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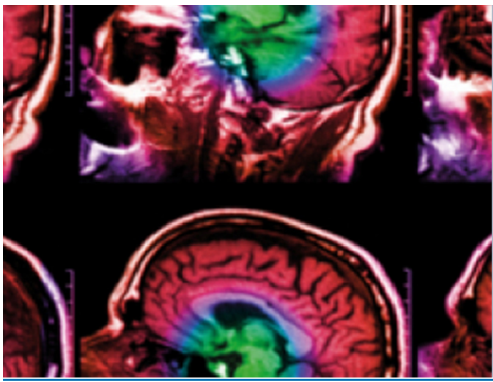

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Intensity Distribution of Focussed Laser Beams in Bio-medical Studies

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ABSTRACT. Universal numerical solutions for the intensity of laser beams focussed by simple lenses having various amounts of primary spherical aberration have been obtained and are presented graphically in the form of lines of constant intensity (isophotes). The solutions demonstrate the increasingly non-uniform nature of the isophotes as the lens aberration function is increased. The existence of a multiplicity of intensity maxima, both along the optic axis and in off-axis positions, extending over comparatively large distances ($\sim$ mm) from the lens Gaussian focus, is disclosed. These must be taken into account in any quantitative assessment of the effects of laser beam radiation upon biological material.

1. Introduction

Laser beams are finding increasing application not only as therapeutic devices but as an essential part of the equipment used in a wide variety of medical, dental and biological studies. In much of this work, the laser beams are focussed in order to make use of the very large energy and power densities which can be achieved over quite small pre-selected areas.

The ability to focus laser radiation into localized regions enables it to be used for the treatment of retinal tears, for cauterization and bloodless surgery and for massive tissue destruction (Goldman and Rockwell 1967). The concentrated power of focussed beams has been used recently in studies of the nature of tooth enamel and studies are now in progress on the use of laser beams as dental drills (Varner, Jarabak, Phatak and Kinersly 1967, Nixon 1968). In biological investigations, advantage may be taken of the very large local electric fields ($\sim 10^7$ v cm^{-1}) created by the focussed Q-switched laser beam to produce large potential differences ($\sim$several hundred volts) across transparent biological specimens and thus separate electrical from thermal effects. A focussed beam may also be used to destroy individual cells or puncture cell-walls on a micro-scale in genetic studies (Peppers 1965, Elion 1967).

In all these applications, the interaction between the laser beam and the target material proceeds initially via the influence of the intense laser-produced electromagnetic field upon electrons and ions in the material. These charged particles gain energy from the optical-frequency field by making electron–molecule and ion–molecule collisions which can result in excitation and ionization. The energy acquired by the electrons and ions is subsequently degraded thermally and is transmitted to neighbouring material as a result of molecule–molecule collisions, thus giving rise to macroscopically observable changes. There may be several intermediate processes in the case of pulsed focussed radiation including, for example, shock-wave generation, re-radiation, and absorption, before the conduction of heat plays the final role in producing observable biological changes.

In order to make a quantitative assessment of the effects created by laser beams it is necessary to know the radiation wavelength, the duration of irradiation, the energy absorbed, and, especially in micro-biological studies, the intensity of the electromagnetic field and its spatial distribution. This paper shows that the distribution is complex; the spatial variation of energy in the laser beam profile and interference effects produced by the focussing system cause substantial changes in the positions of regions of maximum intensity. In addition, several axial and off-axis intensity maxima arise and the radiation pattern is found to vary markedly over surprisingly large distances of the order of millimetres. These effects may have important consequences in the techniques used in bio-medical applications of focussed laser beams. It is thus of interest to examine in detail the intensity distribution obtained with lenses of prescribed aberration functions which may be used in many medical and biological investigations.

