Microcontroller Realization of an Induction Motors Fault Detection Method based on FFT and Statistics of Fractional Moments

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***Abstract*— Induction Motors (IM) are the most widely used motors in industry. Although this type of machines is very reliable, different faults may occur. It is extremely important to detect faults at the earliest stages to reduce financial and energy losses and to overcome catastrophic accidents. Monitoring of mechanical vibrations and electrical currents are widely applied for that purpose. Recently, an effective method for fault detection and identification combining Fast Fourier Transform (FFT) and Statistics of Fractional Moments (SFM), thereby called FFT+SFM, was proposed by some of the authors. This paper shows how the FFT+SFM method can be implemented in microcontroller-based systems for real-time condition monitoring of IM. The method is optimized with respect to microcontroller features and hence it's implemented on a microcontroller. The optimized method is implemented by a C-code algorithm that is run on a microcontroller from the STM32F722 series by STMicroelectronics. The execution time and results of each algorithm iteration are measured to show the efficiency and effectiveness of the proposed algorithm. Results prove that the realization of the FFT+SFM method can be used in real-time condition monitoring systems of IM.**

I. Introduction

Induction motors (IMs) are widely spread in industry and consume nearly 40% of electric energy in the world [1]. However, due to their wide application, the faults problem solution of IMs is very important. Timely detection of the incipient faults overcomes expensive maintenance, plant shutdown, and even catastrophic accidents. The IMs fault consequences lead to essential economic losses. Therefore, IM faults should be detected as early as possible, possibly by automatic systems. Other important motivations of early fault detection are:

- A faulty IM has lower efficiency and performance and financial losses add to a waste of energy because of the 40% share of the world energy consumption.
- Maintenance of IM with small defects at early stages is much cheaper than maintenance with significant faults and full replacement of IM.

The mentioned motivations increase the actuality of reliable IM condition monitoring systems. Condition monitoring is the preferable IM maintenance strategy because it allows to use energy and motor components for the full lifetime, i.e., more effectively than by scheduled or corrective maintenance. Then, it is also the most profitable maintenance strategy [2]. Condition monitoring systems consist of small, low-power, cheap devices, which are frequently based on microcontrollers or low-cost processors, or even a Field Programmable Gate Array (FPGA). Thus, the developed signal processing techniques, which are applied in such systems, should be compact, effective, occupy little memory, and perform few computations.

IM faults can be divided into the following groups [3]:

- stator faults (winding open-phase or short-circuit of a few turns in a phase winding);
- rotor electrical faults (rotor winding open-phase for wound rotor machines and broken bar(s) or cracked end-ring for squirrel-cage machines);
- rotor mechanical faults (bearing damage, eccentricity, bent shaft, axis misalignment);
- other types of faults (load fault, gearbox fault, failure in a power supply system).

Although rotor faults are about 10% of all possible faults and this type of fault does not cause immediately an IM failure, this occurrence significantly degrades IM performance. Also, it can lead to consequences such as: rotor bar and core wear, bearing failures, high motor's vibration. Likewise, broken rotor bars (BRB) can lift out of a slot (especially copper bars) and dig into the stator, causing the motor failure.

Generally, IM fault detection techniques can be classified by the measured variables. Namely, it is possible to monitor vibrations [4], torques [5], fluxes [6], acoustic emissions [7], temperatures [8], and electric currents [9]. The most applied techniques in commercial solutions are vibrations and current monitoring systems. In this study, the focus is on current monitoring, which has been widely used and already employed for protection purposes. Moreover, another advantage of using current measurements for monitoring of the machine state is that this does not require direct access to the motor, which can be complicated in industrial applications.

However, from the industrial application point of view, signal processing techniques are attractive when imply low computational cost and memory usage, because, in these cases, the condition monitoring techniques can be implemented in low-cost embedded systems with limited performance. Then,

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our aim is to contribute with a novel signal processing technique that can be implemented at reduced cost but can however effectively detect faults in IM.

In this field, Motor Current Signature Analysis (MSCA) [3] is a well-known approach for fault detection. It is based on Fast Fourier Transform (FFT). In [10], the authors implemented MCSA in an embedded system for real time condition monitoring. However, MSCA has some significant limitations such as spectral leakage, significant effect of noise, requirements of a long acquisition time interval and information about motor's slip. In [11], the author suggests a multirate technique, which allows to overcome the spectrum leakage problem. But a long acquisition time interval is still required and problems with noise and slip dependence remain unsolved.

