have high intensity.
<ul style="list-style-type: none"> Example is ordinary light. 	<ul style="list-style-type: none"> Example is laser light.

Question(51): Define population inversion. Write the characteristics of laser and mention the various applications of laser.

Solution:

Population inversion

In normal (general) condition, more number of atoms present in lower energy level than higher energy level as shown in fig.

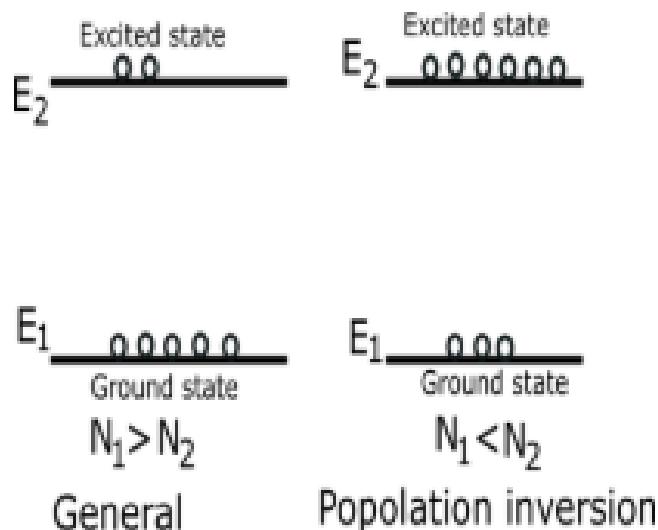


Figure 33: Energy Level system.

When energy applied, atoms move to the higher energy level makes more number of atoms in higher energy level than lower energy level. This process is called population inversion. The process of achieving population inversion is called pumping process.

Characteristics of Laser

Laser has certain unique properties when compared to ordinary sources of light. They are as follows

1. High Monochromaticity
2. High Directionality
3. High Intensity
4. High Coherence

Application of Lasers The most significant applications of lasers include

(i) Lasers in medicine

- Lasers are used for bloodless surgery.
- Lasers are used to destroy kidney stones.

(ii) Lasers in communications

- Laser light is used in underwater communication networks.
- Lasers are used in space communication, radars and satellites.

(iii) Lasers in Industries

- Lasers are used to cut glass and quartz.
- Lasers are used in electronic industries for trimming the components of Integrated Circuits (ICs).

(iV) Lasers in Science and Technology

- A laser helps in studying the Brownian motion of particles.
- Lasers are used in computers to retrieve stored information from a Compact Disc (CD).

(V) Lasers in Military

- Laser range finders are used to determine the distance to an object.
- Lasers are used to dispose the energy of a warhead by damaging the missile.

Question(52): Explain the classification of optical fibers in detail.

Solution:

Optical fiber are classified into two types based on refractive index profile, namely

- (a) Step index optical fibers
- (b) Graded index optical fibers

(a) Step index optical fibers:

- The optical fiber whose refractive index of core is constant and the change of refractive index from core to cladding is in a form of step, is known as step index optical fiber.

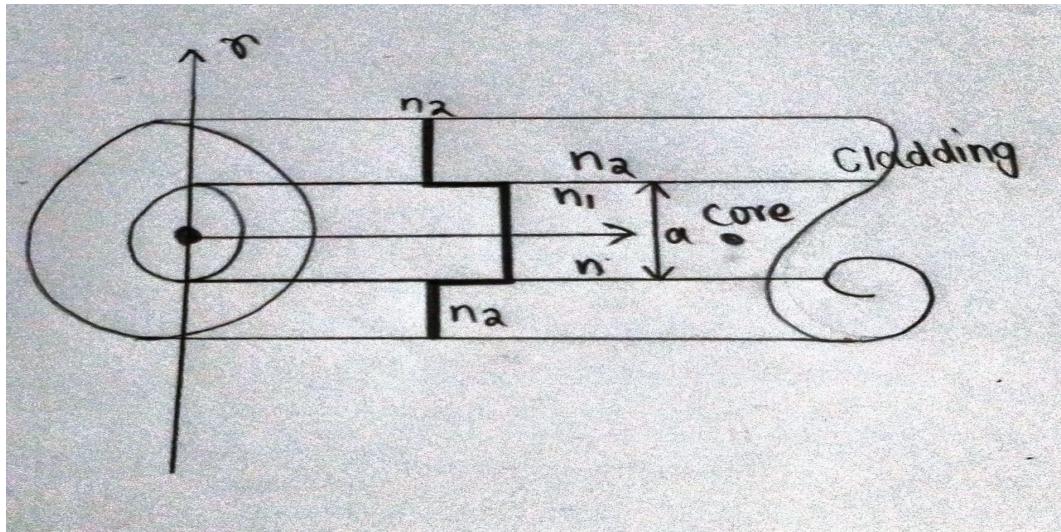
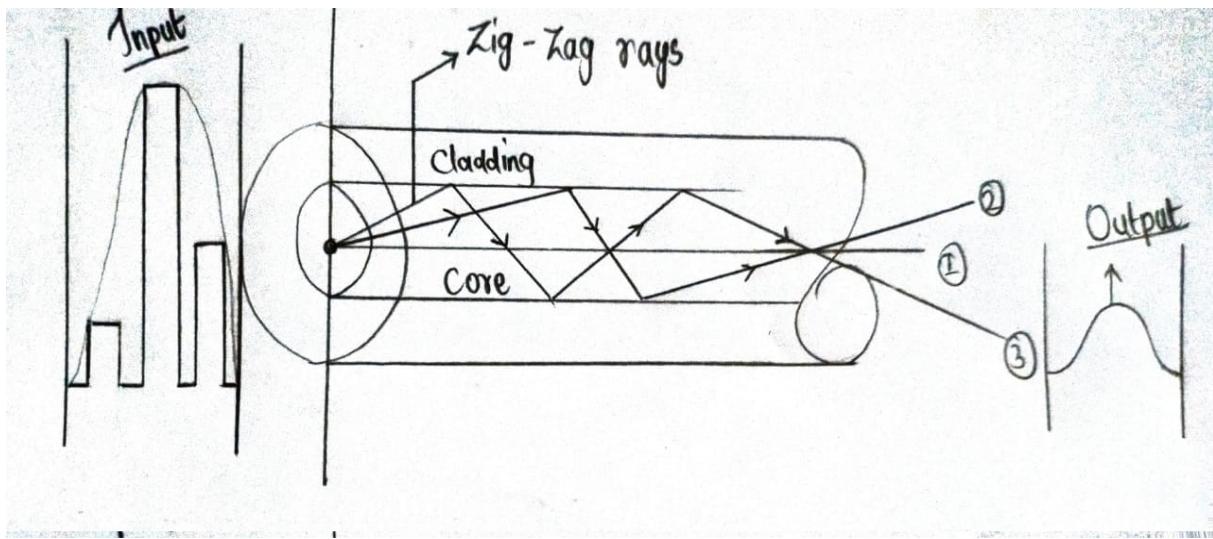


Figure 34: Step index optical fiber.

- The mathematical representation of refractive index is given by

$$n(r) = \begin{cases} n_1, & r < a \\ n_2, & r > a \end{cases}$$

- The numerical aperture of these optical fibers is high due to the greater diameter of core.
- The light signal propagate through step index optical fiber is as shown in figure.



- Here due to delay in signals all the light rays do not reach at the same time on the other side of optical fiber.
- Hence, the output signal gets broaden, which is known as intermodel dispersion.
- This limits the rate of transmission through these optical fibers.

(b) Graded index optical fibers:

- The optical fiber, whose refractive index of core is not constant but refractive index of cladding is constant.
- The refractive index of core is maximum at the axis of the core and decreases gradually towards core and cladding interface and hence called graded index optical fiber.

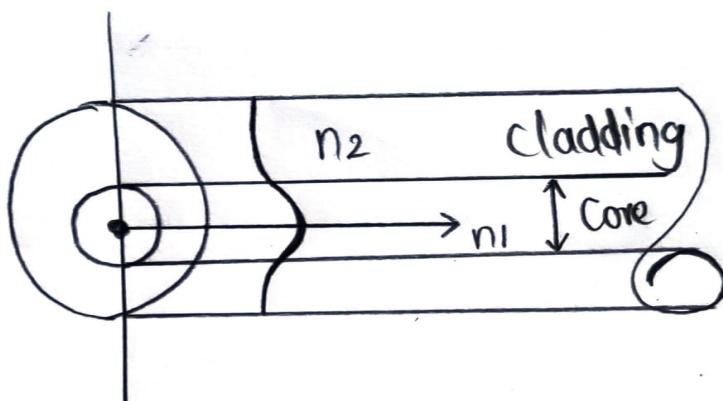
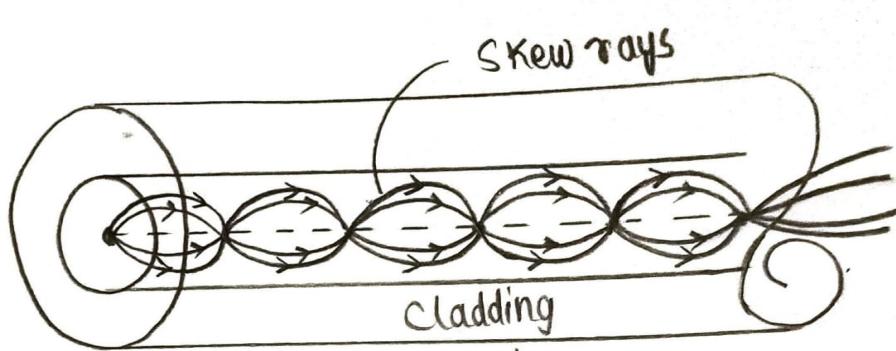


Figure 35: Graded index optical fiber.

- The propagation of light rays through graded optical fiber is shown in Fig.

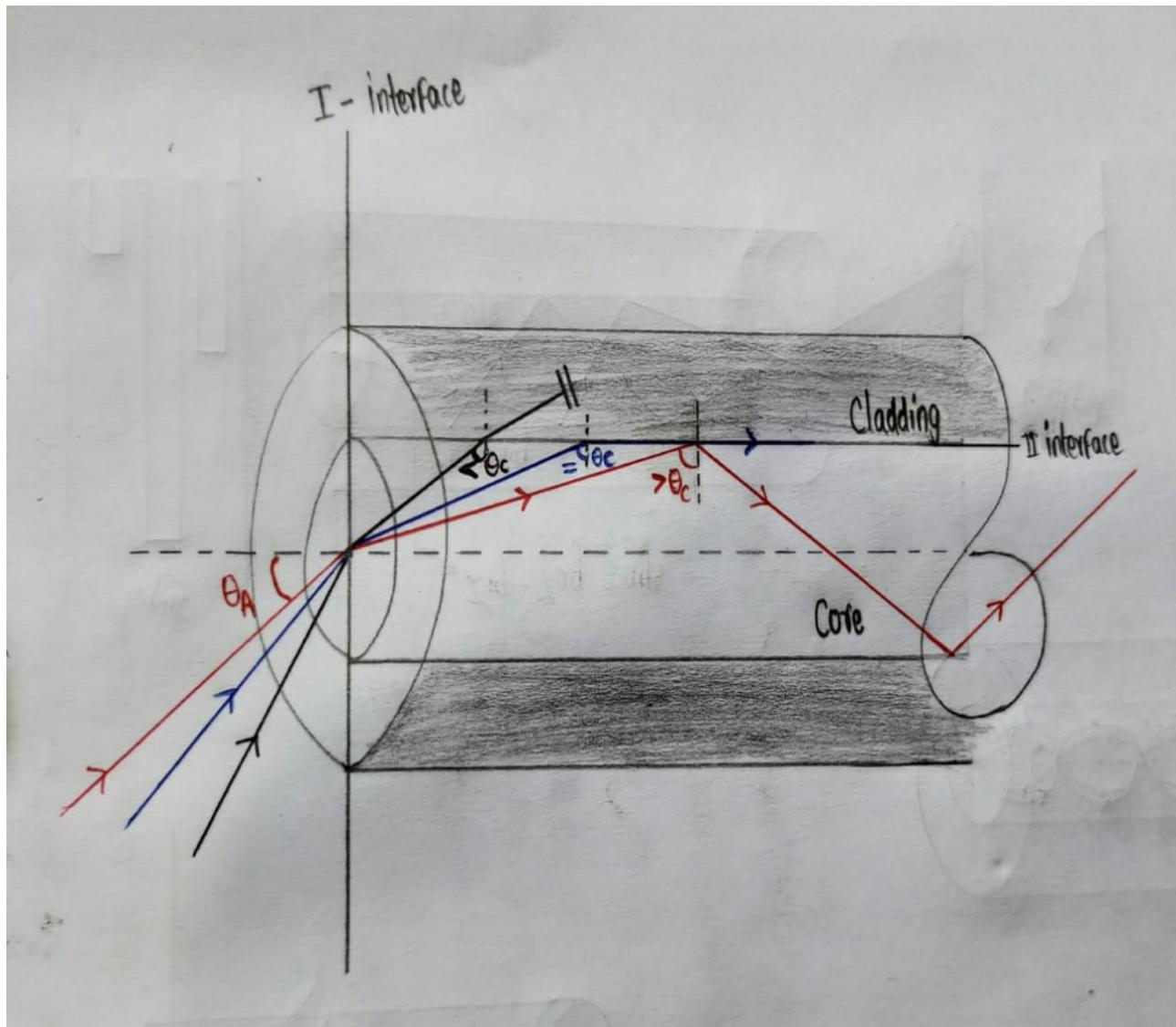


- The optical fiber allows only digital light signals.
- All the light rays undergo multiple refractions and total internal reflection, hence reach at the same time on the other side of the optical fiber.
- This is how the intermodel dispersion is overcome by varying the refractive index of the core as shown in Fig.
- Due to this reason, this optical fiber band width and transmission rate is high, but the numerical aperture is small due to the small core diameter.

Question(53): Derive an expression for the Numerical Aperture of an optical fiber.

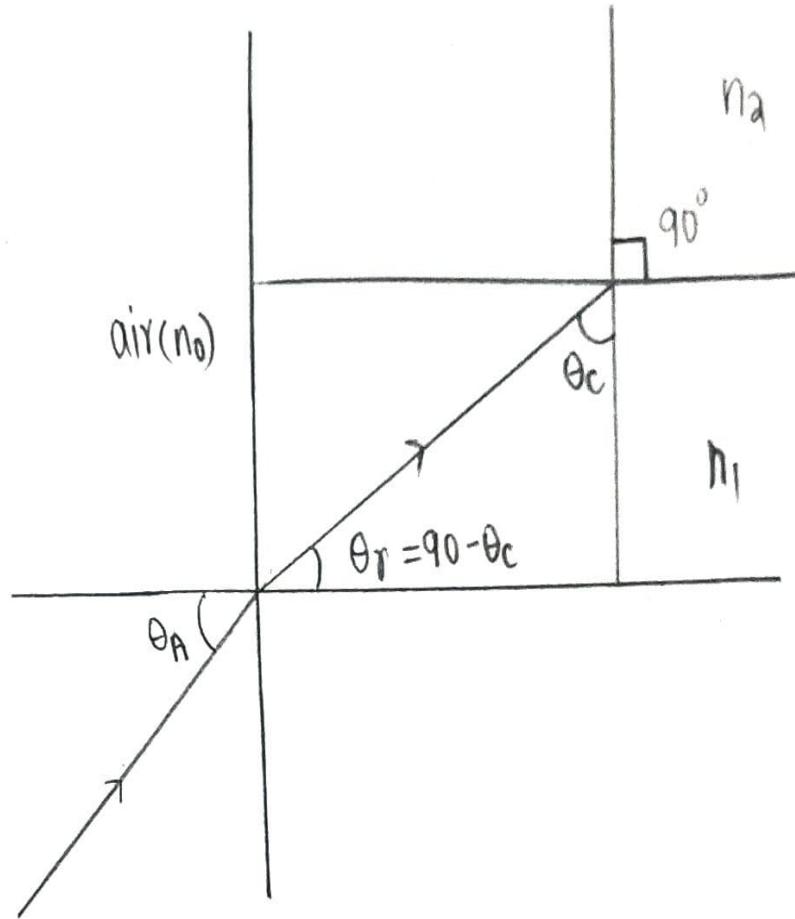
Solution:

- The angle of incidence at which the refracted ray undergo total internal reflection is known as acceptance angle. The acceptance angle is related with the Numerical Aperture.
- When light incident at one end of the optical fiber, assume that the three rays propagating through optical fiber as shown in Fig.



- (a) $\theta_i > \theta_A$ $\theta_i < \theta_C$ (Refraction)
- (b) $\theta_i = \theta_A$ $\theta_i = \theta_C$ (Acceptance)
- (c) $\theta_i < \theta_A$ $\theta_i > \theta_C$ (Total internal reflection (Acceptance))

Let n_0 , n_1 and n_2 are the refractive indices of air, core and cladding materials, respectively.



By applying Snell's law

$$n_0 \sin \theta_i = n_1 \sin \theta_r$$

$$\text{If } \theta_i = \theta_A \quad \theta_r = 90 - \theta_c$$

$$\begin{aligned} n_0 \sin \theta_A &= n_1 \sin(90 - \theta_c) \\ &= n_1 \cos \theta_c \\ &= n_1 \sqrt{1 - \sin^2 \theta_c} \end{aligned}$$

By applying Snell's law

$$\begin{aligned} n_1 \sin \theta_c &= n_2 \sin 90 \\ n_1 \sin \theta_c &= n_2 \\ \sin \theta_c &= \frac{n_2}{n_1} \end{aligned}$$

The above equation becomes

$$\begin{aligned} n_0 \sin \theta_A &= n_1 \sin(90 - \theta_c) \\ &= n_1 \cos \theta_c \\ &= n_1 \sqrt{1 - \sin^2 \theta_c} \\ &= n_1 \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2} \\ &= n_1 \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}} \\ &= \sqrt{n_1^2 - n_2^2} \\ \sin \theta_A &= \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \end{aligned}$$

Here n_0 is the refractive index of the air ($n_0 = 1$). Thus, the acceptance angle is given by

$$\theta_A = \sin^{-1} \left[\sqrt{n_1^2 - n_2^2} \right]$$

The Numerical Aperture is defined as

$$\begin{aligned} \text{NA} &= \sin \theta_A \\ \text{NA} &= \sqrt{n_1^2 - n_2^2} \end{aligned}$$

where n_1 and n_2 are the refractive indices of core and cladding materials, respectively.

Question(54): What is the principle of optical fibers? Explain various attenuation mechanisms in optical fibers.

Solution:

Principle of an optical fiber: Optical fiber works on the principle of total internal reflection. When a light ray incident with an angle greater than the critical angle then the ray totally reflected back into the same medium, this phenomena is known as total internal reflection.

Various attenuation (losses) mechanisms in optical fibers Loss in optical fiber

(i) Loss of light through an optical fiber is known as attenuation.

(ii) It is defined as the ratio of input power to the output power

$$\alpha = \frac{10}{h} \log_{10} \left(\frac{P_i}{P_o} \right) \quad \frac{\text{dB}}{\text{Km}}$$

where h : length of optical fiber and P_i and P_o : input and output powers, respectively.

(iii) It is measured in terms of $\frac{\text{dB}}{\text{Km}}$.

Types of losses in optical fiber There are three types of losses in optical fiber namely:

(a) Absorption losses

(b) Scattering losses

(a) Bending losses

(a) Absorption losses The light through an optical fiber can be absorbed by the following factors

- Absorption by defects/ imperfections
- Extrinsic absorption by impurity atoms (Mn, Cu, Fe, Ni)
- Intrinsic absorption by basic constituents of atom.

(b) Scattering losses They are of two types:

(1) Linear scattering losses

(2) Non Linear scattering losses

(1) Linear scattering losses These losses occur when light power linearly transfer from one propagation mode to another. Again, there are two linear scattering losses,

(i) Rayleigh scattering losses

(ii) Mie scattering losses

(2) Non Linear scattering losses When we use high optical power, non linear scattering losses arise. They are classified into two types:

(i) Brillouin scattering

(ii) Raman scattering

(c) Bending losses These losses arise when optical fiber bend. There are two bending losses

(i) Macro bending losses

(ii) Micro bending losses

(i) Macro bending losses: Macro bending (fiber bends large enough to be seen with the naked eye) losses occur when fibers are physically bent beyond the point at which the critical angle is exceeded.

(ii) Micro bending losses: Micro bending (bends too small to be seen with the naked eye) occur when pressure is applied to the surface of an optical fiber. The pressure applied to the surface results in deformation of the fiber core at the core-cladding interface.

Question(55): Explain types of optical fibers based on refractive index profiles.

Solution:

Optical fiber are classified into two types based on refractive index profile, namely

(a) Step index optical fibers

(b) Graded index optical fibers

(a) Step index optical fibers:

- The optical fiber whose refractive index of core is constant and the change of refractive index from core to cladding is in a form of step, is known as step index optical fiber.

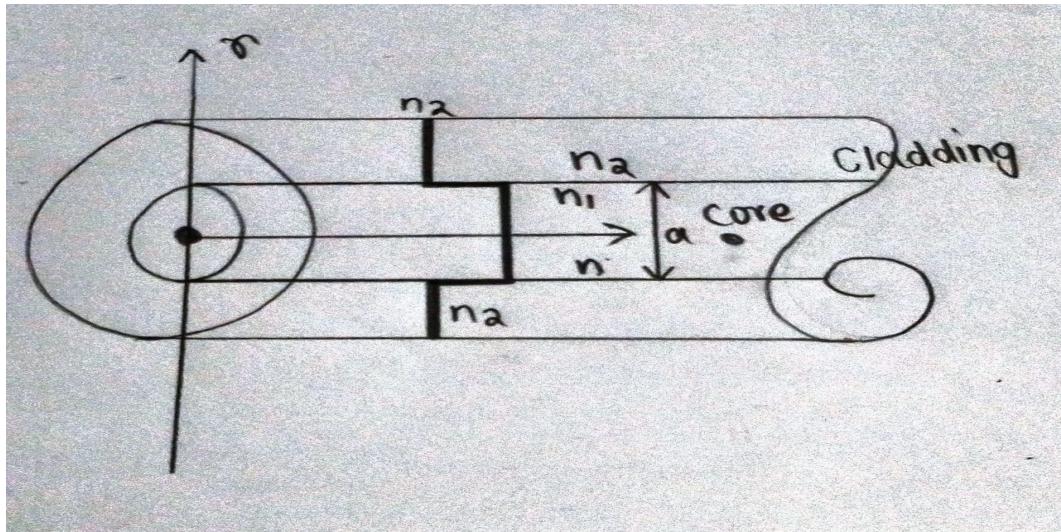
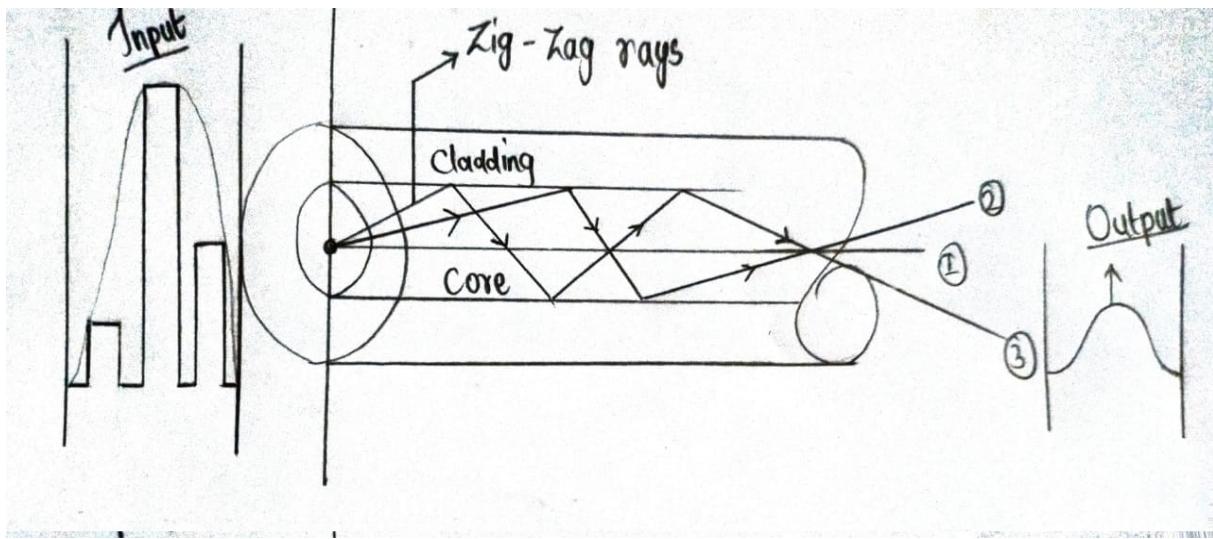


Figure 36: Step index optical fiber.

- The mathematical representation of refractive index is given by

$$n(r) = \begin{cases} n_1, & r < a \\ n_2, & r > a \end{cases}$$

- The numerical aperture of these optical fibers is high due to the greater diameter of core.
- The light signal propagate through step index optical fiber is as shown in figure.



- Here due to delay in signals all the light rays do not reach at the same time on the other side of optical fiber.
- Hence, the output signal gets broaden, which is known as intermodel dispersion.
- This limits the rate of transmission through these optical fibers.

(b) Graded index optical fibers:

- The optical fiber, whose refractive index of core is not constant but refractive index of cladding is constant.
- The refractive index of core is maximum at the axis of the core and decreases gradually towards core and cladding interface and hence called graded index optical fiber.

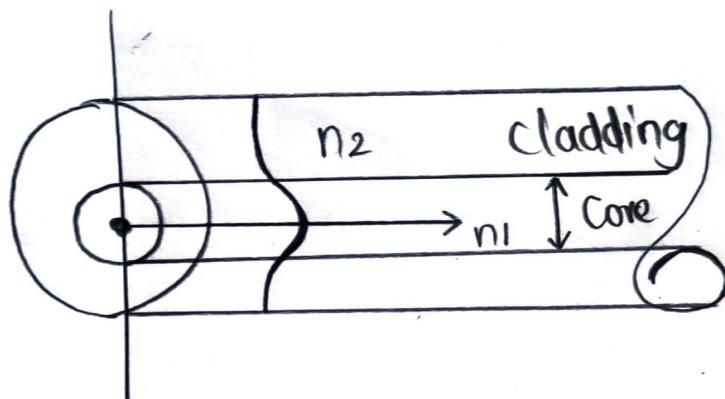
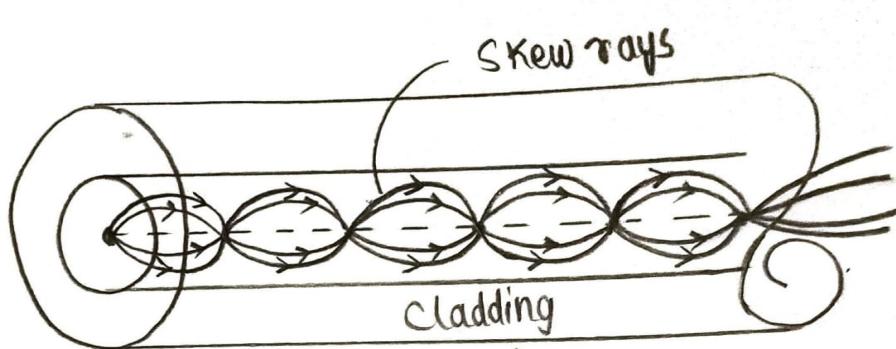


Figure 37: Graded index optical fiber.

- The propagation of light rays through graded optical fiber is shown in Fig.



- The optical fiber allows only digital light signals.
- All the light rays undergo multiple refractions and total internal reflection, hence reach at the same time on the other side of the optical fiber.
- This is how the intermodel dispersion is overcome by varying the refractive index of the core as shown in Fig.
- Due to this reason, this optical fiber band width and transmission rate is high, but the numerical aperture is small due to the small core diameter.

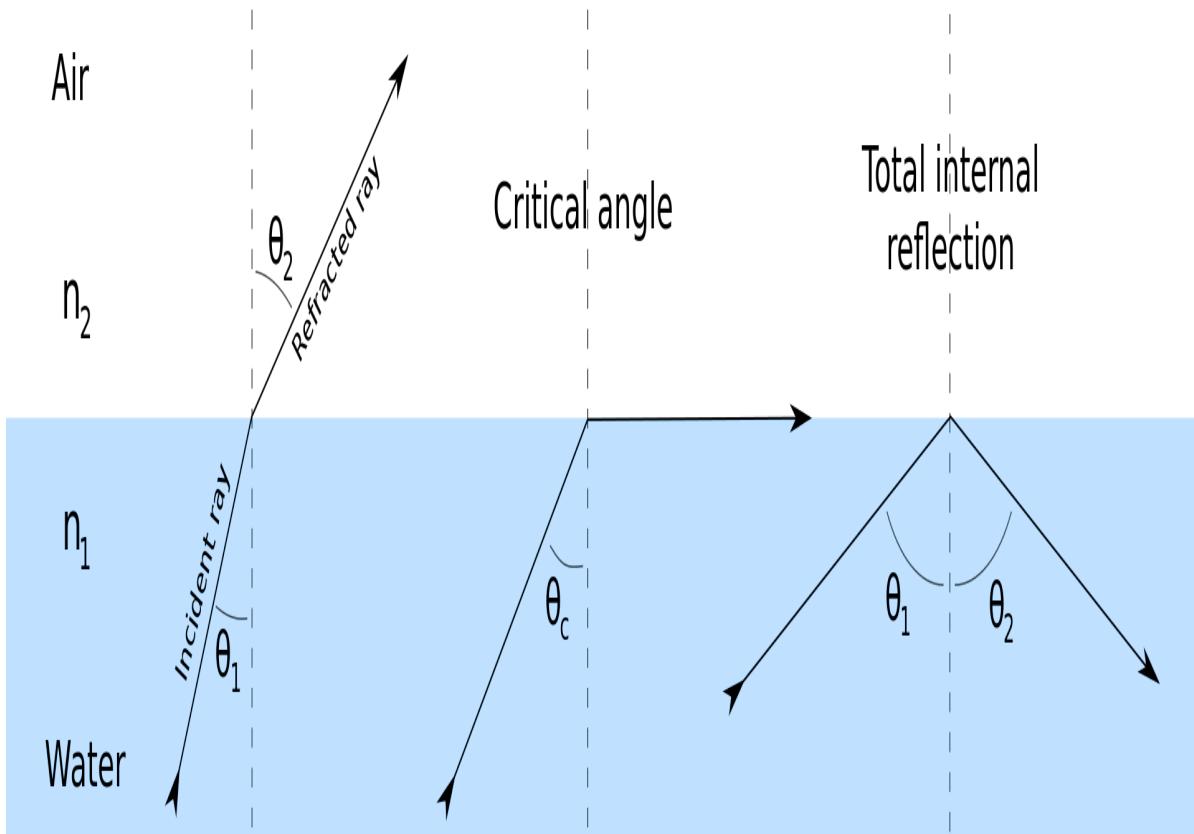
Question(56): Explain the propagation of light through optical fiber by total internal reflection.