2. Theory

The intensity distribution in the region of the Gaussian (i.e. geometrical) image point of a perfect, aberration-free lens is described by the diffraction pattern of the converging spherical wavefront produced by the lens aperture. When the lens is not perfect, the wavefront is distorted from the perfectly spherical shape, i.e. it undergoes aberration to some arbitrary shape. The deformation of the wavefront by the exit pupil can be described in terms of an aberration function Φ (Born and Wolf 1964). This is defined as the number of wavelengths displacement between the perfect undistorted spherical wavefront and the distorted wavefront at any point on the exit pupil. For primary spherical aberration, the maximum value of the aberration function, Φ_{max}, is the greatest distance between the distorted and undistorted waves and is a maximum at the boundary of the exit pupil. For a given aberration function, the amplitude of the optical disturbance in the converging wave is conveniently expressed by introducing polar co-ordinates (r, θ, z) and by defining the so-called optical co-ordinates u and v, given by

$$u = \frac{2\pi}{\lambda}\left(\frac{a}{f}\right)^2 z \quad \text{and} \quad v = \frac{2\pi}{\lambda}\frac{a}{f}\, r, \tag{1}$$

where a is the laser beam radius at the lens, f is the geometrical focal length of the lens, and z and r represent distances measured from the Gaussian focus in the axial and radial directions respectively.

The wave amplitude in u, v space is represented, in accordance with the Fresnel–Kirchoff development of the Huygens' principle, as a double integral over the surface of the focussing lens exit pupil. Thus, the amplitude at any point $\mathrm{M}(u, v)$ in the neighbourhood of the Gaussian focus is given by

$$U(\mathrm{M}) = -\frac{i}{\lambda}\frac{a^2}{f^2}\exp\left\{i\left(\frac{f}{a}\right)^2 u\right\} . \int_0^1\int_0^{2\pi} A(\rho) \exp\left[i\{\mathrm{k}\Phi(\rho) - v\rho\cos\theta - \tfrac{1}{2}u\rho^2\}\right]\rho\, \mathrm{d}\rho\, \mathrm{d}\theta, \tag{2}$$

where $A(\rho)$ is the wave amplitude across the exit pupil of radius a, and $\mathrm{k}=2\pi/\lambda$ is the wave number. The coefficient ρ, introduced to simplify notation, is the ratio (r/a).

The intensity distribution is obtained by taking the square of the modulus of $U(\mathrm{M})$, i.e.

$$I(\mathrm{M})=\left\{\frac{a^2}{f^2}\right\}^2\left|\int_0^1\int_0^{2\pi} A(\rho)\exp\left[i\{\mathrm{k}\Phi(\rho)-v\rho\cos\theta-\tfrac{1}{2}u\rho^2\}\right]\rho\,\mathrm{d}\rho\,\mathrm{d}\theta\right|^2, \tag{3}$$

and the absolute value of the intensity expressed in w cm^{-2} is given by

$$I(\mathrm{M})=\frac{1{\cdot}66Pa^2}{\pi\lambda^2f^2}\left|\int_0^1\int_0^{2\pi}\exp\left(-\frac{\rho^2}{2}\right).\exp\left[i\left\{\mathrm{k}\Phi(\rho)-v\rho\cos\theta-\frac{u\rho^2}{2}\right\}\right]\rho\,\mathrm{d}\rho\,\mathrm{d}\theta\right|^2, \tag{4}$$

when the beam profile is Gaussian, i.e. when

$$A(\rho)=\frac{P^{1/2}f}{a\sqrt{(\pi)}(1-\mathrm{e}^{-1})^{1/2}}\;.\exp\left(-\frac{\rho^2}{2}\right)=A_0\exp\left(-\frac{\rho^2}{2}\right).$$

In this expression P is the laser beam power in watts, and is, of course, a measurable quantity.

In the absence of aberrations, i.e. for very well corrected lenses, the intensity distribution will have a maximum at the Gaussian image point, and is symmetrical on either side of the focal plane and optic axis (Zernike and Nijboer 1949, Linfoot and Wolf 1956, Taylor and Thompson 1958). However, in the presence of aberrations, this will not be the case; the point of maximum intensity is displaced towards the lens, and is then called the diffraction focus.

For lenses with small aberrations† (less than one wavelength) and for uniform plane waves ($A(\rho)=\text{constant}$), the integral has been solved analytically by Zernike and Nijboer (1949) by expanding the integrand as a power series and neglecting all but the first few terms in the expansion. Their method is not readily applicable to determine the intensity distribution when the lens aberrations are not small compared to a wavelength, since the integrand oscillates very rapidly. In many studies using relatively short focal length simple lenses, there will be large amounts of spherical aberration present. It is thus necessary to obtain numerical solutions of the type developed for uniform plane waves by Marechal (see Linfoot 1955) but taking into account the spatial variation in amplitude at the exit pupil.