High resolution methods such as Multiple Signal Classification (MUSIC), root-MUSIC, Estimation of the Signal Parameters via Rotational Invariance Technique (ESPRIT) have also been developed for fault detection of IM [2], [9]. These methods require calculation of the autocorrelation matrix, which is an operation very computationally demanding.

Recently, some of the authors of this work have developed an approach that combines the Fast Fourier Transform and the so-called Statistics of Fractional Moments (SFM) – from which the acronym FFT+SFM – for fault detection of IM [12]. The proposed method allows to significantly reduce the acquisition time interval and is still able to detect broken rotor bars. It is mainly based on the properties of the General Mean Value (GMV) function, which was developed in the frame of the Statistics of Fractal Moments in [13]–[16].

This study shows how to properly implement the FFT+SFM approach onto a microcontroller by developing a suitable code. The aim is to check the effectiveness and efficiency of the approach by evaluating both the capability to distinguish faulty from healthy motors and the execution time of the detection algorithm. The aim is to show the possibility to use the FFT+SFM algorithm in low-cost microcontroller units (MCU), with Cortex-M core, for real time condition monitoring of IM. In particular, the FFT step of the approach is optimized compared to the original version of the FFT+SFM approach to use the well-known and reliable CMSIS-DSP library of the Cortex Microcontroller Software Interface Standard (CMSIS). This library provides a suitable function for obtaining FFT under strict memory and computational constraints of the MCU.

To verify the implementation of the IM condition monitoring system, stator current signals from a healthy IM and an IM with two broken rotor bars or an eccentricity fault were acquired. These signals were processed by the original FFT+SFM algorithm coded in a high-level language (Matlab) that can be easily executed on a PC and then by the MCU implementation. Results were compared and discussed. Also, to measure the execution time, the MCU implemented code was run on a real MCU from the STM32F722 series. The obtained results are shown.

The rest of the paper is organized as follows. Section II briefly describes the basics of the FFT+SFM fault detection approach. Section III describes the microcontroller implementation by specifying characteristics, special features, and implementation and communication issues of the microcontroller unit that is used for processing data and testing the detection method. Section IV shows the results of the microcontroller-based fault detection. The comparison between fault detection results using the original FFT+SFM approach in a high-level computer-based processing environment and fault detection results by the MCU implementation is shown and commented. Moreover, the results of the algorithm running on MCU are separately shown, including the measurement of the execution time of each algorithm iteration. Finally, Section 4 summarizes the results of this study and concludes the paper.

II. THE FFT+SFM METHOD

In [12], the FFT+SFM method was developed and tested for detection of broken rotor bars. Here below a synthetical description is given.

It is known that FFT determines a spectrum with an accuracy and resolution depending on the sampling frequency f_s (larger than the highest analyzed signal frequency) and the acquisition time interval T that is used to acquire measurements. The selection of T comes from a compromise between a high resolution (long T) and the demand for fast computation and reduced memory occupation (short T). The frequency resolution is $\Delta f = f_s/N = 1/T$, where N is the number of points after applying FFT. A poor resolution would create problems in identification of harmonics determined by faults.

To allow fault detection with very short T, the FFT+SFM method proposed a deep statistical analysis of the FFT spectra with poor resolution. Fractional order moments replace the classical integer order moments [14],[15]. One approach, as described in [14], could take the amplitudes $|y_1|, |y_2|, \ldots, |y_N|$ of the stator current spectrum (FFT provides complex values) and firstly normalize them with respect to $\|y\|_\infty$. Then, the normalized amplitudes (between 0 and 1) define a sequence that can be characterized by a normalized moment with fractional order mom_p:

$$\overline{\Delta}_N^{(mom_p)} = \sum_{i=1}^{N} \left(\frac{|y_i|}{\|y\|_\infty}\right)^{mom_p} \tag{1}$$

If the order is 0, then the moment coincides with N. If the order goes to infinity, then the moment goes to zero. By developments in [16], some orders are not important to detect statistical differences between sequences of data coming out from different measurements, such that the orders that are necessary and sufficient to detect these differences are computed by the following relation:

$$mom_p = e^{\left[min + \frac{p}{P}(max - min)\right]} \tag{2}$$

where $p = 0, 1, \ldots, P$, $min = -15$ and $max = +15$ are the limits in the logarithmic scale, $50 \leq P \leq 100$ is a resolution factor.