Solution:

Propagation of light energy in the form of optical signals inside the core-cladding arrangement and throughout the length of the fiber takes place by a phenomenon called the Total Internal Reflection (TIR) of light. This phenomenon occurs only when the refractive index of core is greater than the refractive index of cladding and so the cladding is made from glass of lower refractive index. By multiple total internal reflections at the core-cladding interface the light propagates throughout the fiber over very long distances with low attenuation.

Total internal reflection The explanation of total internal reflection is as follows:

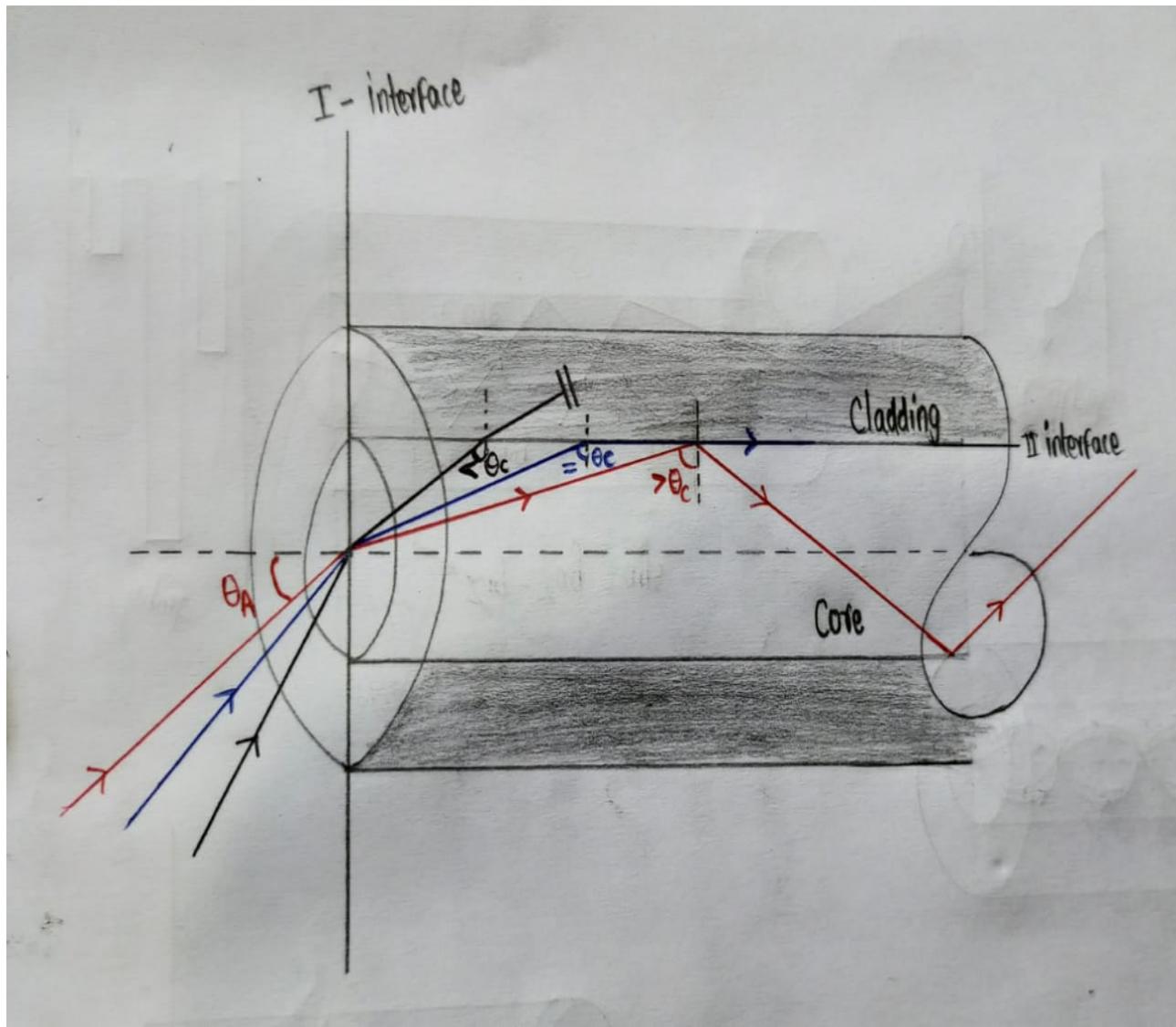
- When a light ray moving from denser to rarer medium and if it makes an angle less than the critical angle, then the ray refracted into the rarer medium as shown in Fig.
- When a light ray incident with critical angle, then it propagate along the interface as shown in Fig.
- When a light ray incident with an angle greater than the critical angle then the ray totally reflected back into the same medium, this phenomena is known as total internal reflection which is shown in Fig.



Question(57): Derive an expression for the acceptance angle in optical fibers.

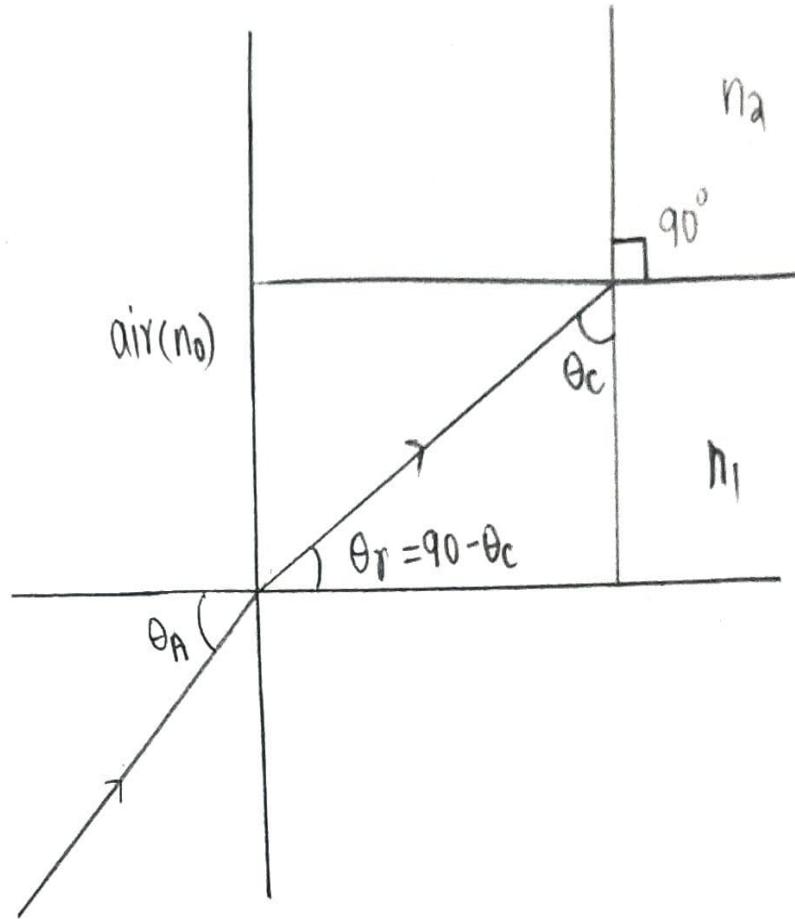
Solution:

- The angle of incidence at which the refracted ray undergo total internal reflection is known as acceptance angle.
- When light incident at one end of the optical fiber, assume that the three rays propagating through optical fiber as shown in Fig.



- (a) $\theta_i > \theta_A$ $\theta_i < \theta_C$ (Refraction)
- (b) $\theta_i = \theta_A$ $\theta_i = \theta_C$ (Acceptance)
- (c) $\theta_i < \theta_A$ $\theta_i > \theta_C$ (Total internal reflection (Acceptance))

Let n_0 , n_1 and n_2 are the refractive indices of air, core and cladding materials, respectively.



By applying Snell's law

$$n_0 \sin \theta_i = n_1 \sin \theta_r$$

If $\theta_i = \theta_A$ $\theta_r = 90 - \theta_c$

$$\begin{aligned} n_0 \sin \theta_A &= n_1 \sin(90 - \theta_c) \\ &= n_1 \cos \theta_c \\ &= n_1 \sqrt{1 - \sin^2 \theta_c} \end{aligned}$$

By applying Snell's law

$$\begin{aligned} n_1 \sin \theta_c &= n_2 \sin 90 \\ n_1 \sin \theta_c &= n_2 \\ \sin \theta_c &= \frac{n_2}{n_1} \end{aligned}$$

The above equation becomes

$$\begin{aligned} n_0 \sin \theta_A &= n_1 \sin(90 - \theta_c) \\ &= n_1 \cos \theta_c \\ &= n_1 \sqrt{1 - \sin^2 \theta_c} \\ &= n_1 \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2} \\ &= n_1 \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}} \\ &= \sqrt{n_1^2 - n_2^2} \\ \sin \theta_A &= \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \end{aligned}$$

Here n_0 is the refractive index of the air ($n_0 = 1$). Thus, the acceptance angle is given by

$$\theta_A = \sin^{-1} \left[\sqrt{n_1^2 - n_2^2} \right]$$

where n_1 and n_2 are the refractive indices of core and cladding materials, respectively.

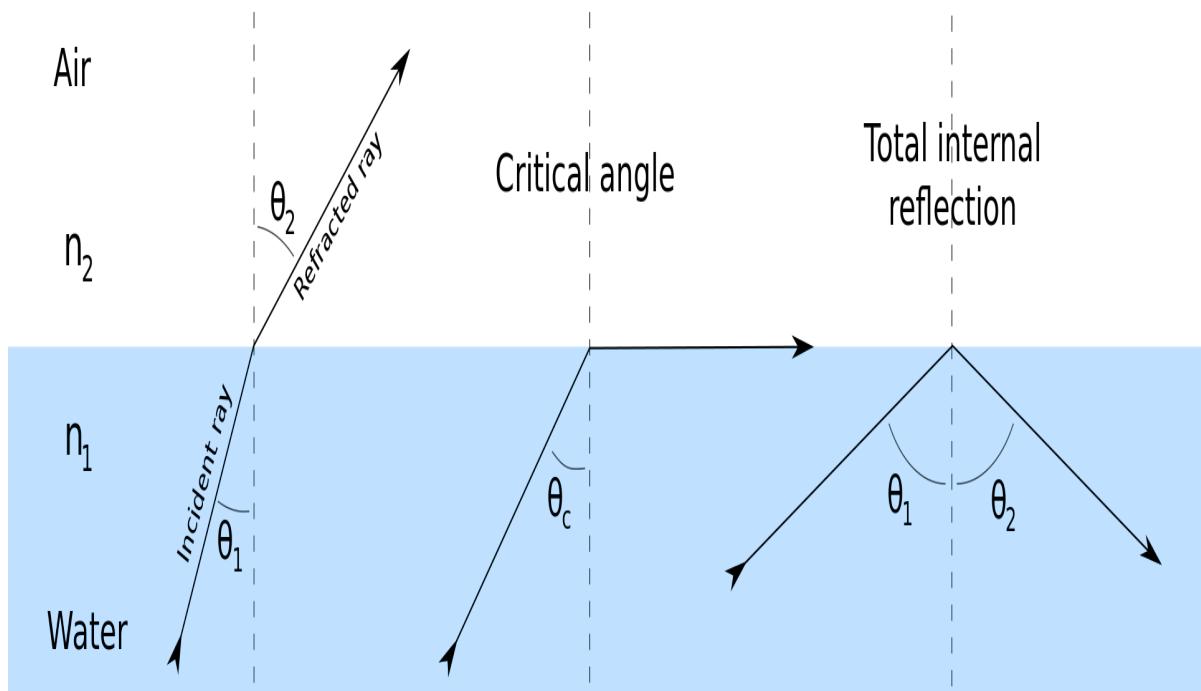
Question(58): Discuss the principle of an optical fiber. Write applications of optical fibers.

Solution:

Principle of an optical fiber: Optical fiber works on the principle of total internal reflection. The explanation of total internal reflection is as follows:

- When a light ray moving from denser to rarer medium and if it makes an angle less than the critical angle, then the ray refracted into the rarer medium as shown in Fig.

- When a light ray incident with critical angle, then it propagate along the interface as shown in Fig.
- When a light ray incident with an angle grater than the critical angle then the ray totally reflected back into the same medium, this phenomena is known as total internal reflection which is shown in Fig.



Applications of Optical Fibers

- **Medical industry:** In several instruments, it is utilised to observe interior body components by entering into hollow places in the body because of its thin and flexible nature. Surgical lasers, endoscope lasers, microscope lasers, and biomedical lasers all make use of fiber lasers.
- **Communication:** Optical fibre cables play an important role in the principle of optical fibre communication systems because they are used for both transmission and reception. Speed and accuracy can be improved by using it in many networking applications.
- **Defense:** High-level data security sectors of military and aerospace applications use fiber optics for data transfer. Hydrophones for SONAR and seismic applications, as well as aviation wiring, make use of this material.

- **Broadcasting:** Such cables deliver high-speed, high-bandwidth HDTV signals. When compared to the same amount of copper cables, fibre optic cable is less expensive. Broadcasters employ optical fibers to connect HDTV, CATV, VOD, and other services.
- **Decorations and lighting:** Now that we know what optical fibre is, we can see why it's so popular in Christmas decorations and trees: it's inexpensive, easy to use, and beautiful.

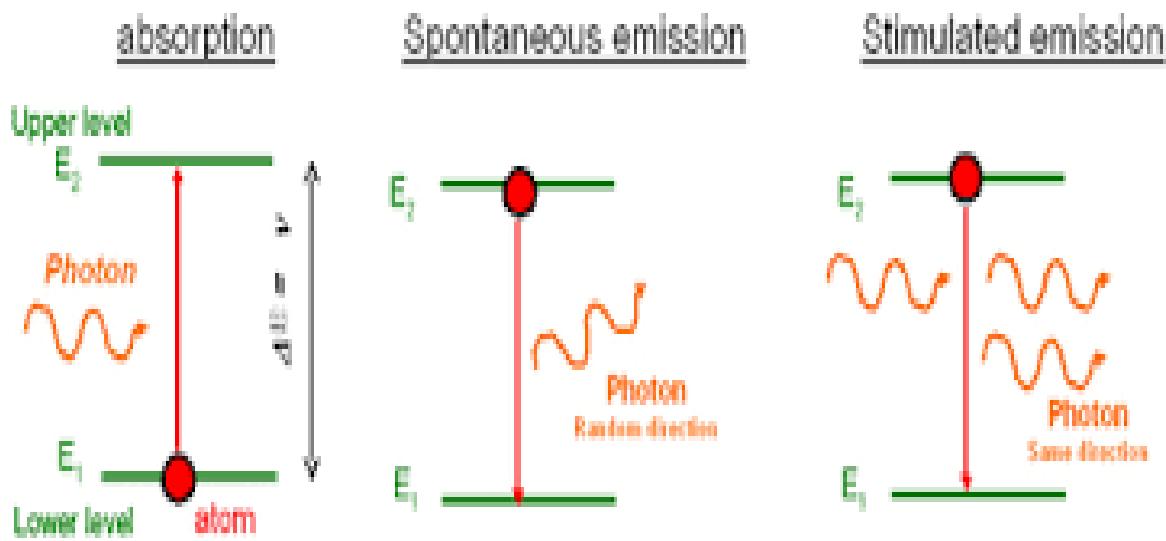
Question(59): Elaborate absorption, spontaneous emission and stimulated emissions with diagrams.

Solution:

Absorption: When a photon of energy ($h\nu$) is absorbed, this energy may cause an atom of energy (E_1) to go from a lower level to a higher energy level, as a result the energy of the atom in higher energy level will be $E_2 = E_1 + h\nu$. This process is known as absorption of radiation.

Spontaneous emission: An atom in an upper level can decay spontaneously to the lower level and emit a photon of energy $h\nu = E_2 - E_1$, if the transition between E_2 and E_1 is radiative. This photon has a random direction and phase.

Stimulated emission: An incident photon causes an upper level atom to decay, emitting a “stimulated” photon whose properties are identical to those of the incident photon. The term “stimulated” underlines the fact that this kind of radiation only occurs if an incident photon is present.



Question(60): Discuss the losses in optical fibers.

Solution:

Loss in optical fiber

- (i) Loss of light through an optical fiber is known as attenuation.
- (ii) It is defined as the ratio of input power to the output power

$$\alpha = \frac{10}{h} \log_{10} \left(\frac{P_i}{P_o} \right) \quad \frac{\text{dB}}{\text{Km}}$$

where h : length of optical fiber and P_i and P_o : input and output powers, respectively.

- (iii) It is measured in terms of $\frac{\text{dB}}{\text{Km}}$.

Types of losses in optical fiber There are three types of losses in optical fiber namely:

- (a) Absorption losses
- (b) Scattering losses
- (c) Bending losses

(a) Absorption losses The light through an optical fiber can be absorbed by the following factors

- Absorption by defects/ imperfections
- Extrinsic absorption by impurity atoms (Mn , Cu , Fe , Ni)
- Intrinsic absorption by basic constituents of atom.

(b) Scattering losses They are of two types:

- (1) Linear scattering losses
- (2) Non Linear scattering losses

(1) Linear scattering losses These losses occur when light power linearly transfer from one propagation mode to another. Again, there are two linear scattering losses,

- (i) Rayleigh scattering losses
- (ii) Mie scattering losses

(2) Non Linear scattering losses When we use high optical power, non linear scattering losses arise. They are classified into two types:

- (i) Brillouin scattering
- (ii) Raman scattering

(c) Bending losses These losses arise when optical fiber bend. There are two bending losses

- (i) Macro bending losses
- (ii) Micro bending losses

(i) Macro bending losses: Macro bending (fiber bends large enough to be seen with the naked eye) losses occur when fibers are physically bent beyond the point at which the critical angle is exceeded.

(ii) Micro bending losses: Micro bending (bends too small to be seen with the naked eye) occur when pressure is applied to the surface of an optical fiber. The pressure applied to the surface results in deformation of the fiber core at the core-cladding interface.

UNIT-III (SAQs)

Question(61): Write any two properties of wave function.

Solution:

1. It is variable quantity i.e., associated with a moving particle at any position (x, y, z) and at any time t and it relates probability of finding the particle at that point and at that time. It is generally represented by ψ .
2. It must be well behaved, that is single valued and continuous everywhere.

Question(62): List any two properties of matter waves.

Solution:

1. The de-Broglie wavelength of particle of mass (m) moving with a velocity (v) is given by $\lambda = \frac{h}{mv}$.
If $v = 0$, then $\lambda = \infty$ and if $v = \infty$, then $\lambda = 0$.
This implies that matter waves are associated with material particles only if they are in motion.
2. Lesser the velocity of the particle, larger the wave length associated with it.

Question(63): What is the physical significance of wave function?

Solution:

- ψ is the wave function.
- Wave function gives displacement of matter waves.
- Its not a physical quantity.
- It is a complex quantity $\psi = a + ib$, its complex conjugate is $\psi^* = a - ib$

$$\begin{aligned}\psi\psi^* &= (a + ib)(a - ib) \\ |\psi|^2 &= a^2 + b^2\end{aligned}\tag{57}$$

which is real and positive. This can be denoted with $|\psi|^2 = \psi\psi^*$ known as probability density, which means probability of finding a particle at a point or volume or space. Hence $|\psi|^2$ is called physical significance of ψ .

Question(64): Explain de-Broglie's wavelength.

Solution: According to de-Broglie hypothesis, every moving particle has always associated with a wave. The particle is controlled by this wave in every aspect. The wave associated with moving particle of matter (like electrons, protons etc) are known as matter wave or pilot waves or de-Broglie waves.



If a particle of mass m moving with velocity v , then the wavelength of matter wave is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Where

h : Planck's constant

m : mass of particle

v : velocity of the particle

Question(65): Find the wavelength of an electron when accelerated by a potential of 1600V.

Solution:

Given

$$V = 1600V$$

The wavelength of electron is given by

$$\begin{aligned}\lambda &= \frac{12.27}{\sqrt{V}} \text{ \AA} \\ &= \frac{12.27}{\sqrt{1600}} \text{ \AA} \\ &= \frac{12.27}{40} \text{ \AA} \\ \lambda &= 0.306 \text{ \AA}\end{aligned}$$

Question(66): What is the objective of Davisson-Germer experiment with electrons and which hypothesis is supported by this experiment.

Solution: The Davisson-Germer experiment was carried out to confirm the wave character of electrons. It is the first experiment that provides evidence for the wave nature of matter. The Davisson-Germer experiment demonstrates the wave nature of matter particles. The Davisson-Germer experiment gives crucial evidence of the de-Broglie hypothesis, which states that particles such as electrons have a dual nature.

Question(67): Matter waves are not electromagnetic waves. Justify?

Solution:

Matter Waves	EM waves
Matter waves are independent of charge	EM waves depends on charge
$\lambda = \frac{h}{mv}$	$\lambda = \frac{hc}{E}$
Matter waves velocity is not constant	EM waves velocity is constant.

Question(68): Find the wavelength of a proton moving with a velocity of 2200 m/s. (mass of the proton = 1.67×10^{-27} kg)

Solution:

Given

$$\begin{aligned}m &= 1.67 \times 10^{-27} \text{ kg} \\v &= 2200 \text{ m/s}\end{aligned}$$

The wavelength of proton is given by

$$\begin{aligned}\lambda &= \frac{h}{mv} \\&= \frac{6.626 \times 10^{-34}}{1.67 \times 10^{-27} \times 2200} \\&= \frac{6.626 \times 10^{-9}}{36.74} \\&= 0.18 \times 10^{-9} \text{ m} \\&= 1.8 \times 10^{-10} \text{ m} \\&= 1.8 \text{ \AA}\end{aligned}$$

Question(69): State Bloch's theorem.

Solution:

Bloch theorem states that the solutions of Schrodinger wave equation for an electron moving in a periodic potential are the plane waves modulated by a function having the same periodicity as that of the lattice. According to Bloch theorem, the general solution of wave can be represented as

$$\psi(x) = e^{\pm iKx}u_K(x)$$

where $u_K(x) = u_K(x + a)$, is a function having complete periodicity of the lattice and K is the wave vector.

Question(70): Sketch energy band structures for conductors, semiconductors and insulators.

Solution:

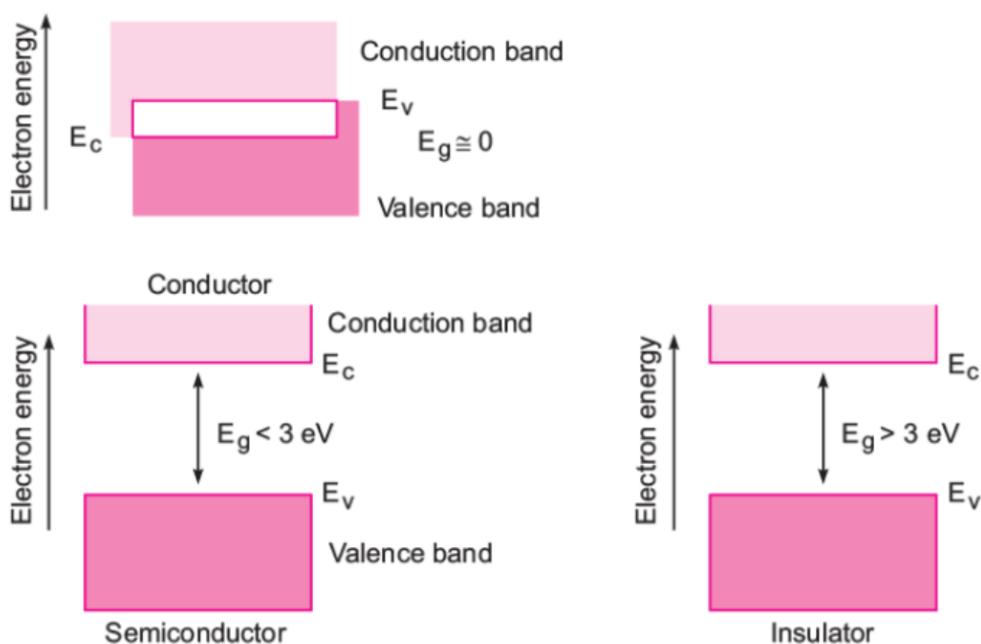


Figure 38: Energy band diagrams for conductor, semiconductor and insulator.

Question(71): Write any two differences between intrinsic and extrinsic semiconductors.

Solution:

Intrinsic Semiconductor:

- It is pure, natural semiconductor, such as pure Ge and pure Si .

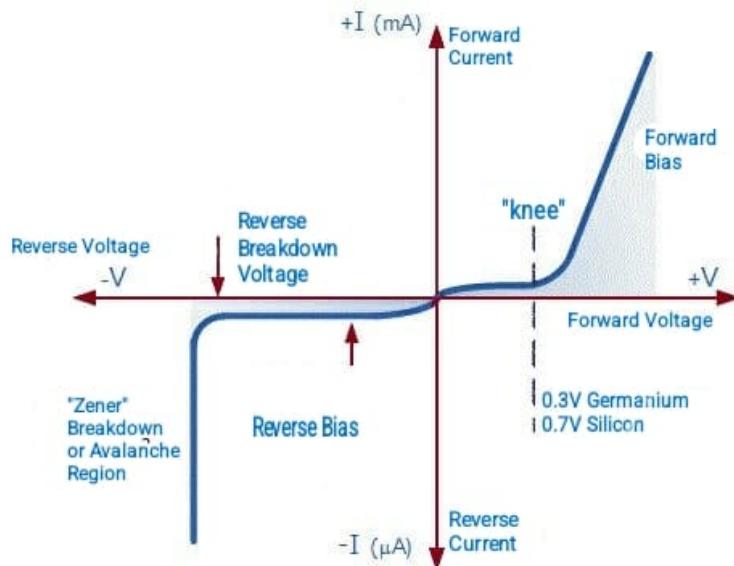
- The concentrations of electrons and holes are equal.

Extrinsic Semiconductor:

- It is prepared by adding a small quantity of impurity to a pure semiconductor, such as n-type and p-type semiconductors.
- The two concentrations are unequal. There is an excess of electrons in n-type semiconductors and an excess of holes in p-type semiconductors.

Question(72): Draw I-V characteristics of p-n junction diode.

Solution:



Question(73): List any two applications of Hall Effect.

Solution: The following are the applications of Hall effect:

- Hall effect is used for determining carrier concentrations (electron or hole concentration) in semiconductor.
- Hall effect is used to determine the type of semiconductor.

Question(74): Define Solar cell and mention the applications of solar energy.

Solution:

Solar Cell Solar cells, also called photovoltaic cells, directly transform energy into electricity from the sun. Renewable energy is provided by solar cells, and they are durable, compact and low-maintenance.

Applications: Sunlight, or solar energy, can be used specifically for residential and enterprise heating and lighting, for power generation, for hot water heating, for solar cooling, and for a range of other commercial and industrial applications

Question(75): Write down the various applications of LED.

Solution: Following are the various applications of Light Emitting Diode (LED):

- TV Backlighting
- Smartphone Backlighting
- LED displays
- Automotive Lighting
- Dimming of lights

UNIT-III (LAQs)

Question(76): Derive an expression for the Schrodinger's time independent wave equation.

Solution: Let us consider a particle of mass m moving with a velocity v . Let ψ be the wave function of a particle along x -direction at any time t (as shown in Fig).

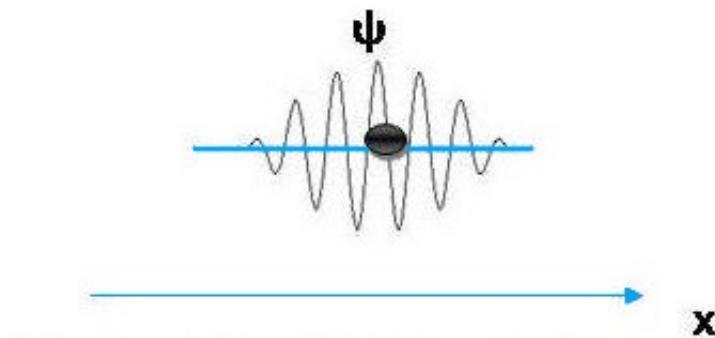


Figure 39: Wave function of a particle.

The wave function is given by

$$\psi = \psi_0 \sin(\omega t - kx) \quad (58)$$

where ψ_0 is the amplitude of the wave at any point and ω is the angular frequency. Differentiating above equation with respect to x , we get

$$\frac{d\psi}{dx} = \psi_0 \cos(\omega t - kx) \times (-k) \quad (59)$$

Differentiating once again with respect to x , we get

$$\begin{aligned} \frac{d^2\psi}{dx^2} &= -k\psi_0[-\sin(\omega t - kx) \times (-k)] \\ &= -k^2 \psi_0 \sin(\omega t - kx) \\ &= -k^2 \psi \end{aligned}$$

$$\frac{d^2\psi}{dx^2} + k^2 \psi = 0 \quad (60)$$

$$\begin{aligned} k &= \frac{2\pi}{\lambda} \\ k^2 &= \frac{4\pi^2}{\lambda^2} \end{aligned} \quad (61)$$

where λ is the wavelength. After substituting, we get

$$\frac{d^2\psi}{dx^2} + \frac{4\pi^2}{\lambda^2}\psi = 0 \quad (62)$$

According to de-Broglie relation, we can write

$$\lambda = \frac{h}{mv} \quad (63)$$

$$\frac{d^2\psi}{dx^2} + \frac{4\pi^2 m^2 v^2}{h^2}\psi = 0 \quad (64)$$

If E is the total energy of a particle

$$\begin{aligned} E &= \text{Kinetic Energy} + \text{Potential Energy} \\ E &= \frac{1}{2}mv^2 + V \\ 2(E - V) &= mv^2 \\ 2m(E - V) &= m^2v^2 \end{aligned} \quad (65)$$

Substituting, we get

$$\begin{aligned} \frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V)\psi &= 0 \\ \frac{d^2\psi}{dx^2} + \frac{2m}{(\frac{h}{2\pi})^2}(E - V)\psi &= 0 \\ \frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi &= 0 \end{aligned} \quad (66)$$

where $\hbar = \frac{h}{2\pi}$ is the reduced Plank's constant. This equation is known as Schrodinger time independent one-dimensional wave equation.

Question(77): What are Matter waves? Write properties of matter waves. Explain physical significance of wavefunction.

Solution:

Matter/de-Broglie waves:

According to de-Broglie hypothesis, every moving particle has always associated with a wave. The particle is controlled by this wave in every aspect. The wave associated with moving particle of matter (like electrons, protons etc) are known as matter wave or pilot waves or de-Broglie waves.



Figure 40: Motion of a particle (electron).

If a particle of mass m moving with velocity v , then the wavelength of matter wave is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Where

h : Planck's constant

m : mass of particle

v : velocity of the particle

Properties of de-Broglie/matter waves

The following are the properties of matter waves.