For a simple lens of focal length f, having only primary spherical aberration, the aberration function is given by the following expression (Born and Wolf 1964):

$$\Phi(\rho)=-\frac{1}{4}(a\rho)^4\left[\frac{n^2}{8f^3(n-1)^2}-\frac{n}{2(n+2)}.\frac{\gamma^2}{f}+\frac{2(n+1)^2}{n(n+2)}.\frac{\gamma^2}{f}\right], \tag{5}$$

† We note that a good microscope objective lens has an aberration function of the order of half a wavelength.

where $\gamma = \frac{1}{2}f$ for plane waves and n is the refractive index of the lens material at the laser radiation wavelength λ. Table 1 gives typical combinations of focal lengths f of simple biconvex lenses of refractive index 1·5 used with single mode laser beams of radius a which correspond to the aberration functions used in the numerical analysis.

Table 1. Typical combinations of focal length f and beam radius a corresponding to aberration functions used

Φ_{max}	a cm	f cm	a cm	f cm	a cm	f cm
$0{\cdot}5\lambda$	0·25	4	0·5	10	1	25
$1{\cdot}8\lambda$	0·25	2·5	0·5	6	1	15
6λ	0·25	1·6	0·5	4	1	10

3. Results

Assuming a Gaussian beam profile at the exit pupil, i.e. $A(\rho) = A_0 \exp(-\rho^2/2)$, a computer program using eqns. (4) and (5) was prepared and used to determine the intensity distribution for the following cases of focussed single mode laser radiation: (1) $\Phi_{max} = 0$, no primary spherical aberration, (2) $\Phi_{max} = 0{\cdot}5\lambda$, (3) $\Phi_{max} = 1{\cdot}8\lambda$, (4) $\Phi_{max} = 6\lambda$.

The method of numerical integration used in the program was the well-known trapezium rule. The mesh size was chosen so that convergence of the integral was ensured to 1 part in 10^4 at successive steps.

The corresponding results of the numerical analyses using an I.C.T. 1905 and the CERN, CDC 6600 computers, are shown in figs. 1–4, in which the lines of constant intensity (isophotes), normalized to 100 at the first diffraction maximum, are drawn. The optic axis of the beam coincides with the u axis. These are universal curves in image space and the data can readily be converted into real space using eqn. (1).

To obtain the absolute value of the intensity at any point in real space, it is necessary to multiply the absolute value of the square of the modulus of the integral in eqn. (4) by $1{\cdot}66Pa^2/\pi\lambda^2 f^2$ and by the relative intensity which is quoted on each isophote. For convenience, the value of the square of the modulus of the integral at the 100% isophote is given in the legend of the figures.

Fig. 1 is similar to the results obtained by Linfoot and Wolf. It should be noted that the Gaussian focus and the principal diffraction focus coincide. The other diffraction maxima lie on the optic axis on either side of the Gaussian focus and are very weak. The first zero of intensity in the focal plane coincides with that given by the Airy formula and the isophotes are symmetrical about the focal plane.

Fig. 2 demonstrates the influence of a very small amount ($\Phi_{max} = 0{\cdot}5\lambda$) of spherical aberration. The principal diffraction focus no longer coincides with