However, to reduce memory requirements, in this paper the data size is reduced by a sort of decimation process. Namely, the measured signal x has $N = T\, f_s$ points, while the decimated signal x_d is:

$$x_d = \left[\frac{\sum_{j=0}^{k} x_j}{k} \quad \frac{\sum_{j=k}^{2k} x_j}{k} \quad \ldots \quad \frac{\sum_{j=N-k}^{N} x_j}{k}\right] \tag{3}$$

where k is the decimation factor, and the size of the decimated signal is N/k. Hence FFT is applied to x_d.

After FFT, an 'informative' frequency range is determined to select only the L harmonics that are necessary for computing the subsequent General Mean Value function (GMV-function). This range is related to the specific fault type because different faults have different effects on the stator current.

In the next step, the GMV-function is computed for the FFT spectrum defined in the previously selected frequency range. This index is determined as follows [13]–[16]:

$$GMV_{(mom_p)} = \left(\sum_{i=1}^{L} FFT_i^{mom_p}\right)^{\frac{1}{mom_p}} \quad (4)$$

where L is the number of the selected FFT amplitudes and mom_p is the moment order, which is calculated by (2).

To compare different sequences produced by stator currents from different motors, the slopes of the so-called 'moments curves' can be determined and compared. A moment curve plots the moments associated to an analyzed motor versus the moments that are associated to a reference, healthy motor. The resulting curve is monotonically increasing. The slope is determined by applying the Linear Least Squares Method. Then, if the slope of the curve is unitary, the compared sequences show statistically no difference and the examined motor is healthy; otherwise, the motor is subject to a fault.

In the same way, the computed GMV-functions are compared to a reference GMV-function that is obtained from a healthy IM. In particular, the slopes of the curves describing the GMV-functions are calculated by using the Linear Least Squares Method. Then, the slopes of the GMV-functions of the tested motor are compared with that of the GMV-function of the healthy motor.

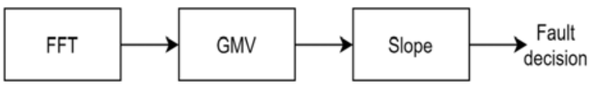

Figure 1. Conceptual scheme of the FFT+SFM method.

To synthesize, the general scheme of the FFT+SFM method is shown in Figure 1. Firstly, the Fast Fourier Transform of the acquired decimated signal is applied. Note that, in the specific case of the analysis performed for processing stator currents, the 'alpha-beta' coordinates transformation of the three-phase current signals should be made before applying FFT (see [12]). Secondly, the General Mean Value function should be calculated for the obtained FFT spectrum, in a selected frequency range. Finally, the slope difference between the obtained GMV and the reference GMV, which is associated to a healthy motor without any applied load, is calculated. The decision on fault appearance is made based on the value of this slope. Values of slope close to the unit value are considered as those of a healthy IM, other values are considered as those of a faulty IM.

An earlier version of the method, without decimation and selection of the informative frequency range, was tested by computer simulation and processing of experimental data [12]. The proposed method showed good fault detection results with a very short acquisition time interval. This peculiarity made the authors think about a possible implementation of FFT+SFM in low-cost and low-performance microcontrollers, especially those with relatively small RAM memory storage.

III. MICROCONTROLLER IMPLEMENTATION

The objective of this work is to evaluate the efficiency and performance of the detection algorithm when implemented on an embedded system. To this aim, an hardware-in-the-loop test configuration was realized to speed-up the tuning and performance analysis process. The considered configuration included a microcontroller, used for the diagnosis computations, which was connected to a personal computer storing the data to be analyzed. In a first step, data from the real motors were suitably collected by performing experimental tests. Then, the measurements were transferred on a personal computer which was used to feed the microcontroller running the fault detection algoritms.

A. *Data acquisition*

The tested motors were identical, directly powered from power lines. Namely, they were three-phase IM Siemens UD 1010/71980410-10, with star connection and the rated characteristics P = 1.5 kW, f = 50 Hz, U = 400 V, I = 3.25 A, n = 2860 rpm, cosφ = 0.85. The first motor was a healthy IM; the second one had a rotor with one bar drilled at the end ring (broken rotor bar fault); the third motor had an eccentricity fault provoked by nonuniform diameter bearings and eccentric bushings. No mechanical load was applied.