- The de-Broglie wavelength of particle of mass (m) moving with a velocity (v) is given by $\lambda = \frac{h}{mv}$.
If $v = 0$, then $\lambda = \infty$ and if $v = \infty$, then $\lambda = 0$.

This implies that matter waves are associated with material particles only if they are in motion.

- Lesser the velocity of the particle, larger the wave length associated with it.
- Smaller the mass of the particle, the greater the wavelength associated with it.
- The matter wave of particle is independent of charge (or) nature of the particle.
- De-Broglie waves are pilot waves. The word, pilot means that these waves pilot (or) guide the particle.
- Mater waves are not electromagnetic waves.

Physical significance of wave function

- ψ is the wave function.
- Wave function gives displacement of matter waves.
- Its not a physical quantity.
- It is a complex quantity $\psi = a + ib$, its complex conjugate is $\psi^* = a - ib$

$$\begin{aligned}\psi\psi^* &= (a + ib)(a - ib) \\ |\psi|^2 &= a^2 + b^2\end{aligned}$$

which is real and positive. This can be denoted with $|\psi|^2 = \psi\psi^*$ known as probability density, which means probability of finding a particle at a point or volume or space. Hence $|\psi|^2$ is called physical significance of ψ .

Question(78): Derive an expression for the Schrodinger's time dependent wave equation.

Solution: The wave function ψ can be written as

$$\psi = \psi_0 e^{-i\omega t} \quad (67)$$

Where ψ_0 is the amplitude of the wave at any point and ω is the angular frequency. Differentiating ψ with respect to t , we get

$$\begin{aligned}\frac{d\psi}{dt} &= \psi_0 e^{-i\omega t} \times (-i\omega) \\ &= -i\omega \psi_0 e^{-i\omega t} \\ &= -i\omega \psi\end{aligned}\tag{68}$$

We know, $\omega = 2\pi\nu$ or $\nu = \frac{\omega}{2\pi}$. So, we can write $E = h\nu$ as

$$\begin{aligned}E &= h\frac{\omega}{2\pi} \\ E &= \hbar\omega \\ \omega &= \frac{E}{\hbar}\end{aligned}\tag{69}$$

Substituting, we get

$$\begin{aligned}\frac{d\psi}{dt} &= -i\frac{E}{\hbar}\psi \\ E\psi &= \frac{-\hbar}{i}\frac{d\psi}{dt} \\ E\psi &= \frac{i^2\hbar}{i}\frac{d\psi}{dt} \quad (\text{where } i^2 = -1) \\ E\psi &= i\hbar\frac{d\psi}{dt}\end{aligned}\tag{70}$$

Now we write the Schrodinger time independent wave equation

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E\psi - V\psi) = 0\tag{71}$$

After substitution, we get

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(i\hbar\frac{d\psi}{dt} - V\psi) = 0$$

Multiplying $\frac{\hbar^2}{2m}$ on both sides, we get

$$\begin{aligned}
 \frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + \frac{\hbar^2}{2m} \frac{2m}{\hbar^2} (i\hbar \frac{d\psi}{dt} - V\psi) &= 0 \\
 \frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + i\hbar \frac{d\psi}{dt} - V\psi &= 0 \\
 \frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} - V\psi &= -i\hbar \frac{d\psi}{dt} \\
 -\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V\psi &= i\hbar \frac{d\psi}{dt} \\
 \left(-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V \right) \psi &= i\hbar \frac{d\psi}{dt} \\
 H\psi &= E\psi
 \end{aligned} \tag{72}$$

where $H = \left(-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V\right)$ is the Hamiltonian operator and $E = i\hbar \frac{d}{dt}$ is the energy operator. This equation known as Schrodinger time dependent one-dimensional wave equation.

Question(79): Derive an expression for the energy values for a particle in 1-D box.

Solution: Let us consider a particle of mass m moving with velocity v along x -direction and is confined between infinite potential rigid walls, so that the particle has no chance of escaping from the box. Therefore, the particle bounces back and forth between the two walls as shown in Fig.

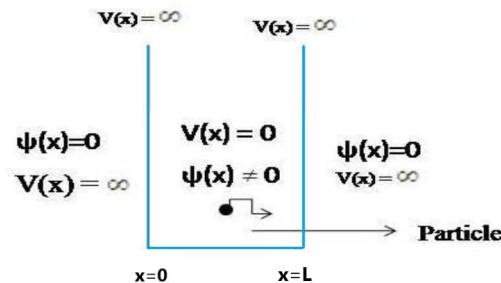


Figure 41: Particle moving in 1D box

Outside the box: ($x \leq 0$ and $x \geq L$)

The particle cannot exist outside and on the walls of the box, therefore the probability of finding the particle outside the wall is zero and the potential energy is ∞ :

$$\begin{aligned} V(x) &= \infty \\ \psi(x) &= 0 \end{aligned}$$

Inside the box: ($0 < x < L$)

The particle exists inside the box; therefore the probability of finding the electron inside the box is not equal to zero and the particle is moving freely, so potential energy is zero:

$$\begin{aligned} V(x) &= 0 \\ \psi(x) &\neq 0 \end{aligned}$$

As we have just seen that the wavefunction of particle outside the box is zero, so we can not solve the time independent Schrodinger equation for this region. However, for inside the box we can solve the time independent Schrodinger equation, which can be written as

$$\begin{aligned} \frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - 0)\psi &= 0 \\ \frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi &= 0 \\ \frac{d^2\psi}{dx^2} + k^2\psi &= 0 \end{aligned} \tag{73}$$

where the k^2 is defined as follows

$$\begin{aligned} k^2 &= \frac{2mE}{\hbar^2} \\ &= \frac{2mE}{\frac{\hbar^2}{4\pi^2}} \\ &= \frac{8\pi^2 m E}{\hbar^2} \end{aligned} \tag{74}$$

Now we can write the general solution as follows

$$\psi(x) = A \sin(kx) + B \cos(kx) \tag{75}$$

where A and B are the arbitrary constants, which can be found by applying the boundary conditions.

Boundary Condition at $x = 0$:

$$\begin{aligned}\psi(x = 0) &= 0 \\ A \sin 0 + B \cos 0 &= 0 \\ B &= 0\end{aligned}$$

which gives

$$\psi(x) = A \sin(kx) \quad (76)$$

Boundary Condition at $x = L$:

$$\begin{aligned}\psi(x = L) &= 0 \\ A \sin(kL) &= 0\end{aligned} \quad (77)$$

Since $A \neq 0$,

$$\sin(kL) = 0 \quad (78)$$

We know that,

$$\sin n\pi = 0 \quad (\text{where } n = 1, 2, 3, \dots) \quad (79)$$

After combining, we get

$$\begin{aligned}\sin(kL) &= \sin n\pi \\ kL &= n\pi \\ k &= \frac{n\pi}{L}\end{aligned} \quad (80)$$

Energy of the Particle

We have defined

$$\begin{aligned}k^2 &= \frac{8\pi^2 m E}{h^2} \\ \frac{n^2 \pi^2}{L^2} &= \frac{8\pi^2 m E}{h^2} \quad (\text{where we used } k^2 = \frac{n^2 \pi^2}{L^2}) \\ E_n &= \frac{n^2 h^2}{8m L^2}\end{aligned} \quad (81)$$

This equation is an expression for the energy of the particle.

- If $n = 1$, $E_1 = \frac{h^2}{8ml^2}$
- If $n = 2$, $E_2 = \frac{4h^2}{8ml^2} = 4E_1$
- If $n = 3$, $E_3 = \frac{9h^2}{8ml^2} = 9E_1$

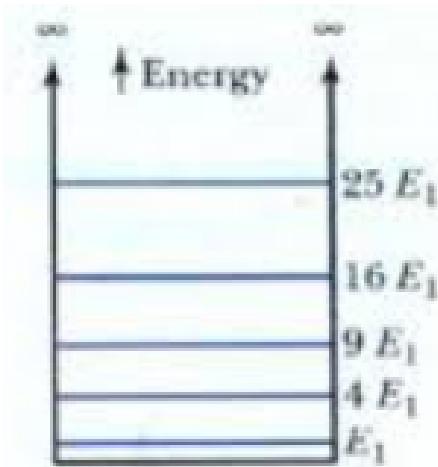


Figure 42: Energy levels of a particle enclosed in a 1D box.

Question(80): Derive an expression for the energy values and wave-functions for a particle in 1-D box.

Solution: Let us consider a particle of mass m moving with velocity v along x -direction and is confined between infinite potential rigid walls, so that the particle has no chance of escaping from the box. Therefore, the particle bounces back and forth between the two walls as shown in Fig.

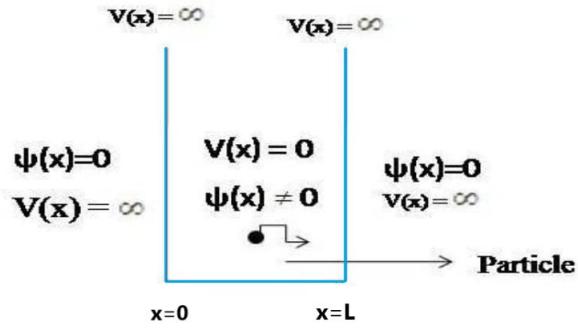


Figure 43: Particle moving in 1D box

Outside the box: ($x \leq 0$ and $x \geq L$)

The particle cannot exist outside and on the walls of the box, therefore the probability of finding the particle outside the wall is zero and the potential energy is ∞ :

$$\begin{aligned} V(x) &= \infty \\ \psi(x) &= 0 \end{aligned}$$

Inside the box: ($0 < x < L$)

The particle exists inside the box; therefore the probability of finding the electron inside the box is not equal to zero and the particle is moving freely, so potential energy is zero:

$$\begin{aligned} V(x) &= 0 \\ \psi(x) &\neq 0 \end{aligned}$$

As we have just seen that the wavefunction of particle outside the box is zero, so we can not solve the time independent Schrodinger equation for this region. However, for inside the box we can solve the time independent Schrodinger

equation, which can be written as

$$\begin{aligned}\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - 0)\psi &= 0 \\ \frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi &= 0 \\ \frac{d^2\psi}{dx^2} + k^2\psi &= 0\end{aligned}\tag{82}$$

where the k^2 is defined as follows

$$\begin{aligned}k^2 &= \frac{2mE}{\hbar^2} \\ &= \frac{2mE}{\frac{\hbar^2}{4\pi^2}} \\ &= \frac{8\pi^2 m E}{\hbar^2}\end{aligned}\tag{83}$$

Now we can write the general as follows

$$\psi(x) = A \sin(kx) + B \cos(kx)\tag{84}$$

where A and B are the arbitrary constants, which can be found by applying the boundary conditions.

Boundary Condition at $x = 0$:

$$\begin{aligned}\psi(x = 0) &= 0 \\ A \sin 0 + B \cos 0 &= 0 \\ B &= 0\end{aligned}$$

which gives

$$\psi(x) = A \sin(kx)\tag{85}$$

Boundary Condition at $x = L$:

$$\begin{aligned}\psi(x = L) &= 0 \\ A \sin(kL) &= 0\end{aligned}\tag{86}$$

Since $A \neq 0$,

$$\sin(kL) = 0 \quad (87)$$

We know that,

$$\sin n\pi = 0 \quad (\text{where } n = 1, 2, 3, \dots) \quad (88)$$

After combining, we get

$$\begin{aligned} \sin(kL) &= \sin n\pi \\ kL &= n\pi \\ k &= \frac{n\pi}{L} \end{aligned} \quad (89)$$

After substitution, we get

$$\psi_n(x) = A \sin\left(\frac{n\pi x}{L}\right) \quad (90)$$

This equation represents the wave function associated with moving free particle inside the box of length L .

By applying normalization condition

$$\begin{aligned} \int_0^L |\psi_n(x)|^2 dx &= 1 \\ \int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx &= 1 \\ A^2 \int_0^L \sin^2\left(\frac{n\pi x}{L}\right) dx &= 1 \\ \frac{A^2}{2} \int_0^L \left[1 - \cos\left(\frac{2n\pi x}{L}\right)\right] dx &= 1 \end{aligned} \quad (91)$$

where we have used $\cos 2x = 1 - 2 \sin^2 x$. After simplification, we will get

$$\begin{aligned} \frac{A^2}{2} \left[x - \frac{\sin\left(\frac{2n\pi x}{L}\right)}{\frac{2\pi n}{L}} \right]_0^L &= 1 \\ \frac{A^2}{2} \left[L - \frac{\sin 2n\pi}{\frac{2\pi n}{L}} \right] &= 1 \\ \frac{A^2}{2} [L] &= 1 \quad (\text{because } \sin 2n\pi = 0) \\ A &= \sqrt{\frac{2}{L}} \end{aligned} \quad (92)$$

Now the normalized wave function is written as

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \quad (93)$$

Energy of the Particle

We have defined

$$\begin{aligned} k^2 &= \frac{8\pi^2 m E}{h^2} \\ \frac{n^2 \pi^2}{L^2} &= \frac{8\pi^2 m E}{h^2} \quad (\text{where we used } k^2 = \frac{n^2 \pi^2}{L^2}) \\ E_n &= \frac{n^2 h^2}{8m L^2} \end{aligned} \quad (94)$$

This equation is an expression for the energy of the particle.

- If $n = 1$, $E_1 = \frac{h^2}{8ml^2}$
- If $n = 2$, $E_2 = \frac{4h^2}{8ml^2} = 4E_1$
- If $n = 3$, $E_3 = \frac{9h^2}{8ml^2} = 9E_1$

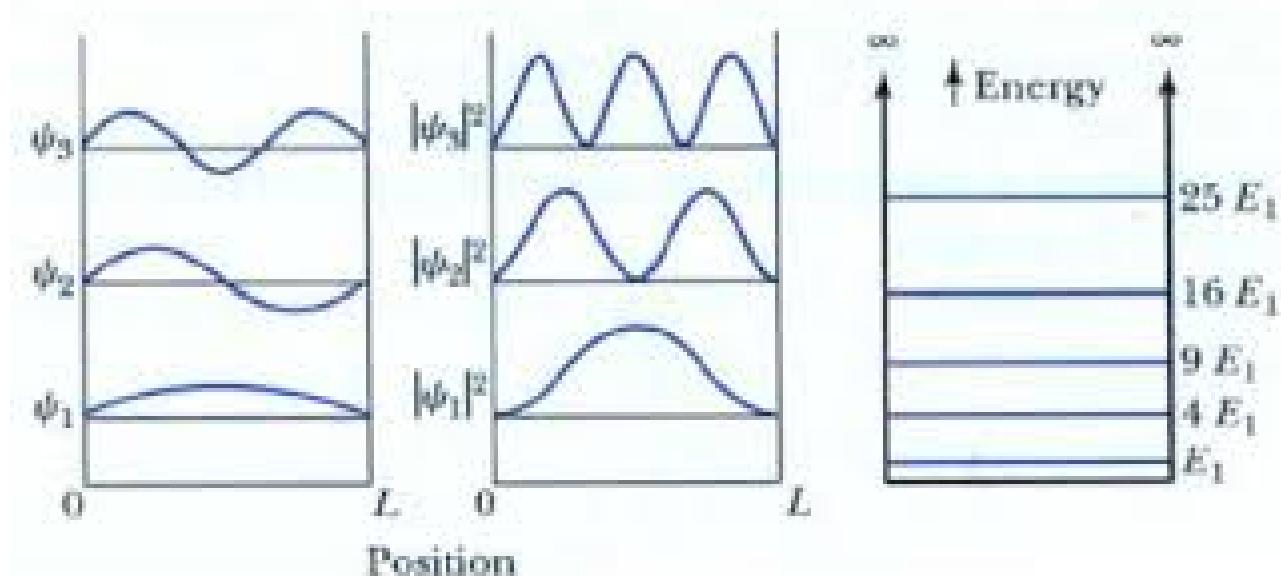


Figure 44: ψ , $|\psi|^2$ and Energy graphs of a particle enclosed in a 1D box.

Question(81): Apply Schrodinger's time independent wave equation to obtain energy values of a particle in 1-D box.

Solution: Let us consider a particle of mass m moving with velocity v along x -direction and is confined between infinite potential rigid walls, so that the particle has no chance of escaping from the box. Therefore, the particle bounces back and forth between the two walls as shown in Fig.

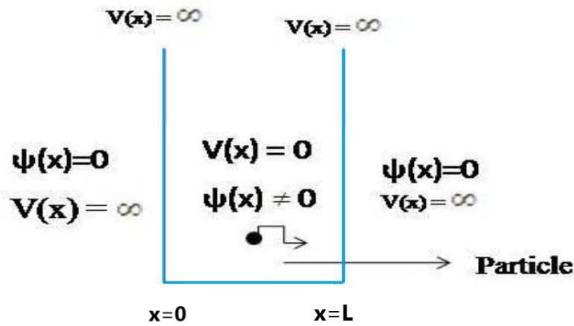


Figure 45: Particle moving in 1D box

Outside the box: ($x \leq 0$ and $x \geq L$)

The particle cannot exist outside and on the walls of the box, therefore the probability of finding the particle outside the wall is zero and the potential energy is ∞ :

$$\begin{aligned} V(x) &= \infty \\ \psi(x) &= 0 \end{aligned}$$

Inside the box: ($0 < x < L$)

The particle exists inside the box; therefore the probability of finding the electron inside the box is not equal to zero and the particle is moving freely, so potential energy is zero:

$$\begin{aligned} V(x) &= 0 \\ \psi(x) &\neq 0 \end{aligned}$$

As we have just seen that the wavefunction of particle outside the box is zero, so we can not solve the time independent Schrodinger equation for this region. However, for inside the box we can solve the time independent Schrodinger equation, which can be written as

$$\begin{aligned}\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - 0)\psi &= 0 \\ \frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi &= 0 \\ \frac{d^2\psi}{dx^2} + k^2\psi &= 0\end{aligned}\tag{95}$$

where the k^2 is defined as follows

$$\begin{aligned}k^2 &= \frac{2mE}{\hbar^2} \\ &= \frac{2mE}{\frac{\hbar^2}{4\pi^2}} \\ &= \frac{8\pi^2 m E}{\hbar^2}\end{aligned}\tag{96}$$

Now we can write the general solution as follows

$$\psi(x) = A \sin(kx) + B \cos(kx)\tag{97}$$

where A and B are the arbitrary constants, which can be found by applying the boundary conditions.

Boundary Condition at $x = 0$:

$$\begin{aligned}\psi(x = 0) &= 0 \\ A \sin 0 + B \cos 0 &= 0 \\ B &= 0\end{aligned}$$

which gives

$$\psi(x) = A \sin(kx)\tag{98}$$

Boundary Condition at $x = L$:

$$\begin{aligned}\psi(x = L) &= 0 \\ A \sin(kL) &= 0\end{aligned}\tag{99}$$

Since $A \neq 0$,

$$\sin(kL) = 0 \quad (100)$$

We know that,

$$\sin n\pi = 0 \quad (\text{where } n = 1, 2, 3, \dots) \quad (101)$$

After combining, we get

$$\begin{aligned} \sin(kL) &= \sin n\pi \\ kL &= n\pi \\ k &= \frac{n\pi}{L} \end{aligned} \quad (102)$$

Energy of the Particle

We have defined

$$\begin{aligned} k^2 &= \frac{8\pi^2 m E}{h^2} \\ \frac{n^2 \pi^2}{L^2} &= \frac{8\pi^2 m E}{h^2} \quad (\text{where we used } k^2 = \frac{n^2 \pi^2}{L^2}) \\ E_n &= \frac{n^2 h^2}{8m L^2} \end{aligned} \quad (103)$$

This equation is an expression for the energy of the particle.

- If $n = 1$, $E_1 = \frac{h^2}{8ml^2}$
- If $n = 2$, $E_2 = \frac{4h^2}{8ml^2} = 4E_1$
- If $n = 3$, $E_3 = \frac{9h^2}{8ml^2} = 9E_1$

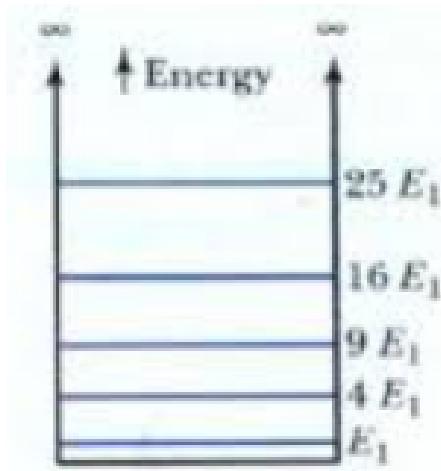


Figure 46: Energy levels of a particle enclosed in a 1D box.

Question(82): What is de-Broglie's wavelength? Find the velocity and kinetic energy of an electron whose wavelength is 1.66×10^{-10} m.

Solution:

de-Broglie's wavelength: According to de-Broglie hypothesis, every moving particle has always associated with a wave. The particle is controlled by this wave in every aspect. The wave associated with moving particle of matter (like electrons, protons etc) are known as matter wave or pilot waves or de-Broglie waves.



Figure 47: Motion of a particle (electron).

If a particle of mass m moving with velocity v , then the wavelength of matter wave is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Where

h : Planck's constant

m : mass of particle

v : velocity of the particle

Solution of numerical problem

Given

$$\begin{aligned}\lambda &= 1.66 \times 10^{-10} m \\ m &= 9.1 \times 10^{-31} kg\end{aligned}$$

The velocity of electron is calculated by using the following equation

$$\begin{aligned}\lambda &= \frac{h}{mv} \\ v &= \frac{h}{m\lambda} \\ &= \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 1.66 \times 10^{-10}} \\ &= \frac{6.626 \times 10^7}{15.106} \\ v &= 0.438 \times 10^7 \text{ m/s}\end{aligned}$$

Now, the kinetic energy is given by

$$\begin{aligned}K &= \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 9.1 \times 10^{-31} \times (0.438 \times 10^7)^2 \\ K &= 0.869 \times 10^{-17} \text{ Joule}\end{aligned}$$

We know that ($1 \text{ eV} = 1.6 \times 10^{-19} \text{ Joule}$), So the kinetic energy will be

$$\begin{aligned}K &= \frac{0.869 \times 10^{-17}}{1.6 \times 10^{-19}} \text{ eV} \\ K &= 0.543 \times 10^2 \text{ eV} \\ K &= 54.3 \text{ eV}\end{aligned}$$

Question(83): What are salient features of Kronig-Penney model? Explain the formation of allowed and forbidden energy bands based on Kronig-Penney model.

Solution: According to this model, the electron of mass (m) moves in a periodic potential produced by the positive ion cores. As a result, the potential energy of the electron is zero at the positive ion site and maximum in between two ions. It is not easy to solve Schrodinger equation with these potential. So Kronig and Penney approximated these potential to the shape of rectangular steps as shown in Fig.

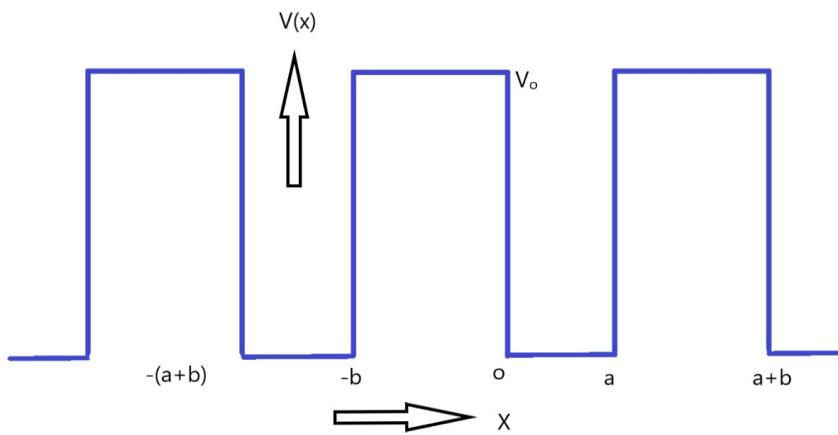


Figure 48: Kronig Penney potential.

Now, we can define the potential regions as follows:

Region-I, $V = 0, \quad (0 < x < a)$

We can write the time independent Schrodinger equation

$$\begin{aligned} \frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - 0)\psi &= 0 \\ \frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi &= 0 \\ \frac{d^2\psi}{dx^2} + \alpha^2\psi &= 0 \end{aligned} \tag{104}$$

where $\alpha^2 = \frac{2mE}{\hbar^2}$

Region-II, $V = V_0, \quad (-b < x < 0)$

We can write the time independent Schrodinger equation

$$\begin{aligned}\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\psi &= 0 \\ \frac{d^2\psi}{dx^2} - \frac{2m}{\hbar^2}(V_0 - E)\psi &= 0 \\ \frac{d^2\psi}{dx^2} - \beta^2\psi &= 0\end{aligned}\quad (105)$$

where $\beta^2 = \frac{2m(V_0-E)}{\hbar^2}$

The solutions are given by Bloch and can be written as follows

$$\psi_1(x) = e^{ikx}u_1(x) \quad (106)$$

$$\psi_2(x) = e^{ikx}u_2(x) \quad (107)$$

Here u_1 and u_2 are known as Bloch functions. After solving using the boundary conditions, we get the final governing equation in Kronig-Penney model as follows

$$P \frac{\sin(\alpha a)}{\alpha a} + \cos(\alpha a) = \cos(ka) \quad (108)$$

where $P = \frac{mV_0ba}{\hbar^2}$ is the scattering power of barrier and V_0b is the barrier strength. To verify at what conditions above Eq is satisfied by a particle, let us draw a plot as follows:

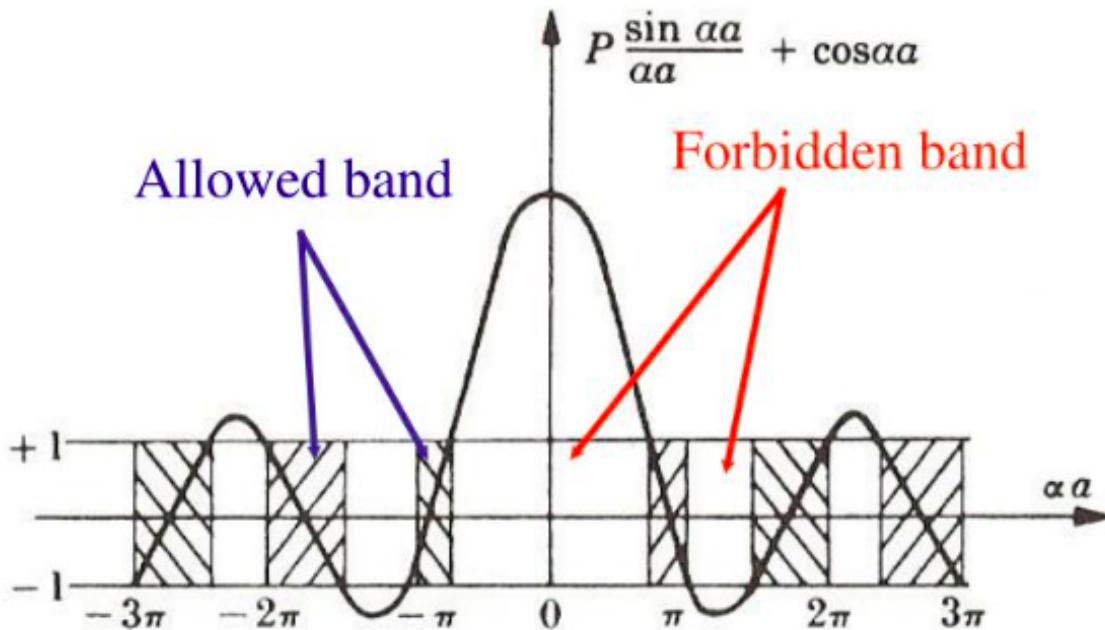
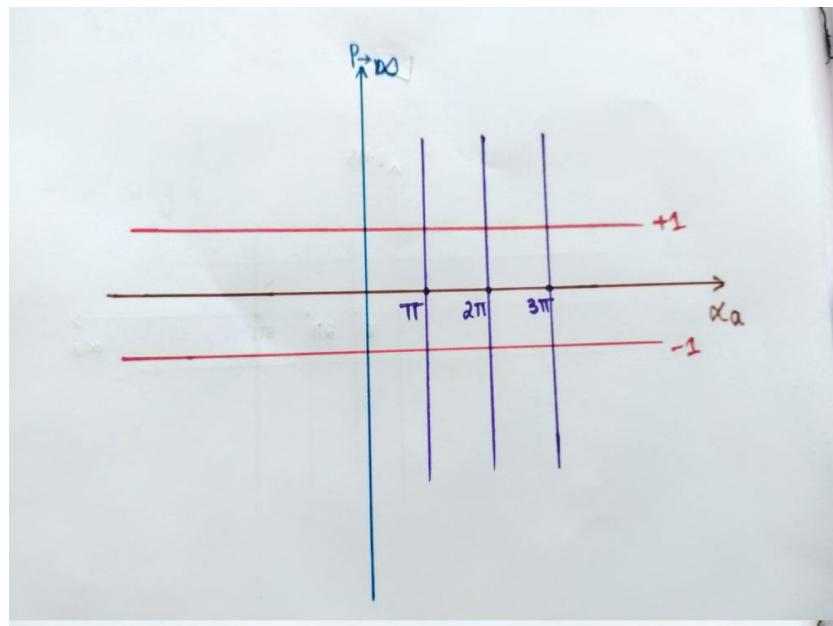


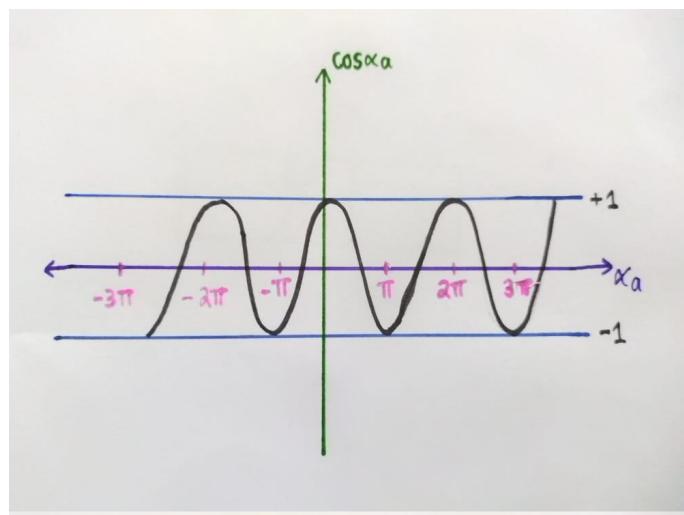
Figure 49: Allowed and forbidden bands in Kronig Penney potential.

Salient features of Kronig-Penney model

1. The particle has allowed and forbidden energy regions. The allowed energy region width increases and forbidden energy region width decreases with αa as shown in Fig.
2. With increasing P value, the width of an allowed band decreases.
3. For ($P \rightarrow \infty$), The allowed energy regions converted into single energy levels and is represented by single line spectrum.