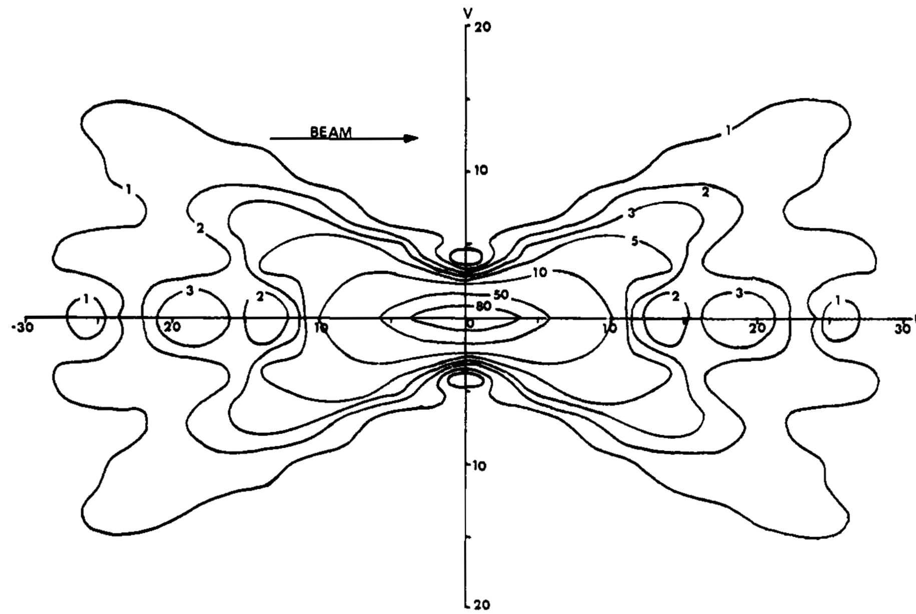

Fig. 1. Isophotes in the neighbourhood of the Gaussian focus of an aberration-free lens. The intensity is normalized to 100 at the focus. The absolute value of the square of the modulus of the integral is 6·11.

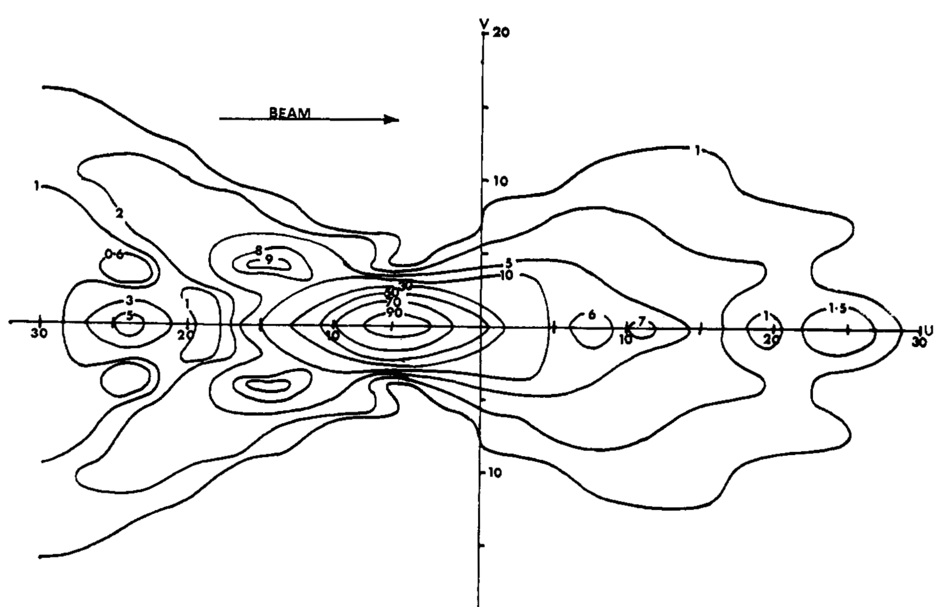

Fig. 2. Isophotes for a lens with aberration function $\Phi = 0{\cdot}5\lambda\rho^4$. The absolute value of the square of the modulus of the integral is 5·81.

the Gaussian focus: it is displaced towards the lens by about five units in u-space. In addition, appreciably large off-axis diffraction maxima appear and the isophotes are no longer symmetrical about the focal plane. The absolute