The current was measured in all three phases, using current clamps. All sensors were connected to a digital oscilloscope. Signals were acquired for 100 seconds for each measurement cycle, with a 100 kHz sampling frequency. The FFT+SFM method analyzed stator current signals in stationary conditions.

B. *Microcontroller specifications*

The microcontroller chosen to implement and test the detection algorithm is an STM32F722ZE mounted on a NUCLEO144 board, which also includes an ST-LINK programmer and provides a direct USB connection to the MCU's UART3 (universal asynchronous receiver-transmitter) peripheral. This MCU was mainly chosen because was more likely than other MCUs in the same price range to produce good results, thanks to its high clock speed, combined with 256 kB of SRAM and an FPU (Floating Point Unit). Table 1 provides the MCU specifications.

TABLE I. MCU SPECIFICATIONS

Specification	***Value***
Core	ARM 32-bit Cortex-M7 CPU with FPU
Flash	512 KB
SRAM	256 KB
SysTick Timer frequency	Up to 216 MHz
Cost	5.0469 US$/10k

C. Implementation features and issues

The first task to be accomplished is the calculation of the FFT under the strict memory and computational constraints of the MCU. A suitable solution is represented by CMSIS-DSP, a software library including mathematical, filtering and transform functions for Cortex-M and Cortex-A processor-based devices [17]. CMSIS-DSP is part of the CMSIS (Cortex Microcontroller Software Interface Standard), which is a Software Interface Standard for Arm Cortex-based Microcontrollers, a vendor-independent hardware abstraction layer for microcontrollers based on Arm Cortex processors. It provides software interfaces to the processor and the peripherals to reduce the time to market for new devices.

More specifically, the library implements a split-radix algorithm to calculate the Discrete Fourier Transform (DFT) [18], enabling high-speed computations even at MCU-level computing speed, but with the drawback of limited flexibility. In fact, the arrays of data samples must contain a number of elements strictly in powers of 2, up to 2^{12}, which could affect the detection method efficiency. Therefore, for the proposed case study, the number of data points under FFT is changed from 1000 (as it was in [12]) to 1024. However, a different number of samples results in a different sampling frequency for the DFT, eventually causing a loss of information due to the introduction of spurious harmonic components.

The operating environment of the device was designed as bare metal, which implies that firmware directly runs on the processor's hardware, to prove the feasibility of an embedded implementation of the algorithm. Figure 2 shows the high-level structure of the main loop. The algorithm is structured as in the following:

- The first step aims at converting the input data (currents), which are supplied by the caller in three arrays (phase1, phase2 and phase3) of 100 elements, to an alpha-beta representation. The results are stored in the alpha-beta array of 200 elements. All the arrays containing complex data used in the firmware are of float-32 type.
- The second step averages the real and imaginary parts of the input arrays, and then writes them at the (2j)-th and (2j+1)-th indexes of the output array. The original decimation function required to store 512,000 floating point elements simultaneously in the SRAM, resulting in an approximately 2 MB amount of data, which greatly exceeds the SRAM capacity. The adopted approach permitted to keep only 500 floating point elements in memory by moving the iteration to fill the decimated data array from the decimation function to the outer loop containing communication, alpha-beta transformation, and decimation.
- Thirdly, a library function computes the complex FFT of data, then the algorithm computes the GMV function of the input data, which contains the FFT spectrum.
- The last step performs a linear regression to compute the slope of the GMV function with respect to the reference function referring to the healthy motor, stored in the constant array.

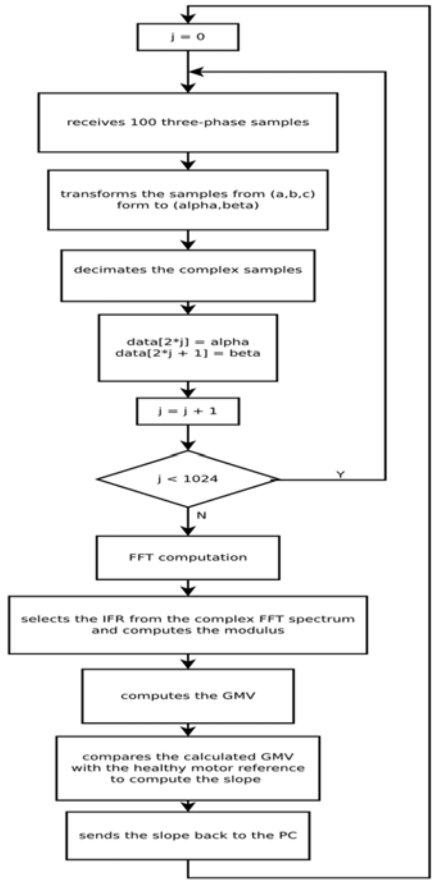

Figure 2. Main algorithm implemented on microcontroller.