4. For ($P \rightarrow 0$), then all the energy values are allowed by the particle and it is the case of a free particle. Thus, the electrons will have continuous range of energy from 0 to ∞ .



Question(84): Classify the materials into conductors, semiconductors and insulators based on the formation of energy bands and write any four properties for each.

Solution: Conductors The valence band and conduction bands are overlapped with each other and the energy gap E_g is zero. At room temperatures, free electrons already exist in huge number at conduction band. Hence these solids are good electrical conductors as well as good thermal conductors. The electrical resistivity increases at high temperature by collisions among the free electrons.

Examples: Al, Cu, Ag, Au etc.

Properties:

- A conductor always allows the free movement of electrons or ions.
- The electric field inside a conductor must be zero to permit the electrons or ions to move through the conductor.
- Charge density inside a conductor is zero i.e. the positive and negative charges cancel inside a conductor.
- As no charge inside the conductor, only free charges can exist only on the surface of a conductor.

Semiconductors The valence band and conduction bands are separated with a small energy band gap $E_g \approx 1\text{eV}$. At low temperatures ($0K$), free electrons are not available in conduction band. Hence, they behave like insulators at low temperatures. The electrical conductivity increases at high temperatures by the transition of free electrons from valence band to conduction band. Thus, these solids behave like electrical conductors at high temperatures.

Examples: Silicon $E_g = 1.1\text{eV}$, Germanium $E_g = 0.7\text{eV}$.

Properties:

- Semiconductor acts like insulator at $0K$ and conductor at high temperatures.
- Their resistivity is higher than conductors but lesser than insulators.

- It's temperature coefficient of resistivity is negative, i.e. resistance decrease with increase in temperature.
- Resistivity: 10^{-5} to 10^6 ohm-m.
- Conductivity: 10^5 to 10^{-6} $\text{ohm}^{-1} \cdot \text{m}^{-1}$.

Insulators The valence band and conduction bands are separated by a very large energy gap $E_g \geq 3\text{eV}$. At room temperatures conduction band is empty and valence band is full of electrons. Hence, these solids are electrical insulators. Even at high temperatures valence electrons are unable to jump in to conduction band.

Examples: Glass, Mica, Rubber etc.

Properties:

- An insulator is a material which is a poor conductor of electricity or heat due to tight binding of electrons in their atoms and molecules.
- Some common insulators include wood, plastic, glass widely use in household applications.
- Floors, roofs and basements often get urethane foam insulation, as it reduces heating costs by retaining heat in homes.

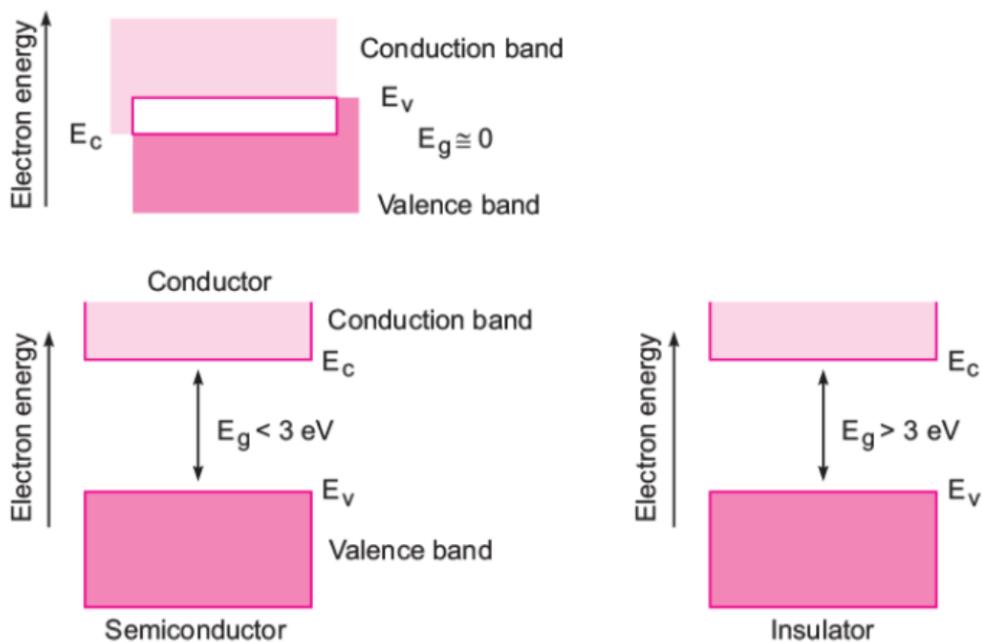


Figure 50: Energy band diagrams for conductor, semiconductor and insulator.

Question(85): Derive an expression for intrinsic carrier concentration.

Solution: The number of electron per unit volume of the conduction band of given intrinsic semiconductor is called electron concentration. It is denoted by 'n'.

$$n = \int_{E_C}^{\infty} Z(E) dE f(E) \quad (109)$$

where $f(E)$ is the Fermi-Dirac distribution function, which is represented as

$$f(E) = \frac{1}{1 + e^{\frac{(E-E_F)}{kT}}} \quad (110)$$

where K is the Boltzmann constant, T is the absolute temperature and E_F is the Fermi energy.

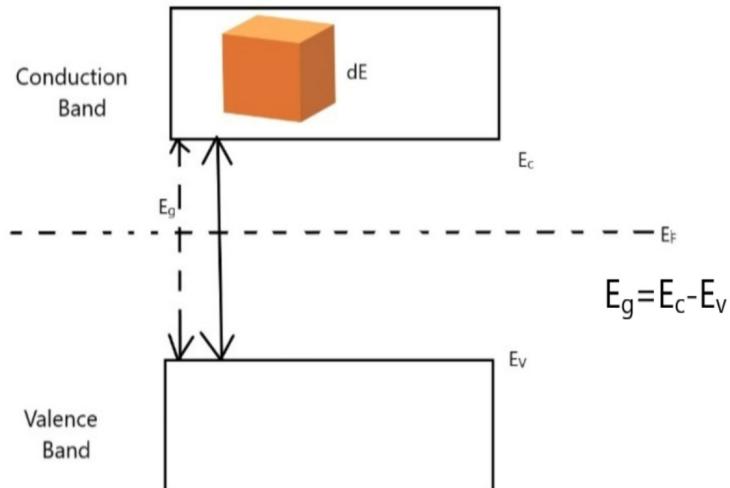


Figure 51: Energy band diagram for semiconducotor.

If $(E > E_F)$, then compare to $e^{\frac{(E-E_F)}{kT}}$, we can neglect 1. Thus, the above Equation becomes

$$\begin{aligned} f(E) &= \frac{1}{e^{\frac{(E-E_F)}{kT}}} \\ f(E) &= e^{\frac{(E_F-E)}{kT}} \end{aligned} \quad (111)$$

The free electron density of states per unit volume of the conduction band having energy E and $E + dE$ is given by

$$Z(E)dE = \frac{\pi}{2} \left[\frac{8m_e^*}{h^2} \right]^{3/2} (E - E_C)^{1/2} dE \quad (112)$$

where m_e^* is the effective mass of electron and h is the Plank's constant. After substitution, we get the value of n as

$$\begin{aligned} n &= \int_{E_C}^{\infty} \frac{\pi}{2} \left[\frac{8m_e^*}{h^2} \right]^{3/2} (E - E_C)^{1/2} e^{\frac{(E_F-E)}{kT}} dE \\ &= \frac{\pi}{2} \left[\frac{8m_e^*}{h^2} \right]^{3/2} \int_{E_C}^{\infty} (E - E_C)^{1/2} e^{\frac{(E_F-E)}{kT}} dE \end{aligned} \quad (113)$$

Let us assume $E - E_C = x$, then the above equation becomes

$$n = \frac{\pi}{2} \left[\frac{8m_e^*}{h^2} \right]^{3/2} e^{\frac{(E_F-E_C)}{kT}} \int_0^{\infty} x^{1/2} e^{-\frac{x}{kT}} dx \quad (114)$$

where the integration is given by $\int_0^{\infty} x^{1/2} e^{-\frac{x}{kT}} dx = \frac{\sqrt{\pi}}{2} (kT)^{3/2}$. After using the integration result, the above becomes

$$\begin{aligned} n &= \frac{\pi}{2} \left[\frac{8m_e^*}{h^2} \right]^{3/2} e^{\frac{(E_F-E_C)}{kT}} \frac{\sqrt{\pi}}{2} (kT)^{3/2} \\ n &= \frac{1}{4} \left[\frac{8m_e^* \pi kT}{h^2} \right]^{3/2} e^{\frac{(E_F-E_C)}{kT}} \end{aligned}$$

where $8^{3/2} = (4 \times 2)^{3/2} = 4^{3/2} \times 2^{3/2} = 8 \times 2^{3/2}$.

$$n = 2 \left[\frac{2m_e^* \pi kT}{h^2} \right]^{3/2} e^{\frac{(E_F-E_C)}{kT}} \quad (115)$$

Similarly, we can write the concentration of holes having effective mass (m_h^*)

$$p = 2 \left[\frac{2m_h^* \pi kT}{h^2} \right]^{3/2} e^{\frac{(E_V-E_F)}{kT}} \quad (116)$$

According to the mass action law

$$n_i^2 = np \quad (117)$$

After substituting n and p , we get

$$\begin{aligned}
 n_i^2 &= \left\{ 2 \left[\frac{2m_e^* \pi kT}{h^2} \right]^{3/2} e^{\frac{(E_F - E_C)}{kT}} \right\} \times \left\{ 2 \left[\frac{2m_h^* \pi kT}{h^2} \right]^{3/2} e^{\frac{(E_V - E_F)}{kT}} \right\} \\
 &= 4 \left[\frac{2\pi kT}{h^2} \right]^3 (m_e^* m_h^*)^{3/2} e^{\frac{-(E_C - E_V)}{kT}} \\
 &= 4 \left[\frac{2\pi kT}{h^2} \right]^3 (m_e^* m_h^*)^{3/2} e^{-\frac{E_g}{kT}}
 \end{aligned} \tag{118}$$

where the energy gap value $E_g = E_C - E_V$. Now from above equation, we can write n_i as

$$n_i = 2 \left[\frac{2\pi kT}{h^2} \right]^{3/2} (m_e^* m_h^*)^{3/4} e^{-\frac{E_g}{2kT}} \tag{119}$$

The n_i is the intrinsic carrier concentration for intrinsic semiconductor.

Question(86): Discuss intrinsic and extrinsic semiconductors with neat diagrams.

Solution:

Intrinsic Semiconductor: The semiconductor which do not have impurities or they are in pure form are known as intrinsic semiconductor.

Examples: Silicon (*Si*) and Germanium (*Ge*).

- The structure of pure silicon semiconductor and its band diagram are shown in Fig-5. At $0^\circ K$, all the valence electrons in a pure *Si* semiconductor are under covalent bonds, hence no free electrons available which leads to zero current.
- Due to this reason a pure semiconductor acts as an insulator at $T = 0^\circ K$.
- When $T > 0^\circ K$, if the thermal energy $(KT) \geq E_g$, of a pure semiconductor then current flows, and it will act as a conductor.
- At $T > 0^\circ K$, number of electrons in conduction band is equal to the number of holes in valence band ($n = p$). Hence the fermi level lies in the middle of the two bands.

Extrinsic Semiconductor: When a pure semiconductor added with impurities then impure semiconductor can be obtained. The process of adding of impurities is known as doping. Extrinsic semiconductor can be classified into two types based on the types of impurities, namely:

1. N-type semiconductor
2. P-type semiconductor

N-type semiconductor When a small amount of pentavalent impurity is added to a pure semiconductor crystal during the crystal growth, the resulting crystal is called as N-type semiconductor. Let us consider the case when pentavalent arsenic (*As*, atomic number: 33) is added to pure silicon (*Si*, atomic number: 14) crystal as shown in Fig. The *As* atom fits in the *Si* crystal in such a way that its four valence electrons form covalent bonds with the four *Si* atoms. The fifth electron of the *As* is available as a carrier of current. Such a liberated valence electron is then free to move in the crystal lattice in the same way as free electrons. Thus, in N-type the majority charge carriers are electrons, so the fermi level lies close to the conduction band as shown in Fig.

P-type semiconductor When a small amount of trivalent impurity is added to a pure semiconductor crystal during the crystal growth, the resulting crystal is called as P-type semiconductor. Let us consider the case when trivalent boron (*B*, atomic number: 5) is added to pure *Si* crystal as shown in Fig. Each atom of *B* fits into the *Si* crystal with only three covalent bonds. This is because the three valence electrons of *B* atom form covalent bonds with the valence electrons of *Si* atom. In the fourth covalent bond, only *Si* atom contributes one valence electron and there is deficiency of one electron which is called a hole. So, for each boron atom added, one hole is created. A small amount of boron provides millions of holes. Thus, in P-type the majority charge carriers are holes, so the fermi level lies close to the valence band as shown in Fig.

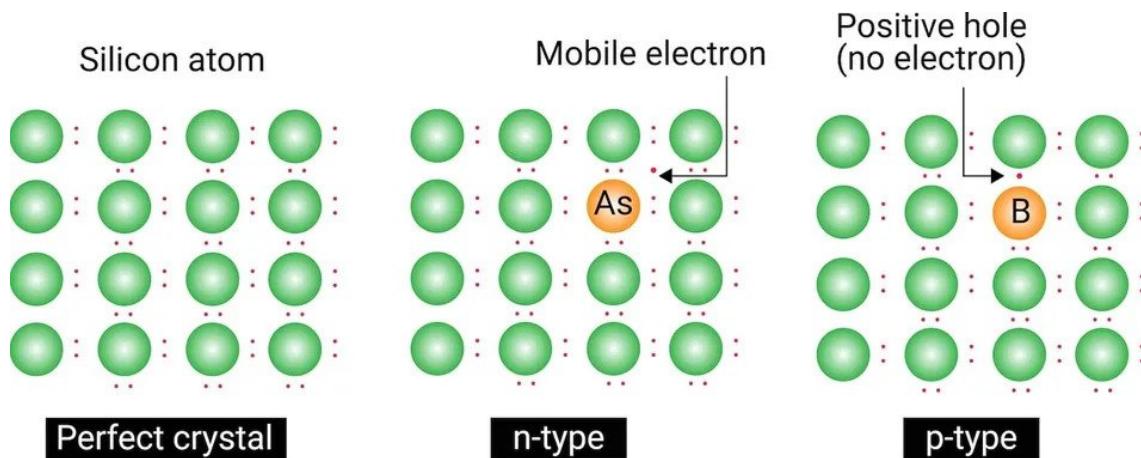


Figure 52: Perfect (Intrinsic semiconductor), n-type and p-type semiconductors.



Figure 53: Energy band diagram for Extrinsic semiconductor.

Question(87): Explain the formation of p-n junction diode along with its I-V characteristics.

Solution: Formation of P-N junction diode:

- Firstly take a pure semiconductor and add trivalent impurities at one side and pentavalent impurities on the other side. As a result, we get P and N types semiconductors, respectively, as shown in Fig-50.
- Holes are the majority carriers on one side and electrons on the other side, respectively.
- Holes diffuse from *P* to *N* side and electrons in the opposite direction.

- During diffusion, electrons and holes meet at a point known as a P-N junction.
- A P-N junction can be defined as an interface between a P and N type semiconductors.

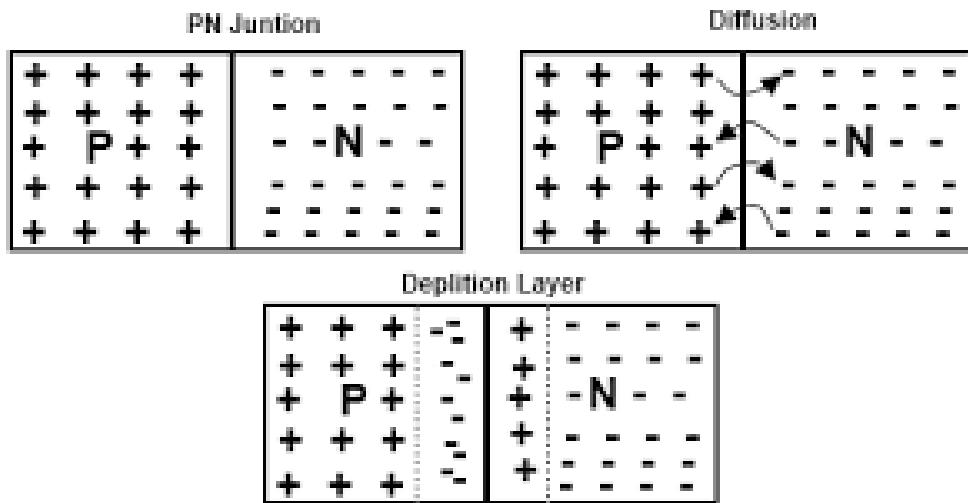


Figure 54: Formation of P-N junction.

- The immobile ions gather near the junction and create an electric field known as built-in field (E_B or E_i) or build-in potential V_B (barrier voltage) as shown in Fig.

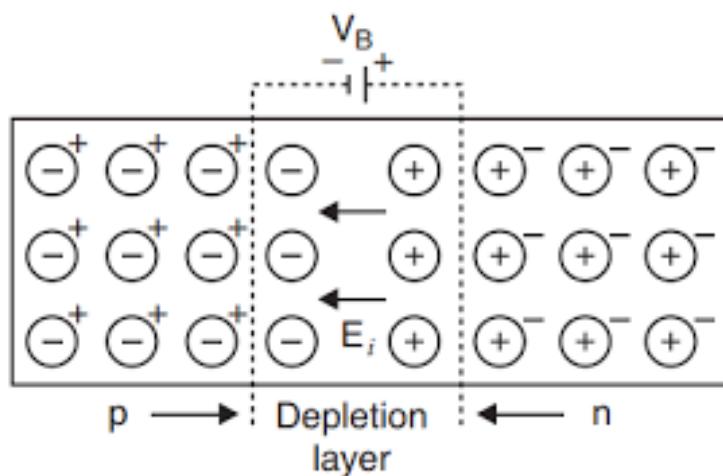


Figure 55: Formation of P-N junction.

- This field stops the diffusion of carriers across the P-N junction.

- One region is created across the P-N junction which does not contain any carriers known as space-charge region or depletion region.

Characteristics of a P-N junction diode

Forward Bias

- When positive terminal of the battery connected to the P-side and negative terminal of the battery connected to the N-side, then the biasing is called forward bias.

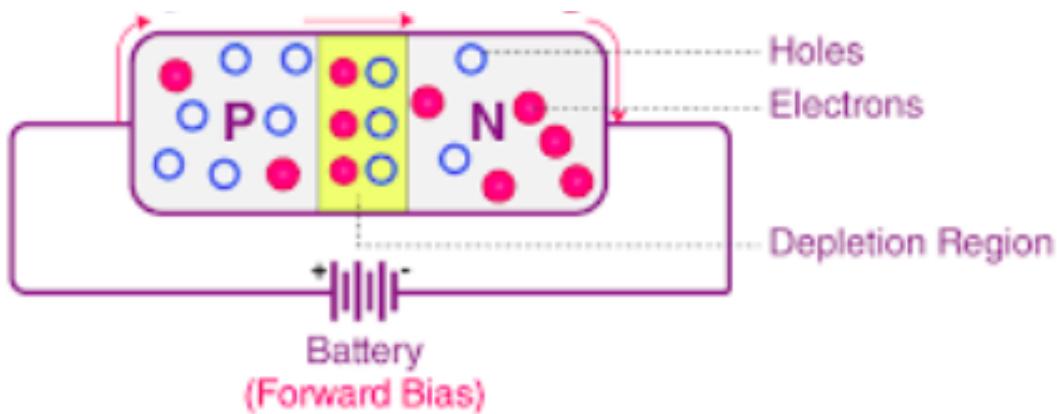


Figure 56: Forward bias of P-N junction diode.

- As long as $V_B > V_F$ where (V_B is the barrier voltage and V_F is the forward voltage), there is no current across the P-N junction diode.
- When $V_B = V_F$, the current starts flowing through P-N junction diode.
- When $V_B < V_F$, a large current flows across the P-N junction diode.

Reverse Bias

- When positive terminal of the battery connected to the N-side and negative terminal of the battery connected to the P-side, then the biasing is called reverse bias.

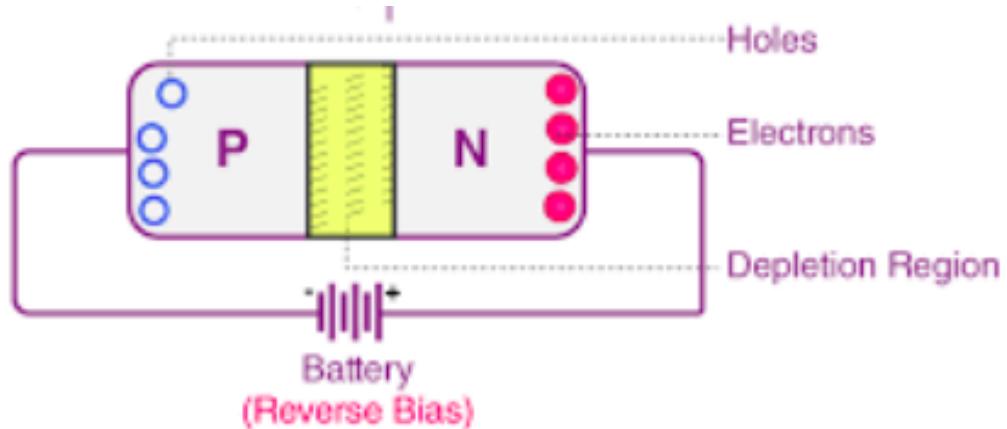


Figure 57: Reverse bias of PN junction diode

- Since, the applied reverse voltage (V_R) is in the direction of the barrier voltage (V_B), so, the depletion region width will increase and there is no current across the P-N junction diode.
- But in practicals, a small current flows due to the minority carriers.

I-V characteristics of P-N junction diode

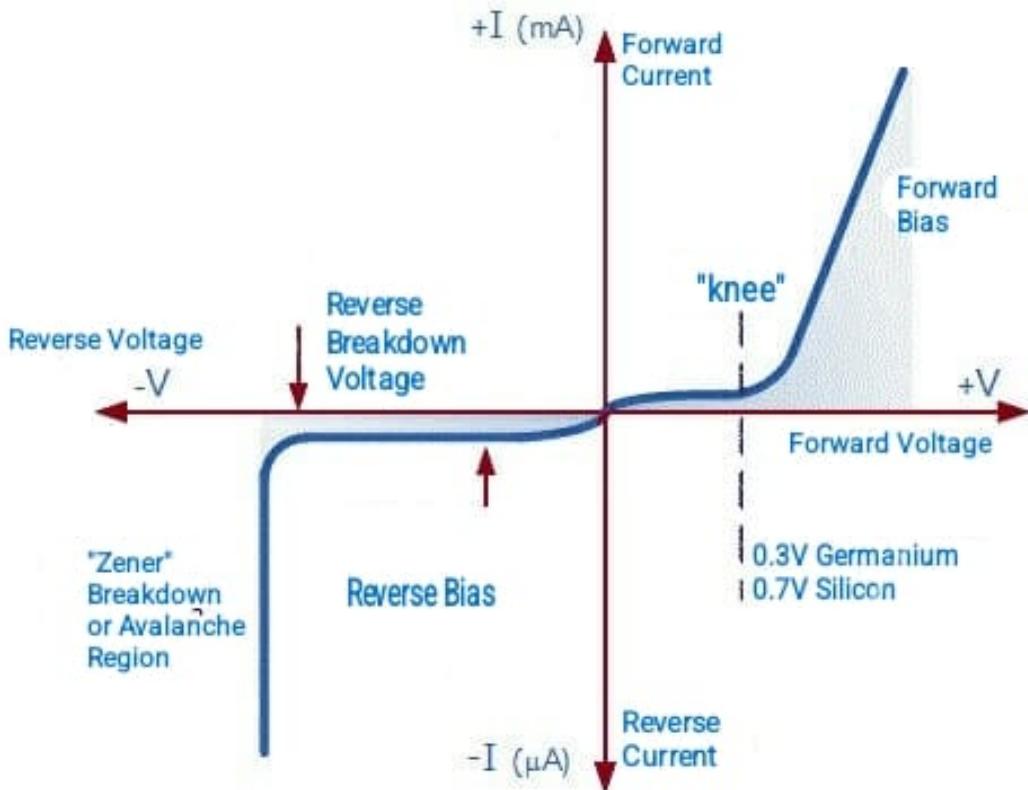


Figure 58: I-V characteristics of a P-N junction diode.

Question(88): What is thermistor, explain its characteristics with neat diagrams.

Solution: A thermistor is a special type of resistor whose resistance strongly depends on temperature. The word thermistor is a combination of words “thermal” and “resistor”. A thermistor is a temperature-sensing element composed of mixture of metallic oxides such as *Mn*, *Ni*, *Co*, *Cu* and *Fe*. Thermistor is a heat sensitive device. The thermistor exhibits a highly non-linear characteristic of resistance versus temperature.

Depending upon the variation of resistance value with respect to the variation of temperature there are two types of thermistors.

- Negative temperature coefficient (NTC) thermistor
- Positive temperature coefficient (PTC) thermistor

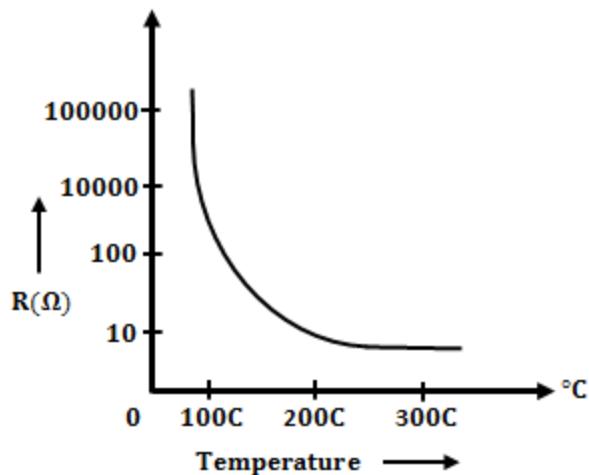
Negative Temperature Coefficient or NTC Thermistor NTC thermistors have a negative temperature coefficient, there will be an inverse relationship between resistance and temperature i.e. if the temperature of the thermistor increases the resistance decreases.

This type of thermistors are widely used for measuring temperature. The resistance temperature relation can be expressed by the following relationship,

$$R_T = R_0 e^{\beta \left(\frac{1}{T} - \frac{1}{T_0} \right)} \quad (120)$$

where

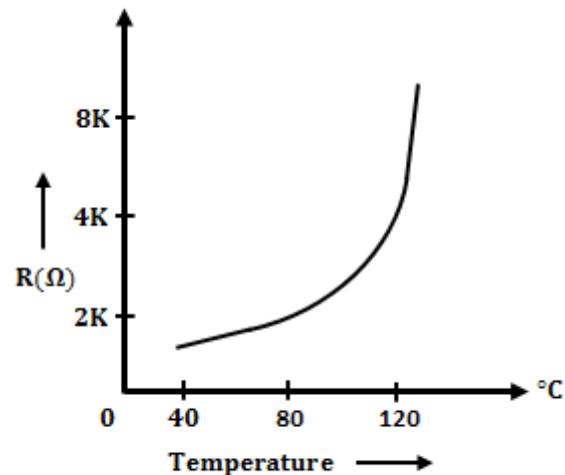
- R_T = Resistance at temperature (T) Kelvin
- R_0 = Resistance at temperature (T_0) Kelvin
- β = A constant (usually lying between 3400 to 3900)



**Resistance Temperature Characteristics
of NTC Thermistor**

The curve above shows the resistance variation with the temperature of an NTC thermistor. The change is non-linear. They are available in values ranging from few ohms to megaohms.

Positive Temperature Coefficient or PTC Thermistor This type of thermistors has a positive temperature coefficient i.e., their resistance increases with an increase in temperature. It is shown in the below figure.

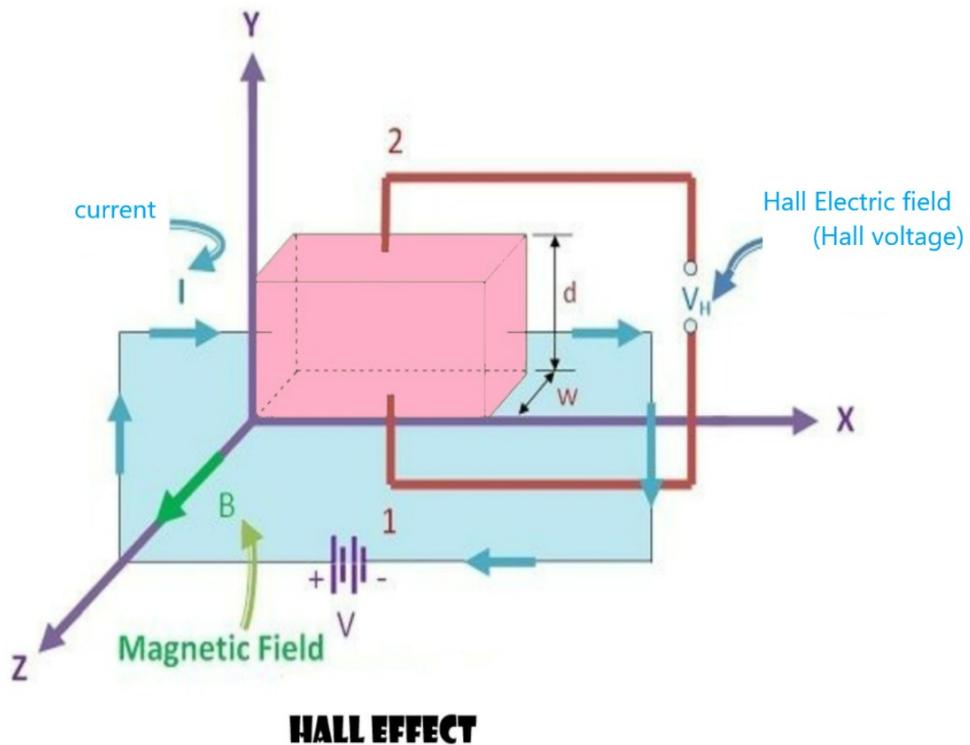


**Resistance Temperature Characteristics
of PTC Thermistor**

PTC thermistors are used as a device to protect all kinds of electrical apparatus against overheating. If the apparatus is overheated PTC thermistor resistance raises sharply. The relay coil would be de-energized and the apparatus is disconnected from the supply.

Question(89): Explain Hall effect and calculate the Hall coefficient. List few applications of Hall effect.

Solution: If a semiconductor (conductor) carrying a current placed in a magnetic field which acts perpendicular to the direction of current, then a potential difference (or electric field) is developed in the direction perpendicular to both magnetic field and current. This phenomena is known as “Hall effect” and generated voltage is “Hall voltage” and generated electric field is “Hall electric field”. The setup of Hall effect is depicted in Fig.