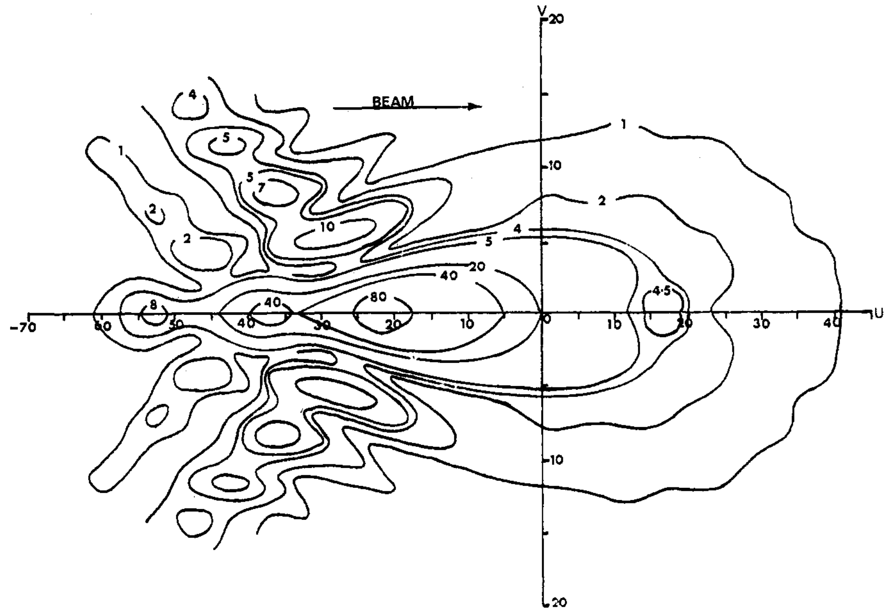

Fig. 3. Isophotes for lens with aberration function $\Phi = 1{\cdot}8\lambda\rho^4$. The absolute value of the square of the modulus of the integral is 2·54.

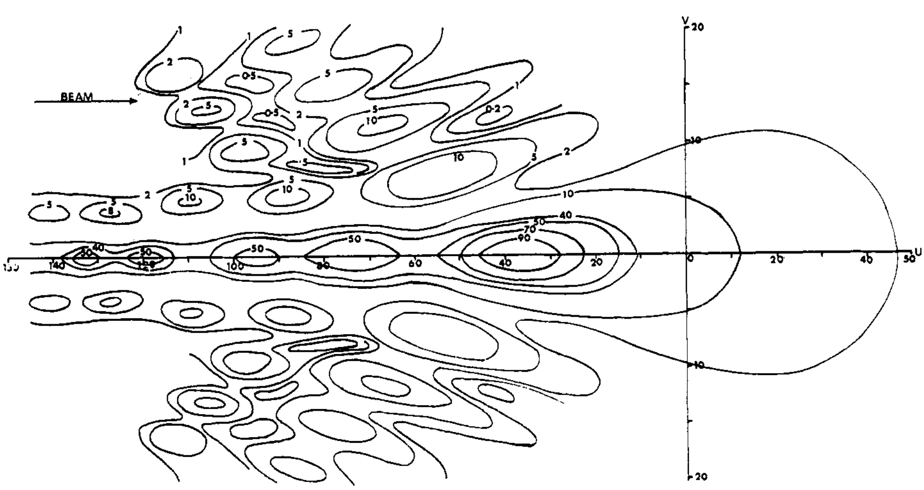

Fig. 4. Isophotes for a lens with aberration function $\Phi = 6\lambda\rho^4$. The absolute value of the square of the modulus of the integral is 0·83.

value of the power density at the principal diffraction focus, for a 25 cm lens and 100 Mw, 1 cm radius ruby laser beam is $\sim 8 \times 10^{13}$ w cm^{-2}. With a 4 cm lens and a 0·25 cm radius beam, the corresponding power density is $\sim 2{\cdot}5 \times 10^{14}$ w cm^{-2}.

Fig. 3, for the case of $\Phi_{max} = 1{\cdot}8\lambda$, shows a very marked departure from symmetry about the focal plane and still further displacement of the axial maxima towards the focussing lens. The intensity of the axial and off-axis maxima has increased considerably. The absolute intensity at the principal diffraction focus for a 15 cm lens and 100 Mw, 1 cm beam is $\sim 8 \times 10^{13}$ w cm^{-2}.

Finally, fig. 4, for the case of $\Phi_{max} = 6\lambda$, shows how highly distorted the intensity distribution has become even though the spherical aberration is still relatively small.

Intense axial maxima are now found extending well beyond 150 units in u-space. This corresponds to a distance of a few mm from the Gaussian focus for a biconvex 10 cm focal length lens. Evidence for the existence of these maxima has been demonstrated experimentally in studies of laser induced breakdown in gases (Bleeker and Grey Morgan 1965, Evans and Grey Morgan 1968).