D. Communication system

Since this work is aimed at evaluating only the performance of the fault detection algorithm, the efficiency of the communication between the microcontroller and the computer was not an issue. Therefore, the choice fell on serial communication through the UART 3 peripheral, which is already connected to the USB interface of the NUCLEO board. The main setback of this approach is latency, but in normal operation, the data should be supplied by an analog sensor connected through ADC or a serial bus making the communication fast enough. Conversely, here the analysis is performed offline by the microcontroller: it receives the data directly from the database and then proceeds with the necessary computations. Consequently, latency does not affect in any way the efficiency of the detection algorithm. Future developments will be devoted to select the most appropriate communication system for timely identification of the fault.

The UART was used to send the data to the MCU and to collect small amounts of output data, such as the GMV slopes, while the SWV was used to read larger amounts or time-sensitive data, e.g. the computation times. For maximum flexibility, a serial terminal was implemented from scratch using POSIX (Portable Operating System Interface) standard general terminal interface, implementing a master-slave protocol. The MCU acted as the master. The use of a protocol was made necessary by the unstructured nature of UART that made synchronization particularly difficult.

IV. EXPERIMENTAL RESULTS

The issues described in subsection III.C implied some differences regarding the implementation of the algorithm on the microcontroller with respect to that in [12]. To evaluate the degradation of detection performance, the results were compared with those obtained by the FFTW library [19] that is used to cmpute the DFT on a personal computer. The FFTW library shows versatility, performance, and ease of use, thus representing an appropriate benchmark for comparison. Three different cases considered the healthy motor, the motor with a broken rotor bar, and the motor affected by rotor eccentricity.

Figure 3 shows the results obtained by processing the healthy motor current measurements both on the personal computer and MCU. The upper subplot depicts the GMV functions, while the lower represents the GMV slopes corresponding to each dataset. It is evident that the sub-optimal sampling frequency adopted for FFT computation with CMSIS-DSP library introduces a substantial amount of noise, which affects detection accuracy. Analogous considerations can be drawn from Figures 4 and 5 sketching the results of motors with broken rotor bar and affected by eccentricity, respectively. However, despite the noise introduced by MCU computation, the plots show that it is still possible to distinguish between the healthy motor state and the two faults by applying the proposed approach. This result is more evident in Figure 6, which plots the GMV functions and the GMV slopes obtained by the MCU analysis comparing the three different operating conditions.

Finally, a practical real-time application of the proposed detection method requires a reasonable computational time. An analysis was carried out to measure the execution time of each algorithm iteration, by using a timer with a counting period of 10 µs. Figure 7 shows that the execution was timed in about 47 ms, which enables a timely detection of faults.

V. Conclusion

This paper proposed the implementation of a peculiar method for detecting broken rotor bars and rotor eccentricity faults in induction motors by the analysis of stator currents. The method combines Fast Fourier Transform and the so-called Statistics of Fractional Moments to easily distinguish healthy and faulty motors. The amount of processed data is reduced with respect to data from measurements by a decimation technique. Moreover, only part of the FFT spectrum is used to perform detection. Such low memory requirement and reduced amount of computations allow an efficient implementation on low-cost microcontroller units.

The results obtained by the microcontroller implementation clearly indicate that the proposed FFT+SFM method can be successfully implemented on low-cost microcontrollers (with ARM Cortex-M core). Hence this implementation can be very useful in real time condition monitoring systems of IM.