Let us consider the n -type semiconductor, then the majority charge carriers will be electrons. Because of the presence of magnetic field (B), a magnetic force (F_m) will be applied on the electron moving with drift velocity (v_d)

$$F_m = -eBv_d \quad (121)$$

The generated Hall electric field (E_H) will also apply a electrostatic force (F_e) on the electron

$$F_e = -eE_H \quad (122)$$

At equilibrium, the two forces will be equal

$$\begin{aligned} F_e &= F_m \\ -eE_H &= -eBv_d \\ E_H &= Bv_d \end{aligned} \quad (123)$$

The relation between the Hall potential difference V_H and Hall electric field E_H can be written as

$$E_H = \frac{V_H}{d} \quad (124)$$

where d is the distance or separation between the positive and negative charges. After comparing, we can write

$$\begin{aligned} \frac{V_H}{d} &= Bv_d \\ V_H &= Bd v_d \end{aligned} \quad (125)$$

Calculation of Hall coefficient The Hall coefficient is the most important quantity, that we can calculate from the experimental setup. Hall coefficient (R_H) is defined as the ratio of the Hall electric field (E_H) to the product of the current density (J) and the applied magnetic field (B)

$$R_H = \frac{E_H}{JB} \quad (126)$$

As we know, the current density of electron having electron concentration (n) is given by

$$J = -nev_d \quad (127)$$

Substituting E_H and J , we get

$$\begin{aligned} R_H &= \frac{Bv_d}{-nev_d B} \\ R_H &= -\frac{1}{ne} \end{aligned} \quad (128)$$

Similarly, if we take *p*-type semiconductor having the majority charge concentrations as hole concentrations (*p*), we will get the Hall coefficient as follows

$$R_H = \frac{1}{pe} \quad (129)$$

As we also know, current density $J = \frac{\text{current}}{\text{Area}} = \frac{I}{A} = \frac{I}{wd}$, (where the area $A = wd$), and $E_H = \frac{V_H}{d}$, thus R_H can be rewritten as

$$\begin{aligned} R_H &= \frac{V_H \text{ wd}}{dB} \\ R_H &= \frac{V_H \text{ w}}{BI} \end{aligned} \quad (130)$$

Applications of Hall effect

1. Hall effect is used for determining carrier concentrations (electron or hole concentration) in semiconductor.
2. Hall effect is used to determine the type of semiconductor.
3. Hall effect is used to determine the mobility of electrons and holes.
4. Hall effect is used to determine the strength of magnetic field.

Question(90): Define Hall coefficient, Carrier concentration and mobility. The Hall coefficient of certain silicon specimen was found to be $-7.35 \times 10^{-5} \text{ m}^3 \text{ C}^{-1}$ from $100K$ to $400K$. Determine the nature of the semiconductor. Further, the electrical conductivity was found to be $200 \Omega^{-1} \text{ m}^{-1}$. Calculate the concentration and mobility of charge carriers.

Solution:

- **Hall Coefficient:** Hall coefficient (R_H) is defined as the ratio of the Hall electric field (E_H) to the product of the current density (J) and the applied magnetic field (B)

$$R_H = \frac{E_H}{JB} \quad (131)$$

As we know, the current density of electron having electron concentration (*n*) is given by

$$J = -nev_d \quad (132)$$

Substituting E_H and J , we get

$$\begin{aligned} R_H &= \frac{Bv_d}{-nev_d B} \\ R_H &= -\frac{1}{ne} \end{aligned} \quad (133)$$

- **Carrier Concentration:** The intrinsic carrier concentration is the number of electrons in the conduction band or the number of holes in the valence band in intrinsic material. This number of carriers depends on the band gap of the material and on the temperature of the material.
- **Mobility:** In semiconductor physics, electron mobility describes how fast an electron can move through a semiconductor when charges are pulled by an electric field.

Solution of numerical problem

Given

$$\begin{aligned} R_H &= -7.35 \times 10^{-5} m^3 C^{-1} \\ \sigma &= 200 \Omega^{-1} m^{-1} \\ T &= 300 K \end{aligned}$$

The negative sign of the Hall coefficient indicates that the nature of the semiconductor is n-type. Hence the electron concentration can be obtained from the equation

$$\begin{aligned} n &= -\frac{1}{eR_H} \\ &= \frac{1}{1.6 \times 10^{-19} \times 7.35 \times 10^{-5}} \\ n &= 8.5 \times 10^{22} / m^3 \end{aligned}$$

and the electron mobility

$$\begin{aligned} \mu_e &= \frac{\sigma}{n e} \\ &= \frac{200}{8.5 \times 10^{22} \times 1.6 \times 10^{-19}} \\ \mu_e &= 14.7 \times 10^{-3} m^2 V^{-1} s^{-1} \end{aligned}$$

UNIT-IV (SAQs)

Question(91): List the types of magnetic materials.

Solution: The following are the types of magnetic materials

1. Diamagnetic material
2. Paramagnetic material
3. Ferromagnetic material
4. Antiferromagnetic material
5. Ferrimagnetic material

Question(92): Summarize magnetic domains.

Solution: Below Curie temperature, the individual dipole moments in a ferromagnetic material align in one particular direction due to spin-spin coupling and forms regions, known as magnetic domains.

Question(93): Sketch Hysteresis curve.

Solution:

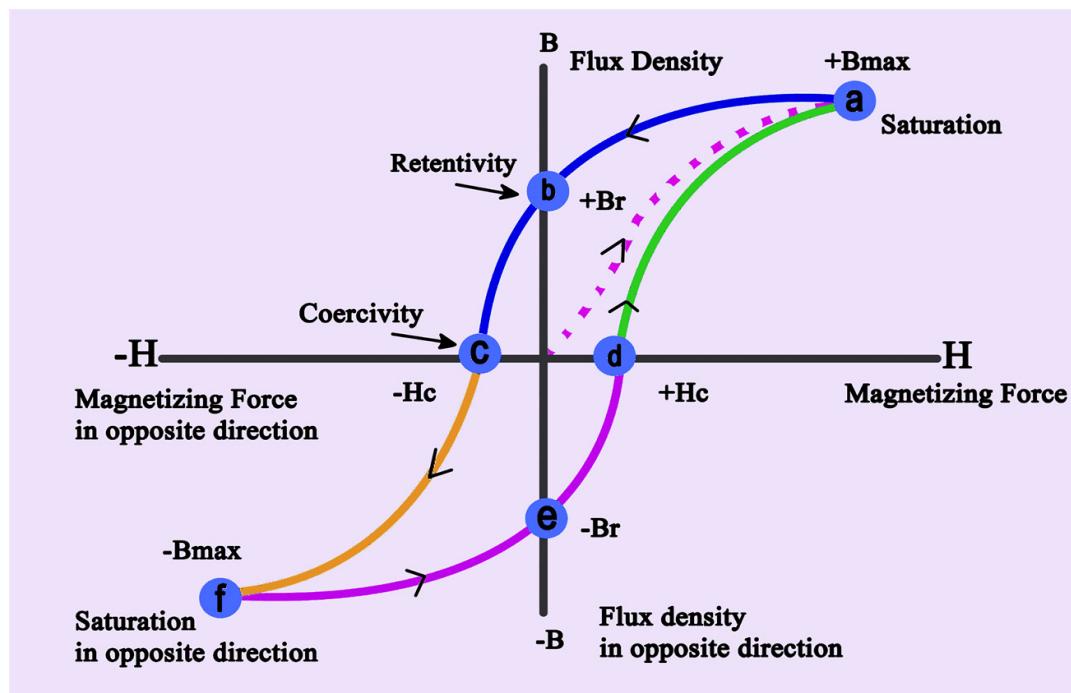


Figure 59: Hysteresis curve (BH curve).

Question(94): Give any two differences between soft and hard ferromagnetic materials.

Solution:

Soft ferromagnetic material

1. These materials are easily magnetised and demagnetized.
2. They have small hysteresis area.

Hard ferromagnetic material

1. These materials are not easily magnetised and demagnetized.
2. They have large hysteresis area

Question(95): Write any two applications of ferrites.

Solution: Ferrites are used in

1. Magnetic storage devices
2. Generators
3. Motors
4. Telephones

Question(96): List any two general properties of superconductors.

Solution: The materials which exhibit the phenomena of superconductivity are known as superconductors. The following are the general properties of superconductors

1. Superconductivity is a low temperature phenomena.
2. The critical temperature is different for different superconductor. Critical temperature depends on
 - Impurities
 - Pressure

Question(97): Define relative permeability.

Solution: The ratio of permeability of a magnetic medium (μ) to the permeability of free space (μ_0) is defined as the relative permeability (μ_r) of that magnetic medium. It has no unit. Mathematically, it can be expressed as follows:

$$\mu_r = \frac{\mu}{\mu_0}$$

Question(98): Explain high T_c superconductors.

Solution: High-temperature superconductors (abbreviated high- T_c or HTS) are materials that behave as superconductors at unusually high temperatures. The first high- T_c superconductor was discovered in 1986 by researchers Georg Bednorz and K. Alex Muller, who were awarded the 1987 Nobel Prize in Physics “for their important break-through in the discovery of superconductivity in ceramic materials”. Any superconductor with a transition temperature above 10 K is in general called High T_c superconductor.

Question(99): Define SQUIDs with suitable example.

Solution: A SQUID (superconducting quantum interference device) is a very sensitive magnetometer used to measure extremely weak magnetic fields, based on superconducting loops containing Josephson junctions. They are used in mine detection equipment to help in the removal of land mines. There are two main types of SQUID: direct current (DC) and radio frequency (RF).

Question(100): What are the critical conditions of a superconductors?

Solution: The critical conditions for superconductors are

- Critical temperature (T_c)
- Critical magnetic field (H_c)

$$H_c = H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

- Critical current (I_c)

Question(101): Explain Meissner's effect.

Solution: When a superconducting material at temperature $T > T_C$ is placed in external magnetic field, lines of magnetic induction pass through its body, but when it is cooled below the critical temperature $T < T_C$, these lines of induction are pushed out of the superconducting body. So inside the superconducting body, $B = 0$. This phenomenon is known as the “Meissner effect”.

Question(102): Write any two reasons to show superconductors are diamagnetic materials.

Solution: We know the relation $\vec{B} = \mu_0(\vec{H} + \vec{M})$. Now, for superconducting state, $B = 0$,

$$0 = \mu_0(\vec{H} + \vec{M})$$

which gives $\mu_0 \neq 0$. The two to show superconductors are diamagnetic are

$$\vec{M} = -\vec{H}$$

and magnetic susceptibility $\chi_m = -1$.

Question(103) A superconductor Tin has critical temperature of 3.7 K and critical magnetic field of 0.306 Tesla at 0 K temperatures. Find Critical magnetic field at 2 K temperature.

Solution:

Given $T_c = 3.7K$, $H_0 = 0.306$ Tesla, ($T = 0$), we need to calculate $H_c = ?$ at $T = 2K$

Using formula

$$\begin{aligned} H_c &= H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right] \\ H_c &= 0.306 \left[1 - \left(\frac{2}{3.7} \right)^2 \right] \\ &= 0.306 \times 0.7084 \text{ Tesla} \\ &= 0.216 \text{ Tesla} \end{aligned}$$

Question(104): Write any four applications of superconductors.

Solution: Superconductors are used in:

1. Zero loss power transmission lines
2. Memory or storage elements.
3. Generator, motors and transformers.
4. Ore Separation

Question(105): Give the relation between relative permeability and magnetic susceptibility.

Solution: The relation between relative permeability (μ_r) and magnetic susceptibility (χ_m) is expressed as

$$\mu_r = 1 + \chi_m$$

UNIT-IV (LAQs)

Question(106): Classify magnetic materials in to dia, para, ferro, anti-ferro and ferri magnetic materials.

Solution:

Dia magnet	Para magnet	Ferro magnet	Anti-ferro magnet	Ferri magnet
1. These materials have permanent dipole moment.	1. These material have permanent magnetic dipole moment but align randomly. Hence magnetic moment is zero.	1.These material have permanent magnetic dipole moment but align systematically. Hence non-zero magnetic moment.	1. These material have permanent magnetic dipole moment but the neighbouring magnetic moment are in opposite direction. Hence zero magnetic moment.	1. These material have permanent magnetic dipole moment but the neighbouring magnetic moment are opposite with different in magnitude. Hence net magnetic moment is non-zero .
2. They repel the magnetic flux lines only.	2. They weakly allow the magnetic flux lines.	2. They strongly allow the magnetic field lines.	2. They weakly allow the magnetic flux lines.	2. They strongly allow the magnetic field lines.
3. The direction of magnetic field intensity is opposite to the direction of magnetisation. $H\alpha M$ $H=-M$	3. The direction of magnetic field intensity is parallel to the direction of magnetisation. $H\alpha M$ $H=M$	3. This kind of material shows the spontaneous magnetisation.	3. The direction of magnetic field intensity is parallel to the direction of magnetisation. $H\alpha M$ $H=M$	3.The phenomenon of non-zero dipole moment is known as ferri magnetic material. They show spontaneous magnetisation.
4. Magnetic permeability is less than one $\mu < 1$	4. Magnetic permeability is greater than one. $\mu > 1$	4. Magnetic permeability is much greater than one. $\mu \gg 1$	4. Magnetic permeability is greater than one. $\mu > 1$	4. Magnetic permeability is much greater than one. $\mu \gg 1$
5. Magnetic susceptibility is negative $X_m < 0$	5. Magnetic susceptibility is small or positive. $X_m > 0$	5.Magnetic susceptibility is large $X_m \gg 0$	5. Magnetic susceptibility is small or positive. $X_m > 0$	5. Magnetic susceptibility is large $X_m \gg 0$
6. Magnetic susceptibility does not depend on temperature $X_m = C/T$	6. Magnetic susceptibility depend on temperature $X_m = C/(T-T_c)$	6. X_m depends on temperature $X_m = C/(T-T_h)$	6. X_m depends on temperature $X_m = C/(T\pm T_h)$	6. X_m depends on temperature $X_m = C/(T\pm T_h)$
7. Examples: Cu, Ag, Hg	7. Examples: Al, Cr, Na	7. Examples: Fe, Co, Ni	7. Examples: $Mn_2Fe_3O_4$	7. Examples: Ferrites

Question (107): Explain the formation of domains. Discuss the Hysteresis curve.

Solution: Domain theory of ferromagnetism

- The phenomena of spontaneous magnetisation is known as ferromagnetism.
- The magnetization in absence of external magnetic field is known as spontaneous magnetisation.
- Below Curie temperature, the individual dipole moment align in one particular direction due to spin-spin coupling and forms regions known as domains.
- The domains are different in sizes and shapes.

Effect of external magnetic field:

- (a) In the absence of external magnetic field, the domains of a ferromagnetic material are randomly oriented. In other words, within the domain, all magnetic dipole moments are aligned, but the direction of alignment varies from domain to domain. The result is that there is no net magnetic dipole moment. Therefore, a Ferro magnetic material does not exhibit magnetism in the normal state.

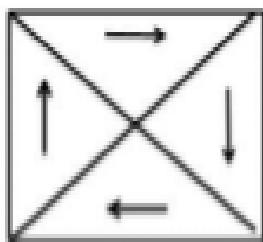


Figure 60: Without magnetic field

- (b) When a Ferro magnetic material is placed in an external magnetic field below T_c , a net magnetic dipole moment develops. This can occur in two ways:
- (i) *By the movement of domain walls:*

- The movement of domain walls takes place in weak magnetic fields.
- Due to weak magnetic field applied to the material the magnetic dipole moments increases and hence the boundary of domains displaced, so that the volume of the domains changes as shown in fig.

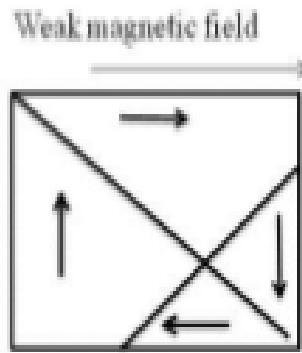


Figure 61: Displacement of domain walls with magnetic field.

(ii) *By the rotation of domain walls:*

- The rotation of domain wall takes place in strong magnetic fields.
- Due to strong magnetic field applied to the material the magnetic dipole moments increases enormously and hence the domains rotate, so that the magnetic dipole moments are aligned in the direction of applied magnetic field as shown in fig.

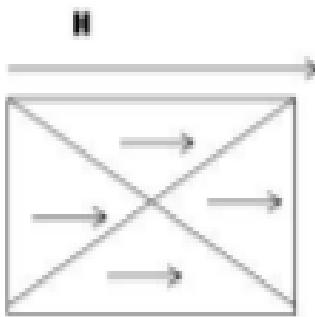


Figure 62: Rotation of domain walls in strong magnetic field.

Hysteresis curve: When a Ferro magnetic substance (e.g. iron) is subjected to a cycle of magnetization, it is found that magnetic field induction (flux density) B in the material lags behind the applied magnetizing force H . This phenomenon is known as hysteresis. If a piece of ferromagnetic material is subjected to one cycle of magnetization, the resultant B-H curve is a closed loop “a b c f e d a” is called hysteresis loop.

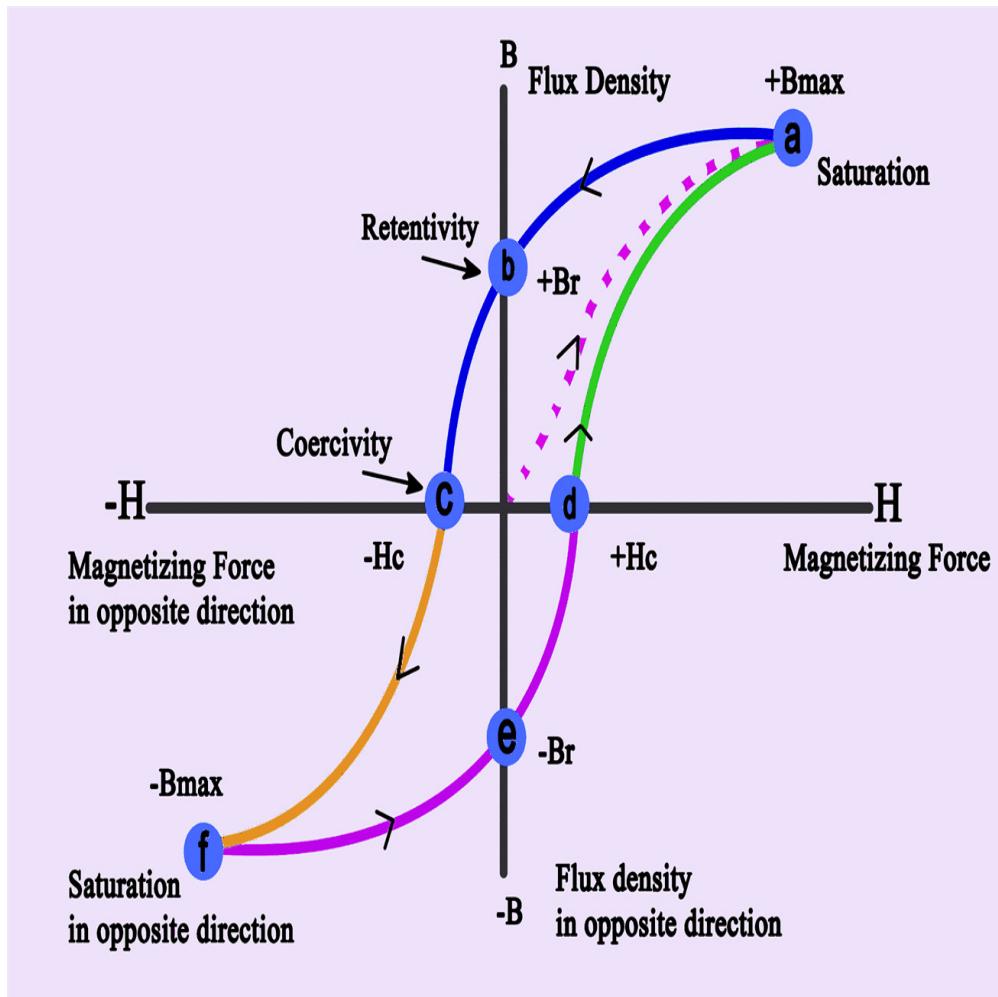


Figure 63: B-H curve/ Hysteresis loop

- In Ferromagnetic materials below T_C domain dipole moment are aligned in random direction, hence magnetization is zero.
- Under this condition, magnetic field is applied then the magnetization increases slowly in the beginning then rapidly till saturation value.
- When the direction of magnetic field is change, the magnetization does not follow its original path.

- The magnetization at zero magnetic field induction is known as remanent magnetization (Retentivity).
- If the magnetic field is still increases, then the magnetization becomes zero. This property is known as Coercivity.
- Finally, the magnetization reaches to saturation value in opposite direction.

Question (108): Distinguish between hard and soft magnetic materials.

Solution: Solution(a): Soft and hard magnetic materials

S.No.	Soft magnetic materials	Hard magnetic materials
1.	Can be easily magnetized or demagnetized	Can not be magnetized or demagnetized easily.
2.	Thin and long hysteresis loop	wide hysteresis loop.
3.	High permeability and low coercive field.	Low permeability and high coercive field.
4.	Large magnetic susceptibility and low remanent magnetic field	Small magnetic susceptibility and high remanent magnetic field
5.	As area of loop is small, magnetic energy loss per volume is less during magnetization and demagnetization.	Large area of loop indicates, magnetic energy loss per volume is high during magnetization and demagnetization.
7.	<i>Applications:</i> <ul style="list-style-type: none"> • electromagnets • in motors • dynamos • switching circuits 	<i>Applications:</i> <ul style="list-style-type: none"> • For permanent magnets in speakers, • clocks
8.	<i>Examples:</i> Fe-Si alloy, Fe-Co-Mn alloy and Fe-Ni alloy	<i>Examples:</i> Rare earth alloys with Mn, Fe, Co, Ni

Question (109): What is superconductivity? Discuss the general properties of superconductors. Write applications of superconductors.

Solution: Superconductivity: Superconductivity is a set of physical properties observed in certain materials where electrical resistance vanishes and magnetic flux fields are expelled from the material. Any material exhibiting these properties is a superconductor.

Properties of superconductors: The following are the properties of superconductors:

- Superconductors show the zero resistivity at low temperature.
- As temperature decrease, the electrical resistivity becomes zero at particular low temperature known as critical temperature (T_c).
- The T_c 's are different for different superconductors.
- T_c of a superconductor depends on impurities, pressure, structure and isotopic mass.
- In superconducting state materials exhibit Meissner's effect.
- Superconductor shows the persistent current.
- The current at which superconductor lost its properties is known as critical current.
- The Critical magnetic field (H_c) for superconductor is given by

$$H_c = H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right] \quad (134)$$

If $T = 0$, $H_c = H_0$ and $T = T_c$, $H_c = 0$

- By passing proper current through the superconductor we can destroy the property of superconductivity.
- The elements between (2-8) groups are superconductor.
- Thermal properties (Entropy and heat capacity)

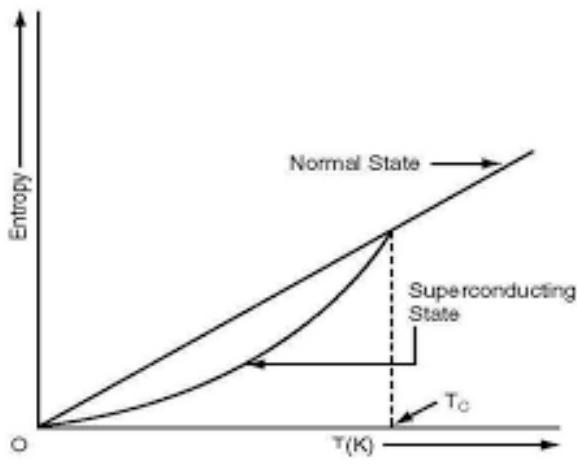


Figure 64: Entropy Vs Temperature

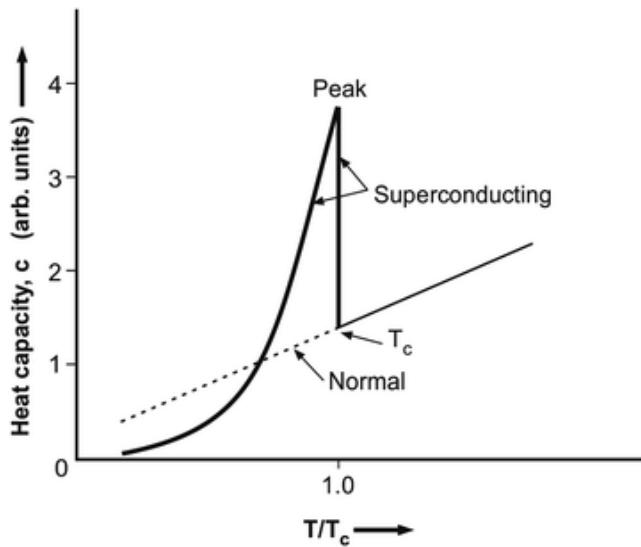


Figure 65: Heat capacity Vs temperature

Applications of superconductors: The following are the various applications of superconductors:

- Zero loss transmission lines.
- Superconducting magnets.
- Permanent magnets.
- Wires and cables for motors and power distribution systems.
- Ore separation (impurity separation).

Question (110): Explain Ferrites and write applications of Ferrites.

Solution: Ferrites: Ferrimagnetism is a typical behaviour of a class of materials called “ferrites”. The following are properties are characteristics of ferrites:

1. These material have permanent magnetic dipole moment but the neighbouring magnetic moment are opposite with different in magnitude. Hence net magnetic moment is non-zero.
2. They strongly allow the magnetic field lines.
3. The phenomenon of non-zero dipole moment is known as ferrimagnetic material. They show spontaneous magnetisation.

4. Magnetic permeability is much greater than one, $\mu \gg 1$.
5. Magnetic susceptibility is large, $\chi_m \gg 0$
6. χ_m depends on temperature as $\chi_m = \frac{C}{(T \pm T_N)}$.

Applications of Ferrites: Due to their unique electrical and magnetic characteristics, ferrites are useful for a wide range of technological applications:

- Ferrite is used in radio receivers to increase the sensitivity and selectivity of the receiver.
- Ferrites are used as cores in audio and TV transformers.
- Ferrites are used in digital computers and data processing circuits.
- Ferrites are used to produce low frequency ultrasonic by magnetostriction effect.
- Ferrites are used in the design of ferromagnetic amplifier of microwave signals.
- Hard Ferrites are used to make permanent magnets.

Question (111): Explain Meissner's effect. Distinguish Type-I and Type-II superconductors.

Solution: Explanation of Meissner's effect When a superconducting material at temperature $T > T_C$ is placed in external magnetic field, lines of magnetic induction pass through its body, but when it is cooled below the critical temperature $T < T_C$, these lines of induction are pushed out of the superconducting body. So inside the superconducting body, $B = 0$. This phenomenon is known as the "Meissner effect".

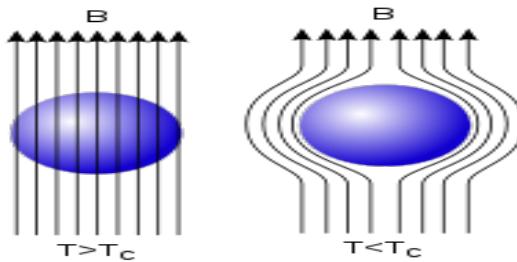


Figure 66: Behaviour of superconductor in magnetic field

- Let us consider a superconducting sphere placed in a magnetic field as shown in fig.
- When $T > T_c$ (normal conducting state), the magnetic lines pass through the material as shown in figure.
- When $T < T_c$ (superconducting state), it repels the magnetic flux lines and hence the magnetic field inside the sphere is zero.

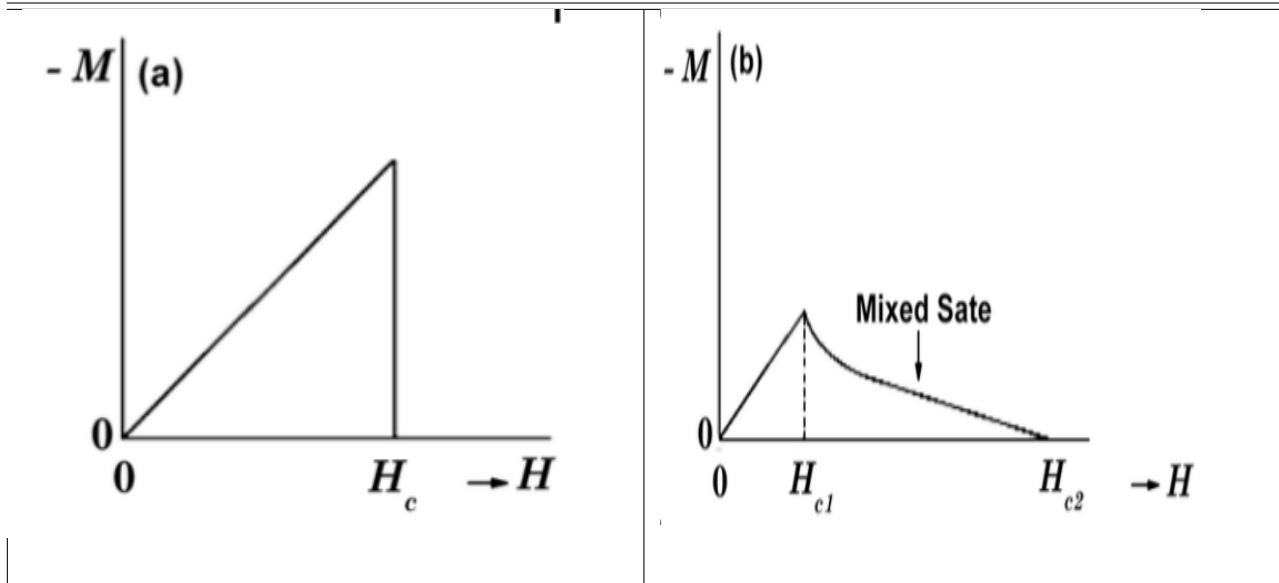
As we know the relation $\vec{B} = \mu_0(\vec{H} + \vec{M})$. Now, for superconducting state ($B = 0$), which gives, $0 = \mu_0(\vec{H} + \vec{M})$

$$(\mu_0 \neq 0) \quad \frac{\vec{M}}{\vec{H}} = -1 \Rightarrow \chi_m = -1 \quad (135)$$

The only magnetic materials whose magnetic susceptibility is negative are diamagnetic materials. Therefore, we can conclude that in superconducting state, the material exhibits diamagnetism. Hence superconductor is a perfect diamagnet.