4. Conclusions

The isophotes shown in figs. 1–4 demonstrate the important role of primary spherical aberration in determining the spatial intensity distribution of focussed laser beams, and thus the distribution of the radiation delivered to a given target. The isophotes enable the absolute value of the power density and thus the value of the electric field at any point, to be computed. They show that when highly concentrated doses are required, lenses having very small aberration functions must be used. In this case, the irradiated area is well defined, since the cross-section of target material in which the beam intensity exceeds a given value is sharply located. Irradiation is symmetrical about the optic axis and secondary axial and off-axis maxima are either non-existent or negligibly small. This state of affairs is best achieved with simple lenses by using narrow laser beams. Ideally, of course, one would use very well corrected lenses but these are very expensive and may not withstand very high power laser radiation.

In contrast, with lenses displaying only modest spherical aberration, ($\Phi_{max} = 6\lambda$), the irradiation pattern is highly asymmetrical and complicated by the depth over which intense secondary maxima can arise. These can easily extend over several mm along the optic axis. The cross-section of target material in which the intensity is in excess of say 10% of the principal maximum is more than double that for a very well corrected lens. It follows that simple short focal length lenses can be used only when relatively large areas are to be irradiated.

In addition to specifying the type and duration of radiation, it is also necessary to specify the lens aberration when comparing the results of various investigations: the isophotes given in figs. 1 to 4 then enable a more quantitative assessment of the effects of irradiation to be made. The aberration function of a particular lens can be evaluated using a Twyman–Green interferometric method (Kingslake 1927).

We wish to thank the Science Research Council for a grant in support of the study of the interaction of radiation with matter and for the award of a Studentship to one of us (L.R.E.). We also wish to thank Dr. C. J. Evans for helpful discussions on computational methods.

Résumé

Distribution d'intensité de faisceaux de laser focalisés dans les études bio-médicales

On a obtenu des solutions universelles numériques pour l'intensité de faisceaux de laser, focalisés au moyen de lentilles simples ayant des valeurs différentes d'aberration sphérique primaire, et ces solutions sont présentées graphiquement comme lignes d'intensité constante (isophotes). Les solutions démontrent la nature de plus en plus non-uniforme de ces isophotes quand la fonction d'aberration de la lentille augmente. On découvre l'existance d'une multiplicité de maximums d'intensité tant le long de l'axe optique qu'en positions hors de l'axe, qui s'étend sur des distances relativement grandes ($\sim$mm) du foyer Gauss de la lentille. Ces maximums doivent être pris en considéretion dans n'importe quelle évaluation quantitative des effets de la radiation de faisceaux laser sur un matériel biologique.

Zusammenfassung

Intensitätsverteilung fokussierter Laserstrahlen in biomedizinischen Untersuchungen

Es sind allgemeine numerische Lösungen für die Intensität der mittels einfacher Linsen mit verschiedenen Betragen der primären sphärischen Aberration fokussierten Laserstrahlen erhalten und graphisch als Linien konstanter Intensität dargestellt worden. Die Lösungen zeigen die mit der Linsen-Aberrationsfunktion zusammen anwachsende ungleichförmige Beschaffenheit der Isophoten. Es wird die Existenz einer Multiplizität der Intensitätsmaxima sowohl entlang der optischen Achse als auch in Lagen ausserhalb der Achse gezeigt, wobei sich diese Multiplizität über verhältnismässig grosse Abstände ($\sim$mm) von dem Gauss'schen Linsenbrennpunkt erstreckt. Diese Maxima müssen in einer beliebigen Schätzung der Effekte der Laserstrahlen auf biologisches Material berücksichtigt werden.

Резюме

Распределение интенсивности фокусированных лазерных пучков в био-медицинских исследованиях

Универсальные числовые решения, полученные для интенсивности лазерных пучков, фокусированных простыми линзами с разными величинами первичной сферической аберрации, представлены в виде линий постоянной интенсивности (изофот). Решения указывают на увеличение неравномерной природы изофот по мере того, как функция аберрации линзы возрастает. Обнаруживается существование многочисленных максимумов интенсивности, как вдоль оптической оси, так и в положениях вдали от оси; эти максимумы простираются на сравнительно большие расстояния ($\sim$мм) от гауссовского фокуса линзы. Этот факт должен приниматься во внимание при любой количественной оценке эффектов лазерного излучения, падающего на биологический материал.

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