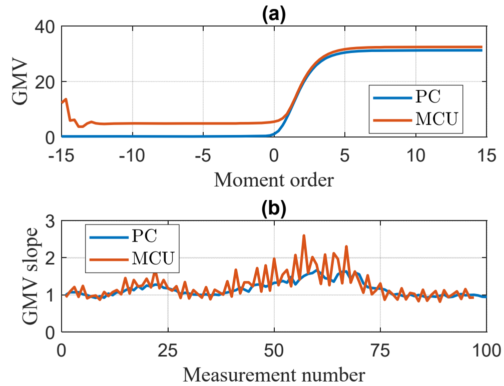

Figure 3. (a) The GMV function obtained for a healthy motor by the original computer version of the FFT+SFM algorithm (blue color) and the microcontroller algorithm (red color). (b) Slopes for the healthy motor by the original computer algorithm (blue color) and the microcontroller algorithm (red color).

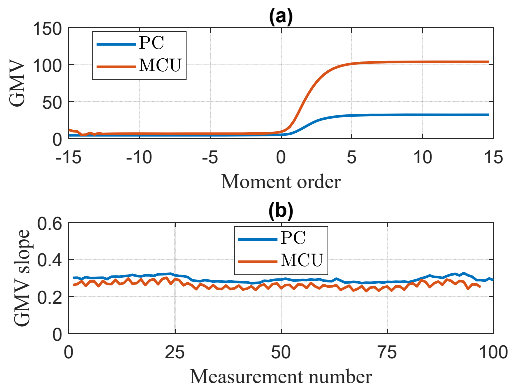

Figure 4. (a) The GMV function obtained for a faulty motor with a broken rotor bar by the original computer version of the FFT+SFM algorithm (blue color) and the microcontroller algorithm (red color). (b) Slopes for the faulty motor obtained by the original computer algorithm (blue color) and the microcontroller algorithm (red color).

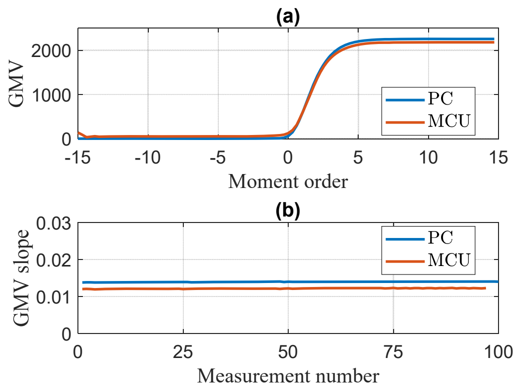

Figure 5. The GMV function obtained for a faulty motor with rotor eccentricity by the original computer version of the FFT+SFM algorithm (blue color) and the microcontroller algorithm (red color). (b) Slopes for the faulty motor obtained by the original computer algorithm (blue color) and the microcontroller algorithm (red color).

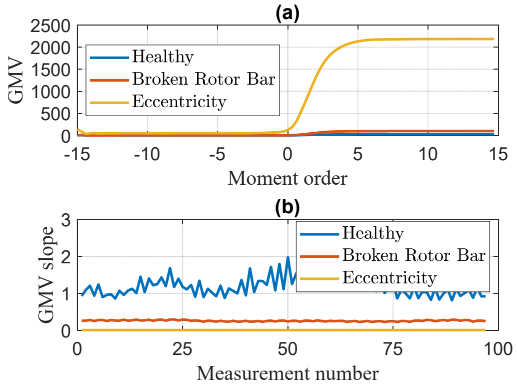

Figure 6. The GMV functions obtained for a healthy motor and for faulty motors (with broken rotor bar and rotor eccentricity) by the microcontroller algorithm. (b) Slopes for a healthy motor and for faulty motors obtained by the microcontroller algorithm.

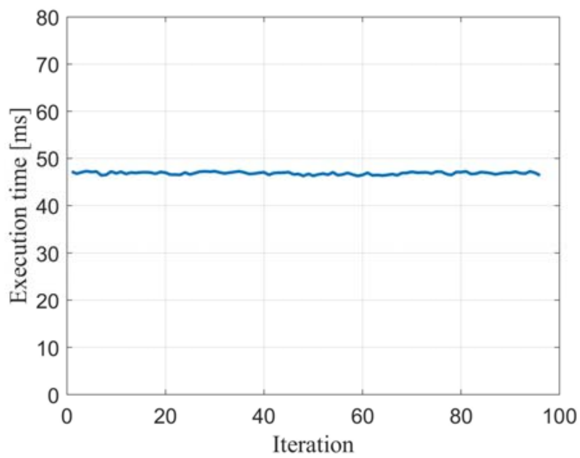

Figure 7. Execution time of each iteration of the algorithm on the MCU.

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