Distinguish Type-I and Type-II superconductors

Type-I Superconductor	Type-II Superconductor
The materials loses magnetization suddenly.	The materials loses magnetization gradually.
They exhibit complete Meissner effect i.e., they are completely diamagnetic.	They do not exhibit complete Meissner effect.
There is only critical magnetic field H_C .	There are two critical magnetic fields i.e., lower critical magnetic field H_{C1} and higher critical magnetic field H_{C2} .
The value of H_C is always too low. It is about 0.1 Tesla.	The value of H_C is high and it is about 30 Tesla.



Below H_C , it is a superconductor. Above H_C , it is a normal conductor.	Below H_{C1} , it is a superconductor. Above H_{C2} , it is a normal material. Between H_{C1} and H_{C2} , it has mixed state.
It requires low magnetic field to destroy the superconductivity.	It requires large magnetic field to destroy the superconductivity.
They carry small current and hence called Soft superconductors.	They carry large current and hence called Hard superconductors.

Question(112) Describe BCS theory.

Solution: The phenomena of superconductivity was explained by Bardeen, Cooper, and Schrieffer, theoretically. According to these scientists, the quantum mechanics help in explaining superconductivity as follows:

- (a) Electron-Electron interaction by lattice deformation.
- (b) Cooper pairs
- (c) Superconducting energy gap

(a) Electron-Electron interaction:

- In a superconductor below T_C , the ions are perfectly in a order.
- Under this condition, when one electron passing through the ions it will be attracted by them equally. So instead the electron moves, all the positive ions displayed towards the electron.
- At this point, the effective charge of electron is invisible.
- If another electron passing by them it will also be attracted towards this positive ions.
- In this way, the electrons attract with each other with the help of lattice deformations.
- In other words, when electron of energy K_1 passing by positive ions it loses the energy (q) due to the attraction with ions.
- The other electron at the same time absorbs the energy as shown in Fig.

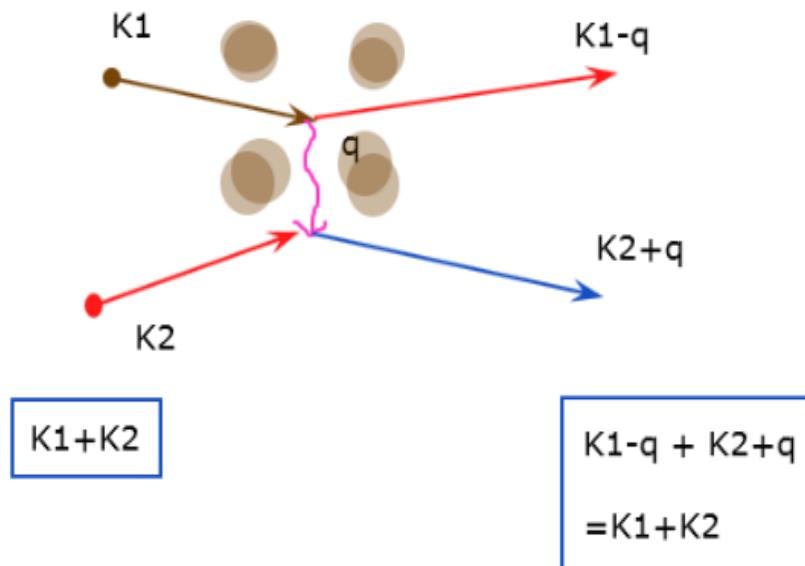


Figure 67: Electron-phonon interaction

- This is how the two electron attract with each other by exchanging a virtual phonon.

(b) Cooper pairs

- The pair of electrons after exchanging virtual photon's are known as Cooper pairs.
- In superconducting state the electrons are always in pairs.
- Due to these pair of electrons, the superconductor exhibits zero resistivity.

(c) Superconductor energy gap

- In superconducting state, the super electrons do not offer any resistance.
- Due to the attraction between these electrons, they occupy lower energy levels and energy gap is exists between superelectron and normal electrons energy levels.

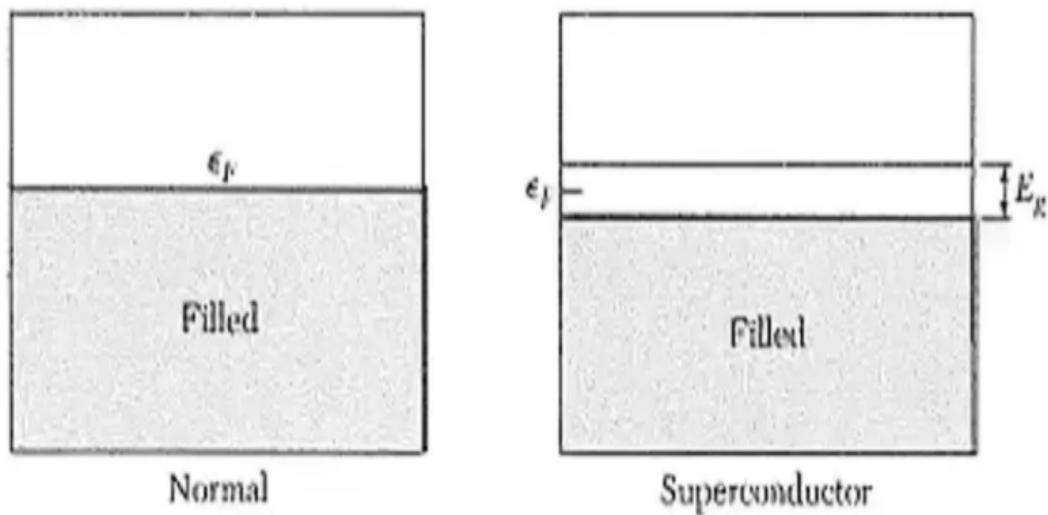


Figure 68: Energy level diagram

Question(113) Explain high T_c superconductors. Write applications of superconductors.

Solution:

High T_c superconductors: High-temperature superconductors (abbreviated high- T_c or HTS) are materials that behave as superconductors at unusually high temperatures. The first high-Tc superconductor was discovered in 1986 by researchers Georg Bednorz and K. Alex Muller, who were awarded the 1987 Nobel Prize in Physics “for their important break-through in the discovery of superconductivity in ceramic materials”. They discovered the phenomenon of superconductivity in *Cu – Oxide* compounds of lanthanum and barium at the temperature $T_c = 35\text{ K}$. Any superconductor with a transition temperature above 10 K is in general called High T_c superconductor.

Examples:

- Nb_3Ge (Neobium Germanium)
- $YBCo$ (Yttrium barium copper oxide)
- $BHCo$ (Bimath mercury calcium copper oxide)

Properties of High T_c superconductors:

- The high- T_c superconductors are type-II superconductors
- They exhibit zero resistance, strong diamagnetism, the Meissner effect, magnetic flux quantization, the Josephson effects, an electromagnetic penetration depth, an energy gap for the superconducting electrons, and the characteristic temperature dependencies of the specific heat and the thermal conductivity.
- High-temperature superconductors have unique physical properties both in the normal state and the superconducting one.

Applications of superconductors: The following are the various applications of superconductors:

- Zero loss transmission lines.
- Superconducting magnets.
- Permanent magnets.

- Wires and cables for motors and power distribution systems.
- Ore separation (impurity separation).

Question(114) Discuss Meissner's effect. Write general properties of superconductors.

Solution: Meissner's effect When a superconducting material at temperature $T > T_C$ is placed in external magnetic field, lines of magnetic induction pass through its body, but when it is cooled below the critical temperature $T < T_C$, these lines of induction are pushed out of the superconducting body. So inside the superconducting body, $B = 0$. This phenomenon is known as the "Meissner effect".

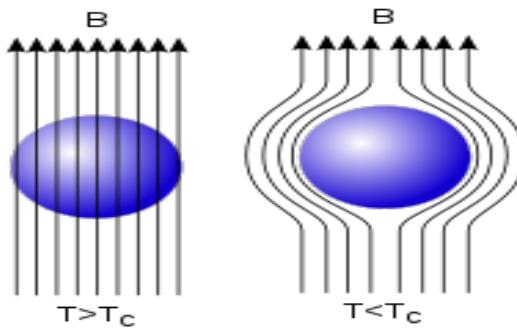


Figure 69: Behaviour of superconductor in magnetic field

- Let us consider a superconducting sphere placed in a magnetic field as shown in fig.
- When $T > T_c$ (normal conducting state), the magnetic lines pass through the material as shown in figure.
- When $T < T_c$ (superconducting state), it repels the magnetic flux lines and hence the magnetic field inside the sphere is zero.

As we know the relation

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) \quad (136)$$

Now, for superconducting state, we can write the above Eq. as

$$0 = \mu_0(\vec{H} + \vec{M}) \quad (137)$$

$$(\mu_0 \neq 0) \quad \frac{\vec{M}}{\vec{H}} = -1 \Rightarrow \chi_m = -1 \quad (138)$$

The only magnetic materials whose magnetic susceptibility is negative are diamagnetic materials. Therefore, we can conclude that in superconducting state, the material exhibit diamagnetism. Hence superconductor is a perfect diamagnet.

Properties of superconductors: The following are the properties of superconductors:

- Superconductors show the zero resistivity at low temperature.
- As temperature decrease, the electrical resistivity becomes zero at particular low temperature known as critical temperature (T_c).
- The T_c 's are different for different superconductors.
- T_c of a superconductor depends on impurities, pressure, structure and isotopic mass.
- In superconducting state materials exhibit Meissner's effect.
- Superconductor shows the persistent current.
- The current at which superconductor lost its properties is known as critical current.
- The Critical magnetic field (H_c) for superconductor is given by

$$H_c = H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

If $T = 0$, $H_c = H_0$ and $T = T_c$, $H_c = 0$

- By passing proper current through the superconductor we can destroy the property of superconductivity.
- The elements between (2-8) groups are superconductor.
- Thermal properties (Entropy and heat capacity)

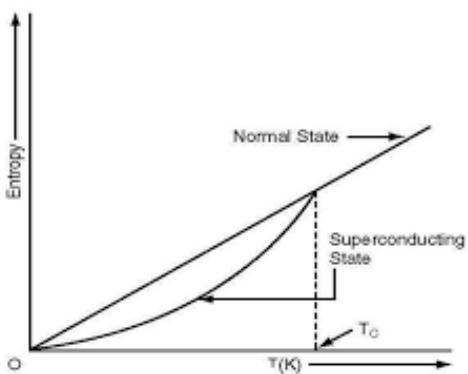


Figure 70: Entropy Vs Temperature

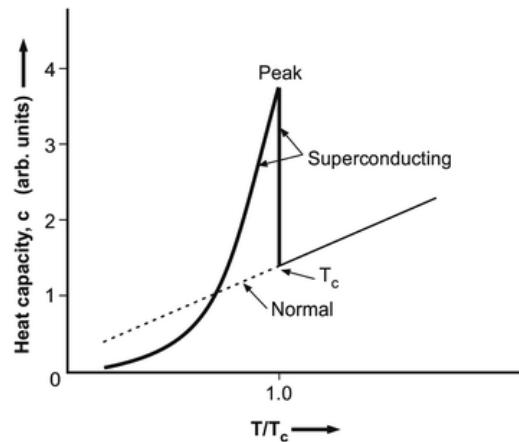


Figure 71: Heat capacity Vs temperature

Question(115): Elaborate BCS theory to explain superconductivity.

Solution: The phenomena of superconductivity was explained by Bardeen, Cooper, and Schrieffer, theoretically. According to these scientists, the quantum mechanics help in explaining superconductivity as follows:

- (a) Electron-Electron interaction by lattice deformation.
- (b) Cooper pairs
- (c) Superconducting energy gap

(a) Electron-Electron interaction:

- In a superconductor below T_C , the ions are perfectly in a order.
- Under this condition, when one electron passing through the ions it will be attracted by them equally. So instead the electron moves, all the positive ions displayed towards the electron.
- At this point, the effective charge of electron is invisible.
- If another electron passing by them it will also be attracted towards this positive ions.
- In this way, the electrons attract with each other with the help of lattice deformations.

- In other words, when electron of energy K_1 passing by positive ions it loses the energy (q) due to the attraction with ions.
- The other electron at the same time absorbs the energy as shown in Fig.

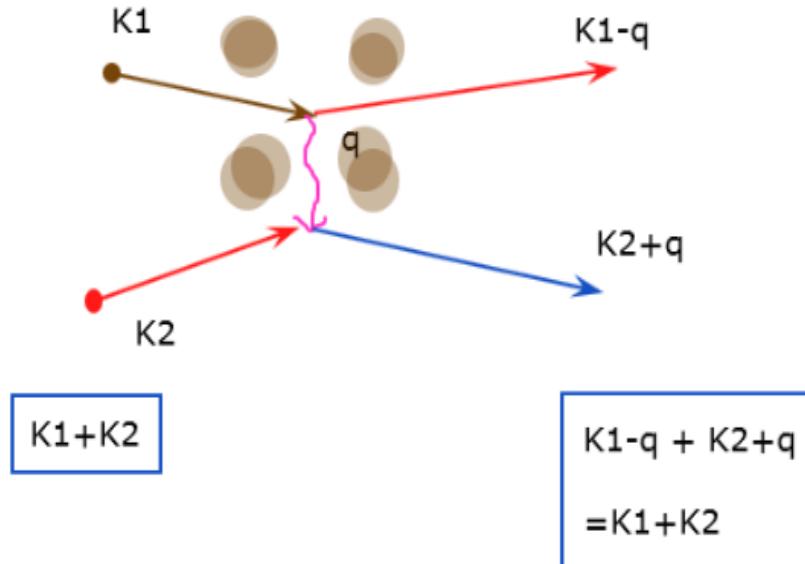


Figure 72: Electron-phonon interaction

- This is how the two electrons attract with each other by exchanging a virtual phonon.

(b) Cooper pairs

- The pair of electrons after exchanging virtual photon's are known as Cooper pairs.
- In superconducting state the electrons are always in pairs.
- Due to these pair of electrons, the superconductor exhibits zero resistivity.

(c) Superconductor energy gap

- In superconducting state, the super electrons do not offer any resistance.
- Due to the attraction between these electrons, they occupy lower energy levels and energy gap exists between superelectron and normal electrons energy levels.

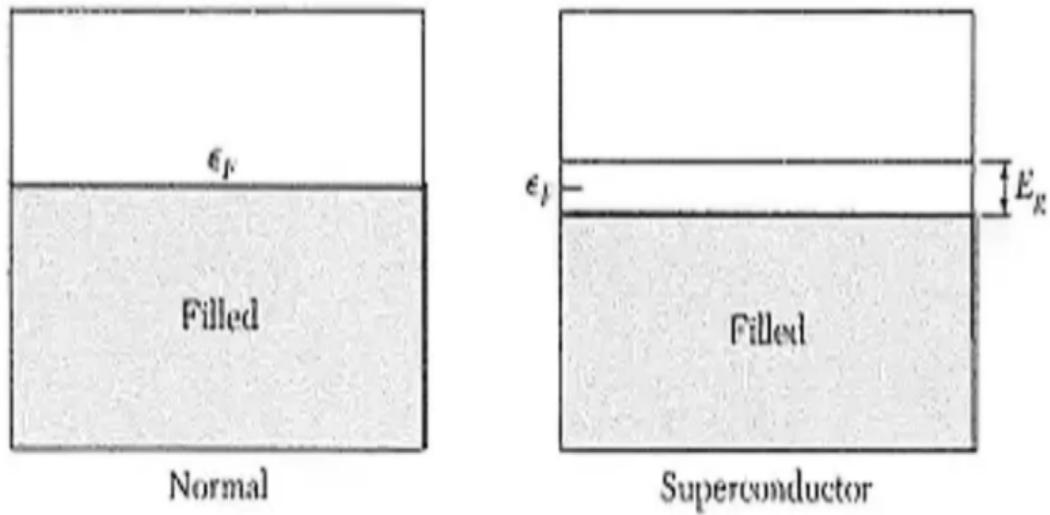


Figure 73: Energy level diagram

Question(116) Discuss Meissner's effect. Write applications of superconductors.

Solution: Meissner's effect When a superconducting material at temperature $T > T_C$ is placed in external magnetic field, lines of magnetic induction pass through its body, but when it is cooled below the critical temperature $T < T_C$, these lines of induction are pushed out of the superconducting body. So inside the superconducting body, $B = 0$. This phenomenon is known as the “Meissner effect”.

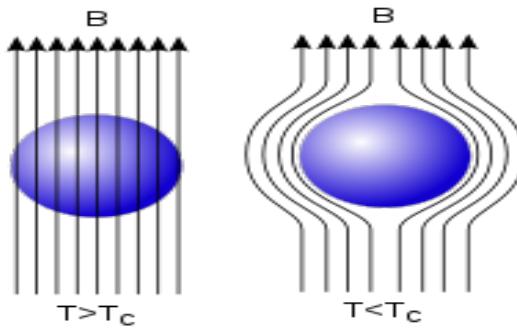


Figure 74: Behaviour of superconductor in magnetic field

- Let us consider a superconducting sphere placed in a magnetic field as shown in fig.

- When $T > T_c$ (normal conducting state), the magnetic lines pass through the material as shown in figure.
- When $T < T_c$ (superconducting state), it repels the magnetic flux lines and hence the magnetic field inside the sphere is zero.

As we know the relation

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) \quad (139)$$

Now, for superconducting state, we can write the above Eq. as

$$0 = \mu_0(\vec{H} + \vec{M}) \quad (140)$$

$$(\mu_0 \neq 0) \quad \frac{\vec{M}}{\vec{H}} = -1 \Rightarrow \chi_m = -1 \quad (141)$$

The only magnetic materials whose magnetic susceptibility is negative are diamagnetic materials. Therefore, we can conclude that in superconducting state, the material exhibit diamagnetism. Hence superconductor is a perfect diamagnet.

Applications of superconductors The following are the various applications of superconductors:

- Zero loss transmission lines.
- Superconducting magnets.
- Permanent magnets.
- Wires and cables for motors and power distribution systems.
- Ore separation (impurity separation).

Question (117): What are the critical conditions of a superconductor. Explain high T_c superconductors.

Solution: Critical conditions of a superconductor:

- **Critical temperature (T_c):** The electrical resistivity for a superconductor becomes zero at particular low temperature known as critical temperature (T_c). The T_c 's are different for different superconductors.

- **Critical magnetic field (H_c):** The superconductor shows the critical magnetic field (H_c), which is given by

$$H_c = H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

If $T = 0$, $H_c = H_0$ and $T = T_c$, $H_c = 0$

- **Critical current (I_c):** The current at which superconductor lost its properties is known as critical current (I_C), which is defined as

$$I_c = 2\pi r H_c$$

High T_c superconductors: High-temperature superconductors (abbreviated high- T_c or HTS) are materials that behave as superconductors at unusually high temperatures. The first high-Tc superconductor was discovered in 1986 by researchers Georg Bednorz and K. Alex Muller, who were awarded the 1987 Nobel Prize in Physics “for their important break-through in the discovery of superconductivity in ceramic materials”. They discovered the phenomenon of superconductivity in *Cu – Oxide* compounds of lanthanum and barium at the temperature $T_c = 35$ K. Any superconductor with a transition temperature above 10 K is in general called High T_c superconductor.

Examples:

- Nb_3Ge (Neobium Germanium)
- $YBCo$ (Yttrium barium copper oxide)
- $BHCo$ (Bimath mercury calcium copper oxide)

Properties of High T_c superconductors:

- The high- T_c superconductors are type-II superconductors
- They exhibit zero resistance, strong diamagnetism, the Meissner effect, magnetic flux quantization, the Josephson effects, an electromagnetic penetration depth, an energy gap for the superconducting electrons, and the characteristic temperature dependencies of the specific heat and the thermal conductivity.
- High-temperature superconductors have unique physical properties both in the normal state and the superconducting one.

Question (118): Explain concept of domains. Discuss the hysteresis in ferromagnetic materials.

Solution: Domain theory of ferromagnetism

- The phenomena of spontaneous magnetisation is known as ferromagnetism.
- The magnetization in absence of external magnetic field is known as spontaneous magnetisation.
- Below Curie temperature, the individual dipole moment align in one particular direction due to spin-spin coupling and forms regions known as domains.
- The domains are different in sizes and shapes.

Effect of external magnetic field:

- (a) In the absence of external magnetic field, the domains of a ferromagnetic material are randomly oriented. In other words, within the domain, all magnetic dipole moments are aligned, but the direction of alignment varies from domain to domain. The result is that there is no net magnetic dipole moment. Therefore, a Ferro magnetic material does not exhibit magnetism in the normal state.

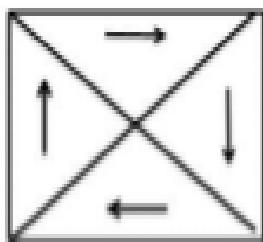


Figure 75: Without magnetic field

- (b) When a Ferro magnetic material is placed in an external magnetic field below T_c , a net magnetic dipole moment develops. This can occur in two ways:
- (i) *By the movement of domain walls:*

- The movement of domain walls takes place in weak magnetic fields.
- Due to weak magnetic field applied to the material the magnetic dipole moments increases and hence the boundary of domains displaced, so that the volume of the domains changes as shown in fig.

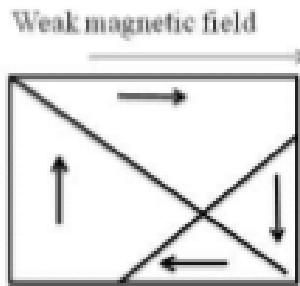


Figure 76: Displacement of domain walls with magnetic field.

(ii) *By the rotation of domain walls:*

- The rotation of domain wall takes place in strong magnetic fields.
- Due to strong magnetic field applied to the material the magnetic dipole moments increases enormously and hence the domains rotate, so that the magnetic dipole moments are aligned in the direction of applied magnetic field as shown in fig.

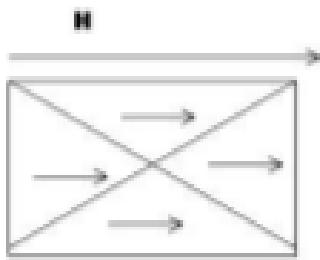


Figure 77: Rotation of domain walls in strong magnetic field.

Hysteresis curve: When a Ferro magnetic substance (e.g. iron) is subjected to a cycle of magnetization, it is found that magnetic field induction (flux density) B in the material lags behind the applied magnetizing force H . This phenomenon is known as hysteresis. If a piece of ferromagnetic material is subjected to one cycle of magnetization, the resultant B-H curve is a closed loop “a b c f e d a” is called hysteresis loop.

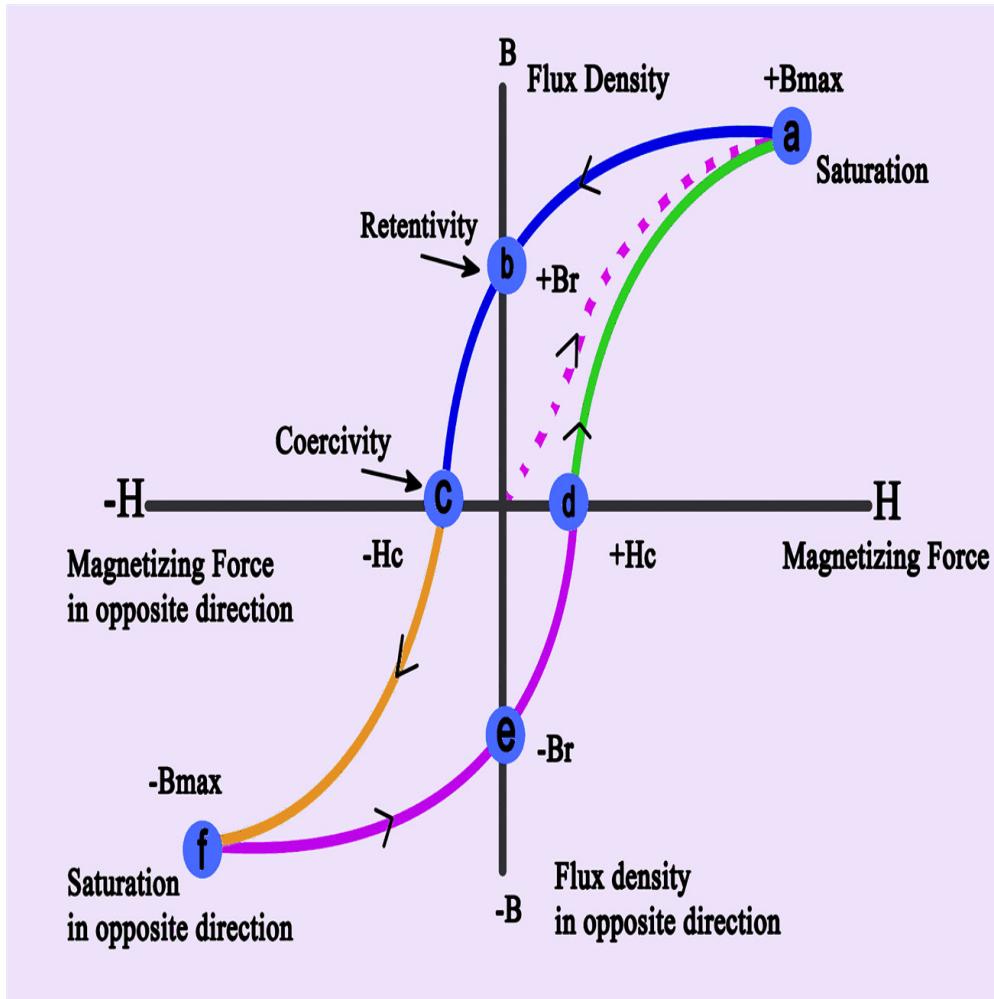


Figure 78: B-H curve/ Hysteresis loop

- In Ferromagnetic materials below T_C domain dipole moment are aligned in random direction, hence magnetization is zero.
- Under this condition, magnetic field is applied then the magnetization increases slowly in the beginning then rapidly till saturation value.
- When the direction of magnetic field is change, the magnetization does not follow its original path.
- The magnetization at zero magnetic field induction is known as remanent magnetization (Retentivity).
- If the magnetic field is still increases, then the magnetization becomes zero. This property is known as Coercivity.
- Finally, the magnetization reaches to saturation value in opposite direction.

Question (119): Distinguish between dia, para and ferro magnetic materials.

Solution:

Dia magnet	Para magnet	Ferro magnet
1. These materials have permanent dipole moment.	1. These material have permanent magnetic dipole moment but align randomly. Hence magnetic moment is zero.	1.These material have permanent magnetic dipole moment but align systematically. Hence non-zero magnetic moment.
2. They repel the magnetic flux lines only.	2. They weakly allow the magnetic flux lines.	2. They strongly allow the magnetic field lines.
3. The direction of magnetic field intensity is opposite to the direction of magnetisation. $H = -M$	3. The direction of magnetic field intensity is parallel to the direction of magnetisation. $H = M$	3. This kind of material shows the spontaneous magnetisation.
4. Magnetic permeability is less than one $\mu < 1$	4. Magnetic permeability is greater than one. $\mu > 1$	4. Magnetic permeability is much greater than one. $\mu \gg 1$
5. Magnetic susceptibility is negative $X_m < 0$	5. Magnetic susceptibility is small or positive. $X_m > 0$	5.Magnetic susceptibility is large $X_m \gg 0$
6. Magnetic susceptibility does not depend on temperature	6. Magnetic susceptibility depend on temperature $X_m = C/T$	6. X_m depends on temperature $X_m = C/(T-T_c)$
7. Examples: Cu, Ag, Hg	7. Examples: Al, Cr, Na	7. Examples: Fe, Co, Ni

Question(120) Write any six general properties of superconductors. Explain Meissner's effect.

Solution:

Properties of superconductors: The following are the properties of superconductors:

- (1) Superconductors show the zero resistivity at low temperature.
- (2) As temperature decrease, the electrical resistivity becomes zero at particular low temperature known as critical temperature (T_c).
- (3) In superconducting state materials exhibit Meissner's effect.
- (4) The current at which superconductor lost its properties is known as critical current.
- (5) The Critical magnetic field (H_c) for superconductor is given by

$$H_c = H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

If $T = 0$, $H_c = H_0$ and $T = T_c$, $H_c = 0$

- (6) Thermal properties (Entropy and heat capacity)

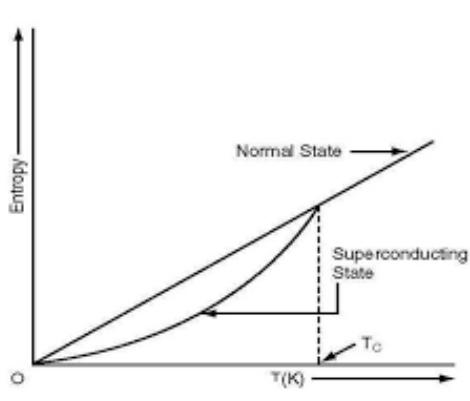


Figure 79: Entropy Vs Temperature

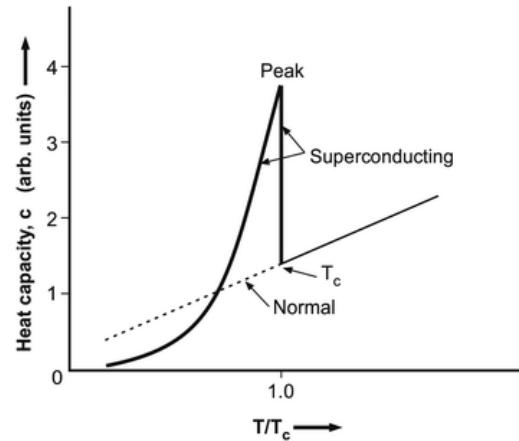


Figure 80: Heat capacity Vs temperature

Meissner's effect When a superconducting material at temperature $T > T_c$ is placed in external magnetic field, lines of magnetic induction pass through

its body, but when it is cooled below the critical temperature $T < T_c$, these lines of induction are pushed out of the superconducting body. So inside the superconducting body, $B = 0$. This phenomenon is known as the “Meissner effect”.

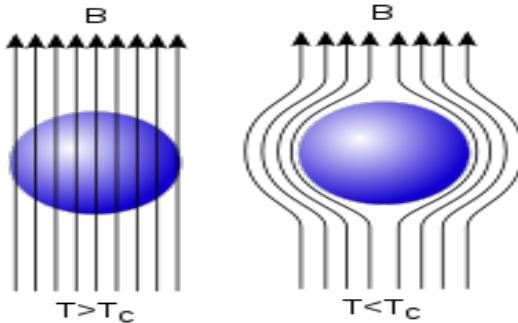


Figure 81: Behaviour of superconductor in magnetic field

- Let us consider a superconducting sphere placed in a magnetic field as shown in fig.
- When $T > T_c$ (normal conducting state), the magnetic lines pass through the material as shown in figure.
- When $T < T_c$ (superconducting state), it repels the magnetic flux lines and hence the magnetic field inside the sphere is zero.

