SHD - Blatt 9

Aufgabe 26 Stichprobenvarianz

a) $E(\hat{\mu}) = E(\bar{X}) = E\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n}\sum_{i=1}^{n} E(X_i) = \frac{1}{n}\sum_{i=1}^{n} \mu$

$= \frac{1}{n} \cdot n\mu = \mu$ ✓

b) $E((\bar{X}-\mu)^2) = Var(\bar{X}) = \frac{\sigma^2}{n}$

$= E(\bar{X}^2) - E^2(\bar{X})$

$= E\left[\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)^2\right] - \left[E\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)\right]^2$

$= E\left[\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)^2\right] - \mu^2$

c) $E(S^2) = E\left(\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2\right) = E\left(\frac{n}{n-1}\frac{1}{n}(X_i - \bar{X})^2\right)$

$= \frac{n}{n-1} E(\hat{\sigma}^2) = \sigma^2$ → unverzerrt

d) $E\left(\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2\right) = \frac{1}{n} E\left(\sum_{i=1}^{n}(X_i - \mu + \mu - \bar{X})^2\right)$

$= \frac{1}{n} E\left(\sum_{i=1}^{n}\left((X_i-\mu)^2 - 2(X_i-\mu)(\bar{X}-\mu) + (\bar{X}-\mu)^2\right)\right)$

$= \frac{1}{n} E\left(\sum_{i=1}^{n}(X_i-\mu)^2 - 2\sum_{i=1}^{n}(X_i-\mu)(\bar{X}-\mu) + \sum_{i=1}^{n}(\bar{X}-\mu)^2\right)$

$= \frac{1}{n} E\left(\sum_{i=1}^{n}(X_i-\mu)^2 - 2n(\bar{X}-\mu)(\bar{X}-\mu) + n(\bar{X}-\mu)^2\right)$

$= \frac{1}{n} E\left(\sum_{i=1}^{n}(X_i-\mu)^2 - n(\bar{X}-\mu)^2\right)$

$= \frac{1}{n}\left(\sum_{i=1}^{n} E(X_i-\mu)^2 - n \cdot E(\bar{X}-\mu)^2\right.$

$= \frac{1}{n}\left(Var(X_i) - n \cdot Var(\bar{X})\right)$

$= \sigma^2 - \frac{\sigma^2}{n} = \frac{n-1}{n}\sigma^2$ → verzerrt

Korrektur: $S_1'^2 = \frac{n}{n-1}\cdot\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$

$= \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$