As we know the relation

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) \quad (142)$$

Now, for superconducting state, we can write the above Eq. as

$$0 = \mu_0(\vec{H} + \vec{M}) \quad (143)$$

$$(\mu_0 \neq 0) \quad \frac{\vec{M}}{\vec{H}} = -1 \Rightarrow \chi_m = -1 \quad (144)$$

The only magnetic materials whose magnetic susceptibility is negative are diamagnetic materials. Therefore, we can conclude that in superconducting state, the material exhibit diamagnetism. Hence superconductor is a perfect diamagnet.

UNIT-V (SAQs)

Question(121): Names the types of polarizations.

Solution: The four types of polarization which occur in dielectrics are:

1. Electronic polarization
2. Ionic polarization
3. Orientational or dipole polarization
4. Space charge or interfacial polarization

Question(122): Define dielectric constant.

Solution: It is defined as the ratio of permittivity of medium to permittivity of free space.

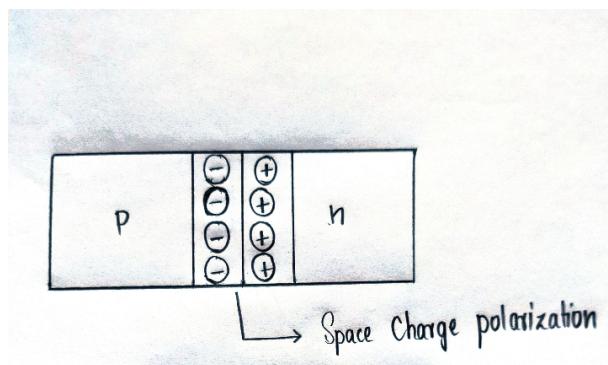
$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

The ϵ_r is called the relative permittivity or dielectric constant of the dielectric material and ϵ_0 is permittivity of free space and is equal to $8.85 \times 10^{-12} \text{ F/m}$.

Question(123): Define space-charge polarization with neat diagram.

Solution: Space charge polarization Space charge polarization arises due to the accumulation of charges at the electrodes or at the interfaces of multiphase dielectric materials.

Examples: Space charge polarization occurs in ferrites and semiconductors.



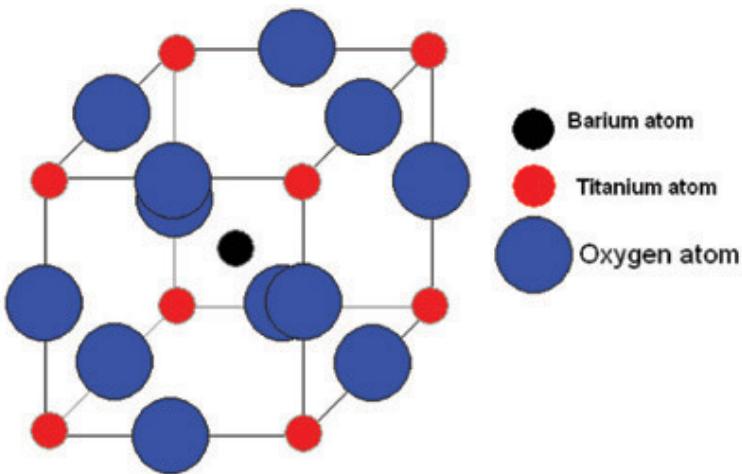
The induced dipole moment in this case can be written as

$$\mu_s = \alpha_s E$$

where α_s is proportionality constant and it is known as space charge polarizability.

Question(124): Draw the structure of Barium titanate.

Solution: The structure of Barium titanate:



Question(125): Define orientational polarization with neat diagram.

Solution:

Orientational polarization Orientation polarization arises in dielectric materials which possess molecules with permanent dipole moment (i.e., in polar molecules).

Examples: Dipolar polarization or orientation takes place only in polar molecules such as H_2O , HCl , NH_3 , CH_3Cl and CO etc.

The induced dipole moment (μ_0) due to orientation polarization is proportional to the electrical field strength (E)

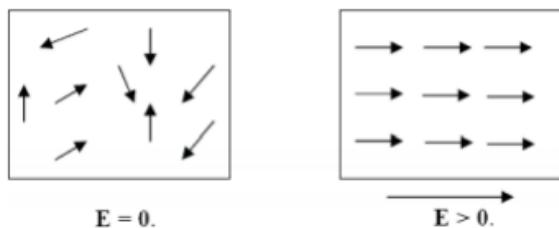


Figure 82: Orientational polarization.

$$\mu_o = \alpha_o E$$

where α_o is proportionality constant and it is known as orientation polarizability

Question(126): Explain the polarization mechanism in dielectrics.

Solution: When a dielectric material is placed in an external dc electric field, it gets polarized. As a result, an alignment of positive and negative charges takes place within the dielectric resulting in no net increase in the charge of the dielectric.

Question(127): Calculate the electronic polarizability of neon. The radius of neon atom is 0.158 nm. (Given, Permittivity of free space = $8.854 \times 10^{-12} Fm^{-1}$)

Solution: Solution:

Given

$$R = 0.158 \text{ nm} = 0.158 \times 10^{-9} \text{ m}$$

$$\epsilon_0 = 8.854 \times 10^{-12} Fm^{-1}$$

The electronic polarizability is given by

$$\begin{aligned}\alpha_e &= 4\pi\epsilon_0 R^3 \\ &= 4 \times 3.14 \times 8.854 \times 10^{-12} \times (0.158 \times 10^{-9})^3 \\ &= 0.436 \times 10^{-39} \\ \alpha_e &= 4.36 \times 10^{-40} Fm^2\end{aligned}$$

Question(128): What are nanomaterials?

Solution: Nanomaterials are materials with structures and properties significantly different from those of bulk materials due to their small size. These materials have dimensions in the nanoscale range, usually between 1 and 100 nanometers, and exhibit unique physical, chemical, and biological properties. They are widely used in various fields due to their high surface area, remarkable biocompatibility, optical and electrical properties.

Question(129): Mention the names of the techniques used for the preparation of Nanomaterials.

Solution: The following two main Technique are used for the preparation of Nanomaterials.

- Top-Down Technique
 - (i) Ball-milling technique

- Bottom-up Technique
- (i) Sol-gel method
 - (ii) Chemical Vapour Deposition (CVD) method

Question(130): Write a short note on Top-down method.

Solution: Top-down approach involves the breaking down of the bulk material into nanosized structures or particles. Top-down synthesis techniques are extension of those that have been used for producing micron sized particles. Top-down approaches are inherently simpler and depend either on removal or division of bulk material or on miniaturization of bulk fabrication processes to produce the desired structure with appropriate properties.

Question(131): Mention some effects of size reduction of nanoparticles.

Solution: As the size of the particle decreases, the ratio of the surface area to the volume of the particle increases. For nanoparticles, this ratio becomes significantly large causing a large portion of the atoms to reside on the surface compared to those in the core of the particle. For example, for a particle of 1 m in diameter, nearly 0.15% of its atoms are on the surface, while for a particle of 6 nm in diameter nearly 20% of its atoms are on the surface.

Question(132): Define quantum well.

Solution: If one dimension of the material is reduced to nano range while the other two dimensions remain large, then the material or structure is called quantum well. Quantum well is also called nanolayers. In nanotechnology, a quantum well is like a potential well in which the carrier particles are trapped and it took place when the thickness of the well is comparable to the de Broglie wavelength of the carriers.

Question(133): Write any two applications of Ferroelectrics.

Solution: Ferroelectric Materials can be used in various applications. Some of the applications are given below:

1. Production of ultrasonics.
2. Magnetic information storage devices.

Question(134): What is meant by quantum confinement?

Solution: Quantum confinement effects describe electrons in terms of energy levels, potential wells, valence bands, conduction bands, and electron energy band gaps. The quantum confinement effect is observed when the size of the particle is too small to be comparable to the wavelength of the electron. Obviously, the confinement of an electron and hole in nanocrystals significantly depends on the material properties.

Question(135): Write the applications of nanomaterials.

Solution: The following are the applications of nanotechnology in various fields.

- Nano-MEMS (Micro-Electro Mechanical Systems) are used in ICS, optical switches, pressure sensors, mass sensors etc.
- Orderly assembled nano materials are used as quantum electronic devices and photonic crystals.
- Nano materials are used to make CD's and Semiconductor laser.
- Consolidated state nano particles are used as catalyst, electrodes in solar and fuel cells.

UNIT-V (LAQs)

Question(136): Obtain an expression for electronic polarizability in terms of radius of atom.

Solution: Here we start with two scenario

Without electric field ($E = 0$):

Let us consider an atom of dielectric material with atomic number Z , then the charge on its nucleus is $+Ze$. The nucleus is surrounded by electron cloud of charge $-Ze$ which is distributed over a sphere of radius R as shown in Fig. The centers of the electron cloud and the positive nucleus are at the same point and hence there is no dipole moment

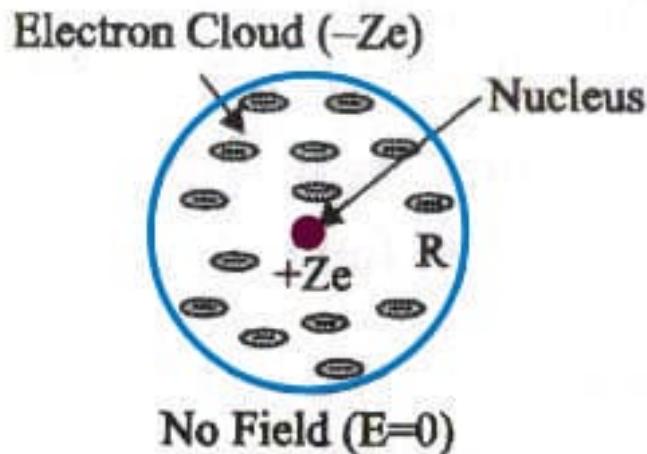


Figure 83: Atom without any electric field ($E = 0$).

Therefore, the charge density for electron cloud is given by

$$\begin{aligned}
 \rho &= \frac{\text{Total negative charge of the electron cloud}}{\text{Volume of the atom}} \\
 &= \frac{-Ze}{\frac{4}{3}\pi R^3} \\
 &= -\frac{3Ze}{4\pi R^3}
 \end{aligned} \tag{145}$$

With electric field ($E \neq 0$):

When the atom of dielectric material is subjected to electric field, two types of phenomena occur:

- Lorentz force arises due to the electric field which separates the nucleus and the electron cloud from their equilibrium positions in opposite directions. Therefore, due to the Lorenz force the electron cloud and the nucleus move in opposite directions and they are separated by a distance x , where there is a formation of electrical dipole in the atom as shown in Fig.

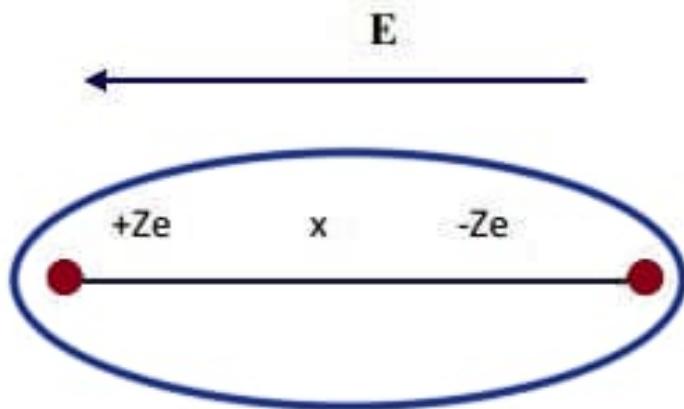


Figure 84: Atom with electric field ($E \neq 0$).

- Coulomb attractive force arises between the nucleus and the electron cloud after separation which tries to maintain the original equilibrium position.

Lorentz force (F_L) between the nucleus and the electron

$$\begin{aligned} F_L &= \text{charge} \times \text{Electric field} \\ &= -ZeE \end{aligned} \quad (146)$$

Coulomb attractive force (F_C) between the nucleus and the electron cloud being separated at a distance x is

$$F_C = \frac{1}{4\pi\epsilon_0} \left(\frac{\text{Charge of nucleus} \times Q'}{x^2} \right) \quad (147)$$

Q' = The total negative charge enclosed in the sphere of radius x

Q' = Charge density of electrons (ρ) \times Volume of the sphere of radius x

$$Q' = -\frac{3Ze}{4\pi R^3} \times \frac{4\pi x^3}{3}$$

$$Q' = -\frac{Zex^3}{R^3} \quad (148)$$

Substituting the value of Q' , the Coulomb force becomes

$$\begin{aligned} F_C &= \frac{1}{4\pi\epsilon_0} \times \left(\frac{Ze \times -\frac{Zex^3}{R^3}}{x^2} \right) \\ &= -\frac{Z^2 e^2 x}{4\pi\epsilon_0 R^3} \end{aligned} \quad (149)$$

At equilibrium, the Coulomb force and Lorentz force are equal. Hence,

$$\begin{aligned} F_L &= F_C \\ -ZeE &= -\frac{Z^2 e^2 x}{4\pi\epsilon_0 R^3} \\ x &= \frac{4\pi\epsilon_0 R^3 E}{Ze} \end{aligned} \quad (150)$$

The induced dipole moment μ_e is the product of magnitude of charge (Ze) and the separation between the charges (x)

$$\mu_e = Ze x \quad (151)$$

Now, after substituting x , we will get

$$\begin{aligned} \mu_e &= Ze \times \frac{4\pi\epsilon_0 R^3 E}{Ze} \\ \mu_e &= 4\pi\epsilon_0 R^3 E \\ \mu_e &= \alpha_e E \end{aligned} \quad (152)$$

where $\alpha_e = 4\pi\epsilon_0 R^3$ is called electronic polarizability, which depends on the radius of the atom (R).

Question(137): Elaborate Ferroelectricity with its properties. Mention applications of Ferroelectrics.

Solution: In general, when the dielectric materials are placed in an external electric field E , they get polarized. But some dielectric materials exhibit spontaneous polarization even in the absence of external electric field. This phenomenon is known as ferroelectricity and these dielectric materials are known as ferroelectrics. The ferroelectric effect was first observed by Valasek in 1921, in the Rochelle salt. This has molecular formula $KNaC_4H_4O_6 \cdot 4H_2O$.

Examples:

- Rochelle salt ($KNaC_4H_4O_6 \cdot 4H_2O$)
- Lithium niobate ($LiNbO_3$)
- Barium titanate ($BaTiO_3$)
- Lithium titanate($LiTiO_3$)

Properties of Ferroelectrics

- They are anisotropic crystals.
- They exhibit spontaneous polarization.
- As the temperature increases the spontaneous polarization decreases and at a particular temperature, the spontaneous polarization vanishes. This temperature is known as Curie temperature (T_C). The variation of spontaneous polarization with temperature as shown in Fig.
- Below Curie temperature, the dielectric constant is function of electric field. Above Curie temperature dielectric constant varies with temperature. The variation of dielectric constant with temperature as shown in Fig. According to Curie- Weiss law,

$$\epsilon_r = \frac{C}{T - T_C} \quad (153)$$

Where C is called curie constant and T_C is called curie temperature.

- Ferroelectric materials exhibit hysteresis as shown in Fig, similar to that of ferro magnetic materials.

Hysteresis: When an electric field is applied on a ferroelectric material then polarization takes place. This polarization always lags behind the applied electric field. This phenomenon is known as hysteresis of ferroelectric material.

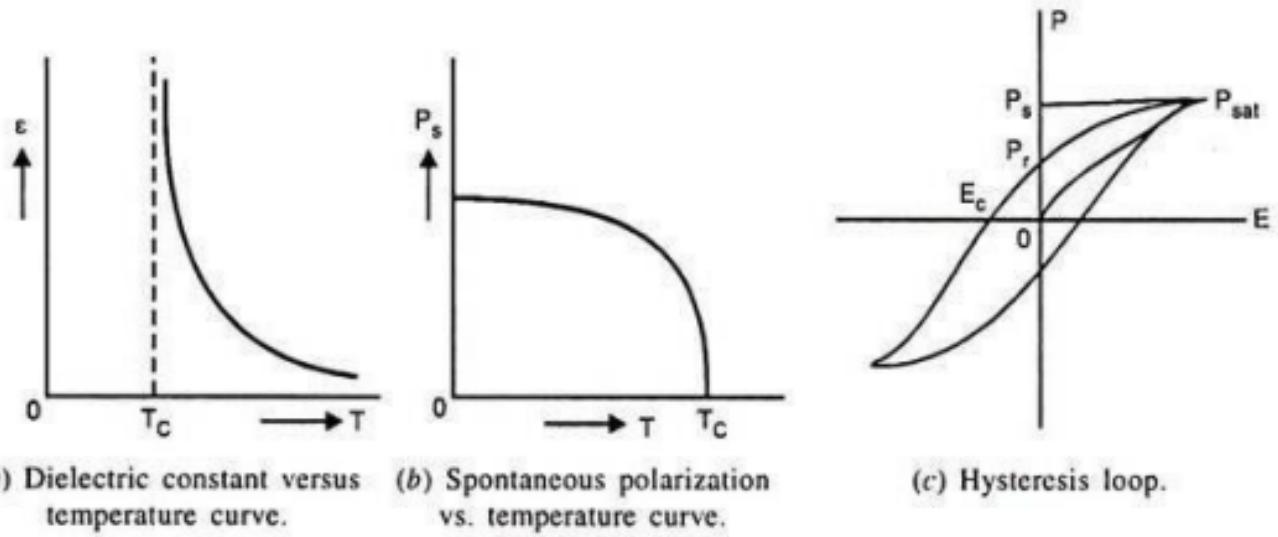


Figure 85: Properties of Ferroelectrics.

- Ferroelectric materials exhibit Piezoelectricity and Pyroelectricity.

Applications of Ferroelectric Materials

Ferroelectric Materials can be used in various applications. Some of the applications are given below:

1. Production of ultrasonics.
2. SONAR, strain gauges, etc.
3. Frequency stabilizers and crystal-controlled oscillators.
4. Magnetic amplifiers and dielectric amplifiers.
5. Magnetic information storage devices.

Question(138): Derive an expression for ionic polarizability.

Solution:

Ionic polarization When an electric field is applied on ionic dielectric material then positive ions move in the field direction where as negative ions move in the opposite direction, hence dipoles will be formed. This phenomenon is known as ionic polarization.

Examples: Ionic solids ($NaCl$) exhibit ionic polarization.

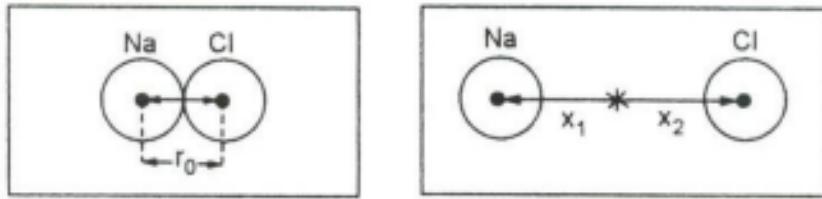


Figure 86: First without E and then with E .

The induced dipole moment (μ_i) due to ionic polarization is given by

$$\begin{aligned}\mu_i &= \text{magnitude of charge} \times \text{displacement} \\ \mu_i &= e(x_1 + x_2)\end{aligned}$$

where x_1 is the shift of positive ion of mass (m) and x_2 is the shift of negative ion of mass (M), from their equilibrium positions. As the field is applied, the restoring force produced is proportional to the displacements. For positive ion, Restoring force F is directly proportional to x_1 ,

$$F = k_1 x_1$$

where $k_1 = m\omega_0^2$, is restoring force constants, which depend on the mass of the ion and the angular frequency of the molecule in which ions are present. Above equation becomes

$$F = m\omega_0^2 x_1$$

As we know in presence of electric field, the force will be $F = eE$. On comparison, we get

$$\begin{aligned}eE &= m\omega_0^2 x_1 \\ x_1 &= \frac{eE}{m\omega_0^2}\end{aligned}$$

Similarly for the negative ion where ($k_2 = M\omega_0^2$), we can write the

$$\begin{aligned}eE &= M\omega_0^2 x_2 \\ x_2 &= \frac{eE}{M\omega_0^2}\end{aligned}$$

Substituting the values of x_1 and x_2 , the above equation for induced dipole moment becomes

$$\mu_i = \frac{e^2 E}{\omega_0^2} \left(\frac{1}{m} + \frac{1}{M} \right)$$

In the case of ionic polarization, the induced dipole moment is given by

$$\mu_i = \alpha_i E$$

On comparing above two equations, we get

$$\alpha_i = \frac{e^2}{\omega_0^2} \left(\frac{1}{m} + \frac{1}{M} \right) \quad (154)$$

Where α_i is known as ionic polarizability and m is the mass of positive ion and M is the mass of negative ion and ω_0 is the angular frequency and e is the charge of the electron.

Question(139): Explain the types of polarizations with their polarizability equations.

Solution: When a dielectric material is placed in an external dc electric field, it gets polarized. The four types of polarization which occur in dielectrics are:

1. Electronic polarization
2. Ionic polarization
3. Orientational or dipole polarization
4. Space charge or interfacial polarization

Electronic Polarization When electric field is applied on dielectric material then all the positive nuclei of atoms move in the field direction and the negative electron cloud of atoms move in opposite direction, hence dipoles will be formed. This phenomenon is known as electronic polarization.

Examples: Mono atomic gases exhibit only electronic polarization: Helium (He), Neon (Ne), Argon (Ar), Krypton (Kr), Xenon (Xe) and Radon (Rn).

The induced dipole moment (μ_e) is proportional to the electrical field strength (E)

$$\mu_e = \alpha_e E \quad (155)$$

where α_e is proportionality constant and it is known as electronic polarizability. The electronic polarizability for a rare or noble gas atom is given by

$$\alpha_e = 4\pi\epsilon_0 R^3 \quad (156)$$

where R is the radius of the atom.

Ionic polarization When an electric field is applied on ionic dielectric material then positive ions move in the field direction where as negative ions move in the opposite direction, hence dipoles will be formed. This phenomenon is known as ionic polarization.

Examples: Ionic solids ($NaCl$) exhibit ionic polarization.

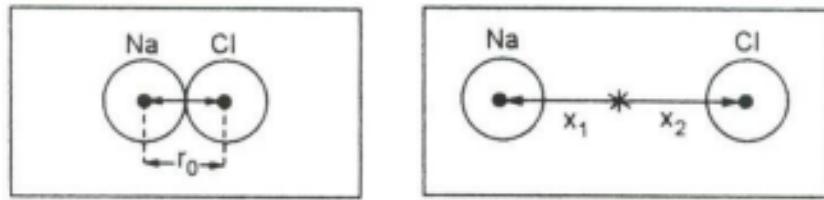


Figure 87: First without E and then with E .

The induced dipole moment (μ_i) due to ionic polarization is proportional to the electrical field strength (E)

$$\mu_i = \alpha_i E \quad (157)$$

Where α_i is proportionality constant and it is known as ionic polarizability. The ionic polarizability for an ionic solid is given by

$$\alpha_i = \frac{e^2}{\omega_0^2} \left(\frac{1}{m} + \frac{1}{M} \right) \quad (158)$$

where m is the mass of positive ion and M is the mass of negative ion and ω_0 is the angular frequency and e is the charge of the electron.

Orientational polarization Orientation polarization arises in dielectric materials which possess molecules with permanent dipole moment (i.e., in polar molecules).

Examples: Dipolar polarization or orientation takes place only in polar molecules such as H_2O , HCl , NH_3 , CH_3Cl and CO etc.

The induced dipole moment (μ_0) due to orientation polarization is proportional to the electrical field strength (E)

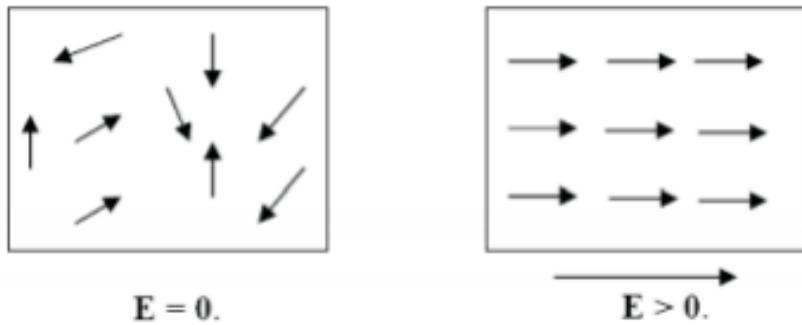


Figure 88: Orientational polarization.

$$\mu_o = \alpha_o E \quad (159)$$

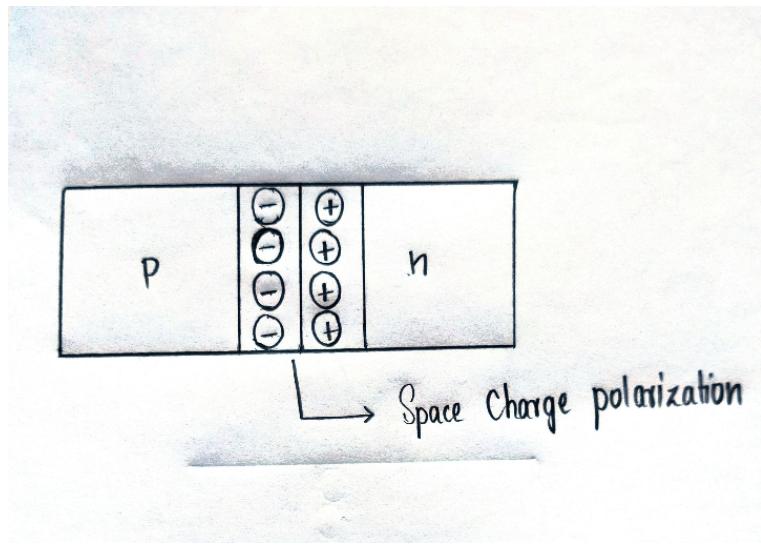
where α_o is proportionality constant and it is known as orientation polarizability. The orientation polarizability for polar dielectric medium is given by

$$\alpha_o = \frac{\mu^2}{3KT} \quad (160)$$

where μ is the average dipole moment, K is the Boltzmann constant and T is the temperature.

Space charge polarization Space charge polarization arises due to the accumulation of charges at the electrodes or at the interfaces of multiphase dielectric materials.

Examples: Space charge polarization occurs in ferrites and semiconductors.



The induced dipole moment in this case can be written as

$$\mu_s = \alpha_s E \quad (161)$$

where α_s is proportionality constant and it is known as space charge polarizability.

The total polarization is the sum of electronic polarization, ionic polarization, orientation polarization and space-charge polarization. But space-charge polarization is very small when compared to other polarization mechanisms and it is not common in most of the dielectrics. So, it can be neglected. Therefore, The total polarizability

$$\alpha_T = \alpha_e + \alpha_i + \alpha_o \quad (162)$$

Question(140): Define the following terms

- (a) Dielectric constant
- (b) Polarization
- (c) Permittivity
- (d) Ferroelectricity

Solution:

- (a) **Dielectric constant:** It is defined as the ratio of permittivity of medium to permittivity of free space.

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

The ϵ_r is called the relative permittivity or dielectric constant of the dielectric material and ϵ_0 is permittivity of free space and is equal to 8.85×10^{-12} F/m.

- (b) **Polarization:** When a dielectric material is placed in an external dc electric field, it gets polarized. As a result, an alignment of positive and negative charges takes place within the dielectric resulting in no net increase in the charge of the dielectric.
- (c) **Permittivity:** Permittivity is the property of every material, which measures the opposition offered against the formation of an electric field. It is represented by the

$$\epsilon = \frac{\vec{D}}{\vec{E}}$$

- (d) **Ferroelectricity** In general, when the dielectric materials are placed in an external electric field E , they get polarized. But some dielectric materials exhibit spontaneous polarization even in the absence of external electric field. This phenomenon is known as ferroelectricity and these dielectric materials are known as ferroelectrics.

Question(141): Define the types of polarization and explain Barium titanate.

Solution: When a dielectric material is placed in an external dc electric field, it gets polarized. The four types of polarization which occur in dielectrics are:

1. Electronic polarization
2. Ionic polarization
3. Orientational or dipole polarization
4. Space charge or interfacial polarization

Electronic Polarization When electric field is applied on dielectric material then all the positive nuclei of atoms move in the field direction and the negative electron cloud of atoms move in opposite direction, hence dipoles will be formed. This phenomenon is known as electronic polarization.

Examples: Mono atomic gases exhibit only electronic polarization: Helium (*He*), Neon (*Ne*), Argon (*Ar*), Krypton (*Kr*), Xenon (*Xe*) and Radon (*Rn*).

Ionic polarization When an electric field is applied on ionic dielectric material then positive ions move in the field direction whereas negative ions move in the opposite direction, hence dipoles will be formed. This phenomenon is known as ionic polarization.

Examples: Ionic solids (*NaCl*) exhibit ionic polarization.

Orientational polarization Orientation polarization arises in dielectric materials which possess molecules with permanent dipole moment (i.e., in polar molecules).

Examples: Dipolar polarization or orientation takes place only in polar molecules such as *H₂O*, *HCl*, *NH₃*, *CH₃Cl* and *CO* etc.

Space charge polarization Space charge polarization arises due to the accumulation of charges at the electrodes or at the interfaces of multiphase dielectric materials.

Examples: Space charge polarization occurs in ferrites and semiconductors.

Barium titanate: Barium titanate (BTO) is an inorganic compound with chemical formula *BaTiO₃*. Barium titanate appears white as a powder and is transparent when prepared as large crystals. It is a ferroelectric material. Above 120°C, barium titanate has a cubic structure. It is therefore centrosymmetric and possesses no spontaneous dipole. With no spontaneous dipole the material behaves like a simple dielectric, such that its polarisation varies linearly with field. *T_C* for barium titanate is 120°C.

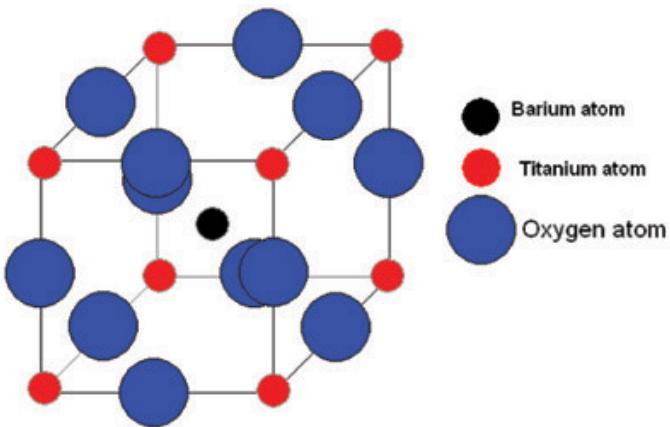


Figure 89: Barium titanate structure.

Below 120°C, it changes to a tetragonal phase, with an accompanying movement of the atoms. The phase which is reached after cooling to 0°C from tetragonal is orthorhombic. And then rhombohedral below -90°C.

Uses of Barium titanate:

- Barium titanate is a dielectric ceramic used in capacitors.
- It is a piezoelectric material used in microphones and other transducers.
- Barium titanate crystals find use in nonlinear optics.
- Due to their elevated biocompatibility, barium titanate nanoparticles (BT-NPs) have been recently employed as nanocarriers for drug delivery.

Question(142): Derive an expression for electronic polarizability in terms of radius of atom.

Solution: Here we start with two scenario

Without electric field ($E = 0$):

Let us consider an atom of dielectric material with atomic number Z , then the charge on its nucleus is $+Ze$. The nucleus is surrounded by electron cloud of charge $-Ze$ which is distributed over a sphere of radius R as shown in Fig. The centers of the electron cloud and the positive nucleus are at the same point and hence there is no dipole moment

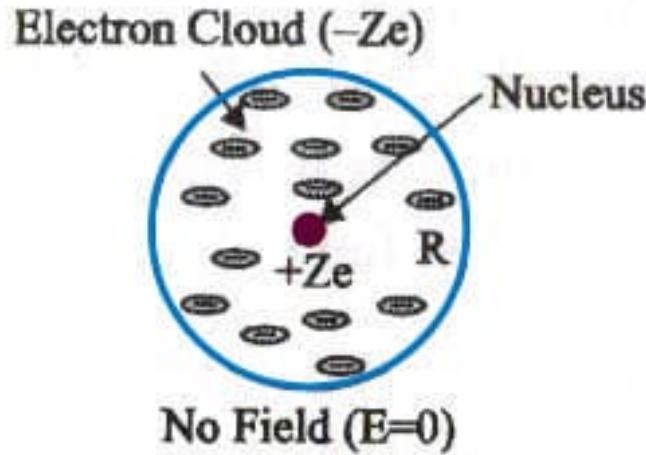


Figure 90: Atom without any electric field ($E = 0$).

Therefore, the charge density for electron cloud is given by

$$\begin{aligned}
 \rho &= \frac{\text{Total negative charge of the electron cloud}}{\text{Volume of the atom}} \\
 &= \frac{-Ze}{\frac{4}{3}\pi R^3} \\
 &= -\frac{3Ze}{4\pi R^3}
 \end{aligned} \tag{163}$$

With electric field ($E \neq 0$):

When the atom of dielectric material is subjected to electric field, two types of phenomena occur:

- Lorentz force arises due to the electric field which separates the nucleus and the electron cloud from their equilibrium positions in opposite directions. Therefore, due to the Lorentz force the electron cloud and the nucleus move in opposite directions and they are separated by a distance x , where there is a formation of electrical dipole in the atom as shown in Fig.

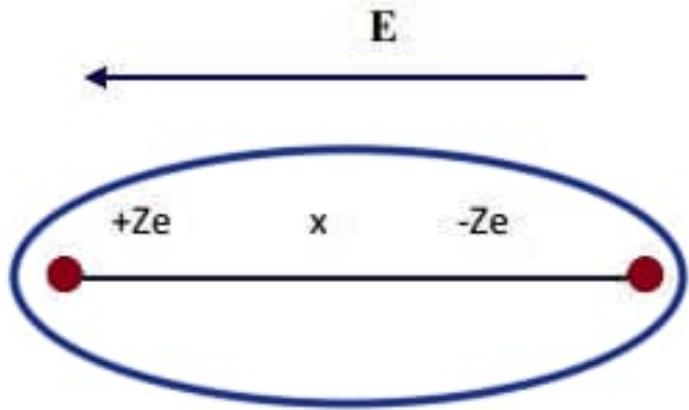


Figure 91: Atom with electric field ($E \neq 0$).

- Coulomb attractive force arises between the nucleus and the electron cloud after separation which tries to maintain the original equilibrium position.

Lorentz force (F_L) between the nucleus and the electron

$$\begin{aligned} F_L &= \text{charge} \times \text{Electric field} \\ &= -ZeE \end{aligned} \quad (164)$$

Coulomb attractive force (F_C) between the nucleus and the electron cloud being separated at a distance x is

$$F_C = \frac{1}{4\pi\epsilon_0} \left(\frac{\text{Charge of nucleus} \times Q'}{x^2} \right) \quad (165)$$

Q' = The total negative charge enclosed in the sphere of radius x

Q' = Charge density of electrons (ρ) \times Volume of the sphere of radius x

$$Q' = -\frac{3Ze}{4\pi R^3} \times \frac{4\pi x^3}{3}$$

$$Q' = -\frac{Zex^3}{R^3} \quad (166)$$

Substituting the value of Q' , the Coulomb force becomes

$$\begin{aligned} F_C &= \frac{1}{4\pi\epsilon_0} \times \left(\frac{Ze \times -\frac{Zex^3}{R^3}}{x^2} \right) \\ &= -\frac{Z^2 e^2 x}{4\pi\epsilon_0 R^3} \end{aligned} \quad (167)$$

At equilibrium, the Coulomb force and Lorentz force are equal. Hence,

$$\begin{aligned} F_L &= F_C \\ -ZeE &= -\frac{Z^2e^2x}{4\pi\epsilon_0 R^3} \\ x &= \frac{4\pi\epsilon_0 R^3 E}{Ze} \end{aligned} \quad (168)$$

The induced dipole moment μ_e is the product of magnitude of charge (Ze) and the separation between the charges (x)

$$\mu_e = Ze \cdot x \quad (169)$$

Now, after substituting x , we will get

$$\begin{aligned} \mu_e &= Ze \times \frac{4\pi\epsilon_0 R^3 E}{Ze} \\ \mu_e &= 4\pi\epsilon_0 R^3 E \\ \mu_e &= \alpha_e E \end{aligned} \quad (170)$$

where $\alpha_e = 4\pi\epsilon_0 R^3$ is called electronic polarizability, which depends on the radius of the atom (R).

Question(143): Explain the main factors which cause the properties of nanomaterials to differ significantly from other materials.

Solution: Two main factors cause the properties of nanomaterials to differ significantly from other materials (i) increase in surface area to volume ratio, and (ii) quantum confinement. These factors can change or enhance the properties such as reactivity, strength and electrical characteristics.

(i) Increase in surface area to volume ratio:

- Nanomaterials have a relatively larger surface area when compared to the larger form of the material of same volume. Let us consider a sphere of radius 'r'.

$$\text{Its surface area} = 4\pi r^2$$

$$\text{Its volume} = \frac{4}{3}\pi r^3$$

$$\begin{aligned} \text{Surface area to its volume ratio} &= \frac{4\pi r^2}{\frac{4}{3}\pi r^3} \\ &= \frac{3}{r} \end{aligned}$$

- Thus, when the radius of the sphere decreases, its surface area to volume ratio increases.
- Let us consider another example. For a one cubic volume shown in fig.
- The surface area is $6m^2$.
- When it is divided in to eight pieces its surface area becomes $12m^2$.
- When the same volume is divided in to 27 pieces its surface area becomes $18m^2$.
- Thus we find that when the given volume is divided in two smaller pieces, the surface area increases.
- Hence as particle size decreases a greater proportion of atoms are found at the surface compared to those in-sides.
- For example, a particle of size 30 nm has 5% of atoms its surface, at 10 nm 20% of its atoms, and at 3 nm has 50% of its atoms.

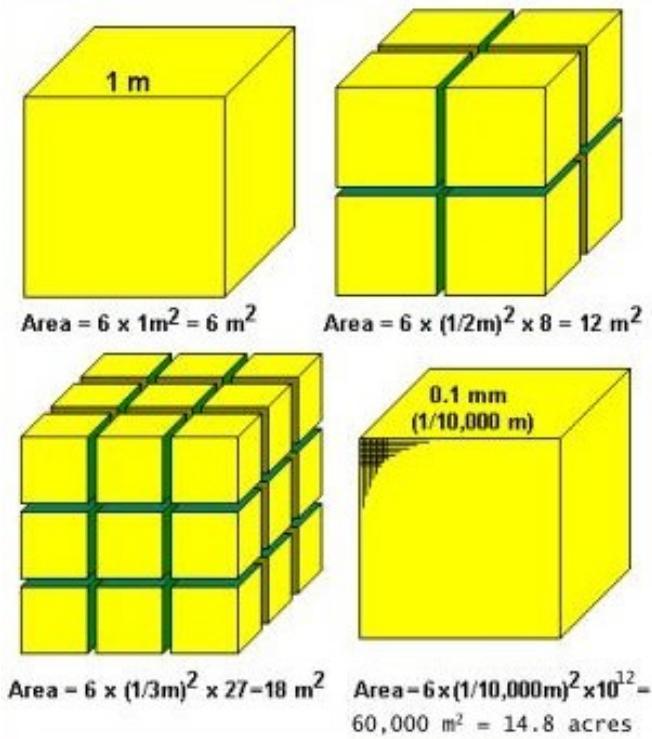


Figure 92: When the given object divided into smaller particles, the surface area increases.

(ii) Quantum Confinement:

- Quantum confinement effects describe electrons in terms of energy levels, potential wells, valence bands, conduction bands, and electron energy band gaps. The quantum confinement effect is observed when the size of the particle is too small to be comparable to the wavelength of the electron. Obviously, the confinement of an electron and hole in nanocrystals significantly depends on the material properties.
- The properties of materials can be studied based on their energy levels.
- When atoms are isolated, their energy levels are discrete. When very large number of atoms closely packed to form a solid, the energy levels split and form bands.
- This affects the optical, electrical and magnetic properties of nanomaterials.

Question(144): Illustrate Top-down Ball milling approach for the preparation of nanomaterials.

Solution: It is a ball milling process where a powder mixture placed in the ball mill is subjected to high-energy collision from the balls. It was found that this method, termed mechanical alloying, could successfully produce fine, uniform dispersions of oxide particles.

- The ball milling method is a typical example of top-down method of fabrication of nano materials.
- A ball mill is a type of grinder used to grind materials into extremely fine powder in order of nm.
- In this method, small balls of the material are allowed to rotate around the inside of a drum and then fall on a solid with gravity force.
- The balls are broken into nano crystallites. This is also known as mechanical crushing.
- This method is used to prepare a wide range of elemental powders. For example iron nano particles of sizes 13 to 30nm can be formed.
- This method is used for produce metal oxide nano crystals (cerium oxide CeO_2 and Zinc oxide ZnO_2).

The figure below shows the motions of the balls and the powder. Since the rotation directions of the bowl and turn disc are opposite, the centrifugal forces are alternately synchronized. Thus friction resulted from the hardened milling balls and the powder mixture being ground alternately rolling on the inner wall of the bowl and striking the opposite wall. The impact energy of the milling balls in the normal direction attains a value of up to 40 times higher than that due to gravitational acceleration. Hence, the planetary ball mill can be used for high-speed milling.

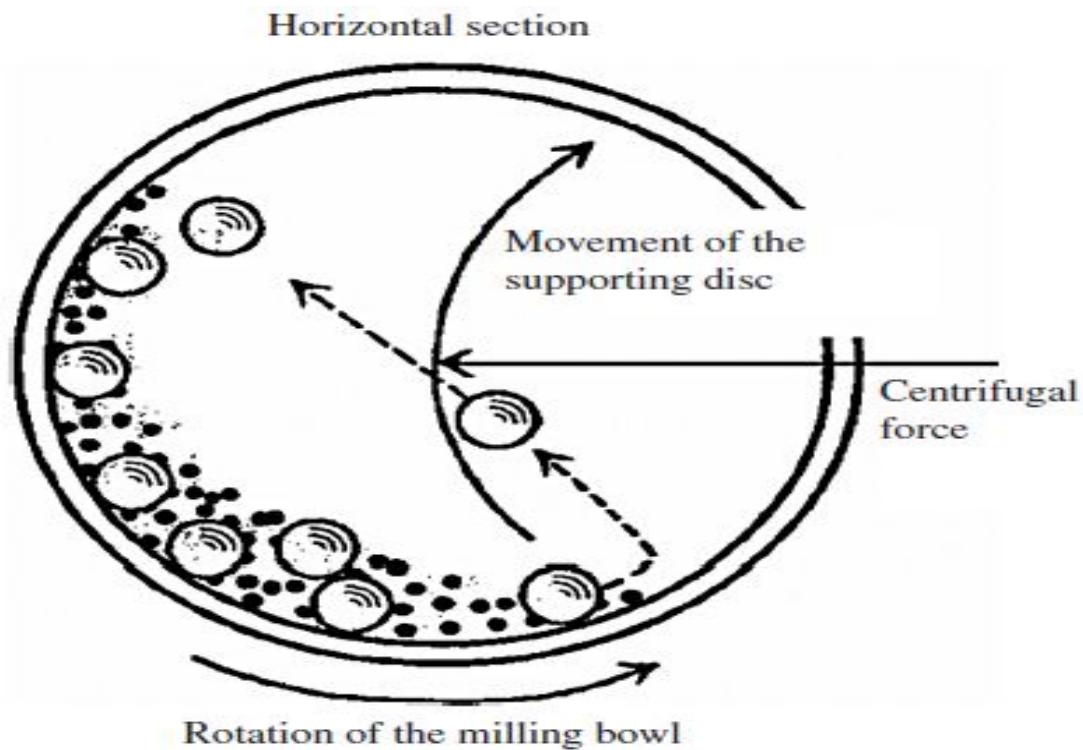


Figure 93: Schematic view of motion of the ball and powder mixture.

Question(145): Explain Sol-gel Bottom-up approach for the preparation of nanomaterials.

Solution: Sol gel methods are based on the principle of transition of materials from the solution or sol phase to the gelatinous or gel phase. It is a process of oxide network formation via polycondensation reaction in the solution phase. Sol-gel process, one of the bottom-up approaches, has been in use for many years for producing metal oxide and ceramic powders with high purity and high homogeneity.

- A sol is a solution with particles suspended in it.

- When particles in the sol form long polymers (chains) that span the entire sol, a gel is formed.
- In this process, the starting material is processed to form a dispersible oxide and a colloidal suspension (sol) of the particles of the metal compound is prepared first and then converted into a gel.
- The gel so formed is network in continuous liquid phase.
- Removal of the liquid from the sol yields the gel, and the sol/gel transition controls the particle and shape.
- Calcination of the gel produces the oxide.
- The sol-gel formation occurs in four stages
 - (i) Hydrolysis
 - (ii) Condensation
 - (iii) Growth of particles
 - (iv) Agglomeration of particles
- Production of SiO_2 is an example of this process.
- The sol gel method is an interesting cheap and low temperature technique which is used to produce a range of nanoparticles with controlled chemical composition.
- The sol-gel process may be summarized in fig.

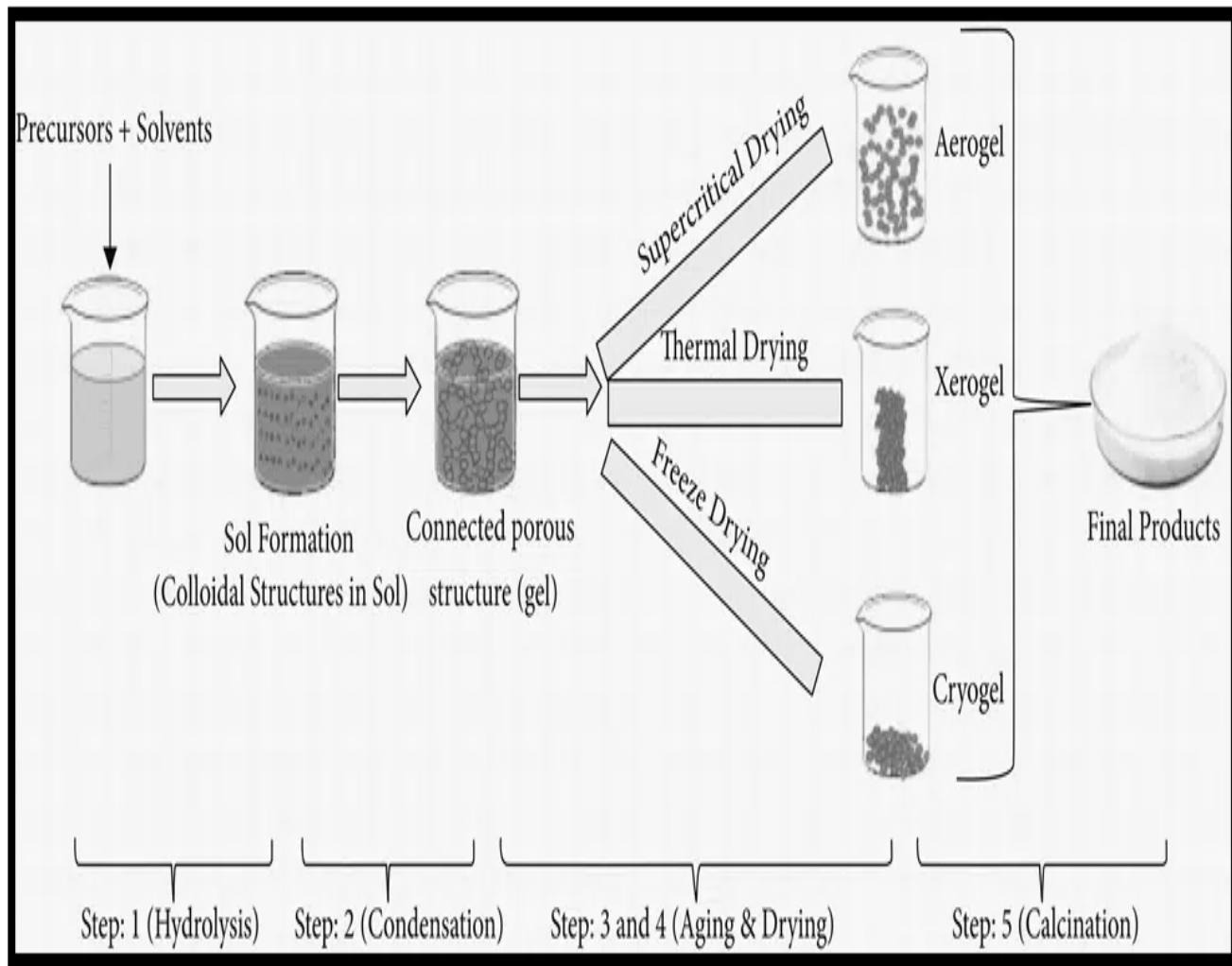


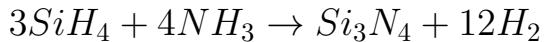
Figure 94: Schematic representation of sol-gel process of synthesis of nanomaterials.

Question(146): Describe the CVD Bottom-up approach for the preparation of nanoparticles.

Solution: Chemical vapor deposition (CVD), a bottom-up approach, is one of the most frequently used methods for the synthesis of thin films and nanoparticles. It is a process where one or more gaseous adsorption species react or decompose on a hot surface to form stable solid products.

- In Chemical vapor deposition (CVD), a thin layer of material is deposited on a substrate by supplying a mixture of reacting gas (volatile compounds such as SiH_4 , $SiCl_4$, WF_6 , etc.) and carrier gas (H_2 , Ar , etc.). For instance, thin film of Si_3N_4 can be deposited using SiH_4 , NH_3 , and Ar .

as:



- Chemical Vapor deposition process involves the following steps:
 - (i) Transport of reacting gaseous species to the surface
 - (ii) Adsorption of the species on the surface
 - (iii) Heterogenous surface-catalyzed reactions
 - (iv) Surface diffusion of the species to growth sites
 - (v) Nucleation and growth of the film
 - (vi) Desorption of gaseous reaction products and transportation of reaction products away from the surface
- In chemical vapor deposition, the vapor generated from a precursor is thermally broken down or reacts with another precursor vapor or gas that is present inside the evaporation apparatus. Nucleation, condensation, and coagulation of the precursor particles generate solid-state particles.
- Particles size obtained from CVD can range from molecular size to hundreds of micrometers. Some of the advantages of gas to particles conversion method include small particle size (a few nm to mm), narrow size distribution, high purity of the particles, etc.
- CVD has been frequently used to grow carbon nanotubes, GaN nanowires, and so on.

Question(147): Explain how oxide nanoparticles can be obtained by Bottom-up Sol-gel method.

Solution: Sol gel methods are based on the principle of transition of materials from the solution or sol phase to the gelatinous or gel phase. It is a process of oxide network formation via polycondensation reaction in the solution phase. Sol-gel process, one of the bottom-up approaches, has been in use for many years for producing metal oxide and ceramic powders with high purity and high homogeneity.

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- When particles in the sol form long polymers (chains) that span the entire sol, a gel is formed.
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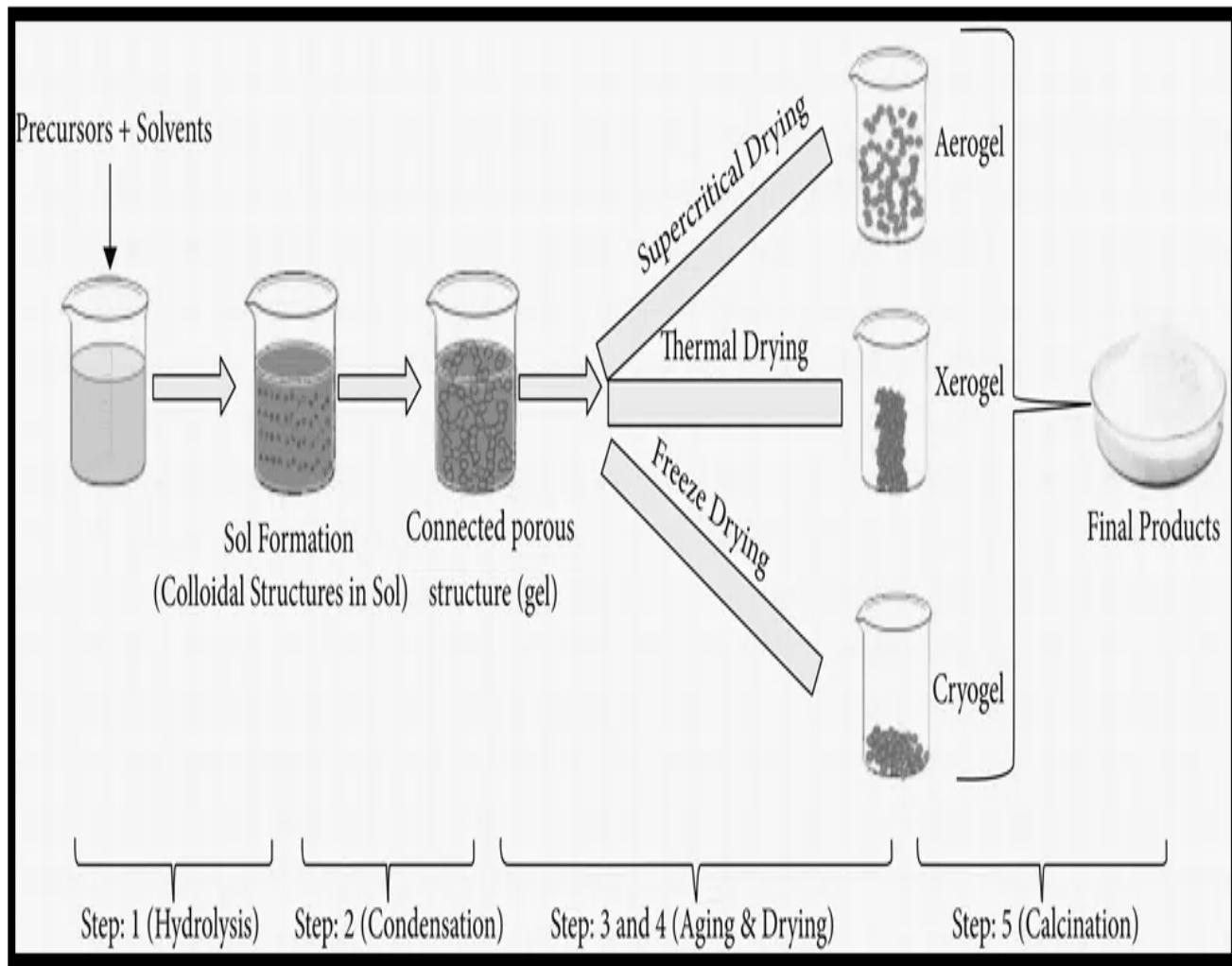


Figure 95: Schematic representation of sol-gel process of synthesis of nanomaterials.

Question(148): Explain Ball milling Top-down approach for the preparation of nanomaterials.

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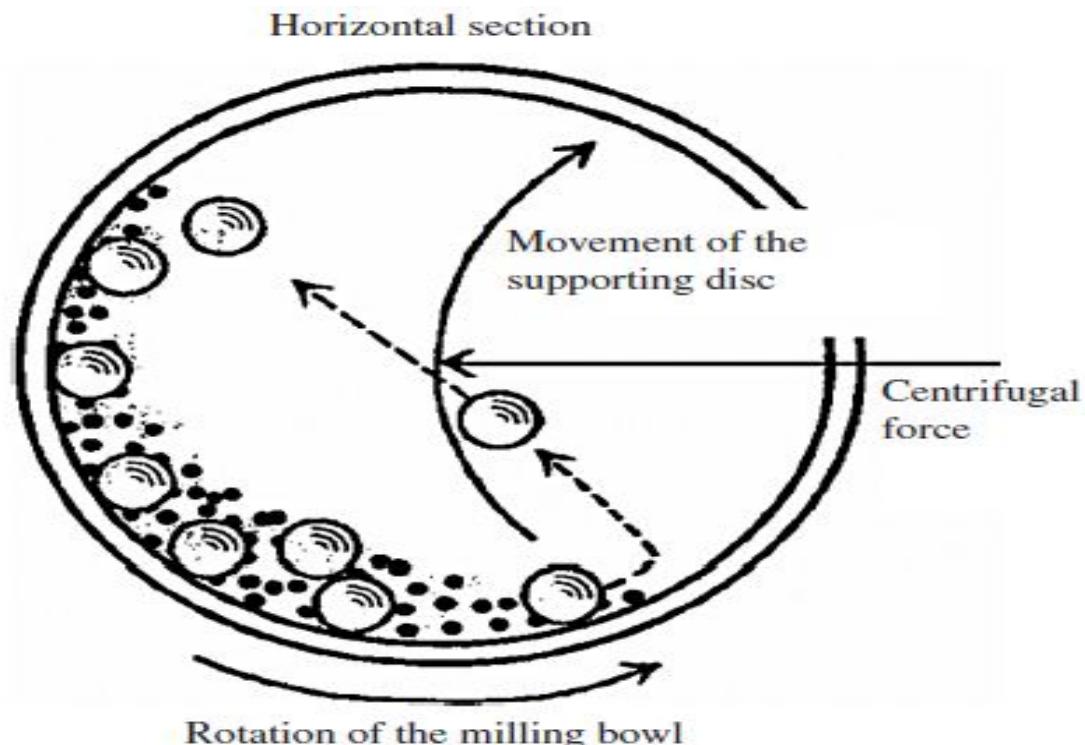


Figure 96: Schematic view of motion of the ball and powder mixture.

Question(149): Discuss the various physical, electronic, magnetic, mechanical and chemical properties of nanomaterials.

Solution: As the size of the nanoparticle is very less, these materials exhibits peculiar physical, electronic, magnetic, mechanical and chemical properties.

Physical Properties:

- Since the size of the particle is very less, the particles are very close to each other and hence the inter particle spacing is very less in nanomaterials.
- Because of its very less size, these nanomaterials cannot be further divided into smaller particles and it does not have any dislocation in it. Thus we can say that they have high strength, hardness and toughness.
- The melting point of nanomaterials will be very less.

Electronic properties:

- The energy bands in these materials will be very narrow.
- The electrical conductivity and energy band width of same materials change when they pass from bulk phase to nanoparticle phase.
- For example, bulk silicon is an insulator; it becomes a conductor in nanophase.
- Nanomaterial semiconductors such as Ge and Si do not show semi conducting property.

Magnetic properties:

- The nano materials shows variation in their magnetic property, when they change from bulk state to nano phase state. Some of the examples are listed in table:

Material	Bulk State	Nano-phase state
Iron, Ni, Cobalt	Ferro-magnetic	Super Para-magnetic
Sodium, Potassium	Para-magnetic	Ferro magnetic
Chromium	Anti-ferromagnetic	Para-magnetic

Mechanical properties:

- Mechanical properties such as hardness, toughness, elastic modulus varies from material to material.
- They exhibit super plastic behaviour.

Chemical properties:

- The nano structures in chemistry are collides and these are formed in a condensed phase having size in the range from 1 to 100 nm.
- The chemical reactivity of a cluster depends on the cluster size.
- This is useful in the preparation of catalytic agents. Some chemically inert bulk materials become good chemical catalyst in the nano phase, for example: Platinum and gold.
- In chemical reactions, the rate of increase in mass transport increases as the particle size decreases.

Question(150): Write a note on nanomaterials and discuss the applications of nanotechnology in various fields.

Solution:

Nanomaterials: Nanomaterials are materials with structures and properties significantly different from those of bulk materials due to their small size. These materials have dimensions in the nanoscale range, usually between 1 and 100 nanometers, and exhibit unique physical, chemical, and biological properties. They are widely used in various fields due to their high surface area, remarkable biocompatibility, optical and electrical properties. Some examples of nanomaterials include nanoparticles, nanofibers, nanotubes, carbon, and quantum dots. The 4 main types of nanomaterials:

- **Nanoparticles:** These are particles with at least one dimension measuring less than 100 nanometers.
- **Nanofibers:** These are long, thin fibers with diameters in the nanoscale range.
- **Nanotubes:** These are tubes with walls made of a single layer of atoms with diameters in the nanoscale range.
- **Nanolaminates:** These are thin layers of different materials stacked on top of each other to form a composite material.

Applications of nanotechnology: The following are the applications of nanotechnology in various fields.

(i) Mechanical Engineering:

- Since they are stronger, lighter etc; they are used to make hard metals.
- Nano-MEMS (Micro-Electro Mechanical Systems) are used in ICS, optical switches, pressure sensors, mass sensors etc.

(ii) Electrical, Electronics and Communication Engineering:

- Orderly assembled nano materials are used as quantum electronic devices and photonic crystals.
- Some of the nano materials are used as sensing elements. Especially the molecular nano materials are used to design the robots, assemblers etc.
- They are used in energy storage devices such as hydrogen storage devices, magnetic refrigeration and in ionic batteries.
- Recently nano robots were designed which are used to remove the damaged cancer cells and also to modify the neuron network in human body.

(iii) Computer Science Engineering and IT:

- Nano materials are used to make CD's and Semiconductor laser.
- These materials are used to store the information in smaller chips.
- They are used in mobiles, laptops etc.
- Nano-dimensional photonic crystals and quantum electronic devices plays a vital role in the recently developed computers.

(iv) Bio-Medical and Chemical Engineering:

- Consolidated state nano particles are used as catalyst, electrodes in solar and fuel cells.
- Bio-sensitive nano particles are used in the production of DNA-chips, bio-sensors etc.
- Nano-structured ceramic materials are used in synthetic bones.
- Few nano materials are also used in adsorbents, self-cleaning glass, fuel additives, drugs, Ferro fluids, paints etc.